Problem 10.26 In this problem, we investigate the effects when transmit and receive filters do not combine to form an ISI-free pulse shape. To be specific, data is transmitted at baseband using binary PAM with an exponential pulse shape $g(t)=\exp(-t/T)u(t)$ where T is the symbol period (see Example 2.2). The receiver detects the data using an integrate-and-dump detector.

(a) With data represented as ±1, what is magnitude of the signal component at the output of the detector.

(b) What is the worst case magnitude of the intersymbol interference at the output of the detector. (Assume the data stream has infinite length.) Using the value obtained in part (a) as a reference, by what percentage is the eye opening reduced by this interference.

(c) What is the rms magnitude of the intersymbol interference at the output of the detector? If this interference is treated as equivalent to noise, what is the equivalent signal-to-noise ratio at the output of the detector? Comment on how this would affect bit error rate performance of this system when there is also receiver noise present.

(Typo in problem statement, there should be minus sign in exponential.)

Solution

(a) For a data pulse

$$g(t) = A\exp(-t/T)u(t)$$

where A is the binary PAM symbol (±1). The desired output of an integrate-and-dump filter in the n^{th} symbol period is

$$\begin{aligned} G_n &= \int_{nT}^{(n+1)T} g(t-nT)dt \\ &= \int_0^T A_n \exp(-t/T)dt \\ &= A_n T\left(1-\exp(-1)\right) \end{aligned}$$

If the data is either ±1, then magnitude of the output is $T(1\text{-}e^{-1})$.

(b) In the n^{th} symbol period the received signal is

$$y(t) = \sum_{k=-\infty}^{\infty} A_n g(t-kT)$$

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The output of the detection filter in the n^{th} symbol period is

$$\begin{aligned} Y_n &= \int_{nT}^{(n+1)T} y(t)dt \\ &= \int_{nT}^{(n+1)T} \sum_{k=-\infty}^{\infty} A_k \exp\left(-(t-kT)/T\right)dt \\ &= \int_{nT}^{(n+1)T} A_n \exp\left(-(t-nT)/T\right)dt + \sum_{k=1}^{\infty} \int_{nT}^{(n+1)T} A_{n-k} \exp\left\{-(t-(n-k)T)/T\right\}dt \end{aligned}$$

where, due to the causality of the pulse shape, the symbols A_{n+1} and later due not cause intersymbol interference into symbol A_n. The first term in the above is the desired signal and the second term is the intersymbol interference. By letting $s = t - (n-k)T$, we can express this interference as

$$\begin{aligned} J_n &= \sum_{k=1}^{\infty} \int_{kT}^{(k+1)T} A_{n-k} \exp\left(-t/T\right)dt \\ &= \sum_{k=1}^{\infty} A_{n-k}T\left(\exp(-k) - \exp(-(k+1))\right) \end{aligned}$$

where each term in the summation corresponds to the interference caused by a previous symbol. For worst case interference we assume that all of the A_{n-k} have the same sign. Then this worst case interference is given by

$$\begin{aligned} J_n &= \sum_{k=1}^{\infty} A_{n-k}T\left(\exp(-k) - \exp(-(k+1))\right) \\ &\le T\left(1-\exp(-1)\right)\sum_{k=1}^{\infty}\exp(-k) \end{aligned}$$

To simplify the notation, we let $\alpha = \exp(-1)$. Then

$$\begin{aligned} J_n^{\max} &= T\left(1-\alpha\right)\sum_{k=1}^{\infty}\alpha^k \\ &= T\left(1-\alpha\right)\frac{\alpha}{1-\alpha} \\ &= \alpha T \end{aligned}$$

Comparing this worst case interference to the desired signal level G_n, the eye-opening is reduced by

$$\frac{J_n^{\max}}{G_n} \times 100 = \frac{T\alpha}{T\left(1-\alpha\right)} \times 100 = 58\%$$

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(c) From part (b), we found that k^{th} preceding symbol contributes an interference

$$I_n^k = A_{n-k}(1-\alpha)\alpha^k$$

The total interference is

$$\begin{aligned} J_n &= \sum_{k=1}^{\infty} I_n^k \\ &= \sum_{k=1}^{\infty} A_{n-k} T(1-\alpha)\alpha^k \end{aligned}$$

Since all symbol intervals are equivalent, we drop the subscript n on J_n. The mean value of this interference is $\mathbf{E}[J] = 0$ since $\mathbf{E}[A_{n-k}] = 0$. The variance of this interference is

$$\begin{aligned} Var(J) &= \mathbf{E}\left[J^2\right] \\ &= \sum_{k=1}^{\infty} \mathbf{E}\left[A_{n-k}^2\right] T^2 (1-\alpha)^2 \alpha^{2k} \\ &= T^2 (1-\alpha)^2 \frac{\alpha^2}{1-\alpha^2} \\ &= (\alpha T)^2 \frac{1-\alpha}{1+\alpha} \end{aligned}$$

where we have assumed the symbols are independent so that $\mathbf{E}[A_i A_j] = 0$ if $i \neq j$. The *rms* interference is given by the square root of the variance so

$$\begin{aligned} J_{rms} &= \alpha T \sqrt{\frac{1-\alpha}{1+\alpha}} \\ &= 0.25T \end{aligned}$$

which is clearly less than the worst case interference $J^{\max}$.

If we represent the signal power by S, the noise power by N, then the equivalent signal-to-noise ratio taking account of the intersymbol interference is

$$SNR = \frac{S}{N + J_{rms}^2}$$

The intersymbol interference will further degrade performance. In fact, if the worst case interference is large enough such that the eye closes, it will result in a lower limit on the bit error rate regardless of how little noise there is.