Problem 9.7 Compute the post-detection SNR in the lower channel for Example 9.2 and compare to the upper channel.

Solution

The SNR of lower channel is, from Eq. (9.59)

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} = \frac{3A_c^2 k_f^2 (P/2)}{2N_0 W^3}$$

where we have assumed that half the power is in the lower channel. Using the approximation to Carson's Rule $B_T = 2\left(k_f P^{1/2} + D\right) \approx 2k_f P^{1/2} = 200 \text{ kHz}$, that is, $k_f^2 P = B_T^2 / 4$ this expression becomes

$$\begin{aligned}\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} &= \frac{A_c^2}{2N_0 B_T} \frac{3}{2} \frac{\left(B_T/2\right)^2}{W} \\ &= \mathrm{SNR}_{\mathrm{pre}}^{\mathrm{FM}} \frac{3}{8}\left(\frac{B_T}{W}\right)^3\end{aligned}$$

With a pre-detection SNR of 12 dB, we determine the post-detection SNR as follows

$$\begin{aligned}\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} &= \mathrm{SNR}_{\mathrm{pre}}^{\mathrm{FM}} \frac{3}{8}\left(\frac{200}{19}\right)^3 \\ &= 10^{12/10} \times 0.375 \times (10.53)^3 \\ &= 6.94 \times 10^3 \\ &\sim 38.4 \text{ dB}\end{aligned}$$

(*The answer in the text for the lower channel is off by factor 0.5 or 3 dB.*) For the upper channel, Example 9.2 indicates this result should be scaled by 2/52 and

$$\begin{aligned}\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} &= \mathrm{SNR}_{\mathrm{pre}}^{\mathrm{FM}} \frac{3}{8}\left(\frac{200}{19}\right)^3 \cdot \frac{2}{52} \\ &\sim 24.3 \text{ dB}\end{aligned}$$

So the upper channel is $10\log_{10}(52/2) \approx 14.1$ dB worse than lower channel.