

Problem 10.29. One of the simplest forms of forward error correction code is the repetition code. With an N -repetition code, the same bit is sent N times, and the decoder decides in favor of the bit that is detected on the majority of trials (assuming N is odd). For a BPSK transmission scheme, determine the BER performance of a 3-repetition code.

Solution

With 3-repetition code, the decoder will output the correct bit if there are one or fewer errors in the 3-bit code. Thus, assuming bit errors are independent, the bit error rate is

$$\begin{aligned} P_b^{coded} &= (1 - P_e)^3 + \binom{3}{1} P_e (1 - P_e)^2, \\ &= (1 - P_e)^2 (1 + 2P_e) \end{aligned}$$

where P_e is the bit error rate of channel bit. With BPSK, the formula for bit error probability is

$$\begin{aligned} P_e &= Q\left(\sqrt{\frac{2E_c}{N_0}}\right) \\ &= Q\left(\sqrt{\frac{2E_b}{3N_0}}\right), \end{aligned}$$

since ratio of channel bit energy to information bit energy is given by $E_c = 1/3E_b$. Therefore, the bit error probability of the 3-repetition code is

$$P_b^{coded} = \left(1 - Q\left(\sqrt{\frac{2E_b}{3N_0}}\right)\right)^2 \left(1 + 2Q\left(\sqrt{\frac{2E_b}{3N_0}}\right)\right)$$