Problem 10.28. A binary FSK system transmits data at the rate of 2.5 megabits per second. During the course of transmission, white Gaussian noise of zero mean and power spectral density 10^{-20} watts per hertz is added to the signal. In the absence of noise, the amplitude of the received signal is 1 μV across 50 ohm impedance. Determine the average probability of error assuming coherent detection of the binary FSK signal.

Solution

The average probability of error for coherent FSK is

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

from Eq. (10.68). For this example, we have noise power spectral density is

$$N_0 = 2\times10^{-20}\,\text{watts / Hz}$$

and the energy per bit is

$$E_b = \frac{1}{2}\frac{A_c^2 T}{R},$$

In the text, we have nominally assumed the resistance is 1 ohm and omitted it. In this problem we use the resistance of R = 50 ohms. The symbol duration is $T = \dfrac{1}{2.5\times10^6}$ seconds and the amplitude of received signal is A_c = 1μV. Therefore,

$$\begin{aligned} E_b &= \frac{1}{2}\times\frac{1\times10^{-12}}{50}\times\frac{1}{2.5\times10^6} \\ &= 4\times10^{-21}\,\text{watts / Hz} \end{aligned}$$

Substituting the above values into the expression for P_e and we have the probability of error is

$$\begin{aligned} P_e &= Q(0.2) \\ &\cong 0.26 \end{aligned}$$