Problem 9.25. Consider a communication system with a transmission loss of 100 dB and a noise density of 10^{-14} W/Hz at the receiver input. If the average message power is P = 1 watt and the bandwidth is 10 kHz, find the average transmitter power (in kilowatts) required for a post-detection SNR of 40 dB or better when the modulation is:

- (a) AM with $k_a = 1$; repeat the calculation for $k_a = 0.1$.
- (b) FM with $k_f = 10$, 50 and 100 kHz per volt.

In the FM case, check for threshold limitations by confirming that the pre-detection SNR is greater that 12 dB.

Solution

(a) In the AM case, the post detection SNR is given by

$$SNR_{post}^{AM} = \frac{A_c^2 k_a^2 P}{2N_o W}$$
$$10^4 = \frac{A_c^2 k_a^2 (1)}{2(2 \times 10^{-14})(10^4)}$$
$$\frac{A_c^2 k_a^2}{2} = 2 \times 10^{-6}$$

where an SNR of 40 dB corresponds to 10^4 absolute and $N_0/2 = 10^{-14}$ W/Hz. For the different values of k_a

$$k_a = 1 \Longrightarrow \quad A_c^2 = 4 \times 10^{-6}$$
$$k_a = 0.1 \Longrightarrow \quad A_c^2 = 4 \times 10^{-4}$$

Average modulated signal power at the input of the detector is $\frac{1}{2}A_c^2(1+k_a^2P)$.

$$k_{a} = 1 \implies \frac{1}{2} A_{c}^{2} (1 + k_{a}^{2} P) = 4 \times 10^{-6}$$

$$k_{a} = 0.1 \implies \frac{1}{2} A_{c}^{2} (1 + k_{a}^{2} P) = 2.02 \times 10^{-4}$$

The transmitted power is 100dB (10^{10}) greater than the received signal power so

 $k_a = 1 \Rightarrow$ transmitted power = $4 \times 10^4 = 40$ kW $k_a = 0.1 \Rightarrow$ transmitted power = $2.02 \times 10^6 = 2020$ kW

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Problem 9.25 continued

(b) In the FM case, the post detection SNR is

$$SNR_{post}^{FM} = \frac{3A_c^2 k_f^2 P}{2N_o W^3}$$
$$10^4 = \frac{3A_c^2 k_f^2 (1)}{2(2 \times 10^{-14})(10^4)^3}$$
$$\frac{A_c^2 k_f^2}{2} = 0.667 \times 10^2$$

For the different values of k_a

$$k_f = 10 \text{ kHz/V} \Rightarrow \frac{A_c^2}{2} = 0.667 \times 10^{-6}$$
$$k_f = 50 \text{ kHz/V} \Rightarrow \frac{A_c^2}{2} = 26.667 \times 10^{-9}$$

$$k_f = 100 \,\mathrm{kHz/V} \Rightarrow \frac{\mathrm{A}_{\mathrm{c}}^2}{2} = 0.667 \times 10^{-8}$$

The transmitted power is 100dB (10^{10}) greater than the received signal power so

$$k_f = 10 \text{ kHz/V} \Rightarrow \text{ transmitted power} = 0.667 \text{ x} 10^4 \text{ W} = 6.67 \text{ kW}$$

 $k_f = 50 \text{ kHz/V} \Rightarrow \text{ transmitted power} = 26.667 \text{ x} 10^1 \text{ W} = 0.27 \text{ kW}$
 $k_f = 100 \text{ kHz/V} \Rightarrow \text{ transmitted power} = 0.667 \text{ x} 10^2 \text{ W} = 0.07 \text{ kW}$

To check the pre-detection SNR, we note that it is given by : $\frac{1}{2}$

$$SNR_{pre}^{FM} = \frac{A_c^2}{2N_0 B_T} = \frac{A_c^2}{4N_0 (k_f P^{1/2} + W)}$$

where from Carson's rule $B_T = 2(k_f P^{1/2} + W)$. From the above $A_c^2 = \frac{4 \times 10^2}{3k_f^2}$, so

$$SNR_{pre}^{FM} = \frac{4 \times 10^2}{3k_f^2 \times 4N_0(k_f P^{1/2} + W)} = \frac{10^2}{3k_f^2 \times 2 \times 10^{-14}(k_f + 10^4)}$$

For the different values of k_f , the pre-detection SNR is

$$\begin{aligned} k_f &= 10kHz \implies SNR_{pre}^{FM} = 10^4 / 12 = 29dB > 12dB \\ k_f &= 50kHz \implies SNR_{pre}^{FM} = 11.11 = 10.45dB < 12dB \\ k_f &= 100kHz \implies SNR_{pre}^{FM} = 1.515 = 1.8dB < 12dB \end{aligned}$$

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Problem 9.25 continued

Therefore, for $k_f = 50$ kHz and 100 kHz, the pre-detection *SNR* is too low and the transmitter power would have to be increased by 1.55 dB and 10.2 dB, respectively.

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