Problem 7.30. The purpose of this experiment is to demonstrate that the simulation of a digitally modulated signal on a computer can be simplified by using the idea of complex envelope that was introduced in Chapter 3. In particular, the experiment compares band-pass and baseband forms of data transmission, supported by the matlab scripts provided in Appendix 7. The scripts provided therein also cater to the generation of binary random sequences needed to perform the experiment and thereby add practical realism to the experiments.

(a) To proceed then, generate two random binary sequences and use them to form the multiplexed band-pass signal

$$s(t) = b_I(t)\cos\ 2\pi f_c t\ \ - b_Q(t)\sin\ 2\pi f_c t$$

Hence, compute the magnitude spectrum of $s(t)$, using the following parameters:

Carrier frequency,	f_c = 10 Hz
Symbol (bit) duration,	T_b = 1s

Assume that the two binary sequences are synchronized and they both use amplitude levels ±1 to represent symbols 0 and 1.

(b) Using complex notations, define the complex envelope

$$\tilde{b}(t) = b_I(t) + jb_Q(t)$$

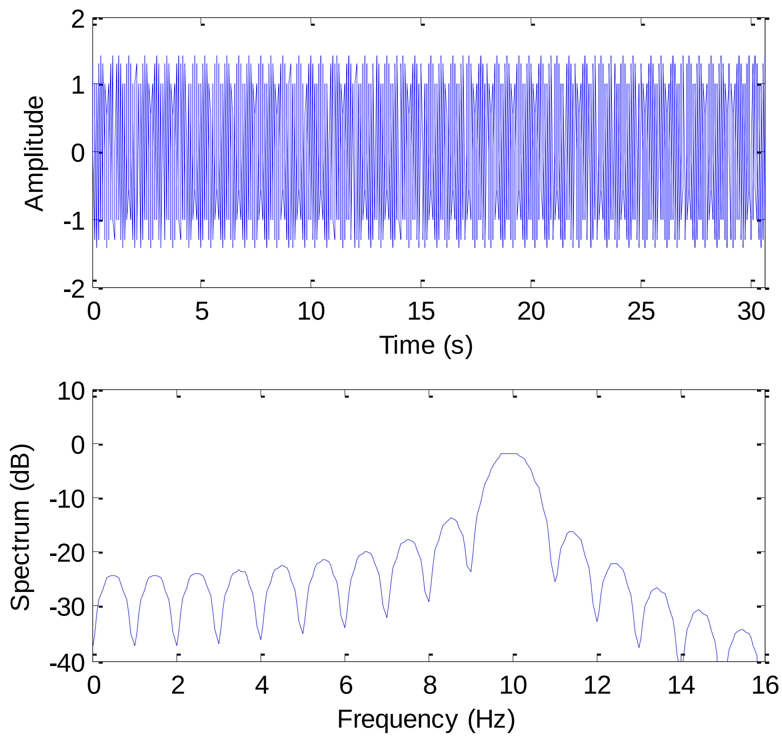
on the basis of which we may reconstruct the original band-pass signal

$$s(t) = \text{Re}\ \ \tilde{b}(t)\exp\ \ j2\pi f_c t$$

Specifically, compute the magnitude spectrum of the complex envelope $b(t)$, and compare it to the magnitude spectrum of computed in part (a). Comment on the computational significance of this comparison.

Solution

(a) Running the Matlab script provided in Appendix 7, the following plots are obtained.



The top plot shows the time domain version of the bandpass signal: the carrier appears to show a small amount of amplitude modulation but this is due to the sampling process; if the sampling rate is increased by a factor of four this amplitude modulation disappears as one would expect with rectangular pulse shaping. The bottom plot shows the frequency-domain version of the bandpass signal: it is a sin *x*/*x* spectrum that is centered at the carrier frequency of 10 Hz, and the first null is offset by the bit rate of 1 Hz. The spectrum is not perfectly symmetric about the carrier due to aliasing which affects the higher frequency components.

(b) We modify the provided script by inserting the statement

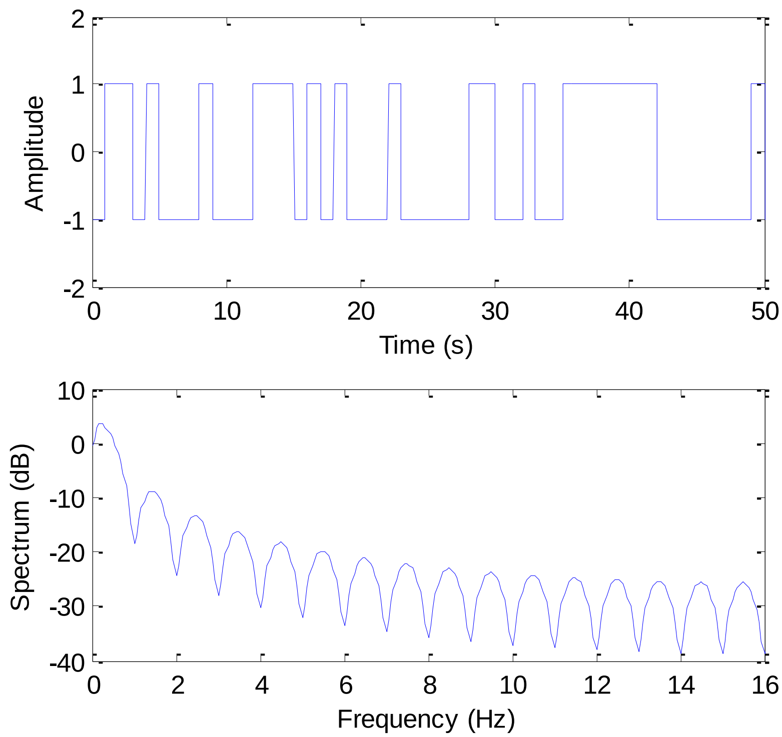
```
b = bIp + j*bQp;
```

and modifying the two statements

```
subplot(2,1,1), plot(t,real(b));      % time display

[spec,freq] = spectrum(b,nFFT,nFFT/4,nFFT/2,Fs);
```

With these changes, we obtain the plots shown below.



The top plot shows the time-domain sequence of random data with rectangular pulse shaping. The bottom graph shows the sin *x* /*x* magnitude spectrum centered at the origin. In part (a), distortion of both the time-domain and frequency domain signals was noted due to the limitations of the sampling rate. In part (b), these distortions are much less evident. Consequently, if we simulate signals at complex baseband then we may use much lower sampling rates (and thus less computational requirements) than for bandpass signals and obtain the same accuracy.

Problem 7.31 Repeat Problem 7.30, this time using a raised-cosine pulse shape of roll-off factor to construct the binary sequences $b_I(t)$ and $b_Q(t)$; Appendix 7 provides the matlab scripts for generating the raised-cosine pulse. Compute the magnitude spectrum of the complex envelope and compare it to the magnitude spectrum of the band-pass signal $s(t)$. Comment on your results.

Solution

We modify the script of Problem 7.30(a) by replacing

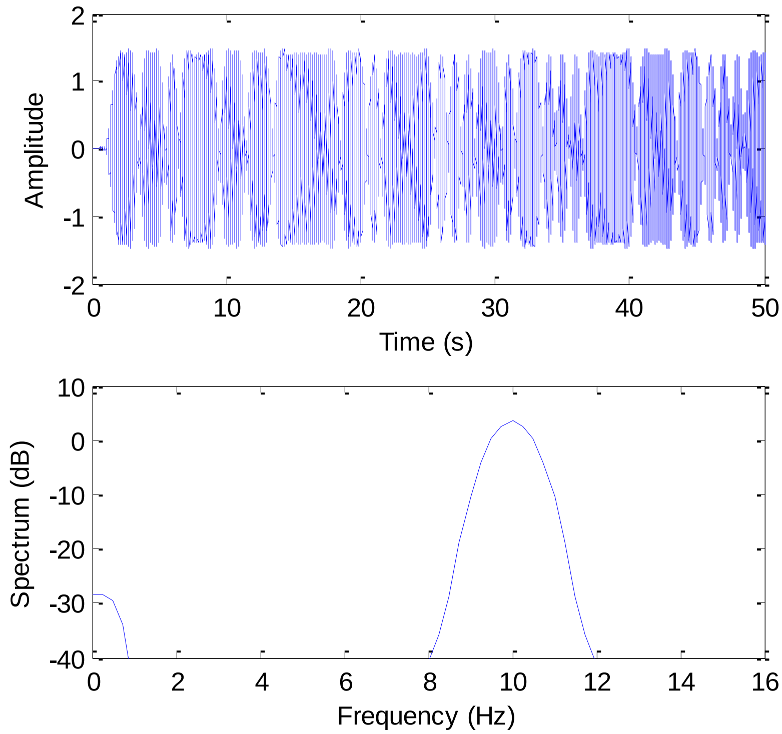
```
PulseShape = ones(1,Fs); % rectangular pulse shape
```

With the lines

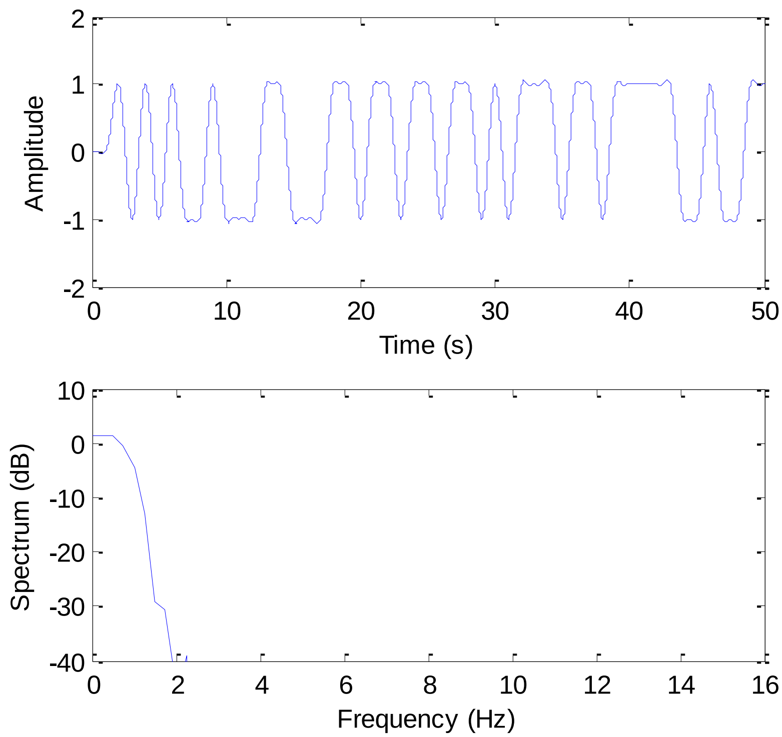
```
B0   = 0.5;                              % (Hz)
t    = [-2.001: 1/Fs : +2.001]           % time scale for pulse shape
```

```
rcos = sinc(4*B0*t) ./ (1-16*B0^2*t.^2);  % from Eq.(6.20)
PulseShape = rcos;
```

Doing so, we obtain the following graphs. The top graph shows the time-domain versio of the bandpass signal including the amplitude modulation that occurs with raised cosine pulse shaping. The bottom graph shows the raised cosine spectrum of the transmitted signal; there presence of the effects of aliasing is evident by the spurious signal present at 0 Hz in the magnitude spectrum.



If we make changes similar to those of Problem 7.30(b), then we obtain the following plots. The top graph shows the baseband *I*-channel consisting of a random data stream with raised cosine pulse shaping. The bottom graph shows the magnitude spectrum of the complex baseband signal. There is no evidence of significant aliasing effects in this figure. The effects of aliasing are less evident in the raised-cosine case because the spectrum is much more constrained than with rectangular pulse shaping.



2
1
0
-1
-2
Amplitude
0
10
20
30
40
50
Time (s)
10
0
-10
-20
-30
-40
Spectrum (dB)
0
2
4
6
8
10
12
14
16
Frequency (Hz)