Problem 10.11. A communication system that transmits single isolated pulses is subject to multipath such that, if the transmitted pulse is $p(t)$ of length T, the received signal is

$$s(t) = p(t) + \alpha p(t-\tau)$$

Assuming that α and τ are known, determine the optimum receiver filter for signal in the presence of white Gaussian noise of power spectral density $N_0/2$. What is the post-detection SNR at the output of this filter?

Solution

We first note that the pulse is non-zero over the interval $0 \le t \le T+\tau$. From Section 10.2 the appropriate linear receiver is

$$Y = \int_0^{T+\tau} g(T+\tau-u) r(u) du$$

and the optimum choice for $g(t)$ is

$$g(T+\tau-t) = c\left(p(t) + \alpha p(t-\tau) \right)$$

where c is chosen such that

$$\int_0^{T+\tau} |g(t)|^2 \, dt = T+\tau$$

With this filtering arrangement, if follows from the modified Eq. (10.9) that

$$\mathbf{E}\left[N^2 \right] = \frac{N_0 \left(T+t \right)}{2}$$

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Problem 10.11 continued

The corresponding signal level S is

$$S = c\int_0^{T+\tau} g(T-t)\big(p(t)+\alpha\, p(t+\tau)\big)\,dt$$
$$= c\int_0^{T+\tau} \big(p(t)+\alpha\, p(t+\tau)\big)^2\,dt$$
$$= T+\tau$$

which follows from the normalization properties of c. The received signal to noise is then

$$\text{SNR} = \frac{S^2}{\mathbf{E}\left[N^2\right]} = \frac{T+\tau}{N_0/2}$$

Although the units on this expression may appear unusual, note that the units of N_0 are $(\text{volt})^2/\text{Hz} = (\text{volt})^2\text{-sec}$. The units of the numerator are also $(\text{volt})^2$-sec, although the $(\text{volt})^2$ has been suppressed. Consequently, the SNR is dimensionless, as it should be.