Chapter 11 Solutions

Problem 11.1 What is the root-mean-square voltage across a 10 Mega-ohm resistor at room temperature if measured over a 1 GHz bandwidth? What is the available noise power?

Solution
Following Example 11.2, the available noise power is

$$\begin{aligned} P_N &= kTB_N \\ &= 1.38\times10^{-23}\times290\times10^{9} \\ &= 4\times10^{-12}\,\text{watts} \end{aligned}$$

The root-mean-square voltage across a 10 mega-ohm resistor is

$$\begin{aligned} V_{\text{rms}} &= \sqrt{P_N R} \\ &= \sqrt{4\times10^{-12}\times10^{7}} \\ &= 6.3\ \text{millivolts} \end{aligned}$$

Problem 11.2 What is the available noise power over 1 MHz due to shot noise from a junction diode that has a voltage differential of 0.7 volts and carries average current of 0.1 milliamperes, if the current source of the Norton equivalent circuit has a resistance of 250 ohms?

Solution
From Eq. (11.9), the saturation current for a junction diode is given by

$$\begin{aligned} I_S &= \frac{I}{\exp\left(\frac{qV}{RT}\right)-1} \\ &= 1.8\times10^{-12}\,I \end{aligned}$$

Consequently, the noise contribution from the saturation current may be ignored. From Eq. (11.10) the expected current variance is then

$$\begin{aligned} \mathbf{E}\left[I_{shot}{}^2\right] &= 2q(I+2I_s)B_N \\ &\approx 2qIB_N \\ &= 2\times\ 1.6\times10^{-19}\ \times\ 0.1\times10^{-3}\ \times10^{6} \\ &= 3.2\times10^{-17}\ Amp^2 \end{aligned}$$

The corresponding noise power with an equivalent resistance of 250 ohms is

$$\begin{aligned} P_N &= \mathbf{E}\left[i_{shot}^2 \right] R \\ &= 8\times 10^{-15} \text{ watts} \end{aligned}$$

Problem 11.3 An electronic device has a noise figure of 10 dB. What is the equivalent noise temperature?

Solution

From Eq. (11.7), the equivalent noise temperature is

$$\begin{aligned} T_e &= T_0 (F-1) \\ &= 290(10-1) \\ &= 2610^o K \end{aligned}$$

Problem 11.4. The device of Problem 11.3 has a gain of 17 dB and is connected to a spectrum analyzer. If the input to the device has an equivalent temperature of 290°K and the spectrum analyzer is noiseless, express the measured power spectral density in dBm/Hz. If the spectrum analyzer has a noise figure of 25 dB, what is the measured power spectral density in this case?

Solution

For the device of Problem 11.3, the total output noise is, from Eq. (11.15),

$$\begin{aligned} N &= Gk(T+T_e)B_N \\ &= GkTFB_N \\ &= 10^{17/10} \quad 1.38\times 10^{-23} \quad 290 \quad 10^{10/10} \quad B_N \\ &= 2.0\times 10^{-18} B_N \end{aligned}$$

The noise spectral density at the device output is approximately

$$\begin{aligned} \frac{N}{B_N} &= 2.0\times 10^{-18} \text{ W/Hz} \\ &\sim \text{-147 dBm/Hz} \end{aligned}$$

If the spectrum analyzer has a noise figure of 25 dB, then we must use the results of the following section. Specifically, Eq. (11.21), to obtain the total noise figure of

$$
\begin{aligned}
F &= F_1 + \frac{F_2}{G_1} \\
&= 10^{10/10} + \frac{10^{25/10}}{10^{17/10}} \\
&= 16.3 \\
&\sim 12.1\,\text{dB}
\end{aligned}
$$

Since the overall noise figure is increased 2.1 dB by the spectrum analyzer, the noise spectral density at the spectrum analyzer output is (assuming unity gain for the spectrum analyzer) -144.9 dBm/Hz. (*There is an error in the second answer given in the text.*)

Problem 11.5 A broadcast television receiver consists of an antenna with a noise temperature of 290°K and a pre-amplifier with a gain of 20 dB and a noise figure of 9 dB. A second-stage amplifier in the receiver provides another 20 dB of gain and has a noise figure of 20 dB. What is the noise figure of the overall system?

Solution

From Eq (11.21), after converting from decibels to absolute

$$
\begin{aligned}
F &= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} \\
&= 2 + \frac{7.94 - 1}{1} + \frac{99}{100} \\
&= 2 + 6.94 + .99 \\
&= 9.93
\end{aligned}
$$

Converting this result back to decibels, the overall noise figure is 9.97 dB.

Problem 11.6 A satellite antenna has a diameter of 4.6 meters and operates at 12 GHz. What is the antenna gain if the aperture efficiency is 60%? If the same antenna was used at 4 GHz what would be the corresponding gain?

Solution

From Eq (11.25), the antenna gain is

$$
G = \frac{4\pi A_{eff}}{\lambda^2}
$$

The effective area is given by Eq.(11.24)

$$\begin{aligned} A_{eff} &= \eta A \\ &= \eta \frac{\pi d^2}{4} \\ &= 9.97 \text{ m}^2 \end{aligned}$$

where the efficiency is 60% and the diameter is 4.6 meters. At 12 GHz, the wavelength $\lambda = c/f = 0.025$ meters. Consequently, the antenna gain is

$$\begin{aligned} G &= 2000458.7 \\ &\sim 53.0 \text{ dB} \end{aligned}$$

With a transmission frequency of 4 GHz, the wavelength $\lambda = 0.075$ m and the antenna gain is

$$\begin{aligned} G &= 22273.2 \\ &\sim 43.5 \text{ dB} \end{aligned}$$

Problem 11.7 A satellite at a distance of 40,000 kilometers transmits a signal at 12 GHz with an EIRP of 10 watts towards a 4.6 meter antenna that has an aperture efficiency of 60%. What is the received signal level at the antenna output?

Solution

From Eq.(11.32), the path loss due to free-space transmission of a 12 GHz signal over 40,000 kilometers is

$$\begin{aligned} L_P &= 20 \log_{10}\left(\frac{4\pi r}{\lambda}\right) \\ &= 20 \log_{10}\left(\frac{4\pi 40 \times 10^6}{0.025}\right) \\ &\sim 206.1 \text{ dB} \end{aligned}$$

Substituting this result in Eq (11.29), the received power is

$$\begin{aligned} P_R &= EIRP - L_P + G_R \\ &= 10 \text{ dBW} - 206.1 \text{ dB} + 53.0 \text{ dB} \\ &= -143 \text{ dBW} \end{aligned}$$

where we have used the antenna gain of 53 dB from Problem 11.6.

Problem 11.8 The antenna of Problem 11.7 has a noise temperature of 70°K and is directly connected to a receiver with an equivalent noise temperature of 50°K and a gain of 60 dB. What is the system noise temperature? If the transmitted signal has a bandwidth of 100 kHz, what is the carrier-to-noise ratio? If the digital signal has a bit rate of 150 kbps, what is the E_b/N_0?

Solution

From Eq. (11.22), the combined system noise temperature is

$$\begin{aligned} T_s &= T_{ant} + \frac{T_{rx}}{1} \\ &= 70^o + 50^o \\ &= 120^o K \end{aligned}$$

where the electrical gain of the antenna is 1. For a bandwidth of 100 kHz the available noise power is

$$\begin{aligned} N &= kT_sB \\ &= 1.38 \times 10^{-23} \cdot 120 \cdot 10^5 \\ &= 1.66 \times 10^{-16} \text{ watts} \\ &\sim -157.8 \text{ dBW} \end{aligned}$$

Comparing to the result for Problem 11.7, we have that the C/N is 14.8 dB.

To convert the C/N_0 to an E_b/N_0, we use the formula

$$\frac{C}{N_0} = \frac{E_b}{N_0} \times R$$

where the bit rate R relates the energy per bit E_b to the power C. In decibels,

$$\left(\frac{C}{N_0}\right)_{dB-Hz} = \left(\frac{E_b}{N_0}\right)_{dB} + 10\log_{10} R$$

Re-arranging, we have

$$\begin{aligned} \left(\frac{E_b}{N_0}\right)_{dB} &= \left(\frac{C}{N_0}\right)_{dB-Hz} - 10\log_{10} R \\ &= \left(\frac{C}{N}\right)_{dB} + 10\log B_N - 10\log_{10} R \\ &= 14.8 + 50 - 51.8 \\ &= 13 \text{ dB} \end{aligned}$$

Problem 11.9 Transmitting and receiving antennas for a 4 GHz signal are located on top of 20 meter towers separated by 2 kilometers. For free-space propagation, what is the maximum height permitted for an object located midway between the two towers?

Solution

The radius of the first Fresnel zone with $d_1 = d_2 = 1$ kilometer and $\lambda = c/f = 0.075$ m is

$$
\begin{aligned}
h &= \sqrt{\frac{\lambda d_1 d_2}{d_1 + d_2}} \\
&= \sqrt{\frac{(.075)(1000)(1000)}{2000}} \\
&= 6.1\,\text{m}
\end{aligned}
$$

Consequently, the maximum height of intermediate object is 20 m – 6.1m = 13.9 m, if we require free-space propagation conditions.

Problem 11.10 A measurement campaign indicates that the median path loss at 900 MHz in a suburban area may be modeled with a path-loss exponent of 2.9. What is the median path loss at a distance of 3 kilometers using this model? How does this loss compare to the free-space loss at the same distance?

Solution

From Eq (11.37), the free-space path loss at one meter, with a transmission frequency of 900 MHz, is

$$
\beta_0 = \left(\frac{\lambda}{4\pi r_0}\right)^2 = \left(\frac{.333}{4\pi}\right)^2 = .000704
$$

with $r_0 = 1$ m. From Eq.(11.37), the path loss with the terrestrial propagation model is

$$
\begin{aligned}
\frac{P_R}{P_T} &= \frac{\beta_0}{(r/r_0)^n} \\
&= \frac{.0007}{(3000)^{2.9}} \\
&= 5.8\times 10^{-14} \\
&\sim -132\,\text{dB}
\end{aligned}
$$

The free-space loss over the same distance is given by

$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi r}\right)^2$$
$$= 7.8\times 10^{-11}$$
$$\sim -101.1\,\text{dB}$$

or 31 dB less.

Problem 11.11 Express the true median of the Rayleigh distribution as a fraction of the R_{rms} value? What is the decibel error in the approximation $R_{\text{median}} \approx R_{\text{rms}}$?

Solution

The median of the distribution satisfies $\mathbf{P}\ R < r\ = 0.5$. Consequently, from Eq. (11.38) we have that the median r satisfies

$$1 - \exp\left\{\frac{-r^2}{R_{rms}^2}\right\} = 1/2$$

Solving for r we obtain

$$-\frac{r^2}{R_{rms}^2} = \ln 1/2$$
$$r = R_{rms}\sqrt{\ln 1/2}$$
$$= 0.83 R_{rms}$$

Consequently, there is a $20\log_{10}(0.83)$ =1.62 dB error when using the rms value of the amplitude instead of the median value.

Problem 11.12 Compute the noise spectral density in watts per hertz of:
(a) an ideal resistor at nominal temperature of 290°K;
(b) an amplifier with an equivalent noise temperature of 22,000°K.

Solution

(a) From Eq. (11.19), the noise power spectral density is

$$N_0 = kT_e$$
$$= 1.38\times 10^{-23}\times 290$$
$$= 4.0\times 10^{-21}\ \text{W/Hz}$$

(b) From Eq. (11.19), the noise power spectral density is

$$
\begin{aligned}
N_0 &= kT_e \\
&= 1.38\times10^{-23}\times22000 \\
&= 3.04\times10^{-19}\ \text{W/Hz}
\end{aligned}
$$

Problem 11.13 For the two cases of Problem 11.12, compute the pre-detection SNR when the received signal power is:
(a) -60 dBm and the receive bandwidth is 1 MHz;
(b) -90 dBm and the receive bandwidth is 30 kHz.
Express the answers in both absolute terms and decibels.

Solution
(a) The signal power is obtained by converting -60 dBm to watts

$$S = 10^{(-60/10)} = 10^{-6}\,\text{mW} = 10^{-9}\,\text{W}$$

The noise power from the ideal resistor is from Eq. (11.13)

$$
\begin{aligned}
N &= kT_eB_N \\
&= 4.0\times10^{-21}\times(10^6) \\
&= 4.0\times10^{-15}\,\text{W}
\end{aligned}
$$

The SNR is the ratio of the two

$$SNR = \frac{S}{N} = \frac{10^{-9}}{4.0\times10^{-15}} = 2.5\times10^5 \sim 54\ \text{dB}$$

A similar calculation for the amplifier of the previous problem results in

$$SNR = \frac{S}{N} = \frac{10^{-9}}{3.04\times10^{-19}\times10^6} = 2.94\times10^3 \sim 34.7\ \text{dB}$$

(b) The signal power is obtained by converting -90 dBm to watts

$$S = 10^{(-90/10)} = 10^{-9}\,\text{mW} = 10^{-12}\,\text{W}$$

The noise power from the ideal resistor is from Eq. (11.13)

$$
\begin{aligned}
N &= kT_e B_N \\
&= 4.0\times10^{-21}\times(30\times10^{3}) \\
&= 1.2\times10^{-16}\,\text{W}
\end{aligned}
$$

The SNR is the ratio of the two

$$SNR = \frac{S}{N} = \frac{10^{-12}}{1.2\times10^{-16}} = 8.3\times10^{3} \sim 39.2\text{ dB}$$

A similar calculation for the amplifier of the previous problem results in

$$SNR = \frac{S}{N} = \frac{10^{-12}}{3.04\times10^{-19}\times\ 30\times10^{3}} = 1.1\times10^{2} \sim 20.4\text{ dB}$$

Problem 11.14 A wireless local area network transmits a signal that has a noise bandwidth of approximately 6 MHz. If the signal strength at the receiver input terminals is –90 dBm and the receiver noise figure is 8 dB, what is the pre-detection signal-to-noise ratio?

<u>Solution</u>

The signal power is obtained by converting -90 dBm to watts

$$S = 10^{(-90/10)} = 10^{-9}\,\text{mW} = 10^{-12}\,\text{W}$$

The noise power with an 8 dB noise figure *F* is from Eqs. (11.15) and (11.16)

$$
\begin{aligned}
N &= kT_0 FB \\
&= 1.38\times10^{-23}\times(290)\times10^{8/10}\times(6\times10^{6}) \\
&= 1.52\times10^{-13}\,\text{W}
\end{aligned}
$$

The pre-detection SNR is the ratio of the two

$$SNR = \frac{S}{N} = \frac{10^{-12}}{1.52\times10^{-13}} = 6.6 \sim 8.2\text{ dB}$$

Problem 11.15 A communications receiver includes a whip antenna whose noise temperature is approximately that of the Earth, that is, 290°K. The receiver pre-amplifier has a noise figure of 4 dB and a gain of 25 dB. What is the equivalent noise temperature of the antenna and the pre-amplifier? What is the combined noise figure?

Solution

(a) Following Example 11.4, the combined noise temperature of the antenna and pre-amplifier is, from Eq. (11.17)

$$\begin{aligned} T_{sys} &= T_{ant} + T_{amp} \\ &= 290 + 290(F-1) \\ &= 728\text{K} \end{aligned}$$

(b) From Eq. (11.16), the combined noise figure is

$$F_{comb} = \frac{T + T_e}{T} = \frac{290 + 728}{290} = 3.51 \sim 5.45\text{dB}$$

Problem 11.16 A parabolic antenna with a diameter of 0.75 meters is used to receive a 12 GHz satellite signal. What is the gain in decibels of this antenna? Assume the antenna efficiency is 60%.

Solution

From Eq.(11.25), the antenna gain is

$$G_R = \frac{4\pi A_{eff}}{\lambda^2} \qquad (1)$$

The signal wavelength is $\lambda = c/f = 3\times10^8 / 12\times10^9 = 0.025\text{m}$, and the effective area is

$$A_{eff} = \eta \frac{\pi d^2}{4} = 0.60 \frac{\pi(.75)^2}{4}$$

Substituting these two results into Eq. (1), the antenna gain is

$$G_R = \frac{4\pi}{0.025^{\ 2}} \times \ 0.6 \ \frac{\pi(0.75)^2}{4} = 5.33\times10^3 \sim 37.3\text{dB}$$

Problem 11.17 If the system noise temperature of a satellite receiver is 300°K, what is the required received signal strength to produce a C/N_0 of 80 dB?

Solution

(There is a typo in problem statement, the units should be "dB-Hz".)

From Eq. (11.19), the noise power spectral density is

$$
\begin{aligned}
N_0 &= kT_s \\
&= 1.38\times10^{-23}\times(300) \\
&= 4.14\times10^{-21}\,\text{W/Hz} \\
&\sim -203.8\,\text{dBW/Hz}
\end{aligned}
$$

In decibels, the carrier to noise density is given by

$$
\begin{aligned}
C/N_0\big|_{dB-Hz} &= (C)_{dBW} - N_0\big|_{dBW-Hz} \\
80 &= C\big|_{dBW} - (-203.8)
\end{aligned}
$$

Solving for C, we obtain C = -123.8 dBW = -93.8 dBm.

Problem 11.18 If a satellite is 40,000 km from the antenna of Problem 11.16, what satellite EIRP will produce a signal strength of –110 dBm at the antenna terminals? Assume the transmission frequency is 12 GHz.

Solution

The received power is given by Eq. (11.29)

$$P_R = EIRP + G_R - L_P \qquad (1)$$

where all quantities are in decibels. From Problem 11.16, the antenna gain is G_R = 37.3 dB. The free-space path loss is given by Eq. (11.32)

$$L_P = 20\log_{10}\left(\frac{4\pi r}{\lambda}\right)$$

From Problem 11.16, the wavelength is λ = 0.025m at 12 GHz. So, at a distance r = 40,000 km, the path loss is

$$L_P = 20\log_{10}\left(\frac{4\pi(40000\times10^3}{0.025}\right) = 206.1\,\text{dB}$$

Substituting these in Eq.(1) with a received power of -110 dBm, we obtain

$$-110\,\text{dBm} = EIRP + 37.3\,\text{dB} - 206.1\text{dB}$$

Solving this equation, we find the require EIRP is 58.8 dBm.

Problem 11.19 Antennas are placed on two 35-meter office towers that are separated by ten kilometers. What is the minimum height of a building between the two towers that would disturb the assumption of free-space propagation?

Solution

From Eq. (11.35), the radius of the first Fresnel zone is

$$h = \sqrt{\frac{\lambda d_1 d_2}{d_1 + d_2}}$$

This radius is maximized midway between the two towers and must be kept clear to approximate free-space propagation. With $d_1 = d_2$ = 5km, the radius in meters is

$$h = \sqrt{2500\lambda} = 50\sqrt{\lambda}$$

The maximum building height (in meters) is

$$\begin{aligned} b &= 35 - h \\ &= 35 - 50\sqrt{\lambda} \end{aligned}$$

For example, at a transmission frequency of 4 GHz, the maximum height is b = 21.3 m.

Problem 11.20 If a receiver has a sensitivity of –90 dBm and a 12 dB noise figure what is minimum pre-detection signal-to-noise ratio of an 8 kHz signal?

Solution

The noise in an 8 kHz bandwidth for a receiver with an 8 dB noise figure is, from Eqs. (11.15) and (11.16),

$$\begin{aligned} N &= kT_0FB \\ &= 1.38\times10^{-23}\times(290)\times\ 10^{12/10}\ \ \times(8\times10^3) \\ &= 5.07\times10^{-16}\,\mathrm{W} \end{aligned}$$

The receiver sensitivity is defined as the minimum received signal power that will provide a demodulated signal with acceptable performance, thus the minimum signal power is S = -90 dBm ~ 10^{-12} W. The minimum pre-detection SNR is the ratio of the two

$$SNR = \frac{S}{N} = \frac{10^{-12}}{5.07\times10^{-16}} = 1.97\times10^3 \sim 32.9\ \mathrm{dB}$$

Problem 11.21 A satellite antenna is installed on the tail of an aircraft and has a noise temperature of 100°K. The antenna is connected by a coaxial cable to a low-noise amplifier in the equipment bay at the front of the aircraft. The cable causes 2 dB attenuation of the signal. The low-noise amplifier has a gain of 60 dB and a noise temperature of 120°K. What is the system noise temperature? Where would a better place for the low-noise amplifier be?

Solution

Following Example 11.4, the system noise temperature is

$$\begin{aligned} T_s &= T_{ant} + \frac{T_{cable}}{G_{ant}} + \frac{T_{amp}}{G_{cable}} \\ &= 100 + \frac{290}{1} + \frac{120}{.631} \\ &= 580\text{K} \end{aligned}$$

where we have used the facts that the antenna does not provide any electrical gain, thus G_{ant} = 1; and the fact the fact that cable causes a 2 dB loss so $G_{cable} = 10^{-2/10} = 0.631$. Locating the low-noise amplifier in the tail of the aircraft, close to the antenna would be a better system design. With the amplifier in the antenna tail, the system noise temperature would be approximately 220 K.

Problem 11.22 A wireless local area network transmitter radiates 200 milliwatts. Experimentation indicates that the path loss may be accurately described by

$$L_p = 31 + 33\log_{10}(r)$$

where the path loss is in decibels and r is the range in meters. If the minimum receiver sensitivity is –85 dBm, what is the range of the transmitter?

Solution

Since the problem says nothing about the transmit and receive antennas, we shall assume they are omni-directional with a gain of 0 dB. In this case, the Friis equation (in decibels) for the received signal strength reduces to

$$\begin{aligned} P_R &= P_T - L_P \\ &= P_T - \; 31 + 33\log_{10} r \end{aligned} \qquad (1)$$

With a transmit power of 200 mW, equivalent to 23 dBm, and a minimum signal strength of -85 dBm, Eq. (1) becomes

$$-85 = 23 - (31 + 33\log_{10} r)$$

Solving this equation for the maximum range, we find *r* is 215.4 meters.

Problem 11.23 A mobile radio transmits 30 watts and the median path loss may be approximated by

$$L_p = 69 + 31\log_{10}(r)$$

where the path loss is in decibels and *r* is the range in kilometers. If the receiver sensitivity is -110 dBm and 12 dB of margin must be included to compensate for variations about the median path loss, what is the range of the transmitter?

Solution

Since the problem says nothing about the transmit and receive antennas, we shall assume they are omni-directional with a gain of 0 dB. In this case, the Friis equation for the received signal strength reduces to

$$\begin{aligned} P_R &= P_T - L_P - L_0 \\ &= P_T - (69 + 31\log_{10} r) - L_0 \end{aligned} \qquad (1)$$

where L_0 represents the required margin. A transmit power of 30 W is equivalent to 14.8 dBW, and a minimum signal strength of -110 dBm is equivalent to -140 dBW. Thus, Eq. (1) becomes

$$-140 = 14.8 - (69 + 31\log_{10} r) - 12$$

Solving this equation for the maximum range, we find *r* is 240.2 kilometers. In practice, the range will likely be somewhat less than this due to the curvature of the earth and depending on the height of the base station antenna.

Problem 11.24 A cellular telephone transmits 600 milliwatts of power. If the receiver sensitivity is –90 dBm, what would the range of the telephone be under free space propagation? Assume the transmitting and receiving antennas have unity gain and the transmissions are at 900 MHz. If propagation conditions actually show a path-loss exponent of 3.1 with a fixed loss β = 36 dB, what would the range be in this case?

Solution

(a) The Friis equation for the received power in decibels is

$$P_R = P_T + G_R + G_T - L_P \qquad (1)$$

where the antenna gains are $G_R = G_T = 0$ dB. The transmit power of 600 mW is equivalent to or 27.8 dBm. For free-space propagation, the path loss is

$$L_P = 20\log_{10}\left(\frac{4\pi r}{\lambda}\right)$$

At 900 MHz, the wavelength is $\lambda = c/f = 3\times 10^8/900\times 10^6 = 0.33\,\mathrm{m}$. Making these substitutions, we have

$$-90 = 27.8 + 0 + 0 - 20\log_{10}\left(\frac{4\pi r}{0.33}\right)$$

Solving this equation for the maximum range, we find the r is 20.4 kilometers.

(b) In this case, the Friis equation still applies but the path loss is given by Eq. (11.37)

$$\begin{aligned} L_P &= \left(\frac{10^{-36/10}}{r^{3.1}}\right)^{-1} \\ &\sim 36 + 31\log_{10}\ r \end{aligned}$$

Substituting the second line into Eq. (1), we have

$$-90 = 27.8 + 0 + 0 - (36 + 31\log_{10} r)$$

Solving this equation for the maximum range, we find that r is 435 meters. Clearly, the propagation conditions can make a huge difference on the range.

Problem 11.25 A line-of-sight 10-kilometer radio link is required to transmit data at a rate of 1 megabit per second at a center frequency of 4 GHz. The transmitter uses an antenna with 10 dB gain and QPSK modulation with a root-raised cosine pulse shape spectrum having a roll-off factor of 0.5. The receiver also has an antenna with 10 dB gain and has a system noise temperature of 900 K. What is the minimum transmit power required to achieve a bit error rate of 10^{-5}?

__**Solution**__

From the BER performance of QPSK in Fig. 10.16, we find that a BER of 10^{-5} implies an E_b/N_0 of 9.5 dB is required. From this, we obtain the required C/N_0 using knowledge of the transmission rate R = 1 Mbps.

$$\left(\frac{C}{N_0}\right)_{dB-Hz} = \left(\frac{E_b}{N_0}\right)_{dB} + 10\log_{10} R$$
$$= 69.5 \text{ dB-Hz}$$

The system noise temperature of 900 K implies

$$\begin{aligned} N_0 &= kT_e \\ &= 1.38\times10^{-23}\times900 \\ &= 1.24\times10^{-20} \text{ W/Hz} \\ &\sim -199.1 \text{ dBW/Hz} \end{aligned}$$

Using this information, the received power level may be calculated from

$$\begin{aligned} P_R &= C \\ &= \left(\frac{C}{N_0}\right)_{dB-Hz} + \; N_0 \;{}_{dBW-Hz} \\ &= 69.5 + \; -199.1 \\ &= -129.6 \text{ dBW} \end{aligned}$$

We now appeal to the decibel form of the Friis equation:

$$P_R = P_T + G_R + G_T - L_P \qquad (1)$$

where the antenna gains are G_R = G_T = 10 dB. Since the problem sight says line-of-sight transmission, we shall assume free-space propagation, and the path loss is

$$L_P = 20\log_{10}\left(\frac{4\pi r}{\lambda}\right)$$

At 4 GHz, the wavelength is $\lambda = c/f = 3\times10^8 / 4\times10^9 = 0.075$ m. Making all these substitutions into Eq. (1) with a range r = 10 km, we obtain

$$-129.6 = P_T + 10 + 10 - 20\log_{10}\left(\frac{4\pi 10\times10^3}{0.075}\right)$$

Solving this equation for the transmitted power, we find that the required P_T is -25.1 dBW or 4.9 dBm.

Problem 11.26 A land-mobile radio transmits 128 kbps at a frequency of 700 MHz. The transmitter uses an omni-directional antenna and 16-QAM modulation with a root-raised cosine pulse spectrum having a roll-off of 0.4. The receiver has an antenna with 3 dB gain and a noise figure of 6 dB. If the path loss between the transmitter and receiver is given by

$$L_P(r) = 30 + 28\log_{10}(r) \text{ dB}$$

where r is in meters, what is the maximum range at which the bit error rate of 10^{-4} may be achieved?

Solution

From the BER performance of 16-QAM in Fig. 10.16, we find that a BER of 10^{-4} implies an E_b/N_0 of 13 dB. From this, we obtain the C/N_0 by using knowledge of the transmission rate R = 128 kbps.

$$\begin{aligned}\left(\frac{C}{N_0}\right)_{dB-Hz} &= \left(\frac{E_b}{N_0}\right)_{dB} + 10\log_{10} R \\ &= 64.1 \text{ dB-Hz}\end{aligned}$$

The noise figure of 6 dB implies

$$\begin{aligned}N_0 &= kFT_0 \\ &= 1.38\times10^{-23} \times 10^{6/10} \times 290 \\ &= 1.59\times10^{-20} \text{ W/Hz} \\ &\sim -198.0 \text{ dBW/Hz}\end{aligned}$$

and the received power level is

$$\begin{aligned}P_R = C &= \left(\frac{C}{N_0}\right)_{dB-Hz} + N_0\ _{dBW-Hz} \\ &= 64.1 + -198.0 \\ &= -133.9 \text{ dBW}\end{aligned}$$

We now appeal to the decibel form of the Friis equation:

$$P_R = P_T + G_R + G_T - L_P \qquad (1)$$

where the antenna gains are $G_R = G_T = 0$ dB. The path loss is

$$L_P = 30 + 28\log_{10} r$$

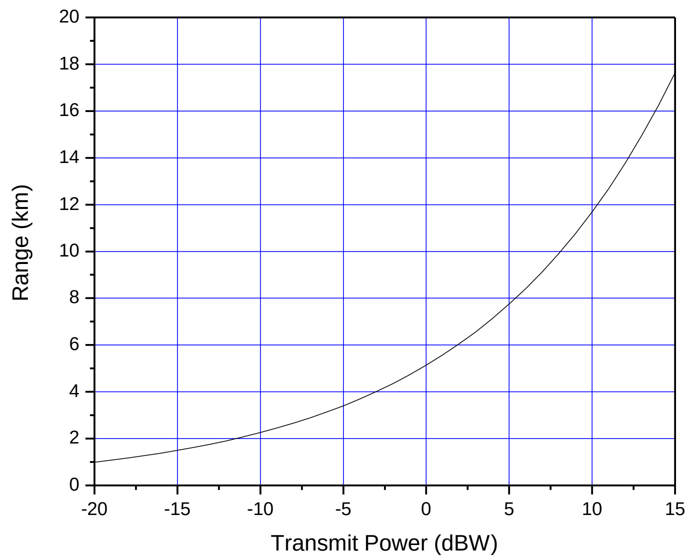
Making all these substitutions into Eq. (1), we obtain

$$-133.9 = P_T + 0 + 0 - \ 30 + 28\log_{10} r$$

or

$$r = 10^{\ P_T + 103.9 \ / 28}$$

In the following figure, we plot the range in kilometres versus the transmit power in dBW.



For example, with a transmit power of 10 W or 10 dBW, we find that range is 11.7 km.