**Problem 10.12**. The impulse response corresponding to a root-raised cosine spectrum, normalized to satisfy Eq.(10.10), is given by

$$g(t) = \frac{4\alpha}{\pi} \frac{\cos\left[\frac{(1+\alpha)\pi t}{T}\right] + \frac{T}{4\alpha t}\sin\left[\frac{(1-\alpha)\pi t}{T}\right]}{1-\left(\frac{4\alpha t}{T}\right)^2}$$

where $T = 1/2B_0$ is the symbol period and $\alpha$ is the roll-off factor. Obtain a discrete-time representation of this impulse response by sampling it at $t = 0.1nT$ for integer $n$ such that $-3T < t < 3T$. Numerically approximate match filtering (e.g. with Matlab) by performing the discrete-time convolution

$$q_k = 0.1\sum_{n=-60}^{60} g_n g_{k-n}$$

where $g_n = g(0.1nT)$. What is the value of $q_k = q(0.1kT)$ for $k = \pm 20, \pm 10$, and 0?

**Solution**

A Matlab script for this problem is shown below. Note the starting time of -3.01 is used to avoid divide-by-zero problems. Using the *filter* function is just one way the discrete convolution can be performed.

```
alpha = 0.5;
B0    = 0.5;
T     = 1/(2*B0);

t = [-3.01: 0.1 :3] * T;

%-- root raised cosine impulse response
g = cos( (1+alpha)*pi*t/T) + (T/4/alpha) ./ t .* sin( (1-
alpha)*pi*t/T);
g = g ./ (1 - (4*alpha*t/T).^2 );
g = 4*alpha/pi * g;

%--- discrete convolution -----
q = 0.1*filter(g,1, [g zeros(1,60)]);
tp = [-6.01:0.1:6] * T;
stem(tp,q)
xlabel('Time (T)')
ylabel('Amplitude')
axis([-4 4 -.2 1]), grid on
```

Continued on next slide

**Problem 10.12 continued**

The plot of $q_k$ is shown below for $\alpha = 0.5$ . At $k = \pm 20$ and $\pm 10$, the amplitude is approximately zero. At $k = 0$ the amplitude is 1.