Problem 9.19. An FM system, operating at a pre-detection SNR of 14 dB, requires a post-detection SNR of 30 dB, and has a message power of 1 watt and bandwidth of 50 kHz. Using Carson's rule, estimate what the transmission bandwidth of the system must be. Suppose this system includes pre-emphasis and de-emphasis network with $f_{3\text{dB}}$ of 10 kHz. What transmission bandwidth is required in this case?

Solution

We are given the pre-detection SNR of 14 dB (~25.1), so

$$SNR_{pre}^{FM} = \frac{A_c^2}{2N_0 B_T} = 25.1$$

and the post-detection SNR of 30 dB (~1000), so

$$SNR_{post}^{FM} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3} = 1000$$

Combining these two expressions, we obtain

$$\frac{SNR_{post}^{FM}}{SNR_{pre}^{FM}} = \frac{3k_f^2 P B_T}{W^3} = 39.8$$

Approximating the Carson's rule for general modulation $B_T = 2\left(k_f P^{1/2} + W\right) \approx 2k_f P^{1/2}$, and if we replace $k_f^2 P$ with $B_T^2/4$ in this last equation, we obtain

$$\frac{SNR_{post}^{FM}}{SNR_{pre}^{FM}} \approx \frac{3B_T^3}{4W^3} = 39.8$$

Upon substituting $W = 50$ kHz, this last equation yields $B_T = 187.9$ kHz.