Chapter 9 Solutions

Problem 9.1 In practice, we often cannot measure the signal by itself but must measure the signal plus noise. Explain how the SNR would be calculated in this case.

Solution
Let $r(t) = s(t) + n(t)$ be the received signal plus noise. Assuming the signal is independent of the noise, we have that the received power is

$$\begin{aligned} R_0 &= \mathbf{E}\left[r^2(t)\right] \\ &= \mathbf{E}\left[(s(t)+n(t))^2\right] \\ &= \mathbf{E}\left[s^2(t)\right] + 2\mathbf{E}\left[s(t)n(t)\right] + \mathbf{E}\left[n^2(t)\right] \\ &= \mathbf{E}\left[s^2(t)\right] + 2\mathbf{E}\left[s(t)\right]\mathbf{E}\left[n(t)\right] + \mathbf{E}\left[n^2(t)\right] \\ &= S + 0 + N \end{aligned}$$

where S is the signal power and N is the average noise power. We then measure the noise alone

$$\begin{aligned} R_1 &= \mathbf{E}\left[n^2(t)\right] \\ &= N \end{aligned}$$

and the SNR is given by

$$\text{SNR} = \frac{R_0 - R_1}{R_1}$$

Problem 9.2 A DSC-SC modulated signal is transmitted over a noisy channel, having a noise spectral density $N_0/2$ of $2\text{x}10^{-17}$ watts per hertz. The message bandwidth is 4 kHz and the carrier frequency is 200 kHz. Assume the average received power of the signal is -80 dBm. Determine the post-detection signal-to-noise ratio of the receiver.

Solution
From Eq. (9.23), the post-detection SNR of DSB-SC is

$$\text{SNR}_{\text{post}}^{\text{DSB}} = \frac{A_c^2 P}{2N_0 W}$$

The average received power is $\frac{A_c^2 P}{2} = -80\ dBm = 10^{-11}$ watts. With a message bandwidth of 4 kHz, the post-detection SNR is

$$\text{SNR}_{\text{post}}^{\text{DSB}} = \frac{10^{-11}}{(4\times 10^{-17})4000} = 62.5 \sim 18.0\ dB$$

Problem 9.3. For the same received signal power, compare the post-detection SNRs of DSB-SC with coherent detection and envelope detection with k_a = 0.2 and 0.4. Assume the average message power is P = 1.

Solution

From Eq. (9.23), the post-detection SNR of DSB-SC with received power $\frac{{}^{DSB}A_c^{\,2}P}{2}$ is

$$SNR_{post}^{DSB} = \frac{{}^{DSB}A_c^2 P}{2N_0 W}$$

From Eq. (9.30), the post-detection SNR of AM with received power $\frac{{}^{AM}A_c^2}{2}\left(1+k_a^{\,2}P\right)$ is

$$SNR_{post}^{AM} = \frac{{}^{AM}A_c^2 k_a^2 P}{2N_0 W}$$

So, by equating the transmit powers for DSB-Sc and AM, we obtain

$$\frac{{}^{DSB}A_c^2 P}{2} = \frac{{}^{AM}A_c^2}{2}\left(1+k_a^{\,2}P\right)$$

$$\Rightarrow \frac{{}^{AM}A_c^2}{2} = \frac{{}^{DSB}A_c^2}{2}\left(\frac{P}{1+k_a^{\,2}P}\right)$$

Substituting this result into the expression for the post-detection SNR of AM,

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{AM}} = \frac{{}^{DSB}A_c^{\,2}P}{2N_0 W}\left(\frac{k_a^{\,2}P}{1+k_a^{\,2}P}\right) = \mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}}\Delta$$

Where the factor Δ is

$$\Delta = \frac{k_a^{\,2}P}{1+k_a^{\,2}P}$$

With k_a= 0.2 and P= 1, the AM SNR is a factor $\Delta = \frac{(.2)^2}{1.04} = .04$ less.

With k_a = 0.4 and P = 1, the AM SNR is a factor $\Delta = \frac{(.4)^2}{1+.16} = \frac{.16}{1.16} \approx 0.14$ less.

Problem 9.4. In practice, there is an arbitrary phase θ in Eq. (9.24). How will this affect the results of Section 9.5.2?

Solution

Envelope detection is insensitive to a phase offset.

Problem 9.5. The message signal of Problem 9.2 having a bandwidth W of 4 kHz is transmitted over the same noisy channel having a noise spectral density $N_0/2$ of $2\text{x}10^{-17}$ watts per hertz using single-sideband modulation. If the average received power of the signal is -80 dBm, what is the post-detection signal-to-noise ratio of the receiver? Compare the transmission bandwidth of the SSB receiver to that of the DSB-SC receiver.

Solution

From Eq. (9.23)

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{SSB}} = \frac{A_c^2 P}{2N_0 W}$$

with $\frac{A_c^{\;2}P}{2} = -80\ \mathrm{dBm}$, $W = 4\ \mathrm{kHz}$, and $N_0 = 4\times 10^{-17}$. The

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{SSB}} = 18\ dB$$

The transmission bandwidth of SSB is 4 kHz, half of the 8 kHz used with DSB-SC.

Problem 9.6 The signal $m(t) = \cos(2000\pi t)$ is transmitted by means of frequency modulation. If the frequency sensitivity k_f is 2 kHz per volt, what is the Carson's rule bandwidth of the FM signal. If the pre-detection SNR is 17 dB, calculate the post-detection SNR. Assume the FM demodulator includes an ideal low-pass filter with bandwidth 3.1 kHz.

Solution

The Carson Rule bandwidth is $B_T = 2(k_f A + f_m) = 2(2(1) + 2) = 8\ kHz$. Then from Eq.(9.59),

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3} = \frac{A_c^2}{2N_0 B_T}\left(\frac{3k_f^2 P}{W^3} B_T\right)$$

We observed that the first factor is the pre-detection SNR, and we may write this as

$$
\begin{aligned}
\text{SNR}_{\text{post}}^{\text{FM}} &= \text{SNR}_{\text{pre}}^{\text{FM}} \left(\frac{3\cdot 2^{2}\ {}^{1}\!\!/_{2}}{3.1\ {}^{3}} 8 \right) \\
&= \text{SNR}_{\text{pre}}^{\text{FM}} \times 1.61 \\
&\sim 19.2\ \text{dB}
\end{aligned}
$$

(There is an error in the answer given in the text.)

Problem 9.7 Compute the post-detection SNR in the lower channel for Example 9.2 and compare to the upper channel.

<u>Solution</u>

The SNR of lower channel is, from Eq. (9.59)

$$
\text{SNR}_{\text{post}}^{\text{FM}} = \frac{3A_c^{\,2} k_f^{\,2} (P/2)}{2N_0 W^3}
$$

where we have assumed that half the power is in the lower channel. Using the approximation to Carson's Rule $B_T = 2\left(k_f P^{1/2} + D\right) \approx 2k_f P^{1/2} = 200\ \text{kHz}$, that is, $k_f^2 P = B_T^2/4$ this expression becomes

$$
\begin{aligned}
\text{SNR}_{\text{post}}^{\text{FM}} &= \frac{A_c^{\,2}}{2N_0 B_T} \frac{3}{2} \frac{B_T/2}{W}^{2} \\
&= \text{SNR}_{\text{pre}}^{\text{FM}} \frac{3}{8} \left(\frac{B_T}{W} \right)^3
\end{aligned}
$$

With a pre-detection SNR of 12 dB, we determine the post-detection SNR as follows

$$
\begin{aligned}
\text{SNR}_{\text{post}}^{\text{FM}} &= \text{SNR}_{\text{pre}}^{\text{FM}} \frac{3}{8} \left(\frac{200}{19} \right)^3 \\
&= 10^{12/10} \times 0.375 \times (10.53)^3 \\
&= 6.94 \times 10^3 \\
&\sim 38.4\ \text{dB}
\end{aligned}
$$

(*The answer in the text for the lower channel is off by factor 0.5 or 3 dB.*) For the upper channel, Example 9.2 indicates this result should be scaled by 2/52 and

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} = \mathrm{SNR}_{\mathrm{pre}}^{\mathrm{FM}} \frac{3}{8}\left(\frac{200}{19}\right)^{3} \square \frac{2}{52}$$
$$\sim 24.3\,\mathrm{dB}$$

So the upper channel is $10\log_{10}(52/2) \approx 14.1$ dB worse than lower channel.

Problem 9.8 An FM system has a pre-detection SNR of 15 dB. If the transmission bandwidth is 30 MHz and the message bandwidth is 6 MHz, what is the post-detection SNR? Suppose the system includes pre-emphasis and de-emphasis filters as described by Eqs. (9.63) and (9.64). What is the post-detection SNR if the $f_{3\mathrm{dB}}$ of the de-emphasis filter is 800 kHz?

Solution

From Eq. (9.59), (see Problem 9.7), the post-detection SNR without pre-emphasis is

$$\begin{aligned}\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} &= \mathrm{SNR}_{\mathrm{pre}}^{\mathrm{FM}} \frac{3}{4}\left(\frac{B_T}{W}\right)^{3} \\ &\sim 15\,\mathrm{dB} + 19.7\,\mathrm{dB} \\ &= 34.7\,\mathrm{dB}\end{aligned}$$

From Eq. (9.65), the pre-emphasis improvement is

$$\begin{aligned}I &= \frac{6/0.8\ ^{3}}{3\left[\ 6/0.8\ - \tan^{-1}\ 6/0.8)\ \right]} \\ &= 23.2 \\ &\sim 13.6\,\mathrm{dB}\end{aligned}$$

With this improvement the post-detection SNR with pre-emphasis is 48.3 dB.

Problem 9.9 A sample function

$$x(t) = A_c \cos(2\pi f_c t) + w(t)$$

is applied to a low-pass RC filter. The amplitude A_c and frequency f_c of the sinusoidal component are constant, and $w(t)$ is white noise of zero mean and power spectral density $N_0/2$. Find an expression for the output signal-to-noise ratio with the sinusoidal component of $x(t)$ regarded as the signal of interest.

Solution

The noise variance is proportional to the noise bandwidth of the filter so from Example 8.16,

$$\mathbf{E}\left[n^2(t)\right] = B_N N_0 = \frac{1}{4RC} N_0$$

and the signal power is $A_c^2/2$ fir a sinusoid, so the signal-to-noise ratio is given by

$$SNR = \frac{A_c^{\,2}}{2\left(\dfrac{N_0}{4RC}\right)} = \frac{2A_c^{\,2}RC}{N_0}$$

Problem 9.10 A DSC-SC modulated signal is transmitted over a noisy channel, with the power spectral density of the noise as shown in Fig. 9.19. The message bandwidth is 4 kHz and the carrier frequency is 200 kHz. Assume the average received power of the signal is -80 dBm, determine the output signal-to-noise ratio of the receiver.

Solution

From Fig. 9.19, the noise power spectral density ate 200 kHz is approximately 5x10^{-19} W/Hz. Using this value for $N_0/2$ (we are assuming the noise spectral density is approximately flat across a bandwidth of 4 kHz), the post-detection SNR is given by

$$\begin{aligned} \text{SNR} &= \frac{A_c^2 P}{2N_0 W} \\ &= \frac{10^{-11}}{4\times10^3 \times 5\times10^{-19}} \\ &= 5\times10^3 \\ &\sim 37 \text{ dB} \end{aligned}$$

where we have used the fact that the received power is -80 dBm implies that $A_c^2 P/2 = 10^{-11}$ watts .

Problem 9.11 Derive an expression for the post-detection signal-to-noise ratio of the coherent receiver of Fig. 9.6, assuming that the modulated signal $s(t)$ is produced by sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

Perform your calculation for the following two receiver types:

(a) Coherent DSB-SC receiver

(b) Coherent SSB receiver.

Assume the message bandwidth is f_m. Evaluate these expressions if the received signal strength is 100 picowatts, the noise spectral density is 10^{-15} watts per hertz, and f_m is 3 kHz.

Solution

(a) The post-detection SNR of the DSB detector is

$$\mathrm{SNR}^{DSB} = \frac{A_c^2 P}{2N_0 W} = \frac{A_c^2 A_m^2}{4N_0 f_m}$$

(b) The post-detection SNR of the SSB detector is

$$\mathrm{SNR}^{SSB} = \frac{A_c^2 P}{4N_0 W} = \frac{A_c^2 A_m^2}{8N_0 f_m}$$

Although the SNR of the SSB system is half of the DSB-SC SNR, note that the SSB system only transmits half as much power.

Problem 9.12 Evaluate the autocorrelation function of the in-phase and quadrature components of narrowband noise at the coherent detector input for the DSB-SC system. Assume the band-pass noise spectral density is $S_N(f) = N_0/2$ for $|f\text{-}f_c| < B_T$.

Solution

From Eg. (8.98), the in-phase power spectral density is (see Section 8.11)

$$\begin{aligned} S_{N_I}(f) &= S_{N_Q}\ \ f \\ &= \begin{cases} S_N\ \ f - f_c\ \ + S_N\ \ f + f_c & |f| < B_T/2 \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} N_0 & |f| < B_T/2 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

From Example 8.13, the autocorrelation function corresponding to this power spectral density is

$$R_{N_Q}\ \ \tau\ \ = R_{N_I}\ \ \tau\ \ = N_0 B_T \mathrm{sinc}\ \ B_T\tau$$

Problem 9.13 Assume a message signal $m(t)$ has power spectral density

$$S_M(f) = \begin{cases} a\dfrac{|f|}{W} & |f| \le W \\ 0 & \text{otherwise} \end{cases}$$

where a and W are constants. Find the expression for post-detection SNR of the receiver when

(a) The signal is transmitted by DSB-SC.
(b) The signal is transmitted by envelope modulation with $k_a = 0.3$.
(c) The signal is transmitted with frequency modulation with $k_f = 500$ hertz per volt.

Assume that white Gaussian noise of zero mean and power spectral density $N_0/2$ is added to the signal at the receiver input.

Solution

(a) with DSB-SC modulation and detection, the post-detection SNR is given by

$$SNR^{DSB} = \frac{A_c^2 P}{2N_0 W}$$

For the given message spectrum, the power is

$$\begin{aligned} P &= \int_{-\infty}^{\infty} S_M(f)df \\ &= 2\int_0^W a\frac{f}{W}df \\ &= aW \end{aligned}$$

where we have used the even-symmetry of the message spectrum on the second line. Consequently, the post-detection SNR is

$$SNR^{DSB} = \frac{A_c^2 a}{2N_0}$$

(b) for envelope detection with $k_a = 0.3$, the post-detection SNR is

$$\begin{aligned}\text{SNR}^{\text{AM}} &= \frac{A_c^2 k_a^2 P}{2N_0 W} \\ &= \frac{A_c^2 a}{2N_0} k_a^2 \\ &= 0.09 \frac{A_c^2 a}{2N_0}\end{aligned}$$

(c) for frequency modulation and detection with k_f = 500 Hz/V, the post-detection SNR is

$$\begin{aligned}\text{SNR}^{\text{FM}} &= \frac{3A_c^2 k_f^2 P}{2N_0 W^3} \\ &= \frac{A_c^2 a}{2N_0} 3\left(\frac{k_f}{W}\right)^2\end{aligned}$$

Problem 9.14 A 10 kilowatt transmitter amplitude modulates a carrier with a tone $m(t) = \sin(2000\pi t)$, using 50 percent modulation. Propagation losses between the transmitter and the receiver attenuate the signal by 90 dB. The receiver has a front-end noise N_0 = -113 dBW/Hz and includes a bandpass filter $B_T = 2W$ = 10 kHz. What is the post-detection signal-to-noise ratio, assuming the receiver uses an envelope detector?

Solution

If the output of a 10 kW transmitter is attenuated by 90 dB through propagation, then the received signal level R is

$$\begin{aligned}R &= 10^4 \times 10^{-90/10} \\ &= 10^{-5} \text{ watts}\end{aligned} \qquad (1)$$

For an amplitude modulated signal, this received power corresponds to

$$R = \frac{A_c^2}{2}\left(1 + k_a^2 P\right) \qquad (2)$$

From Eq. (9.30), the post-detection SNR of an AM receiver using envelope detection is

$$\text{SNR}_{\text{post}}^{\text{AM}} = \frac{A_C{}^2 k_a{}^2 P}{2N_0 W}$$

Substituting for k_a, P, and $A_c^2/2$ (obtained from Eq. (2)), we find

$$\begin{aligned}
\mathrm{SNR}_{\mathrm{post}}^{\mathrm{AM}} &= \frac{R}{1+k_a^2P}\frac{k_a^2P}{N_0W} \\
&= \frac{10^{-5}}{1+0.25\times0.5}\times\frac{0.25\times0.5}{(5\times10^{-12})(5\times10^{3})} \\
&= 44.4
\end{aligned}$$

where $k_a = 0.5$ and $P = 0.5$.

Problem 9.15 The average noise power per unit bandwidth measured at the front end of an AM receiver is 10^{-6} watts per Hz. The modulating signal is sinusoidal, with a carrier power of 80 watts and a sideband power of 10 watts per sideband. The message bandwidth is 4 kHz. Assuming the use of an envelope detector in the receiver, determine the output signal-to-noise ratio of the system. By how many decibels is this system inferior to DSB-SC modulation system?

Solution

For this AM system, the carrier power is 80 watts, that is,

$$\frac{A_C{}^2}{2} = 80 \text{ watts} \tag{1}$$

and the total sideband power is 20 watts, that is,

$$\frac{A_C{}^2}{2}k_a{}^2P = 20 \text{ watts} \tag{2}$$

Comparing Eq.s (1) and (2), we determine that $k_a^2P = \frac{1}{4}$. Consequently, that post-detection SNR of the AM system is

$$\begin{aligned}
SNR_{post}^{AM} &= \frac{A_C{}^2k_a{}^2P}{2N_0W} \\
&= \frac{20}{10^{-6}\times4000} \\
&= 5000 \\
&\sim 37dB
\end{aligned}$$

For the corresponding DSB system the post detection SNR is given by

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}} = \frac{1+k_a^2 P}{k_a^2 P} \quad \mathrm{SNR}_{\mathrm{post}}^{\mathrm{AM}}$$

$$= \frac{1+\frac{1}{4}}{\frac{1}{4}}$$

$$= 5 \times \mathrm{SNR}_{\mathrm{post}}^{\mathrm{AM}}$$

$$\sim 7dB \text{ higher}$$

Problem 9.16 An AM receiver, operating with a sinusoidal modulating wave and 80% modulation, has a post-detection signal-to-noise ratio of 30 dB. What is the corresponding pre-detection signal-to-noise ratio?

<u>Solution</u>

We are given that k_a = 0.80, and for sinusoidal modulation P = 0.5. A post-detection SNR of 30 dB corresponds to an absolute SNR of 1000. From Eq.(9.30),

$$SNR_{post}^{AM} = \frac{A_C{}^2}{2} \frac{k_a{}^2 P}{N_0 W}$$

$$1000 = \frac{A_c^2}{2N_0 W}(0.8)^2 0.5$$

Re-arranging this equation, we obtain

$$\frac{A_c^2}{2N_0 W} = 3125$$

From Eq. (9.26) the pre-detection SNR is given by

$$SNR_{pre}^{AM} = \frac{A_C{}^2 (1 + k_a{}^2 P)}{2N_0 B_T}$$

$$= \frac{A_c^2}{2N_0(2W)}(1 + k_a^2 P)$$

$$= \frac{3125}{2}(1 + (0.8)^2 0.5)$$

$$= 2062.5$$

where we have assumed that $B_T = 2W$. This pre-detection SNR is equivalent to approximately 36 dB.

Problem 9.17. The signal $m(t) = \cos(400\pi t)$ is transmitted via FM. There is an ideal band-pass filter passing $100 \le |f| \le 300$ at the discriminator output. Calculate the post-detection SNR given that $k_f = 1$ kHz per volt, and the pre-detection SNR is 500. Use Carson's rule to estimate the pre-detection bandwidth.

Solution

We begin by estimating the Carson's rule bandwidth

$$\begin{aligned} B_T &= 2(k_f A + f_m) \\ &= 2(1000(1) + 200) \\ &= 2400 \text{ Hz} \end{aligned}$$

We are given that the pre-detection SNR is 500. From Section 9.7 this implies

$$\begin{aligned} SNR_{pre}^{FM} &= \frac{A_c^2}{2N_0 B_T} \\ 500 &= \frac{A_c^2}{2N_0}\frac{1}{2400} \end{aligned}$$

Re-arranging this equation, we obtain

$$\frac{A_c^2}{2N_0} = 1.2 \times 10^6 \text{ Hz}$$

The nuance in this problem is that the post-detection filter is not ideal with unity gain from 0 to W and zero for higher frequencies. Consequently, we must re-evaluate the post-detection noise using Eq. (9.58)

$$\begin{aligned} \text{Avg. post-detection noise power} &= \frac{N_0}{A_c^2}\left[\int_{-300}^{-100} f^2 df + \int_{100}^{300} f^2 df\right] \\ &= \frac{2N_0}{3A_c^2}\left[300^3 - 100^3\right] \\ &= \frac{2N_0}{3A_c^2} 2.6 \times 10^7 \end{aligned}$$

The post-detection SNR then becomes

$$
\begin{aligned}
\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} &= \frac{3A_c^2 k_f^2 P}{2N_0 (2.6\times 10^7)} \\
&= 3\left(\frac{A_c^2}{2N_0}\right)\frac{k_f^2 P}{2.6\times 10^7} \\
&= 3(1.2\times 10^6)\frac{(1000)^2 0.5}{2.6\times 10^7} \\
&= 69230.8
\end{aligned}
$$

where we have used the fact that k_f = 1000 Hz/V and P = 0.5 watts. In decibels, the post-detection SNR is 48.4 dB.

Problem 9.18. Suppose that the spectrum of a modulating signal occupies the frequency band $f_1 \le |f| \le f_2$. To accommodate this signal, the receiver of an FM system (without pre-emphasis) uses an ideal band-pass filter connected to the output of the frequency discriminator; the filter passes frequencies in the interval $f_1 \le |f| \le f_2$. Determine the output signal-to-noise ratio and figure of merit of the system in the presence of additive white noise at the receiver input.

Solution

Since the post detection filter is no longer an ideal brickwall filter, we must revert to Eq. (9.58) to compute the post-detection noise power. For this scenario (similar to Problem 9.17)

$$
\begin{aligned}
\text{Avg. post-detection noise power} &= \frac{N_0}{A_c^2}\left[\int_{-f_2}^{-f_1} f^2 df + \int_{f_1}^{f_2} f^2 df\right] \\
&= \frac{2N_0}{3A_c^2}\left[f_2^3 - f_1^3\right]
\end{aligned}
$$

Since the average output power is still $k_f^2 P$, the post detection SNR is given by

$$
SNR_{post}^{FM} = \frac{3A_c^2 k_f^2 P}{2N_0 (f_2^3 - f_1^3)}
$$

For comparison purposes, the reference SNR is

$$
SNR_{ref} = \frac{A_c^2}{2N_0(f_2 - f_1)}
$$

The corresponding figure of merit is

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{FM}}}{\text{SNR}_{\text{ref}}}$$
$$= \frac{3A_c^2 k_f^2 P}{2N_0 \left(f_2^3 - f_1^3\right)} \Big/ \frac{A_c^2}{2N_0 (f_2 - f_1)}$$
$$= \frac{3k_f^2 P}{f_2^2 + f_2 f_1 + f_1^2}$$

Problem 9.19. An FM system, operating at a pre-detection SNR of 14 dB, requires a post-detection SNR of 30 dB, and has a message power of 1 watt and bandwidth of 50 kHz. Using Carson's rule, estimate what the transmission bandwidth of the system must be. Suppose this system includes pre-emphasis and de-emphasis network with $f_{3\text{dB}}$ of 10 kHz. What transmission bandwidth is required in this case?

<u>Solution</u>

We are given the pre-detection SNR of 14 dB (~25.1), so

$$SNR_{pre}^{FM} = \frac{A_c^2}{2N_0 B_T} = 25.1$$

and the post-detection SNR of 30 dB (~1000), so

$$SNR_{post}^{FM} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3} = 1000$$

Combining these two expressions, we obtain

$$\frac{SNR_{post}^{FM}}{SNR_{pre}^{FM}} = \frac{3k_f^2 P B_T}{W^3} = 39.8$$

Approximating the Carson's rule for general modulation $B_T = 2\left(k_f P^{1/2} + W\right) \approx 2k_f P^{1/2}$, and if we replace $k_f^2 P$ with $B_T^2 / 4$ in this last equation, we obtain

$$\frac{SNR_{post}^{FM}}{SNR_{pre}^{FM}} \approx \frac{3B_T^3}{4W^3} = 39.8$$

Upon substituting W = 50 kHz, this last equation yields B_T = 187.9 kHz.

Problem 9.20. Assume that the narrowband noise $n(t)$ is Gaussian and its power spectral density $S_N(f)$ is symmetric about the midband frequency f_c. Show that the in-phase and quadrature components of $n(t)$ are statistically independent.

Solution

The narrowband noise $n(t)$ can be expressed as:

$$\begin{aligned} n(t) &= n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \\ &= \mathrm{Re}\left[z(t) e^{j2\pi f_c t} \right] \end{aligned},$$

where $n_I(t)$ and $n_Q(t)$ are in-phase and quadrature components of $n(t)$, respectively. The term $z(t)$ is called the complex envelope of $n(t)$. The noise $n(t)$ has the power spectral density $S_N(f)$ that may be represented as shown below

We shall denote $R_{nn}(\tau)$, $R_{n_I n_I}(\tau)$ and $R_{n_Q n_Q}(\tau)$ as autocorrelation functions of $n(t)$, $n_I(t)$ and $n_Q(t)$, respectively. Then

$$\begin{aligned} R_{nn}(\tau) &= \mathbf{E}\ n(t)n(t+\tau) \\ &= \mathbf{E}\ \left[n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \right]\cdot\left[n_I(t+\tau)\cos(2\pi f_c(t+\tau)) - n_Q(t+\tau)\sin(2\pi f_c(t+\tau)) \right] \\ &= \frac{1}{2}\left[R_{n_I n_I}(\tau) + R_{n_Q n_Q}(\tau) \right]\cos(2\pi f_c \tau) + \frac{1}{2}\left[R_{n_I n_I}(\tau) - R_{n_Q n_Q}(\tau) \right]\cos(2\pi f_c(2t+\tau)) \\ &\quad -\frac{1}{2}\left[R_{n_Q n_I}(\tau) - R_{n_I n_Q}(\tau) \right]\sin(2\pi f_c \tau) - \frac{1}{2}\left[R_{n_Q n_I}(\tau) + R_{n_I n_Q}(\tau) \right]\sin(2\pi f_c(2t+\tau)) \end{aligned}$$

Since $n(t)$ is stationary, the right-hand side of the above equation must be independent of t, this implies

$$R_{n_I n_I}(\tau) = R_{n_Q n_Q}(\tau) \qquad (1)$$

$$R_{n_I n_Q}(\tau) = -R_{n_Q n_I}(\tau) \qquad (2)$$

Substituting the above two equations into the expression for $R_{nn}(\tau)$, we have

$$R_{nn}(\tau) = R_{n_I n_I}(\tau)\cos(2\pi f_c \tau) - R_{n_Q n_I}(\tau)\sin(2\pi f_c \tau) \qquad (3)$$

The autocorrelation function of the complex envelope $z(t) = n_I(t) + jn_Q(t)$ is

$$\begin{aligned} R_{zz}(\tau) &= E\left[z^*(t) z(t+\tau) \right] \\ &= 2R_{n_I n_I}(\tau) + j2R_{n_Q n_I}(\tau) \end{aligned} \qquad (4)$$

From the bandpass to low-pass transformation of Section 3.8, the spectrum of the complex envelope z is given bye

$$S_Z(f) = \begin{cases} S_N \quad f + f_c & f > -f_c \\ 0 & \text{otherwise} \end{cases}$$

Since $S_N(f)$ is symmetric about f_c, $S_Z(f)$ is symmetric about $f = 0$. Consequently, the inverse Fourier transform of $S_Z(f) = R_{zz}(\tau)$ must be real. Since $R_{zz}(\tau)$ is real valued, based on Eq. (4), we have

$$R_{n_Q n_I}(\tau) = 0,$$

which means the in-phase and quadrature components of $n(t)$ are uncorrelated. Since the in-phase and quadrature components are also Gaussian, this implies that they are also statistically independent.

Problem 9.21. Suppose that the receiver bandpass-filter magnitude response $|H_{BP}(f)|$ has symmetry about $\pm f_c$ and noise bandwidth B_T. From the properties of narrowband noise described in Section 8.11, what is the spectral density $S_N(f)$ of the in-phase and quadrature components of the narrowband noise $n(t)$ at the output of the filter? Show that the autocorrelation of $n(t)$ is

$$R_N(\tau) = \rho(\tau)\cos(2\pi f_c \tau)$$

where $\rho(\tau) = \mathbf{F}^{-1} \; S_N(f)$; justify the approximation $\rho(\tau) \approx 1$ for $|\tau| < 1/B_T$.

<u>Solution</u>
Let the noise spectral density of the bandpass process be SH(f) then

$$S_H(f) = \frac{N_0}{2}|H_{BP}(f)|^2$$

From Section 8.11, the power spectral densities of the in-phase and quadrature components are given by

$$S_N(f) = \begin{cases} S_H(f-f_c)+S_H(f+f_c), & |f| \le B_T/2 \\ 0, & \text{otherwise} \end{cases}.$$

Since the spectrum $S_H(f)$ is symmetric about f_c, , the spectral density of the in-phase and quadrature components is

$$S_N(f) = \begin{cases} |H_{BP}(f-f_c)|^2 N_0 & |f| < B_T/2 \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

Note that if $|H_{BP}(f)|$ is symmetric about f_c then $|H_{BP}(f\text{-}f_c)|$ will be symmetric about 0. Consequently, the power spectral densities of the in-phase and quadrature components are symmetric about the origin. This implies that the corresponding autocorrelation functions are real valued (since they are related by the inverse Fourier transform). In Problem 9.20, we shown that if the autocorrelation function of the in-phase component is real valued then autocorrelation of $n(t)$ is $R_N(\tau) = R_{n_I n_I}(\tau)\cos(2\pi f_c \tau)$. If we denote

$$\rho(\tau) = R_{n_I n_I}(\tau) = \mathbf{F}^{-1}\ S_N(f)\ = N_O \mathbf{F}^{-1}\left[|H_{BP}(f-f_c)|^2\right]$$

then the autocorrelation of the bandpass noise is

$$R_N(\tau) = \rho(\tau)\cos(2\pi f_c t)$$

For $|\tau| \ll 1/B_T$ (*there is a typo in the text*), we have

$$\begin{aligned} \rho(\tau) &= \int_{-\infty}^{\infty} S_N(f)\exp\ -j2\pi f\tau\ \ df \\ &= \int_0^{\infty} S_N(f)\cos\ 2\pi f\tau\ \ df \end{aligned}$$

due to the real even-symmetric nature of $S_N(f)$. If the signal has noise bandwidth B_T then

$$\begin{aligned}\rho(\tau) &\approx \int_0^{B_T} S_N(f)\cos\left(2\pi f\tau\right) df \\ &\approx \int_0^{B_T} S_N(f)\cos\left(0\right) df \\ &= \int_0^{B_T} S_N(f) df \\ &= \text{a constant}\end{aligned}$$

where the second line follows from the assumption that $|\tau| \ll 1/B_T$. With suitable scaling the constant can be set to one.

Problem 9.22. Assume that, in the DSB-SC demodulator of Fig. 9.6, there is a phase error ϕ in the synchronized oscillator such that its output is $\cos(2\pi f_c t + \phi)$. Find an expression for the coherent detector output and show that the post-detection SNR is reduced by the factor $\cos^2\phi$.

Solution

The signal at the input to the coherent detector of Fig. 9.6 is $x(t)$ where

$$\begin{aligned}x(t) &= s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \\ &= A_c m(t)\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)\end{aligned}$$

The output of mixer2 in Fig. 9.6 is

$$\begin{aligned}v(t) &= x(t)\cos(2\pi f_c t + \phi) \\ &= \left[A_c m(t) + n_I(t)\right]\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) - n_Q(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \phi) \\ &= \frac{1}{2}\left[A_c m(t) + n_I(t)\right]\cos\phi + \frac{1}{2}n_Q(t)\sin\phi + \frac{1}{2}\left[A_c m(t) + n_I(t)\right]\cos(4\pi f_c t + \phi) - \frac{1}{2}n_Q(t)\sin(4\pi f_c t + \phi)\end{aligned}$$

With the higher frequency components will be eliminated by the low pass filter, the received message at the output of the low-pass filter is

$$y(t) = \frac{1}{2}A_c m(t)\cos\phi + \frac{1}{2}n_I(t)\cos\phi + \frac{1}{2}n_Q(t)\sin\phi$$

To compute the post-detection SNR we note that the average output message power in this last expression is

$$\frac{1}{4}A_c^2 P\cos^2\phi$$

and the average output noise power is

$$\frac{1}{4}\cdot 2N_0W\cos^2\phi+\frac{1}{4}\cdot 2N_0W\sin^2\phi=\frac{1}{4}\cdot 2N_0W$$

where $\mathbf{E}[n_I^2(t)]=\mathbf{E}[n_Q^2(t)]=N_0W$. Consequently, the post-detection SNR is

$$\text{SNR}=\frac{1/4A_c^2P\cos^2\phi}{1/4\cdot 2N_0W}=\frac{A_c^2P\cos^2\phi}{2N_0W}$$

Compared with (9.23), the above post-detection SNR is reduced by a factor of $\cos^2\phi$.

Problem 9.23. In a receiver using coherent detection, the sinusoidal wave generated by the local oscillator suffers from a phase error $\theta(t)$ with respect to the carrier wave $\cos(2\pi f_c t)$. Assuming that $\theta(t)$ is a zero-mean Gaussian process of variance σ_θ^2 and that most of the time the maximum value of $\theta(t)$ is small compared to unity, find the mean-square error of the receiver output for DSB-SC modulation. The mean-square error is defined as the expected value of the squared difference between the receiver output and message signal component of a synchronous receiver output.

Solution

Based on the solution of Problem 9.22, we have the DSB-SC demodulator output is

$$y(t)=\frac{1}{2}A_c m(t)\cos\theta(t)+\frac{1}{2}n_I(t)\cos\theta(t)+\frac{1}{2}n_Q(t)\sin\theta(t)$$

Recall from Section 9. that the output of a synchronous receiver is

$$\frac{1}{2}A_c m(t)+\frac{1}{2}n_I(t)$$

The mean-square error (MSE) is defined by

$$\text{MSE}=\mathbf{E}\left[\left(y(t)-\frac{1}{2}A_c m(t)\right)^2\right]$$

Substituting the above expression for $y(t)$, the mean-square error is

$$\text{MSE} = \mathbf{E}\left[\left[\frac{1}{2}A_c m(t)\left[\cos\theta(t) - 1\right] + \frac{1}{2}n_I(t)\cos\theta(t) + \frac{1}{2}n_Q(t)\sin\theta(t)\right]^2\right]$$
$$= \frac{A_c^2}{4}\mathbf{E}\left[m^2(t)\left[\cos\theta(t) - 1\right]^2\right] + \frac{1}{4}\mathbf{E}\left[n_I^2(t)\cos^2\theta(t)\right] + \frac{1}{4}\mathbf{E}\left[n_Q^2(t)\sin^2\theta(t)\right]$$

where we have used the independence of $m(t)$, $n_I(t)$, $n_Q(t)$, and $\theta(t)$ and the fact that $\mathbf{E}\left[n_I(t)\right] = \mathbf{E}\left[n_Q(t)\right] = 0$ to eliminate the cross terms.

$$\text{MSE} = \frac{A_c^2}{4}\mathbf{E}\left[m^2(t)\right]\mathbf{E}\left[\left(1-\cos\theta(t)\right)^2\right] + \frac{1}{4}\mathbf{E}\left[n_I^2(t)\right]\mathbf{E}\left[\cos^2\theta(t)\right] + \frac{1}{4}\mathbf{E}\left[n_Q^2(t)\right]\mathbf{E}\left[\sin^2(\theta(t))\right]$$
$$= \frac{A_c^2 P}{4}\mathbf{E}\left[\left(1-\cos\theta(t)\right)^2\right] + \frac{1}{4}N_0 W\mathbf{E}\left[\cos^2\theta(t)\right] + \frac{1}{4}N_0 W\mathbf{E}\left[\sin^2(\theta(t))\right]$$
$$= \frac{A_c^2 P}{4}\mathbf{E}\left[\left(1-\cos\theta(t)\right)^2\right] + \frac{N_0 W}{2}$$

where we have used the equivalences of $\mathbf{E}[m^2(t)] = P$, and $\mathbf{E}\left[n_I^2(t)\right] = \mathbf{E}\left[n_Q^2(t)\right] = 2N_0 W$. The last line uses the fact that $\cos^2(\theta(t))+\sin^2(\theta(t)) = 1$. If we now use the relation that $1-\cos A = 2\sin^2(A/2)$, this expression becomes

$$\text{MSE} = A_c^2 P\mathbf{E}\left[\sin^4\left(\frac{\theta(t)}{2}\right)\right] + \frac{N_0 W}{2}$$

Since the maximum value of $\theta(t) << 1$, $\sin(\theta(t)) \approx \theta(t)$ and we have

$$\text{MSE} \approx A_c^2 P\mathbf{E}\left[\left(\frac{\theta(t)}{2}\right)^4\right] + \frac{N_0 W}{2}$$
$$= \frac{3}{16}A_c^2 P\sigma_\theta^4 + \frac{N_0 W}{2}$$

where we have used the fact that if θ is a zero-mean Gaussian random variable then

$$\mathbf{E}\left[\theta^4\right] = 3\left(\mathbf{E}\left[\theta^2\right]\right)^2 = 3\sigma_\theta^4$$

The mean square error is therefore $\frac{3}{16}A_c^2 P\sigma_\theta^4 + \frac{1}{2}N_0 W$.

Problem 9.24. Equation (9.59) is the FM post-detection noise for an ideal low-pass filter. Find the post-detection noise for an FM signal when the post-detection filter is a second-order low-pass filter with magnitude response

$$|H(f)| = \frac{1}{\left[1+\left(f/W\right)^4\right]^{1/2}}$$

Assume $|H_{BP}(f+f_c)|^2 \approx 1$ for $|f| < B_T/2$ and $B_T >> 2W$.

Solution

We modify Eq. (9.58) to include the effects of a non-ideal post-detection filter in order to estimate the average post-detection noise power:

$$\begin{aligned}\frac{N_0}{A_c^2}\int_{-W}^{W} f^2 \,|H_{BP}(f)|^2\, df &= \frac{N_0}{A_c^2}\int_{-W}^{W} f^2 \cdot \frac{1}{1+(f/W)^4}\,df \\ &= \frac{2N_0}{A_c^2}\int_{0}^{W} f^2 \cdot \frac{1}{1+(f/W)^4}\,df\end{aligned}$$

This can be evaluated by a partial fraction expansion of the integrand but for simplicity, we appeal to the formula:

$$\int \frac{x^2 dx}{a+bx^4} = \frac{1}{4bk}\left[\frac{1}{2}\log\frac{x^2-2kx+2k^2}{x^2+2kx+2k^2} + \tan^{-1}\frac{2kx}{2k^2-x^2}\right], \quad ab>0,\ k=\sqrt[4]{\frac{a}{2b}}$$

Using this result, we get the average post-detection noise power is

$$\text{Avg. post-detection noise power} = \frac{2N_0}{A_c^2}\cdot\frac{W^3}{4\sqrt{2}}\left[\log\frac{2-\sqrt{2}}{2+\sqrt{2}}+\pi\right] = 0.42\frac{N_0 W^3}{A_c^2}$$

Problem 9.25. Consider a communication system with a transmission loss of 100 dB and a noise density of 10^{-14} W/Hz at the receiver input. If the average message power is $P = 1$ watt and the bandwidth is 10 kHz, find the average transmitter power (in kilowatts) required for a post-detection SNR of 40 dB or better when the modulation is:

(a) AM with $k_a = 1$; repeat the calculation for $k_a = 0.1$.
(b) FM with $k_f = 10$, 50 and 100 kHz per volt.

In the FM case, check for threshold limitations by confirming that the pre-detection SNR is greater that 12 dB.

Solution

(a) In the AM case, the post detection SNR is given by

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{AM}} = \frac{A_c^2 k_a^2 P}{2N_o W}$$

$$10^4 = \frac{A_c^2 k_a^2 (1)}{2(2\times 10^{-14})(10^4)}$$

$$\frac{A_c^2 k_a^2}{2} = 2\times 10^{-6}$$

where an SNR of 40 dB corresponds to 10^4 absolute and $N_0/2 = 10^{-14}$ W/Hz. For the different values of k_a

$$k_a = 1 \Rightarrow \quad A_c^2 = 4\times 10^{-6}$$

$$k_a = 0.1 \Rightarrow \quad A_c^2 = 4\times 10^{-4}$$

Average modulated signal power at the input of the detector is $\frac{1}{2}A_c^2(1+k_a^2 P)$.

$$k_a = 1 \Rightarrow \quad \frac{1}{2}A_c^2(1+k_a^2 P) = 4\times 10^{-6}$$

$$k_a = 0.1 \Rightarrow \quad \frac{1}{2}A_c^2(1+k_a^2 P) = 2.02\times 10^{-4}$$

The transmitted power is 100dB (10^{10}) greater than the received signal power so

$k_a = 1 \Rightarrow$ transmitted power = $4\times 10^4 = 40$ kW

$k_a = 0.1 \Rightarrow$ transmitted power = $2.02\times 10^6 = 2020$ kW

(b) In the FM case, the post detection SNR is

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} = \frac{3A_c^2 k_f^2 P}{2N_o W^3}$$

$$10^4 = \frac{3A_c^2 k_f^2 (1)}{2(2\times 10^{-14})(10^4)^3}$$

$$\frac{A_c^2 k_f^2}{2} = 0.667\times 10^2$$

For the different values of k_a

$$k_f = 10\ \text{kHz/V} \Rightarrow \frac{A_c^2}{2} = 0.667\times 10^{-6}$$

$$k_f = 50\ \text{kHz/V} \Rightarrow \frac{A_c^2}{2} = 26.667\times 10^{-9}$$

$$k_f = 100 \text{ kHz/V} \Rightarrow \frac{A_c^2}{2} = 0.667 \times 10^{-8}$$

The transmitted power is 100dB (10^{10}) greater than the received signal power so

k_f = 10 kHz/V ⇒ transmitted power = 0.667x10^4 W = 6.67 kW
k_f = 50 kHz/V ⇒ transmitted power = 26.667x10^1 W = 0.27 kW
k_f = 100 kHz/V ⇒ transmitted power = 0.667x10^2 W = 0.07 kW

To check the pre-detection SNR, we note that it is given by :

$$SNR_{pre}^{FM} = \frac{A_c^2}{2N_0 B_T} = \frac{A_c^2}{4N_0(k_f P^{1/2} + W)}$$

where from Carson's rule $B_T = 2(k_f P^{1/2} + W)$. From the above $A_c^2 = \frac{4\times 10^2}{3k_f^2}$, so

$$SNR_{pre}^{FM} = \frac{4\times 10^2}{3k_f^2 \times 4N_0(k_f P^{1/2} + W)} = \frac{10^2}{3k_f^2 \times 2\times 10^{-14}(k_f + 10^4)}$$

For the different values of k_f, the pre-detection SNR is

$$k_f = 10kHz \Rightarrow \quad SNR_{pre}^{FM} = 10^4/12 = 29dB > 12dB$$
$$k_f = 50kHz \Rightarrow \quad SNR_{pre}^{FM} = 11.11 = 10.45dB < 12dB$$
$$k_f = 100kHz \Rightarrow \quad SNR_{pre}^{FM} = 1.515 = 1.8dB < 12dB$$

Therefore, for k_f = 50 kHz and 100 kHz, the pre-detection *SNR* is too low and the transmitter power would have to be increased by 1.55 dB and 10.2 dB, respectively.

Problem 9.26 In this experiment we investigate the performance of amplitude modulation in noise. The MatLab script for this AM experiment is provided in Appendix 8 and simulates envelope modulation by a sine wave with a modulation index of 0.3, adds noise, and then envelope detects the message. Using this script:
(a) Plot the envelope modulated signal.
(b) Using the supporting function "spectra", plot its spectrum.
(c) Plot the envelope detected signal before low-pass filtering.
(d) Compare the post-detection SNR to theory.

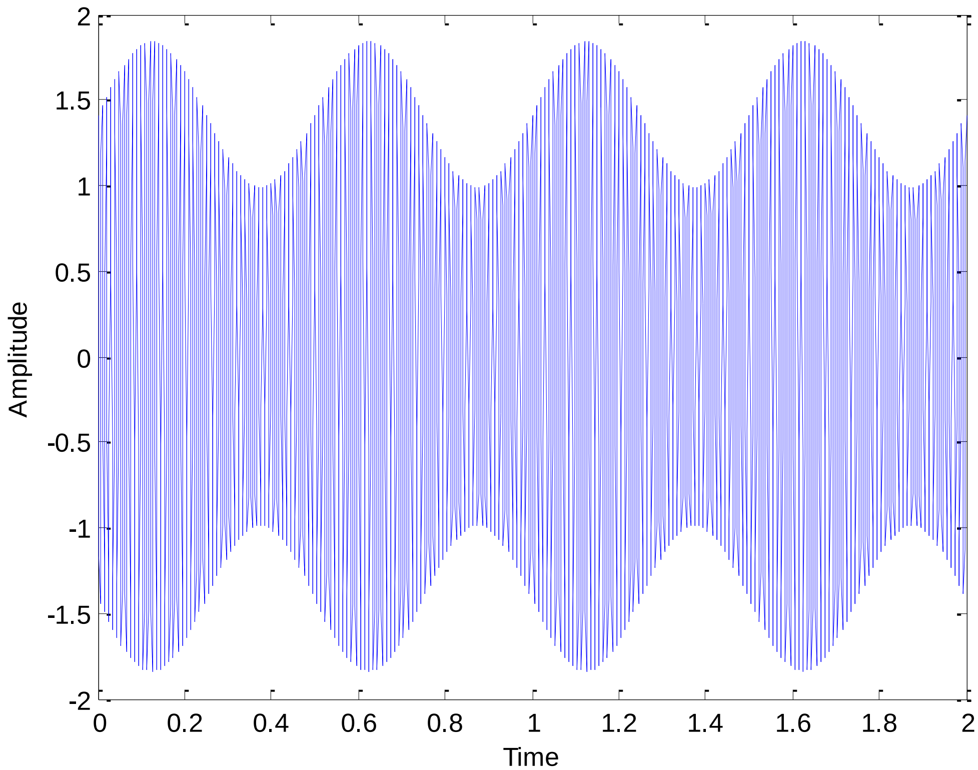
Using the Matlab script given in Appendix 7 we obtain the following plots

(a) By inserting the statements

plot(t,AM)
xlabel('Time')

ylabel('Amplitude')

at the end of Modulator section of the code, we obtain the following plot of the envelope modulated signal:



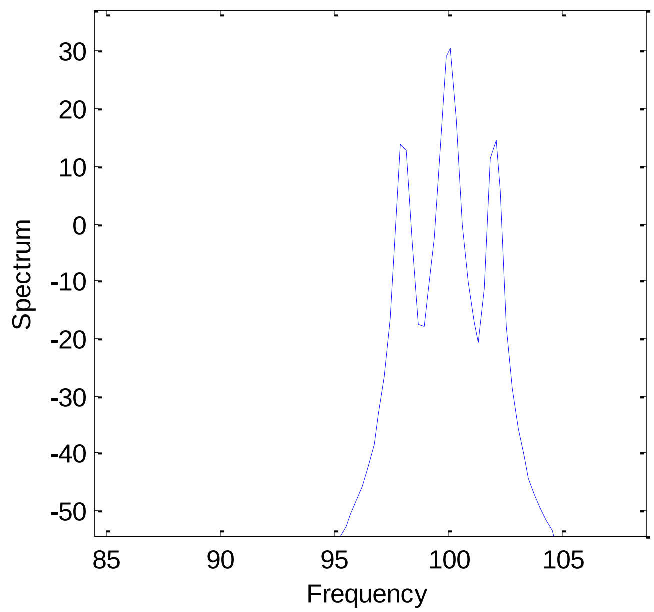
(b) The provided script simulates 2 seconds of the AM signal. Since the modulating signal is only 2 Hz, this is not a sufficient signal length to accurately estimate the spectrum. We extend the simulation to 200 seconds by modifying the statement

t = [0:1/Fs:200];

To plot the spectrum, we insert the following statements after the AM section

[P,F] = spectrum(AM,4096,0,4096,Fs);
*plot(F,10*log10(P(:,1)))*
xlabel('Frequency')
ylabel('Spectrum')

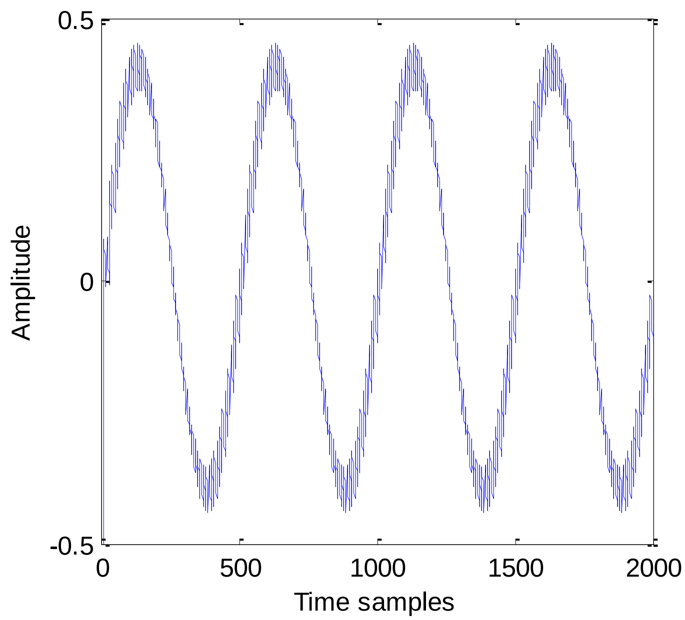
We use the large FFT size of 4096 to provide sufficient frequency resolution. (The resolution is *Fs* (1000 Hz) divided by the FFT size. We plot the spectrum of decibels because it more clearly shows the sideband components. With a linear plot, and this low modulation index, the sideband components would be difficult to see. The following figure enlarges the plot around the carrier frequency of 100 Hz.



(c) To plot the envelope-detected signal before low-pass filtering, we insert the statements (Decrease the time duration to 2 seconds to speed up processing for this part.)

plot(AM_rec)
xlabel('Time samples')
ylabel('Amplitude')

The following plot is obtained and illustrates the tracking of the envelope detector.

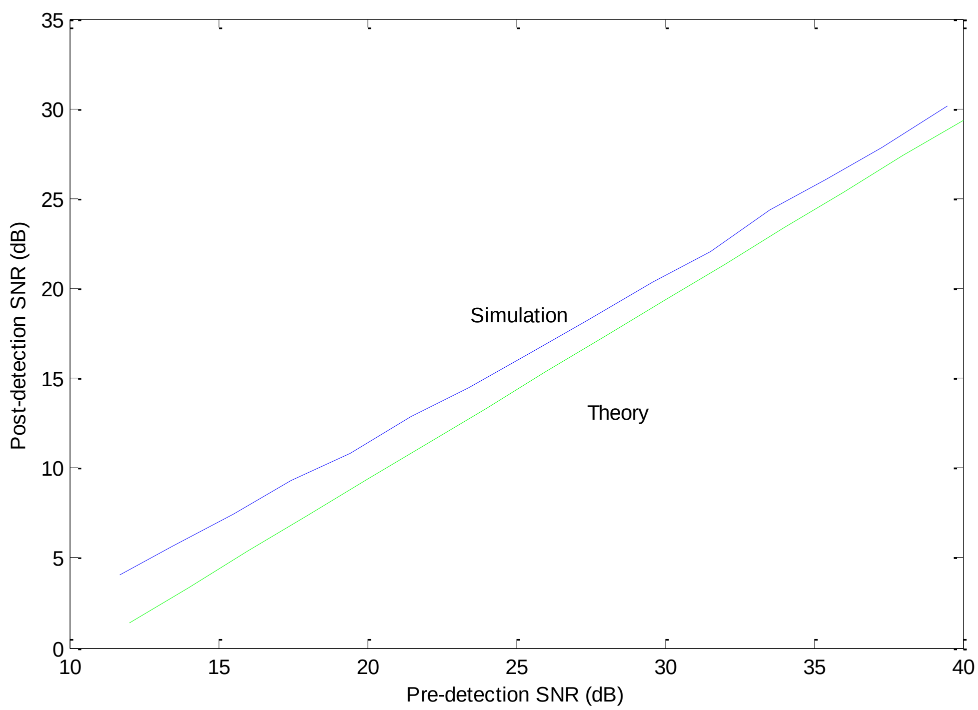


(d) To compare the simulated post detection SNR to theory. Create a loop around the main body of the simulation by adding the following statements

```
for kk = 1:15
        SNRdBr = 10 + 2*kk
            ....
        PreSNR(kk) = 20*log10(std(RxAM)/std(RxAMn-RxAM));
        No(kk) = 2*sigma^2/Fs;
        ....
        SNRdBpost(kk) = 10*log10(C/error);
        W = 50; P = 0.5;
        Theory(kk) = 10*log10 ( A^2*ka^2*0.5 / (2*No(kk)*W));
end

plot(PreSNR, SNRdBpost)
hold on,
plot(PreSNR, Theory,'g');
```

The results are shown in the following chart.



These results indicate that the simulation is performing slightly better than theory? Why? As an exercise try adjusting either the frequency of the message tone or the decay of the envelope detector and compare the results.

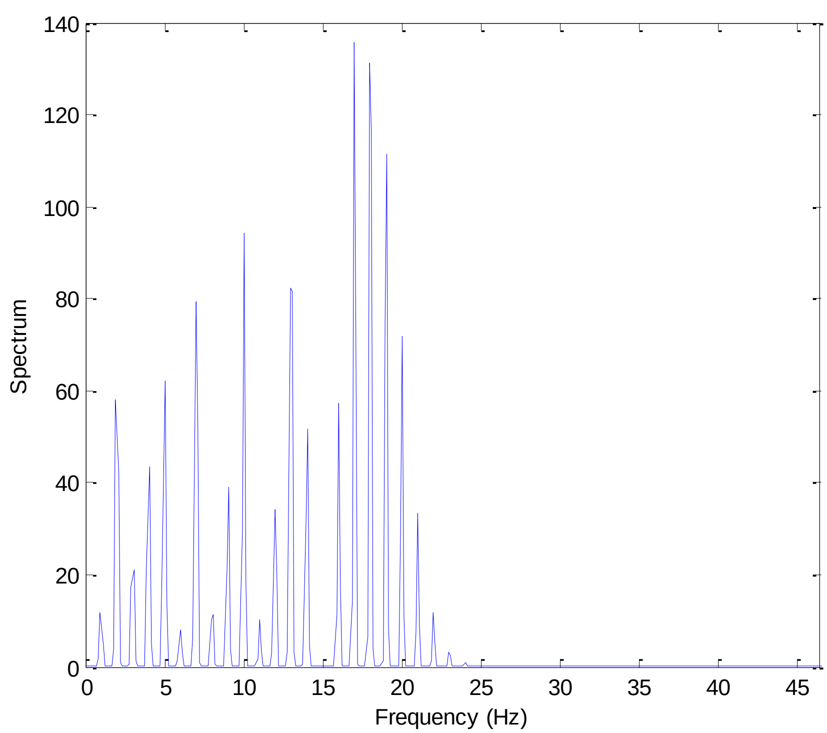
Problem 9.27. In this computer experiment, we investigate the performance of FM in noise. Using the Matlab script for the FM experiment provided in Appendix 8:
(a) Plot the spectrum of the baseband FM phasor.
(b) Plot the spectrum of the band-pass FM plus noise.
(c) Plot the spectrum of the detected signal prior to low-pass filtering.
(d) Plot the spectrum of the detected signal after low pass filtering.
(e) Compare pre-detection and post-detection SNRs for an FM receiver.

In the following parts (a) through (d), set the initial CNdB value to 13 dB in order to be operating above the FM threshold.

(a) By inserting the following statements after the definition of FM, we obtain the baseband spectrum

```
[P,F] = spectrum(FM,4096,0,4096,Fs);
plot(F,P(:,1))
xlabel('Frequency (Hz)')
ylabel('Spectrum')
```

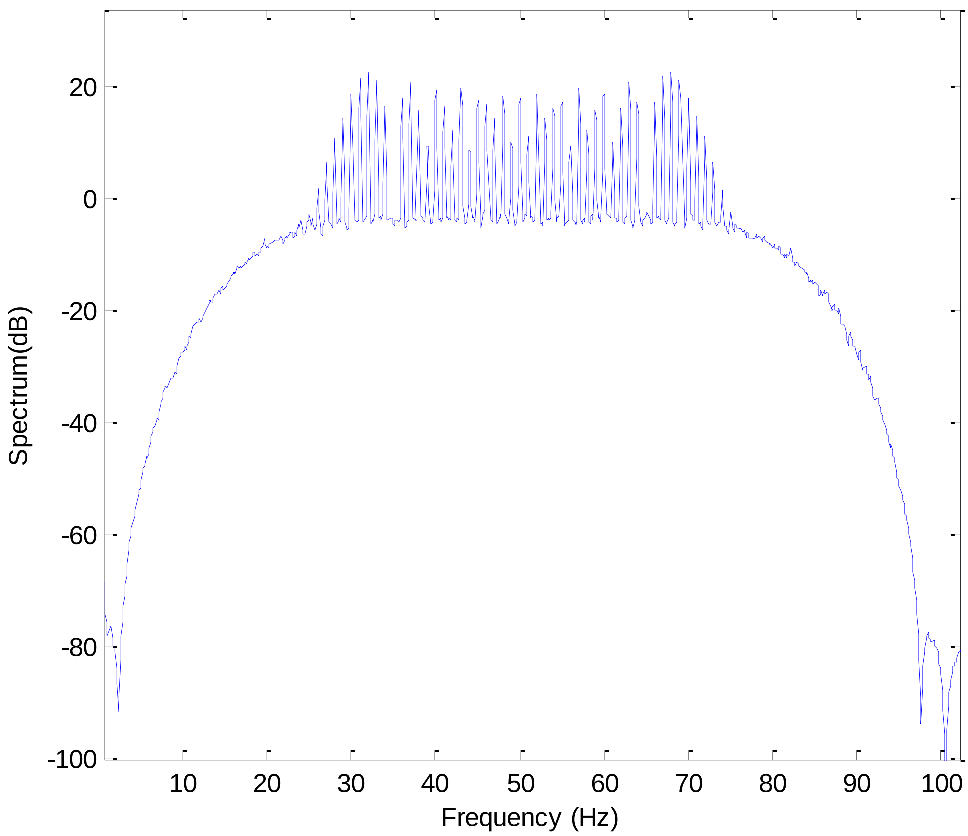
An enlarged snapshot of the spectrum near 0 Hz is shown here. It shows the tones at the regular spacing that one would expect with FM tone modulation. Note that initial plot shows the "negative frequency" portion of the spectrum just below F_s = 500 Hz. This is due to the nature of the FFT and the sampling process.



(b) The spectrum of the bandpass FM plus noise is obtained by inserting the statements

*[P,F] = spectrum((FM+Noise).*Carrier,4096,0,4096,Fs);*
*plot(F,10*log10(P(:,1)))*
xlabel('Frequency (Hz)')
ylabel('Spectrum')

An expanded view of the result around the carrier frequency of 50 Hz is shown below. The spectrum has been plotted on a decibel scale to show both the FM tone spectrum and the noise pedestal.

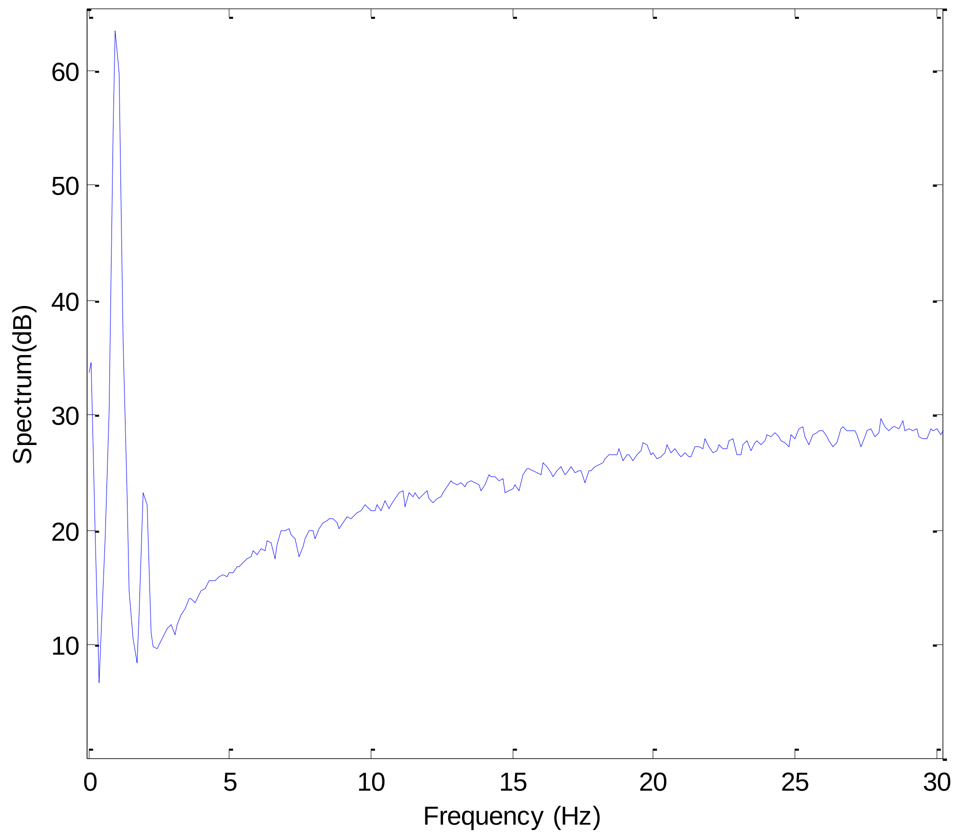


(c) To plot the spectrum of the noisy signal before low-pass filtering, we insert the following statements in the FM discriminator function, prior to the low pass filter

[P,F] = spectrum(BBdec,1024,0,1024,Fsample/4)
*plot(F,10*log10(P(:,1)))*
xlabel('Frequency (Hz)')
ylabel('Spectrum(dB)')

The following plot is obtained when expanded near the origin. We plot the spectrum in decibels in order to show the noise and the non-flat nature of its spectrum more clearly.

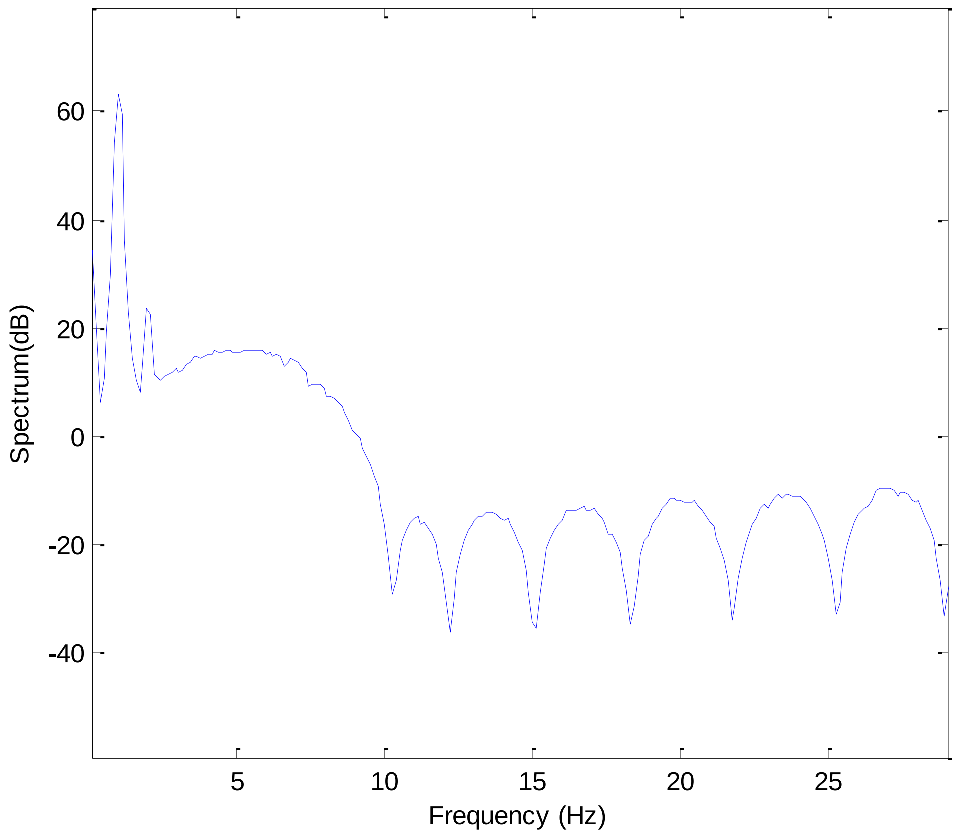
The decibel scale also illustrates some low-level distortion that has been introduced by the demodulation process as exhibited by the small second harmonic at 2 Hz and the low dc level.



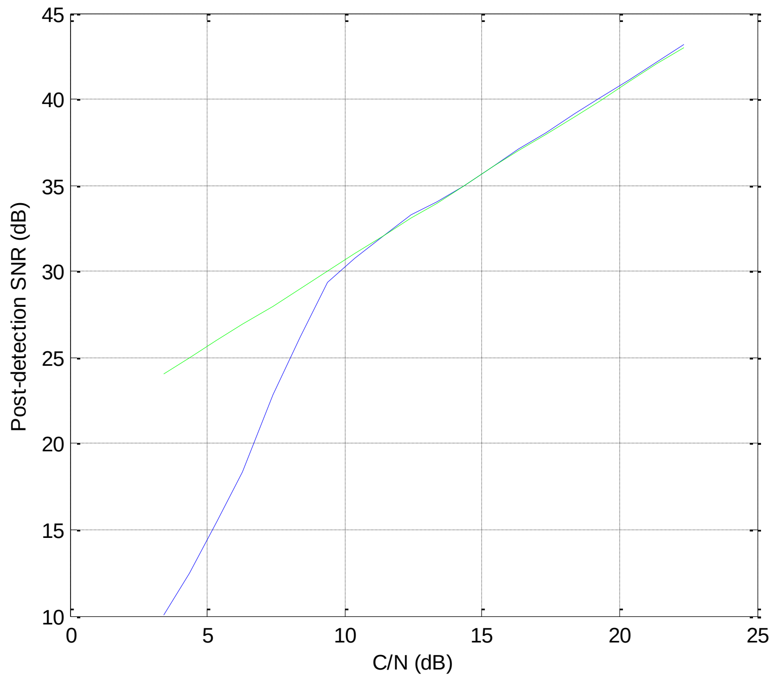
(d) To plot the spectrum of the noisy signal before low-pass filtering, we insert the following statements in the FM discriminator function, after the low-pass filter

```
[P,F] = spectrum(Message,1024,0,1024,Fsample/4)
plot(F,10*log10(P(:,1)))
xlabel('Frequency (Hz)')
ylabel('Spectrum(dB)')
```

The following plot is obtained when expanded near the origin. Again we plot the spectrum in decibels in order to show the noise and, in this case, the effect of the low-pass filtering. The low-pass filtering does not affect the distortion introduced by the demodulator in the passband.



(e) Running the code as provided produces the following comparison of the post-detection and pre-detection SNR.



45
40
35
30
25
20
15
10
Post-detection SNR (dB)
0
5
10
15
20
25
C/N (dB)