Problem 9.9 A sample function

$$x(t) = A_c \cos(2\pi f_c t) + w(t)$$

is applied to a low-pass *RC* filter. The amplitude A_c and frequency f_c of the sinusoidal component are constant, and $w(t)$ is white noise of zero mean and power spectral density $N_0/2$. Find an expression for the output signal-to-noise ratio with the sinusoidal component of $x(t)$ regarded as the signal of interest.

Solution

The noise variance is proportional to the noise bandwidth of the filter so from Example 8.16,

$$\mathbf{E}\left[n^2(t)\right] = B_N N_0 = \frac{1}{4RC} N_0$$

and the signal power is $A_c^2 / 2$ fir a sinusoid, so the signal-to-noise ratio is given by

$$SNR = \frac{A_c^{\,2}}{2\left(\frac{N_0}{4RC}\right)} = \frac{2A_c^{\,2} RC}{N_0}$$