Chapter 6
Baseband Data Transmission

Problem 6.1

The pulse shape $p(t)$ of a baseband binary PAM system is defined by

$$p(t) = \text{sinc}\left(\frac{t}{T_b}\right)$$

where T_b is the bit duration of the input binary data. The amplitude levels at the pulse generator output are +1 V or -1 V, depending on whether the binary symbol at the input is 1 or 0, respectively. Sketch the waveform at the output of the receiving filter in response to the input data 001101001.

Solution

For the input data sequence 001101001, the waveform at the output receiving filter consists of the positive sinc pulse +sinc(t/T_b) every time symbol 1 is transmitted and the negative sinc pulse -sinc(t/T_b) every time symbol 0 is transmitted. Moreover, there will be no intersymbol interference present in this waveform because the sinc pulse for a particularly symbol goes through zero whenever another symbol is transmitted.

Problem 6.2

Show that for positive frequencies, the area under the normalized raised-cosine curve of $P(f)/(\sqrt{E}/2B_0)$ versus f/B_0 is equal to unity for all values of the roll-off factor in the range $0 < \alpha \leq 1$. A similar statement holds for negative frequencies.

Solution

For $\alpha = 0$,the normalized raised-cosine curve reduces to the idealized Nyquist channel, for which the area under this curve for the frequencies is immediately seen to be unity. For nonzero values of α in the range $0 < \alpha < 1$, the raised-cosine curve is odd-symmetric about the value $P(f)/(\sqrt{E}2B_0) = 0.5$. Consequently, the area under this normalized curve remains equal to unity for positive frequencies.

Problem 6.3

Given that $P(f)$ is the Fourier transform of a pulse-like function $p(t)$, we may state the following theorem:[1]

The pulse $p(t)$ decreases asymptotically with time as $1/t^{k+1}$ provided that the following two conditions hold:

1. The first k-1 derivatives of the Fourier transform $P(f)$ with respect to frequency f are all continuous.
2. The kth derivative of $P(f)$ is discontinuous.

Demonstrate the validity of this theorem for the three different values of α plotted in Fig. 6.3(a).

1. For a detailed discussion of this theorem, see Gitlin, Hayes and Weinstein (1992), p.258.

Solution

Consider first the idealized Nyquist channel for which $\alpha = 0$. With the brick-wall characteristic of this limiting case, it is immediately apparent that the Fourier transform $P(f)$ has *no* continuous derivatives with respect to f. Hence, according to the theorem, the inverse Fourier transform $p(t)$ decreases asymptotically as $1/|t|$; this is confirmed by the formula of Eq. (6.14), where the numerator ranges between -1 and +1, whereas the denominator is proportional to t.

Consider next the case of a raised-cosine pulse $p(t)$ defined in Eq. (6.19), rewritten here as

$$p(t) = \sqrt{E}\ \frac{\sin(2\pi B_0 t)}{2\pi B_0 t}\left(\frac{\cos(2\pi\alpha B_0 t)}{1-16\alpha^2 B_0^2 t^2}\right)$$

In this case, we readily see that $p(t)$ decreases asymptotically as $1/|t|^3$, for $0 < \alpha \le 1$. Examining the two plots shown in Fig. 6.3(a), we see that the first derivative of $P(f)$ for this range of values of α is continuous, but the second derivative is discontinuous. Here again validity of the theorem is established.

Problem 6.4

Equation (6.17) defines the raised-cosine pulse spectrum $P(f)$ as real-valued and therefore zero delay. In practice, every transmission system experiences some finite delay. To accommodate this practicality, we may associate with $P(f)$ a linear phase characteristic over the frequency band $0 \le |f| \le 2B_0 - f_1$.

(a) Show that this modification of $P(f)$ introduces a finite delay into its inverse Fourier transform, namely, the pulse shape $p(t)$.

(b) According to Eq. (6.19), $p(t)$ represents a non-causal time response. The delay introduced into $p(t)$ through the modification of $P(f)$ has also a beneficial effect, tending to make $p(t)$ essentially causal. For this to happen however, the delay must not be less than a certain value dependent on the roll-off factor α. Suggest suitable values for the delay for $\alpha = 0$, 1/2, and 1.

Solution

(a) Let the linear phase characteristic appended to $P(f)$ be

$$\theta(f) = 2\pi f\tau$$

where τ is delay to be determined. Then, the modified raised-cosine pulse spectrum is defined by

$$P_{\text{modified}}(f) = P(f)e^{-j\theta(f)}$$
$$= P(f)e^{-j2\pi f\tau}$$

Invoking the time-shifting property, we therefore have

$$p_{\text{modified}}(t) = p(t-\tau)$$

where $p(t)$ is defined by Eq. (6.19).

(b) For $p_{\text{modified}}(t)$ to be causal, it has to be zero for $t < 0$.

From Fig. 6.3(b) in the text, we deduce that we may essentially set

(i) $\tau = 5s$ for $\alpha = 0$

(ii) $\tau = 3s$ for $\alpha = 1/2$

(iii) $\tau = 2.5s$ for $\alpha = 1$

Increasing α corresponds to increasing transmission bandwidth B_T. We therefore find that as the transmission bandwidth B_T is increased, the necessary delay τ is progressively reduced, which is in accord with the inverse relationship that exists between behaviors of a function in the time- and frequency-domains.

(c) The slope of $\theta(f)$ with respect to f is

$$\frac{\partial\theta(f)}{\partial f} = 2\pi\tau$$

Hence,

(i) slope = -10π for $\alpha = 0$

(ii) slope = -6π for $\alpha = 1/2$

(iii) slope = -5π for $\alpha = 1$

Problem 6.5

Starting with the formula of Eq. (6.24) and using the definition of Eq. (6.26), demonstrate the property of Eq. (6.25).

Solution

Using $f' = f - B_0$ in Eq. (6.24), we may express the second line of Eq. (6.24) in the text for positive frequencies

$$P_v(f') = \frac{\sqrt{E}}{4B_0}\left\{1 + \cos\left[\frac{\pi(f' + B_0 - f_1)}{2(B_0 - f_1)}\right]\right\}$$

$$= \frac{\sqrt{E}}{4B_0}\left\{1 + \cos\left[\frac{\pi}{2} + \frac{\pi f'}{2(B_0 - f_1)}\right]\right\}$$

$$= \frac{\sqrt{E}}{4B_0}\left\{1 - \sin\left[\frac{\pi(f')}{2(B_0 - f_1)}\right]\right\} \qquad \text{for } f_1 - B_0 \le f' \le 0 \tag{1}$$

Similarly, we may express the third line of Eq. (6.24) as

$$P_v(f') = \frac{\sqrt{E}}{4B_0}\left\{\sin\left(\left(\left[\frac{\pi(f')}{2(B_0 - f_1)}\right]\right) - 1\right)\right\} \qquad \text{for } 0 \le f' \le B_0 - f_1 \tag{2}$$

From Eqs. (1) and (2), we readily see that

$$P_v(-f') = P_v(f')$$

which is the desired property.

Problem 6.6

Assume the following perfect conditions:

- The residual distortion in the data transmission system is zero.
- The pulse shaping is partitioned equally between the transmitter-channel combination and the receiver.
- The transversal equalizer is infinitely long.

(a) Find the corresponding value of the equalizer's transfer function in terms of infinite the overall pulse spectrum $P(f)$.
(b) For the roll-off factor $\alpha = 1$, demonstrate that a transversal equalizer of length 6 would essentially satisfy the perfect condition found in part (a) of the problem.

Solution

(a) With the pulse-shaping shared equally between the transmit filter-channel combination and receive filter, we may use an equalizer of transfer function $P^{1/2}(f)$ to realize the receive filter, where $P(f)$ is the raised cosine-pulse spectrum.
(b) For a roll-off factor $\alpha = 0$, $P(f)$ reduces to the idealized brick-wall function

$$P(f) = \begin{cases} \dfrac{\sqrt{E}}{2B_0}, & \text{for } B_0 < f < B_0 \\ 0, & \text{otherwise} \end{cases}$$

which defines the Nyquist channel. In light of the transfer function of the equalizer (used to realize the receive filter) is defined by

$$P(f) = \begin{cases} \dfrac{E^{1/4}}{(2B_0)^{1/2}}, & \text{for } B_0 < f < B_0 \\ 0, & \text{otherwise} \end{cases}$$

Correspondingly, the impulse response of the equalizer is required to pass through an infinite number of time instants at $t = \pm 1/(2B_0)$, $\pm 1/B_0$, $\pm 3/(2B_0)$,.... We may satisfy this idealized requirement by using an equalizer of infinite length. Such an equalizer would have an infinite number of adjustable parameters $\ldots W_N, \ldots, W_{-1}, W_0, W_1, \ldots, W_N$ that can be used to satisfy the zero-forcing basis of Eq. (6.43) of the text. In practice, however, the idealized impulse response of the channel reduces effectively to zero at some large enough time t, which, in turn, means that an equalizer of large enough length can be used to satisfy the idealized Nyquist channel.

Note: In the first printing of the book, the following correction in the first line of part (b) of Problem 6.6 should be made: - Roll-off factor $\alpha = 0$.

Problem 6.7

Since $P(f)$ is an even real-valued function, its inverse Fourier transform may be simplified to the formula

$$p(t) = 2\int_0^{\infty} P(f)\cos(2\pi f t)\,df \qquad (1)$$

The $P(f)$ is itself defined by Eq. (6.17) which is reproduced here in the following form (ignoring the scaling factor $\sqrt{E}$ for convenience of presentation)

$$P(f) = \begin{cases} \dfrac{1}{2B_0}, & 0 < |f| f_1 \\ \dfrac{1}{4B_0}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2B_0 - 2f_1}\right]\right\}, & f_1 < f < 2B_0 - f_1 \\ 0, & |f| > 2B_0 - f_1 \end{cases} \tag{2}$$

Hence, using Eq. (2) in (1) and recognizing that $\alpha = (B_0 - f_1)/B_0$, we may write

$$\begin{aligned}
p(t) &= \frac{1}{B_0}\int_0^{f_1} \cos(2\pi f t)\,df + \frac{1}{2B_0}\int_{f_1}^{2B_0 - f_1}\left[1 + \cos\left(\frac{\pi(f - f_1)}{2B_0\alpha}\right)\right]\cos(2\pi f t)\,df \\
&= \left[\frac{\sin(2\pi f t)}{2\pi B_0 t}\right] + \left[\frac{\sin(2\pi f t)}{4\pi B_0 t}\right]_{f_1}^{2B_0 - f_1} \\
&\quad + \frac{1}{4}B_0\left[\frac{\sin\left(2\pi f t + \dfrac{\pi(f - f_1)}{2B_0\alpha}\right)}{2\pi t + \pi/2B_0\alpha}\right]_{f_1}^{2B_0 - f_1} + \frac{1}{4B_0}\left[\frac{\sin\left(2\pi f t - \dfrac{\pi(f - f_1)}{2B_0\alpha}\right)}{2\pi t + \pi/2B_0\alpha}\right]_{f_1}^{2B_0 - f_1} \\
&= \frac{\sin(2\pi f_1 t)}{4\pi B_0 t} + \frac{\sin[2\pi t(2B_0 - f_1)]}{4\pi B_0 t} \\
&\quad - \frac{1}{4B_0}\ \frac{\sin(2\pi f_1 t) + \sin[2\pi t(2B_0 - f_1)]}{2\pi t - \pi/2B_0\alpha} + \frac{\sin(2\pi f_1 t) + \sin[2\pi t(2B_0 - f_1 t)]}{2\pi t - \pi/2B_0\alpha} \\
&= \frac{1}{B_0}[\sin(2\pi f_1 t) + \sin[2\pi t(2B_0 - f_1)]]\left[\frac{1}{4Wt} - \frac{\pi t}{(2\pi t)^2 - (\pi/2B_0\alpha)^2}\right] \\
&= \frac{1}{B_0}[\sin(2\pi B_0 t)\cos(2\pi\alpha B_0)]\left[\frac{-\pi/(2B_0\alpha)^2}{4\pi t[(2\pi t)^2 - \pi/(2B_0\alpha)^2]}\right] \\
&= \operatorname{sinc}(2B_0 t)\cos(2\pi\alpha B_0 t)\left[\frac{1}{1 - 16\alpha^2 B_0{}^2 t^2}\right]
\end{aligned} \tag{3}$$

Equation (3) is a reproduction of Eq. (6.19), except for the scaling factor $\sqrt{E}$ which we ignored in Eq. (2) for convenience of presentation.

Problem 6.8

Starting with Eq. (3) in the solution to Problem 6.7, reproduced here for $0 < \alpha \leq 1$:

$$p(t) = \operatorname{sinc}(2B_0 t)\left(\frac{\cos(2\pi\alpha B_0 t)}{1 - 16\alpha^2 B_0^2 t^2}\right)$$

For $\alpha = 1$, this formula reduces to

$$p(t) = \mathrm{sinc}(2B_0t)\left(\frac{\cos(2\pi B_0 t)}{1 - 16B_0^2t^2}\right) \tag{1}$$

Next, using the trigonometric identity

$$\sin(A)\cos(A) = \frac{1}{2}\sin(2A)$$

and the definition of the sinc function

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

we may go on to write

$$\begin{aligned} \mathrm{sinc}(2B_0t)\cos(2\pi B_0t) &= \frac{\sin(2\pi B_0t)\cos(2\pi B_0t)}{2\pi B_0 t} \\ &= \frac{\sin(4\pi B_0 t)}{4\pi B_0 t} \\ &= \mathrm{sinc}(4B_0t) \end{aligned} \tag{2}$$

Accordingly, using Eq. (2) in (1), we get

$$p(t) = \frac{\mathrm{sinc}(4B_0t)}{1 - 16B_0^2t^2}$$

which is the desired result, except for the scaling factor $\sqrt{E}$.

Problem 6.9

The bandwidth B of a raised cosine pulse spectrum is $2B_0$ - f_1, where $B_0 = 1/2T_b$ and $f_1 = B_0(1 - a)$. Thus $B = B_0(1 + \alpha)$. For a data rate of 56 kilobits per second, B_0 = 28 kHz.

(a) α = 0.25,
B = 28 kHz x 1.25 = 35 kHz

(b) α = 0.5,
B = 28 kHz x 1.5 = 42 kHz

(c) α = 0.75,
B = 28 x 1.75 = 49 kHz

(d) α = 1.0,
B = 28 x 2 = 56 kHz

Problem 6.10

The raised cosine pulse bandwidth $B_T = 2B_0 - f_1$, where $B_0 = 1/2T_b$. For this channel, B_T = 75 kHz. For the given bit duration, B_0 = 50 kHz. Then,

$$\begin{aligned} f_1 &= 2B_0 - B_T \\ &= 25 \text{ kHz} \\ \alpha &= 1 - f_1/B_T \\ &= 0.5 \end{aligned}$$

The design parameters of the required raised-cosine pulse spectrum are f_1 = 25 kHz and α = 0.5.

Problem 6.11

The transmission bandwidth B_T is related to the excess bandwidth f_v by the formula (see Eqs. (6.21) and (6.22))

$$B_T = B_0 + f_v$$

where $B_0 = 1/(2T_b)$. We may therefore express the bit rate $1/T_b$ as a function of the excess bandwidth f_v as follows:

$$\frac{1}{T_b} = 2(B_T - f_v) \tag{1}$$

From Eq. (1), we see that the bit rate $1/T_b$ decreases linearly with the excess bandwidth f_v for a fixed channel bandwidth B_T. Specifically, with B_T = 3 kHz, the bit rate versus excess bandwidth graph takes the form shown in Fig. 1. Note that the excess bandwidth f_v attains its largest value when the roll-off factor α equals unity, in which case f_v = 3 kHz.

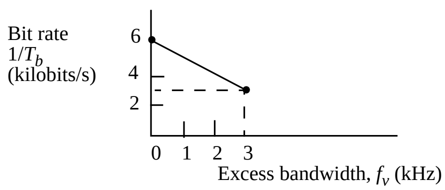


Figure 1

Problem 6.12

We are given the following specifications:

$$B_T = 3 \text{ kHz}$$

$$\frac{1}{T_b} = 4.5 \text{ kilobits/s}$$

(a) The transmission bandwidth is related to the roll-off factor by the formula (see Eq. (6.21))

$$B_T = B_0(1 + \alpha) \tag{1}$$

where

$$B_0 = 1/(2T_b)$$

Therefore, with $(1/T_b)$ = 4.5 kilobits/s, we have

$$B_0 = 2.25 \text{ kHz}$$

Hence, solving Eq. (1) for the roll-off factor, we get

$$\alpha = \frac{B_T}{B_0} - 1$$

$$= \frac{3}{2.25} - 1$$

$$= \frac{1}{3}$$

(b) The excess bandwidth is defined (see Eq. (6.22))

$$\begin{aligned} f_v &= \alpha B_0 \\ &= \frac{1}{3} \times 2.25 \\ &= 0.75 \quad \text{kHz} \end{aligned}$$

Problem 6.13

According to Eq. (6.30), the pulse-shaping criterion for zero-intersymbol interference is embodied in the relation

$$\sum_{m=-\infty}^{\infty} P\left(f - \frac{m}{T}\right) = \text{constant} \qquad (1)$$

where $P(f)$ is pulse-shaping spectrum and $1/T$ is the signaling rate.

(a) The pulse-shaping spectrum of Fig. 6.13(a) is defined by

$$P(f) = \begin{cases} \sqrt{E}/(2B_0) & \text{for} \quad f = 0 \\ \dfrac{\sqrt{E}}{2B_0}\left(1 - \dfrac{f}{B_0}\right) & \text{for} \quad 0 < f < B_0 \\ 0 & \text{for} \quad f = B_0 \end{cases} \qquad (2)$$

Substituting Eq. (2) into (1) leads to the following condition on the signaling rate

$$\frac{1}{T} = \frac{B_0}{2}$$

or, equivalently,

$$B_0 = 2/T \qquad (3)$$

(b) The pulse-shaping spectrum of Fig. 6.12(b) is defined by

$$P(f) = \begin{cases} \sqrt{E}/(2B_0) & \text{for } 0 \le |f| < f_1 \\ \dfrac{\sqrt{E}}{2B_0}\left(1 - \dfrac{f - f_1}{B_0 - f_1}\right) & \text{for } f_1 < f < B_0 \\ 0 & \text{for } f > B_0 \end{cases} \qquad (4)$$

Substituting Eq. (3) into (1) leads to the following condition on the signaling rate

$$\frac{1}{T} = \frac{1}{2}(f_1 + B_0)$$

Equivalently, for a given f_1, we require that

$$B_0 = \frac{2}{T} - f_1 \tag{5}$$

(c) Among the four pulse-shaping spectra described in Figs. 6.2(a), 6.3(a), 6.12(a) and 6.12(b) the prescriptions defined in Fig. 6.3(a) corresponding to the roll-off factor $\alpha = 1/2$ and $\alpha = 1$ are the preferred choices in practice for the following reasons:
- Mathematical simplicity and therefore relative ease of practical realization.
- Improved signaling rate compared to the prescriptions described in Fig.s 6.12(c) and 6.12(b).

Problem 6.14

The transmission bandwidth is maintained at the value

$B_T = 3$ kHz

In using an 8-level PAM system, the signaling rate is raised to

$$\frac{1}{T} = (\log_2 8) \times \left(\frac{1}{T_b}\right), \qquad T_b = \text{bit duration}$$

$$= 3 \times 4.5$$

$$= 13.5 \text{ kilobits/s}$$

However, the symbol rate is maintained at 4.5 x 10^3 symbols/s. Hence, as in Problem 6.12,

(a) The roll-off factor remains at $\alpha = 1/3$.

(b) The excess bandwidth remains at $f_v = 0.75$ kHz.

Problem 6.15

The codeword consists of $\log_2(128) = 7$ bits. With an additional bit added for synchronization, the overall codeword consists of 8 bits. The method of data transmission is quaternary (i.e., 4-level) PAM, and the roll-off factor $\alpha = 1$.

(a) For binary PAM, the signaling rate is defined by (see Eqs. (6.13) and (6.21))

$$\frac{1}{T_b} = \frac{2B_T}{1 + \alpha} \tag{1}$$

For $\alpha = 1$ and $B_T = 13$ kHz, the use of Eq. (1) yields

$$\frac{1}{T_b} = \frac{2 \times 13}{1 + 1}$$

$$= 13 \text{ kilobits/s}$$

The signaling rate of the quaternary PAM system is therefore

$$\frac{1}{T} = \frac{\log_2 4}{T_b}$$

$$= 2 \times 13 \text{ kilosymbols/s}$$

(b) Each element of the overall codeword of the PCM signal must fit into the bit duration

$$T_b = \frac{1}{13 \times 10^3} \text{seconds}$$
$$= 77 \ \mu\text{s}$$

With each code-word consisting of 8 bits, the code-word occupies the duration

$$T_s = 8T_b$$
$$= 8 \times 77 = 616 \ \mu\text{s}$$

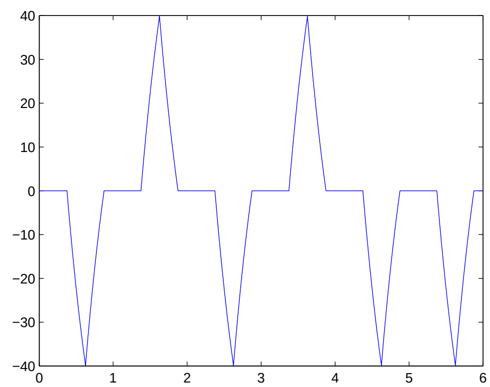
The sampling rate applied to the analog signal is therefore

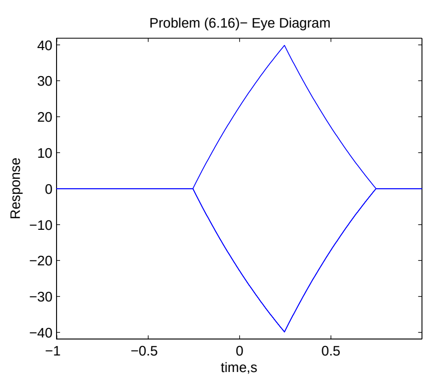
$$f_s = \frac{1}{T_s}$$
$$= \frac{10^6}{616} \text{Hz}$$
$$= 162 \ \text{kHz}$$

The highest frequency component of the analog signal is therefore

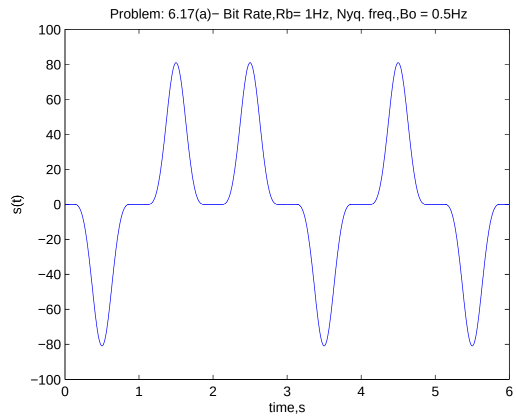
$$W = \frac{f_s}{2} = 81 \ \text{kHz}$$

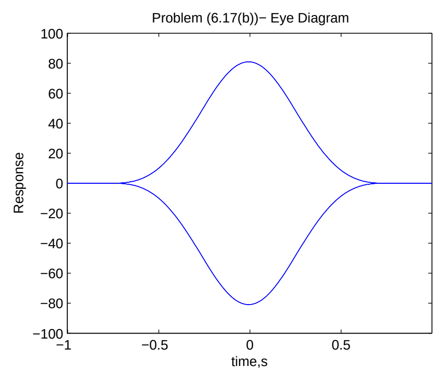
Problem 6.16





Problem 6.17





Problem 6.18

(a) The impulse response of the data-transmission system is defined by (see Fig. 1)

$c_n = \{0.0, 0.15, 0.68, -0.22, 0.08\}$

Using a three-tap transversal filter for zero-forcing equalization, we write in accordance with Eq. (6.43):

$$\begin{bmatrix} 0.68 & 0.15 & 0.0 \\ -0.22 & 0.68 & 0.15 \\ 0.08 & -0.22 & 0.68 \end{bmatrix} \begin{bmatrix} w_{-1} \\ w_0 \\ w_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad (1)$$

In Eq. (1), we have set $\sqrt{E} = 1$ to simplify the presentation. Solving this simultaneous system of three equations, we obtain the tap-weight (parameter) vector,

$$\mathbf{w} = \begin{bmatrix} w_{-1} \\ w_0 \\ w_1 \end{bmatrix} = \begin{bmatrix} 0.68 & 0.15 & 0.0 \\ -0.22 & 0.68 & 0.15 \\ 0.08 & -0.22 & 0.68 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.2825 \\ 1.2805 \\ 0.4475 \end{bmatrix} \tag{2}$$

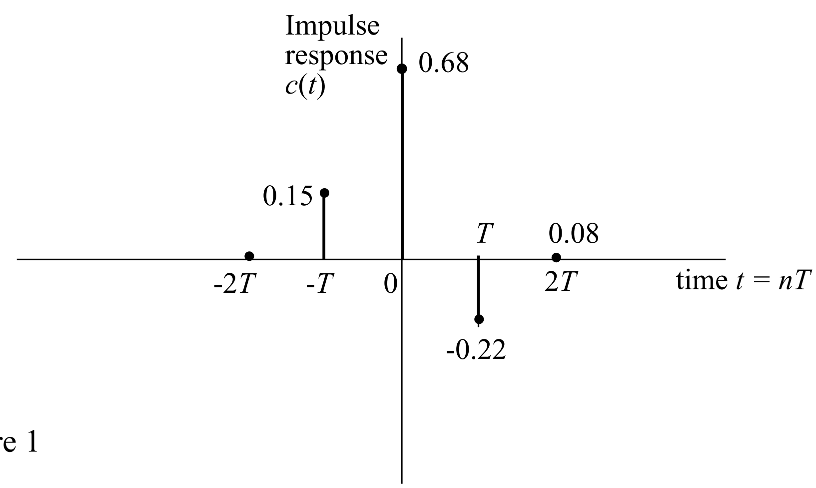


Figure 1

(b) The residual intersymbol interference produced at the equalizer output is given by

$$\mathbf{z} = \mathbf{cw} \tag{3}$$

where

$$\mathbf{c} = \begin{bmatrix} 0.15 & 0.0 & 0.0 \\ 0.68 & 0.15 & 0.0 \\ -0.22 & 0.68 & 0.15 \\ 0.08 & -0.22 & 0.68 \\ 0.0 & 0.08 & -0.22 \end{bmatrix} \tag{4}$$

Therefore, using Eqs. (4) and (2) in (3), we get the residual interference vector

$$\mathbf{z} = \begin{bmatrix} -0.0424 \\ 0 \\ 1 \\ 0 \\ 0.004 \end{bmatrix} \tag{5}$$

(c) From Eq. (5), we see that the largest contribution to the residual interference is

Problem 6.19

In this problem, the transversal zero-forcing equalizer has five adjustable weights. As in Problem 6.18, the unequalized impulse response is defined by

$c_n = \{0.0, 0.15, 0.68, -0.22, 0.08\}$

Accordingly, application of Eq. (6.43) yields (again setting $\sqrt{E} = 1$ to simplify the presentation)

$$\underbrace{\begin{bmatrix} 0.68 & 0.15 & 0.0 & 0.0 & 0.0 \\ -0.22 & 0.68 & 0.15 & 0.0 & 0.0 \\ 0.08 & -0.22 & 0.68 & 0.15 & 0.0 \\ 0.0 & 0.08 & -0.22 & 0.68 & 0.15 \\ 0.0 & 0.0 & 0.08 & -0.22 & 0.68 \end{bmatrix}}_{\mathbf{c}} \underbrace{\begin{bmatrix} w_{-2} \\ w_{-1} \\ w_0 \\ w_1 \\ w_2 \end{bmatrix}}_{\mathbf{w}} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad (1)$$

Solving this system of five simultaneous equations for the tap-weight vector, we get

$$\mathbf{w} = \begin{bmatrix} -0.0581 \\ -0.2635 \\ 1.2800 \\ 0.4465 \\ -0.0061 \end{bmatrix} \qquad (2)$$

Comparing Eq. (1) of this problem with Eq. (1) of the previous problem, we see some basic differences and therefore consequences:

(i) Unlike Problem 6.18, the 5-by-5 metric **c** in Eq. (1) has a row (namely, the third row) which completely describes the unequalized impulse response of the data-transmission system.

(ii) As a consequence of point (i), the 5-by-1 parameter vector **w** produces complete equalization of the system; that is, unlike Problem 6.18, there is no residual intersymbol interference left after equalization.

(iii) The zero residual interference is the result of using a five-tap equalizer which has sufficient degrees of freedom to force each element of the impulse response $\{c_n\}$ down to the desired value of zero.

Problem 6.20

(a) When the two-level sequence embodying

$$a_k = \begin{cases} +1 \text{ if symbol } b_k \text{ is } 1 \\ -1 \text{ if symbol } b_k \text{ is } -1 \end{cases} \qquad (1)$$

is applied to the duobinary conversion filter, the sequence is converted into a three-level output defined by

$$c_k = a_k + a_{k-1} \tag{2}$$

The three levels of c_k are -2, 0, and +2. One effect of transforming Eq. (1) into Eq. (2) is to produce correlated three-level sequence c_k from an uncorrelated two-level sequence a_k.

The overall transfer function of the duobinary conversion filter is therefore defined by

$$\begin{aligned} H(f) &= H_{\text{Nyquist}}(f)[1 + \exp(-j2\pi f T_b)] \\ &= H_{\text{Nyquist}}(f)[\exp(j\pi f T_b) + \exp(-j\pi f T_b)]\exp(-j\pi f T_b) \\ &= 2H_{\text{Nyquist}}(f)\cos(\pi f T_b)\exp(-j\pi f T_b) \end{aligned} \tag{3}$$

For an ideal Nyquist channel, $B_0 = 1/2T_b$. Ignoring the scaling factor $1/T_b$, we may therefore write

$$H_{\text{Nyquist}}(f) = \begin{cases} 1, & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Substituting Eq. (4) into (3), we obtain

$$H(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b), & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

(b) From the first line of Eq. (3) and the defining Eq. (4), we find that the impulse response of the duobinary conversion filter is

$$\begin{aligned} h(t) &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi(t-T_b)/T_b]}{\pi(t-T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[(\pi t/T_b)-\pi]}{\pi(t-T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t-T_b)/T_b} \\ &= \frac{T_b^2\sin(\pi t/T_b)}{\pi t(T_b - t)} \end{aligned} \tag{6}$$

(c) The original sequence may be detected from the duobinary-coded sequence using decision feedback, as shown by

$$\hat{a}_k = c_k - \hat{a}_{k-1} \tag{7}$$

A major drawback of this detection rule is that for the current detection $\hat{a}_k$ to be correct, the previous detection $\hat{a}_{k-1}$ has to be correct. If this requirement is not satisfied, we have error propagation.

Problem 6.21

To overcome the error-propagation problem experienced in Problem 6.20, we use precoding before the duobinary coding, as shown in Fig. 6.14. The precoder is defined by

$$d_k = b_k \oplus d_{k-1} \tag{1}$$

where the symbol $\oplus$ denotes modulo-two addition (i.e., EXCLUSIVE OR) According to Eq. (1), we have

$$d_k = \begin{cases} \text{symbol 1} & \text{if either } b_k \text{ or } d_{k-1} \text{ is 1} \\ \text{symbol 0} & \text{otherwise} \end{cases}$$

As before, the pulse-amplitude modulator output is therefore defined by $a_k = \pm 1$. Applying this sequence to the duobinary conversion filter, we get

$$c_k = a_k + a_{k-1} \tag{2}$$

Note that unlike the linear operation of duobinary coding of Eq. (2), the precoding of Eq. (1) is nonlinear.

The combined use of Eqs. (1) and (2) yields

$$c_k = \begin{cases} 0 & \text{if the original data symbol } b_k \text{ is 1} \\ \pm 2 & \text{if } b_k \text{ is 0} \end{cases} \tag{3}$$

From Eq. (3), we therefore deduce the following decision rule for detecting the original data sequence b_k from c_k, as follows:

If $|c_k| < 1$, say symbol b_k is 1

If $|c_k| > 1$, say symbol b_k is 0 (4)

which can be realized by using a rectifier followed by a threshold device.

The solutions to parts (a), (b) and (c) of the problem in response to the input sequence 0010110 are presented in Table 1.

Table 1: Illustrating Example 3 on Duobinary Coding

Binary sequence $\{b_k\}$		0	0	1	0	1	1	0
Precoded sequence $\{d_k\}$	1	1	1	0	0	1	0	0
Two-level sequence $\{a_k\}$	+1	+1	+2	-1	-1	+1	-1	-1
Duobinary coder output $\{c_k\}$		+2	+2	0	-2	0	0	-2
Binary sequence obtained by applying decision rule of Eq. (7.76)		0	0	1	0	1	1	0

Problem 6.22

(a) For the modified duobinary conversion filter shown in Fig. 6.15, we have

$$c_k = a_k - a_{k-2} \tag{1}$$

Here again, we find that a three-level sequence is generated. Specifically, for $a_k = \pm 1$, we find from Eq. (1) that c_k has three possible values: 2, 0, +2.

The overall transfer function of the modified duobinary conversion filter shown i Fig. 6.15 is therefore given by

$$\begin{aligned} H(f) &= H_{\text{Nyquist}}(f)[1 - \exp(-j4\pi f T_b)] \\ &= H_{\text{Nyquist}}(f)[\exp(j2\pi f T_b) - \exp(-j2\pi f T_b)]\exp(-j2\pi f T_b) \\ &= 2jH_{\text{Nyquist}}(f)\sin(2\pi f T_b)\exp(-j\pi f T_b) \end{aligned} \tag{2}$$

With

$$H_{\text{Nyquist}}(f) = \begin{cases} 1 & \text{for } |f| \le 1/2T_b \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

we may therefore express $H(f)$ as

$$H(f) = \begin{cases} 2j\sin(2\pi f T_b)\exp(-j2\pi f T_b) & \text{for } |f| \le 1/2T_b \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

which is the form of a half-cycle sine function.

(b) The corresponding impulse response of the modified duobinary conversion filter follows from the first line of Eq. (2); specifically,

$$\begin{aligned} h(t) &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi(t-2T_b)/T_b]}{\pi(t-2T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi(t/T_b)-2\pi]}{\pi(t-2T_b)/T_b} \\ &= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin[\pi t/T_b]}{\pi(t-2T_b)/T_b} \\ &= \frac{2T_b^2\sin(\pi t/T_b)}{\pi t(2T_b - t)} \end{aligned} \tag{5}$$

(c) With the precoder in place at the front end of the modified duobinary conversion filter as shown in Fig. 6.15, we have

$$d_k = b_k \oplus d_{k\text{-}1} \tag{6}$$

where b_k is the incoming binary sequence and d_k is the precoder output.

Assuming the use of a polar representation for the precoded sequence d_k, we find that the original data sequence b_k may be detected from the encoded sequence c_k by disregarding the polarity; specifically,

$$\begin{aligned} &\text{If } |c_k| > 1, \quad \text{say symbol } b_k \text{ is } 1 \\ &\text{If } |c_k| < 1, \quad \text{say symbol } b_k \text{ is } 0 \end{aligned} \tag{7}$$

(d) The virtues of modified duobinary coding are two-fold:

- In the absence of channel noise, the detected binary sequence b_k is exactly the same as the original data sequence b_k; this statement also applies to the duobinary coding with precoding.

- The use of Eq. (6) requires the addition of two extra bits to the precoded sequence b_k in accordance with Eq. (6). The composition of the decoded sequence $\hat{b}_k$ using Eq. (7) is invariant to the selection made for these two additional bits.

Note: In the first printing of the book, the delay element of the precoder in Fig. 6.15 should read $2T_b$ to be consistent with Eq. 7.

Problem 6.23

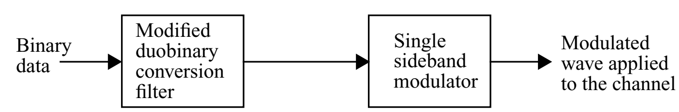
From Eq. (4) in the solution to Problem 6.22 we see that the transfer function of the modified duobinary conversion filter (shown in Fig. 6.15) is zero at $f = 0$. Hence, unlike the ordinary duobinary conversion filter, the modified duobinary conversion filter can be used to handle single-sideband transmission of data.

Specifically, Fig. 1(a) depicts the proposed data transmission system. The transmitter consists of two functional blocks:

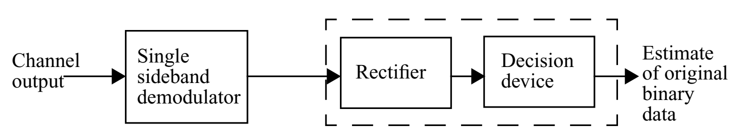
- Modified duobinary conversion filter, which transforms the incoming binary data into a new format whose spectrum has low-frequency content around the origin.
- Single sideband modulator, which upconverts the transformed data to the desired band occupied by the lower or upper sideband of the modulated wave.

Correspondingly, the receiver consists of two functional blocks (see Fig. 1(b))

- Single sideband demodulator.
- Detector, consisting of a rectifier followed by decision device, for recovering the original binary data stream.



(a) Transmitter



(b) Receiver

Figure 1