Problem 10.3 If $g(t) = c \text{ rect}\left[\dfrac{\alpha(t - T/2)}{T}\right]$, determine c such g(t) satisfies Eq. (10.10) where $\alpha > 1$.

Solution

From the definition of the rect(.) function,

$$
\begin{aligned}
g(t) &= c \text{ rect}\left(\frac{\alpha(t - T/2)}{T}\right) \\
&= \begin{cases} c & |t - T/2| < T/(2\alpha) \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

Substituting this into Eq. (10.10)

$$
\begin{aligned}
T &= \int_0^T |g(t)|^2 dt \\
&= c^2 \int_{T/2 - T/(2\alpha)}^{T/2 + T/(2\alpha)} 1^2 dt \\
&= c^2 T/\alpha
\end{aligned}
$$

And so $c = \sqrt{\alpha}$.