Problem 9.17. The signal $m(t) = \cos(400\pi t)$ is transmitted via FM. There is an ideal band-pass filter passing $100 \le |f| \le 300$ at the discriminator output. Calculate the post-detection SNR given that $k_f = 1$ kHz per volt, and the pre-detection SNR is 500. Use Carson's rule to estimate the pre-detection bandwidth.

Solution

We begin by estimating the Carson's rule bandwidth

$$B_T = 2(k_f A + f_m)$$

= 2(1000(1) + 200)
= 2400 Hz

We are given that the pre-detection SNR is 500. From Section 9.7 this implies

$$SNR_{pre}^{FM} = \frac{A_{c}^{2}}{2N_{0}B_{T}}$$
$$500 = \frac{A_{c}^{2}}{2N_{0}}\frac{1}{2400}$$

Re-arranging this equation, we obtain

$$\frac{A_c^2}{2N_0} = 1.2 \times 10^6 \,\mathrm{Hz}$$

The nuance in this problem is that the post-detection filter is not ideal with unity gain from 0 to W and zero for higher frequencies. Consequently, we must re-evaluate the post-detection noise using Eq. (9.58)

Avg. post - detection noise power =
$$\frac{N_0}{A_c^2} \left[\int_{-300}^{-100} f^2 df + \int_{100}^{300} f^2 df \right]$$

= $\frac{2N_0}{3A_c^2} \left[300^3 - 100^3 \right]$
= $\frac{2N_0}{3A_c^2} 2.6 \times 10^7$

The post-detection SNR then becomes

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Problem 9.17 continued

SNR ^{FM}_{post} =
$$\frac{3A_c^2 k_f^2 P}{2N_0 (2.6 \times 10^7)}$$

= $3 \left(\frac{A_c^2}{2N_0} \right) \frac{k_f^2 P}{2.6 \times 10^7}$
= $3 (1.2 \times 10^6) \frac{(1000)^2 0.5}{2.6 \times 10^7}$
= 69230.8

where we have used the fact that $k_f = 1000 \text{ Hz/V}$ and P = 0.5 watts. In decibels, the postdetection SNR is 48.4 dB.