Problem 10.5 Prove the property of root-raised cosine pulse shape $p(t)$ given by Eq. (10.32), using the following steps:

(a) If $R(f)$ is the Fourier transform representation of $p(t)$, what is the Fourier transform representation of $p(t-lT)$?

(b) What is the Fourier transform of $q(\tau) = \int_{-\infty}^{\infty} p(\tau - t)p(t - lT)dt$? What spectral shape does it have?

(c) What $q(\tau)$? What is $q(kT)$?

Use these results to show that Eq. (10.32) holds.

Solution

(a) From the time-shifting property of Fourier transforms (see Section 2.2), we have that

$$\mathbf{F}[p(t-lT)] = R(f)\exp(-j2\pi flT)$$

(b) From the convolution property of Fourier transforms (See Section 2.2) we have that

$$\begin{aligned} Q(f) &= \mathbf{F}[q(\tau)] \\ &= \mathbf{F}[p(t)]\mathbf{F}[p(t-lT)] \\ &= R^2(f)\exp(-j2\pi flT) \end{aligned}$$

(c) Since $R(f)$ is the root-raised cosine spectrum, $R^2(f)$ is the raised cosine spectrum and so $q(\tau)$ corresponds to a raised cosine pulse. In particular, using the time-shifting property of inverse Fourier transforms

$$q(\tau) = m(\tau - lT)$$

where $m(\tau)$ is the raised cosine pulse shape. Using the properties of the raised cosine pulse shape (see Section 6.4)

$$\begin{aligned} q(kT) &= m(kT - lT) \\ &= \begin{cases} 1 & k = l \\ 0 & k \neq l \end{cases} \\ &= \delta(k-l) \end{aligned}$$

and Eq. (10.32) holds.