**Problem 10.12**. The impulse response corresponding to a root-raised cosine spectrum, normalized to satisfy Eq.(10.10), is given by

$$g(t) = \frac{4\alpha}{\pi} \frac{\cos\left[\frac{(1+\alpha)\pi t}{T}\right] + \frac{T}{4\alpha t}\sin\left[\frac{(1-\alpha)\pi t}{T}\right]}{1-\left(\frac{4\alpha t}{T}\right)^2}$$

where  $T = 1/2B_0$  is the symbol period and  $\alpha$  is the roll-off factor. Obtain a discrete-time representation of this impulse response by sampling it at t = 0.1nT for integer *n* such that -3T < t < 3T. Numerically approximate match filtering (e.g. with Matlab) by performing the discrete-time convolution

$$q_k = 0.1 \sum_{n=-60}^{60} g_n g_{k-n}$$

where  $g_n = g(0.1nT)$ . What is the value of  $q_k = q(0.1kT)$  for  $k = \pm 20, \pm 10$ , and 0?

## **Solution**

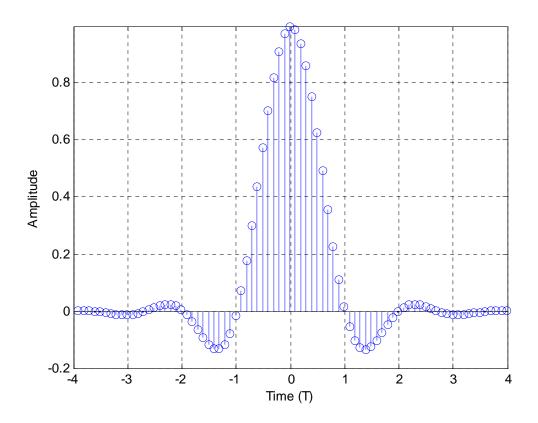
A Matlab script for this problem is shown below. Note the starting time of -3.01 is used to avoid divide-by-zero problems. Using the *filter* function is just one way the discrete convolution can be performed.

$$alpha = 0.5;B0 = 0.5;T = 1/(2*B0);t = [-3.01: 0.1:3] * T;%-- root raised cosine impulse responseg = cos((1+alpha)*pi*t/T) + (T/4/alpha)./t.* sin((1-alpha)*pi*t/T);g = g./(1 - (4*alpha*t/T).^2);g = 4*alpha/pi * g;%--- discrete convolution -----q = 0.1*filter(g,1, [g zeros(1,60)]);tp = [-6.01:0.1:6] * T;stem(tp,q)xlabel('Time (T)')ylabel('Amplitude')axis([-4 4 -.2 1]), grid on$$

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## **Problem 10.12 continued**

The plot of  $q_k$  is shown below for  $\alpha = 0.5$ . At  $k = \pm 20$  and  $\pm 10$ , the amplitude is approximately zero. At k = 0 the amplitude is 1.



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