Problem 10.30 In this experiment, we simulate the performance of bipolar signalling in additive white Gaussian noise. The Matlab script included in Appendix 7 for this experiment:

- generates a random sequence with rectangular pulse shaping
- adds Gaussian noise
- detects the data with a simulated integrate-and-dump detector

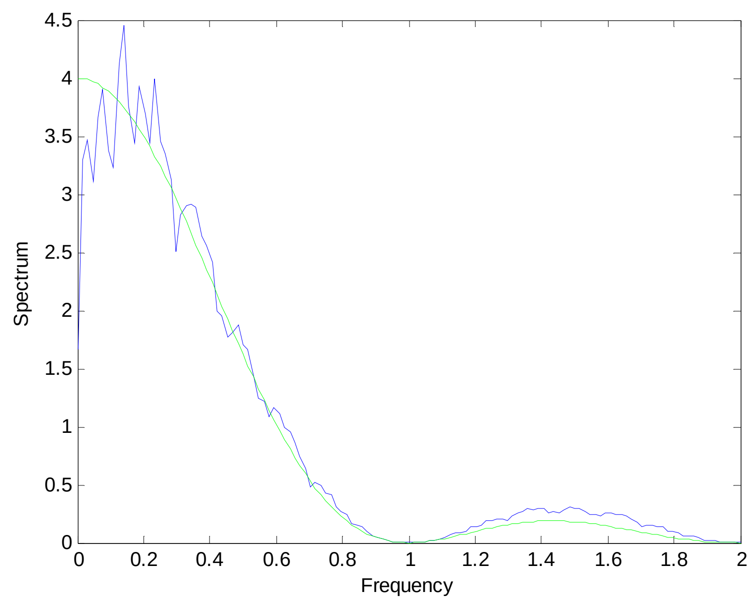
With this Matlab script
(a) Compute the spectrum of the transmitted signal and compare to the theoretical.
(b) Explain the computation of the noise variance given an E_b/N_0 ratio.
(c) Confirm the theoretically predicted bit error rate for E_b/N_0 from 0 to 10 dB.

Solution

(a) The provided script plots the simulated spectrum before noise is added. If we add the statement

*hold on, plot(F, abs(2*sinc(F)).^2,'g'), hold off*

at the same point, we obtain the following comparison graph. The two graphs agree reasonably well. There are two reasons for the differences observed with the simulated spectrum. The first is the relatively short random sequence used for generating the plot and the second is an aliasing effect.



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Problem 10.30 continued

(b) The calculation of the noise variance in a discrete time simulation proceeds as follows. We are given the sampling rate F_s and the required E_b/N_0 to simulation. We then note that

$$\begin{aligned} E_b &= \int_{-\infty}^{\infty} |p(t)|^2 \, dt \\ &\approx \sum |p_k|^2 \, T_s \end{aligned} \qquad (1)$$

where $p(t)$ is the pulse shape, $\{p_k\}$ is its sample version and $T_s = 1/F_s$ is the sample interval. On the other hand, if generate noise of variance σ^2, due to Nyquist considerations this can only be distributed over a bandwidth Fs, thus the noise spectral density is

$$\frac{N_0}{2} = \frac{\sigma^2}{F_{s\;s}} \qquad (2)$$

Re-arranging Eq. (2) and substituting Eq. (1) and the knowns, we have

$$\begin{aligned} \sigma^2 &= \frac{N_0}{2} F_s \\ &= \left(\frac{F_s}{2}\right)\left(\frac{E_b}{N_0}\right)^{-1} E_b \\ &= \left(\frac{F_s}{2}\right)\left(\frac{E_b}{N_0}\right)^{-1} \sum |p_k|^2 \, T_s \\ &= \frac{1}{2}\left(\frac{E_b}{N_0}\right)^{-1} \sum |p_k|^2 \end{aligned}$$

which agrees with what is used in the script (except that in the script we have suppressed F_s and T_s, knowing they would cancel).

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Problem 10.30 continued

(c) To compute the bit error rate for 0 to 10 dB, we add the following statements around the provided script

```
for kk = 0:10
        Eb_N0 = 10^(kk/10);
        Nbits = 100000; %  increase for higher Eb/N0
        …(provided script)
        BER(kk+1)   = Nerrs/Nbits
end
semilogy([0:10], BER)
xlabel('Eb/No (dB)')
ylabel('BER')
grid on
hold on, semilogy([0:10], 0.4*erfc(sqrt(10.^([0:10]/10))),'g')
```

The following plot is then produced by the Matlab script which shows good agreement between theory and simulation.