Problem 9.17. The signal $m(t) = \cos(400\pi t)$ is transmitted via FM. There is an ideal band-pass filter passing $100 \le |f| \le 300$ at the discriminator output. Calculate the post-detection SNR given that k_f= 1 kHz per volt, and the pre-detection SNR is 500. Use Carson's rule to estimate the pre-detection bandwidth.

Solution

We begin by estimating the Carson's rule bandwidth

$$\begin{aligned} B_T &= 2(k_f A + f_m) \\ &= 2(1000(1) + 200) \\ &= 2400 \text{ Hz} \end{aligned}$$

We are given that the pre-detection SNR is 500. From Section 9.7 this implies

$$\begin{aligned} SNR_{pre}^{FM} &= \frac{A_c^2}{2N_0 B_T} \\ 500 &= \frac{A_c^2}{2N_0}\frac{1}{2400} \end{aligned}$$

Re-arranging this equation, we obtain

$$\frac{A_c^2}{2N_0} = 1.2 \times 10^6 \text{ Hz}$$

The nuance in this problem is that the post-detection filter is not ideal with unity gain from 0 to W and zero for higher frequencies. Consequently, we must re-evaluate the post-detection noise using Eq. (9.58)

$$\begin{aligned} \text{Avg. post-detection noise power} &= \frac{N_0}{A_c^2}\left[\int_{-300}^{-100} f^2 df + \int_{100}^{300} f^2 df\right] \\ &= \frac{2N_0}{3A_c^2}\left[300^3 - 100^3\right] \\ &= \frac{2N_0}{3A_c^2} 2.6 \times 10^7 \end{aligned}$$

The post-detection SNR then becomes

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Problem 9.17 continued

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\begin{aligned}
\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} &= \frac{3A_c^2 k_f^2 P}{2N_0\left(2.6\times 10^7\right)} \\
&= 3\left(\frac{A_c^2}{2N_0}\right)\frac{k_f^2 P}{2.6\times 10^7} \\
&= 3\left(1.2\times 10^6\right)\frac{(1000)^2\,0.5}{2.6\times 10^7} \\
&= 69230.8
\end{aligned}
$$

where we have used the fact that $k_f = 1000$ Hz/V and $P = 0.5$ watts. In decibels, the post-detection SNR is 48.4 dB.