Problem 10.21. A Hamming (15,11) block code is applied to a BPSK transmission scheme. Compare the block error rate performance of the uncoded and coded systems. Explain how this would differ if the modulation strategy was QPSK.

Solution

1) For the uncoded system, the probability of a bit error with BPSK is

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

The probability of a block error with block length of 15 bits, assuming independent errors is:

$$P_b^{uncoded} = 1-(1-P_e)^{15}$$

2) For the coded system, with a (15,11) Hamming code, the probability of block error is

$$P_b^{coded} = 1-(1-P_e')^{15} - \binom{15}{1}(1-P_e')^{14}P_e',$$

where P_e' is the bit error probability of coded bit, since the code can correct a single bit error. The probability of bit error in this case is:

$$P_e' = Q\left(\sqrt{\frac{2E_c}{N_0}}\right),$$

where E_c is the coded bit energy, and $E_c = 11/15E_b$. Therefore

$$P_e' = Q\left(\sqrt{\frac{22E_b}{15N_0}}\right)$$

To compare the block error probabilities of uncoded and coded systems, we use Matlab to plot the block error rate curves for $P_b^{uncoded}$ and P_b^{coded} versus E_b/N_0 (dB), as shown below

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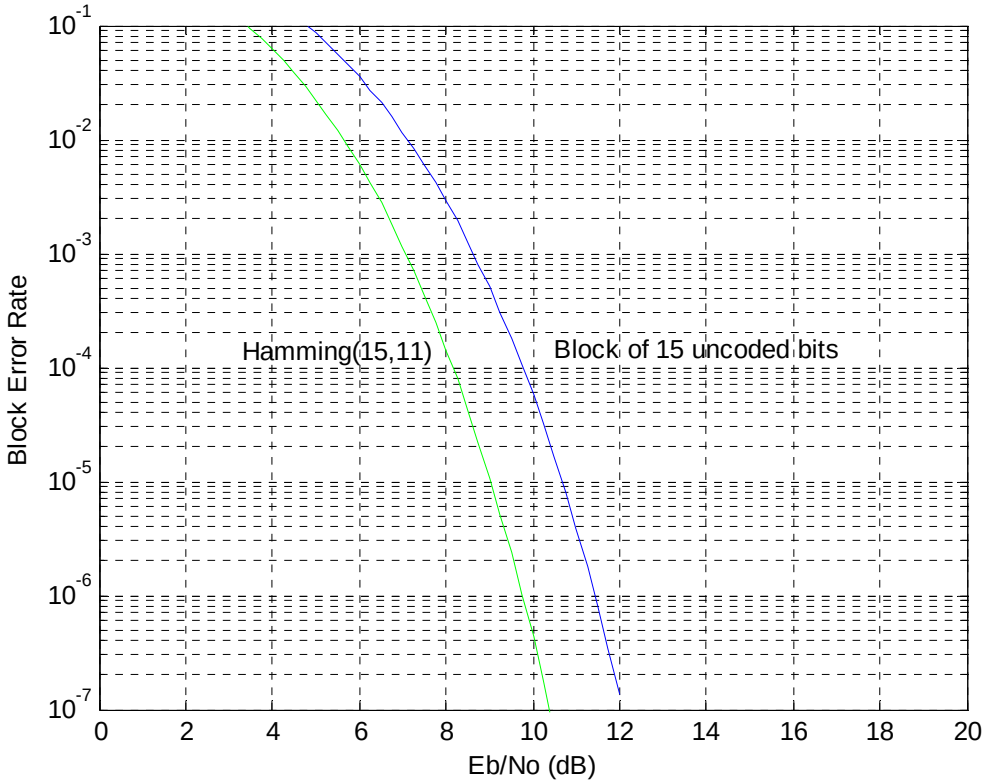
Problem 10.21 continued



The Matlab script that generates the above plot is

```
EbNodB=[0:0.25:12];
EbNo     = 10.^(EbNodB/10);
Pe       = 0.5*erfc(sqrt(EbNo));
Puncoded = 1 - (1-Pe).^15;
EcNo     = 11/15 * EbNo;
Peprime  = 0.5*erfc(sqrt(EcNo));
Pcoded   = 1 - (1-Peprime).^15 - 15*(1-Peprime).^14.*Peprime;
semilogy(EbNodB,Puncoded)
grid
xlabel('Eb/No (dB)')
ylabel('Block Error Rate')
axis([0 20 1E-7 0.1])
hold on, semilogy(EbNodB,Pcoded,'g'), hold off
```

3) Since for QPSK modulation, bit error probabilities of uncoded bits P_e and coded bits $P_e^{'}$ are unchanged compared with BPSK modulation, the block error probabilities of two systems are also the same as those of BPSK modulation.