Problem 9.24. Equation (9.59) is the FM post-detection noise for an ideal low-pass filter. Find the post-detection noise for an FM signal when the post-detection filter is a second-order low-pass filter with magnitude response

$$|H(f)| = \frac{1}{\left(1+(f/W)^4\right)^{1/2}}$$

Assume $|H_{BP}(f + f_c)|^2 \approx 1$ for $|f| < B_T/2$ and $B_T >> 2W$.

Solution

We modify Eq. (9.58) to include the effects of a non-ideal post-detection filter in order to estimate the average post-detection noise power:

$$\begin{aligned}\frac{N_0}{A_c^2}\int_{-W}^{W} f^2\,|H_{BP}(f)|^2\,df &= \frac{N_0}{A_c^2}\int_{-W}^{W} f^2\cdot\frac{1}{1+(f/W)^4}\,df \\ &= \frac{2N_0}{A_c^2}\int_{0}^{W} f^2\cdot\frac{1}{1+(f/W)^4}\,df\end{aligned}$$

This can be evaluated by a partial fraction expansion of the integrand but for simplicity, we appeal to the formula:

$$\int\frac{x^2dx}{a+bx^4} = \frac{1}{4bk}\left[\frac{1}{2}\log\frac{x^2-2kx+2k^2}{x^2+2kx+2k^2} + \tan^{-1}\frac{2kx}{2k^2-x^2}\right], \quad ab>0,\ k=\sqrt[4]{\frac{a}{2b}}$$

Using this result, we get the average post-detection noise power is

$$\text{Avg. post-detection noise power} = \frac{2N_0}{A_c^2}\cdot\frac{W^3}{4\sqrt{2}}\left[\log\frac{2-\sqrt{2}}{2+\sqrt{2}} + \pi\right] = 0.42\frac{N_0W^3}{A_c^2}$$