Problem 9.18. Suppose that the spectrum of a modulating signal occupies the frequency band $f_1 \le |f| \le f_2$. To accommodate this signal, the receiver of an FM system (without pre-emphasis) uses an ideal band-pass filter connected to the output of the frequency discriminator; the filter passes frequencies in the interval $f_1 \le |f| \le f_2$. Determine the output signal-to-noise ratio and figure of merit of the system in the presence of additive white noise at the receiver input.

Solution

Since the post detection filter is no longer an ideal brickwall filter, we must revert to Eq. (9.58) to compute the post-detection noise power. For this scenario (similar to Problem 9.17)

$$\begin{aligned}\text{Avg. post - detection noise power} &= \frac{N_0}{A_c^2}\left[\int_{-f_2}^{-f_1} f^2 df + \int_{f_1}^{f_2} f^2 df\right] \\ &= \frac{2N_0}{3A_c^2}\left[f_2^3 - f_1^3\right]\end{aligned}$$

Since the average output power is still $k_f^2 P$, the post detection SNR is given by

$$SNR_{post}^{FM} = \frac{3A_c^2 k_f^2 P}{2N_0\left(f_2^3 - f_1^3\right)}$$

For comparison purposes, the reference SNR is

$$SNR_{ref} = \frac{A_c^2}{2N_0(f_2 - f_1)}$$

The corresponding figure of merit is

$$\begin{aligned}\text{Figure of merit} &= \frac{\text{SNR}_{\text{post}}^{\text{FM}}}{\text{SNR}_{\text{ref}}} \\ &= \frac{3A_c^2 k_f^2 P}{2N_0\left(f_2^3 - f_1^3\right)} \Big/ \frac{A_c^2}{2N_0(f_2 - f_1)} \\ &= \frac{3k_f^2 P}{f_2^2 + f_2 f_1 + f_1^2}\end{aligned}$$