Problem 9.25. Consider a communication system with a transmission loss of 100 dB and a noise density of 10^{-14} W/Hz at the receiver input. If the average message power is $P = 1$ watt and the bandwidth is 10 kHz, find the average transmitter power (in kilowatts) required for a post-detection SNR of 40 dB or better when the modulation is:

(a) AM with $k_a = 1$; repeat the calculation for $k_a = 0.1$.
(b) FM with $k_f = 10$, 50 and 100 kHz per volt.

In the FM case, check for threshold limitations by confirming that the pre-detection SNR is greater that 12 dB.

Solution

(a) In the AM case, the post detection SNR is given by

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{AM}} = \frac{A_c^2 k_a^2 P}{2N_o W}$$

$$10^4 = \frac{A_c^2 k_a^2 (1)}{2(2\times 10^{-14})(10^4)}$$

$$\frac{A_c^2 k_a^2}{2} = 2\times 10^{-6}$$

where an SNR of 40 dB corresponds to 10^4 absolute and $N_0/2 = 10^{-14}$ W/Hz. For the different values of k_a

$$k_a = 1 \Rightarrow \quad A_c^2 = 4\times 10^{-6}$$

$$k_a = 0.1 \Rightarrow \quad A_c^2 = 4\times 10^{-4}$$

Average modulated signal power at the input of the detector is $\frac{1}{2}A_c^2(1+k_a^2 P)$.

$$k_a = 1 \Rightarrow \quad \frac{1}{2}A_c^2(1+k_a^2 P) = 4\times 10^{-6}$$

$$k_a = 0.1 \Rightarrow \quad \frac{1}{2}A_c^2(1+k_a^2 P) = 2.02\times 10^{-4}$$

The transmitted power is 100dB (10^{10}) greater than the received signal power so

$k_a = 1 \Rightarrow$ transmitted power = $4\times 10^4 = 40$ kW

$k_a = 0.1 \Rightarrow$ transmitted power = $2.02\times 10^6 = 2020$ kW

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(b) In the FM case, the post detection SNR is

$$\mathrm{SNR}^{\mathrm{FM}}_{\mathrm{post}} = \frac{3A_c^2 k_f^2 P}{2N_o W^3}$$

$$10^4 = \frac{3A_c^2 k_f^2 (1)}{2(2\times10^{-14})(10^4)^3}$$

$$\frac{A_c^2 k_f^2}{2} = 0.667\times10^2$$

For the different values of k_a

$$k_f = 10\,\mathrm{kHz/V} \Rightarrow \frac{\mathrm{A_c^2}}{2} = 0.667\times10^{-6}$$

$$k_f = 50\,\mathrm{kHz/V} \Rightarrow \frac{\mathrm{A_c^2}}{2} = 26.667\times10^{-9}$$

$$k_f = 100\,\mathrm{kHz/V} \Rightarrow \frac{\mathrm{A_c^2}}{2} = 0.667\times10^{-8}$$

The transmitted power is 100dB (10^{10}) greater than the received signal power so

k_f = 10 kHz/V ⇒ transmitted power = $0.667\text{x}10^4$ W = 6.67 kW
k_f = 50 kHz/V ⇒ transmitted power = $26.667\text{x}10^1$ W = 0.27 kW
k_f = 100 kHz/V ⇒ transmitted power = $0.667\text{x}10^2$ W = 0.07 kW

To check the pre-detection SNR, we note that it is given by :

$$SNR^{FM}_{pre} = \frac{A_c^2}{2N_0 B_T} = \frac{A_c^2}{4N_0(k_f P^{1/2} + W)}$$

where from Carson's rule $B_T = 2(k_f P^{1/2} + W)$. From the above $A_c^2 = \dfrac{4\times10^2}{3k_f^2}$, so

$$SNR^{FM}_{pre} = \frac{4\times10^2}{3k_f^2 \times 4N_0(k_f P^{1/2} + W)} = \frac{10^2}{3k_f^2 \times 2\times10^{-14}(k_f + 10^4)}$$

For the different values of k_f, the pre-detection SNR is

$$k_f = 10kHz \Rightarrow \quad SNR^{FM}_{pre} = 10^4/12 = 29dB > 12dB$$

$$k_f = 50kHz \Rightarrow \quad SNR^{FM}_{pre} = 11.11 = 10.45dB < 12dB$$

$$k_f = 100kHz \Rightarrow \quad SNR^{FM}_{pre} = 1.515 = 1.8dB < 12dB$$

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Therefore, for k_f = 50 kHz and 100 kHz, the pre-detection *SNR* is too low and the transmitter power would have to be increased by 1.55 dB and 10.2 dB, respectively.