**Problem 9.24**. Equation (9.59) is the FM post-detection noise for an ideal low-pass filter. Find the post-detection noise for an FM signal when the post-detection filter is a second-order low-pass filter with magnitude response

$$|H(f)| = \frac{1}{(1+(f/W)^4)^{1/2}}$$

Assume  $\left|H_{BP}(f+f_c)\right|^2 \approx 1$  for  $\left|f\right| < B_T/2$  and  $B_T >> 2W$ .

## **Solution**

We modify Eq. (9.58) to include the effects of a non-ideal post-detection filter in order to estimate the average post-detection noise power:

$$\frac{N_0}{A_c^2} \int_{-W}^{W} f^2 |H_{BP}(f)|^2 df = \frac{N_0}{A_c^2} \int_{-W}^{W} f^2 \cdot \frac{1}{1 + (f/W)^4} df$$
$$= \frac{2N_0}{A_c^2} \int_{0}^{W} f^2 \cdot \frac{1}{1 + (f/W)^4} df$$

This can be evaluated by a partial fraction expansion of the integrand but for simplicity, we appeal to the formula:

$$\int \frac{x^2 dx}{a + bx^4} = \frac{1}{4bk} \left[ \frac{1}{2} \log \frac{x^2 - 2kx + 2k^2}{x^2 + 2kx + 2k^2} + \tan^{-1} \frac{2kx}{2k^2 - x^2} \right], \quad ab > 0, \ k = \sqrt[4]{\frac{a}{2b}}$$

Using this result, we get the average post-detection noise power is

Avg. post-detection noise power = 
$$\frac{2N_0}{A_c^2} \cdot \frac{W^3}{4\sqrt{2}} \left[ \log \frac{2-\sqrt{2}}{2+\sqrt{2}} + \pi \right] = 0.42 \frac{N_0 W^3}{A_c^2}$$

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