**Problem 10.21.** A Hamming (15,11) block code is applied to a BPSK transmission scheme. Compare the block error rate performance of the uncoded and coded systems. Explain how this would differ if the modulation strategy was QPSK.

## Solution

1) For the uncoded system, the probability of a bit error with BPSK is

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

The probability of a block error with block length of 15 bits, assuming independent errors is:

$$P_b^{uncoded} = 1 - (1 - P_e)^{15}$$

2) For the coded system, with a (15,11) Hamming code, the probability of block error is

$$P_{b}^{coded} = 1 - (1 - P_{e}')^{15} - {\binom{15}{1}} (1 - P_{e}')^{14} P_{e}'$$

where  $P'_{e}$  is the bit error probability of coded bit, since the code can correct a single bit error. The probability of bit error in this case is:

$$P'_e = Q\left(\sqrt{\frac{2E_c}{N_0}}\right),$$

where  $E_c$  is the coded bit energy, and  $E_c = 11/15E_b$ . Therefore

$$P_e' = Q\left(\sqrt{\frac{22E_b}{15N_0}}\right)$$

To compare the block error probabilities of uncoded and coded systems, we use Matlab to plot the block error rate curves for  $P_b^{uncoded}$  and  $P_b^{coded}$  versus  $E_b/N_0$  (dB), as shown below

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The Matlab script that generates the above plot is  $E^{1}N_{12} dP_{12} = 1000.25 \pm 121$ 

*EbNodB*=[0:0.25:12]; EbNo  $= 10.^{(EbNodB/10)};$ = 0.5 \* erfc(sqrt(EbNo));Pe  $Puncoded = 1 - (1-Pe).^{15};$ EcNo = 11/15 \* EbNo;Peprime = 0.5 \* erfc(sqrt(EcNo)); $Pcoded = 1 - (1 - Peprime).^{15} - 15*(1 - Peprime).^{14}.*Peprime;$ semilogy(EbNodB,Puncoded) grid xlabel('Eb/No (dB)') ylabel('Block Error Rate') axis([0 20 1E-7 0.1]) hold on, semilogy(EbNodB,Pcoded,'g'), hold off

3) Since for QPSK modulation, bit error probabilities of uncoded bits  $P_e$  and coded bits  $P_e$  are unchanged compared with BPSK modulation, the block error probabilities of two systems are also the same as those of BPSK modulation.

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