Problem 10.18. Under the bandpass assumptions, determine the conditions under which the two signals $\cos(2\pi f_0 t)$ and $\cos(2\pi f_1 t)$ are orthogonal over the interval from 0 to *T*.

Solution

For two signals to be orthogonal over the interval from 0 to *T*, they must satisfy

$$\int_0^T \cos(2\pi f_0 t)\cos(2\pi f_1 t)dt = 0\,.$$

To verify this we perform the integration as follows:

$$\begin{aligned}\int_0^T \cos(2\pi f_0 t)\cos(2\pi f_1 t)dt &= \frac{1}{2}\int_0^T \left[\cos(2\pi(f_0+f_1)t)+\cos(2\pi(f_0-f_1)t)\right]dt \\ &= \frac{1}{4\pi(f_0+f_1)}\sin(2\pi(f_0+f_1))\Big|_0^T + \frac{1}{4\pi(f_0-f_1)}\sin(2\pi(f_0-f_1))\Big|_0^T \\ &= \frac{1}{4\pi(f_0+f_1)}\sin(2\pi(f_0+f_1)T) + \frac{1}{4\pi(f_0-f_1)}\sin(2\pi(f_0-f_1)T)\end{aligned}$$

By the bandpass assumption $(f_0+f_1) >> 1$ so the first term in the last line is negligible. For the second term to be zero it must satisfy

$$2\pi(f_0 - f_1)T = n\pi$$

where n is an integer. This implies that $(f_0 - f_1) = n/2T$.