Problem 10.25. Show that the noise variance of the in-phase component $n_I(t)$ of the band-pass noise is the same as the band-pass noise $n(t)$ variance; that is, for a band-pass noise bandwidth B_N

$$\mathbf{E}\left[n_I^2(t)\right] = N_0 B_N$$

Solution

Recall the spectra of narrowband noise $n(t)$ and its in-phase component $n_I(t)$ shown in Figure 8.23. The variance of a random process $x(t) = R_x(0) = \int_{-\infty}^{\infty} X(f)df$, where $X(f)$ is the power spectral density of $x(t)$. Therefore,

$$\begin{aligned}\mathrm{Var}[n(t)] &= \mathbf{E}\left[n^2(t)\right] \\ &= \int_{-\infty}^{\infty} S_N(f)df \\ &= 2\cdot\frac{N_0}{2}\cdot 2B \\ &= N_0\cdot 2B\end{aligned}$$

Where we have used the fact that for a bandpass signal $B_T = 2B$, that is twice the lowpass bandwidth. Similarly, the variance of the in-phase noise is

$$\begin{aligned}\mathrm{Var}[n_I(t)] &= \mathbf{E}\left[n_I^2(t)\right] \\ &= \int_{-\infty}^{\infty} S_{n_I}(f)df \\ &= N_0\cdot 2B\end{aligned}$$