**Problem 9.22**. Assume that, in the DSB-SC demodulator of Fig. 9.6, there is a phase error  $\phi$  in the synchronized oscillator such that its output is  $\cos(2\pi f_c t + \phi)$ . Find an expression for the coherent detector output and show that the post-detection SNR is reduced by the factor  $\cos^2 \phi$ .

## **Solution**

The signal at the input to the coherent detector of Fig. 9.6 is x(t) where

$$x(t) = s(t) + n_1(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$
  
=  $A_c m(t)\cos(2\pi f_c t) + n_1(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$ 

The output of mixer2 in Fig. 9.6 is

$$\begin{aligned} v(t) &= x(t)\cos(2\pi f_c t + \phi) \\ &= \left[A_c m(t) + n_I(t)\right]\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) - n_Q(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \phi) \\ &= \frac{1}{2}\left[A_c m(t) + n_I(t)\right]\cos\phi + \frac{1}{2}n_Q(t)\sin\phi + \frac{1}{2}\left[A_c m(t) + n_I(t)\right]\cos(4\pi f_c t + \phi) - \frac{1}{2}n_Q(t)\sin(4\pi f_c t + \phi) \end{aligned}$$

With the higher frequency components will be eliminated by the low pass filter, the received message at the output of the low-pass filter is

$$y(t) = \frac{1}{2}A_{c}m(t)\cos\phi + \frac{1}{2}n_{I}(t)\cos\phi + \frac{1}{2}n_{Q}(t)\sin\phi$$

To compute the post-detection SNR we note that the average output message power in this last expression is

$$\frac{1}{4}A_c^2P\cos^2\phi$$

and the average output noise power is

$$\frac{1}{4} \cdot 2N_0 W \cos^2 \phi + \frac{1}{4} \cdot 2N_0 W \sin^2 \phi = \frac{1}{4} \cdot 2N_0 W$$

where  $\mathbf{E}[n_I^2(t)] = \mathbf{E}[n_Q^2(t)] = N_0 W$ . Consequently, the post-detection SNR is

$$SNR = \frac{1/4A_c^2 P \cos^2 \phi}{1/4 \cdot 2N_0 W} = \frac{A_c^2 P \cos^2 \phi}{2N_0 W}$$

Compared with (9.23), the above post-detection SNR is reduced by a factor of  $\cos^2 \phi$ .

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which the textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.