

Problem 10.1. Let H_0 be the event that a 0 is transmitted and let R_0 be the event that a 0 is received. Define H_1 and R_1 , similarly for a 1. Express the BER in terms of the probability of these events when:

(a) The probability of a 1 error is the same as the probability of a 0 error.

(b) The probability of a 1 being transmitted is not the same as the probability of a 0 being transmitted.

Solution

In both cases, the probability of error may be expressed as

$$\mathbf{P}[\text{error}] = \mathbf{P}(R_0|H_1)\mathbf{P}(H_1) + \mathbf{P}(R_1|H_0)\mathbf{P}(H_0) \quad (1)$$

(a) The BER is the same as the $\mathbf{P}[\text{error}]$ and with $\mathbf{P}(R_0|H_1) = \mathbf{P}(R_1|H_0) = p$ then

$$\mathbf{P}[\text{error}] = p[\mathbf{P}(H_1) + \mathbf{P}(H_0)] = p$$

since $\mathbf{P}(H_1) + \mathbf{P}(H_0) = 1$.

(b) With $\mathbf{P}(H_0) \neq \mathbf{P}(H_1)$, the answer is given by the general result of Eq. (1).