Problem 10.13. Determine the discrete-time autocorrelation function of the noise sequence $\{N_k\}$ defined by Eq. (10.34)

$$N_k = \int_{-\infty}^{\infty} p(kT - t)w(t)dt$$

where $w(t)$ is a white Gaussian noise process and the pulse $p(t)$ corresponds to a root-raised cosine spectrum. How are the noise samples corresponding to adjacent bit intervals related?

Solution

The autocorrelation function of the noise at samples spaced by T is

$$\begin{aligned}
R_N(n) &= \mathbf{E}\left[N_k N_{k+n}\right] \\
&= \mathbf{E}\left[\int_{-\infty}^{\infty} p(kT-t)w(t)dt \cdot \int_{-\infty}^{\infty} p((k+n)T-s)w(s)ds\right] \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(kT-t)\,p((k+n)T-s)\mathbf{E}\left[w(t)w(s)\right]dtds \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(kT-t)\,p((k+n)T-s)\frac{N_0}{2}\delta(t-s)dtds
\end{aligned}$$

where we have interchanged integration and expectation on the third line, and the fourth line follows from the uncorrelated properties of the white noise. We next apply the sifting property of the delta function to obtain

$$\begin{aligned}
R_N(n) &= \int_{-\infty}^{\infty} p(kT-t)\,p((k+n)T-t)\frac{N_0}{2}dt \\
&= \frac{N_0}{2}\int_{-\infty}^{\infty} p(kT-t)\,p(t-(k+n)T)dt \\
&= \frac{N_0}{2}\delta(n)
\end{aligned}$$

where the second line follows from the even symmetry property of the raised cosine pulse, and third line follows from Eq. (10.32). Therefore, noise samples corresponding to adjacent bit intervals are not correlated.