Problem 10.4. Show that with on-off signaling, the probability of a Type II error in Eq.(10.23) is given by

$$P[Y > \gamma \mid H_0] = Q\left(\frac{\gamma}{\sigma}\right)$$

Solution

A Type II error probability is

$$P[Y > \gamma \mid H_0] = \frac{1}{\sqrt{2\pi}\sigma} \int_{\gamma}^{+\infty} \exp\left(-\frac{y^2}{2\sigma^2}\right) dy$$

Let $s = \dfrac{y}{\sigma}$, and then

$$P[Y > \gamma \mid H_0] = \frac{1}{\sqrt{2\pi}} \int_{\gamma/\sigma}^{+\infty} \exp\left(-\frac{s^2}{2}\right) ds = Q\left(\frac{\gamma}{\sigma}\right)$$

using the definition of the Q-function given in Section 8.4.