Problem 9.22. Assume that, in the DSB-SC demodulator of Fig. 9.6, there is a phase error ϕ in the synchronized oscillator such that its output is $\cos(2\pi f_c t + \phi)$. Find an expression for the coherent detector output and show that the post-detection SNR is reduced by the factor $\cos^2\phi$.

Solution

The signal at the input to the coherent detector of Fig. 9.6 is $x(t)$ where

$$\begin{aligned} x(t) &= s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \\ &= A_c m(t)\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \end{aligned}$$

The output of mixer2 in Fig. 9.6 is

$$\begin{aligned} v(t) &= x(t)\cos(2\pi f_c t + \phi) \\ &= \left[A_c m(t) + n_I(t)\right]\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) - n_Q(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \phi) \\ &= \frac{1}{2}\left[A_c m(t) + n_I(t)\right]\cos\phi + \frac{1}{2}n_Q(t)\sin\phi + \frac{1}{2}\left[A_c m(t) + n_I(t)\right]\cos(4\pi f_c t + \phi) - \frac{1}{2}n_Q(t)\sin(4\pi f_c t + \phi) \end{aligned}$$

With the higher frequency components will be eliminated by the low pass filter, the received message at the output of the low-pass filter is

$$y(t) = \frac{1}{2}A_c m(t)\cos\phi + \frac{1}{2}n_I(t)\cos\phi + \frac{1}{2}n_Q(t)\sin\phi$$

To compute the post-detection SNR we note that the average output message power in this last expression is

$$\frac{1}{4}A_c^2 P\cos^2\phi$$

and the average output noise power is

$$\frac{1}{4}\cdot 2N_0 W\cos^2\phi + \frac{1}{4}\cdot 2N_0 W\sin^2\phi = \frac{1}{4}\cdot 2N_0 W$$

where $\mathbf{E}\left[n_I^2(t)\right] = \mathbf{E}\left[n_Q^2(t)\right] = N_0 W$. Consequently, the post-detection SNR is

$$\text{SNR} = \frac{1/4 A_c^2 P\cos^2\phi}{1/4\cdot 2N_0 W} = \frac{A_c^2 P\cos^2\phi}{2N_0 W}$$

Compared with (9.23), the above post-detection SNR is reduced by a factor of $\cos^2\phi$.