BIRZEIT UNIVERSITY

Faculty of Engineering and Technology

Electrical and Computer Engineering Department

Electromagnetics 1 (ENEE 3408)

Assignment #2 Surface and Volume Integrals

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Section No. 1

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Question: The surfaces $r = 0$, $r = 2$, $\varphi = 45°$, $\varphi = 90°$, $\theta = 45°$ and $\theta = 90°$ define a closed surface.

Find:

a) The enclosed volume

b) The area of the closed surface *S*.

c) Write a MATLAB program to verify your answer.

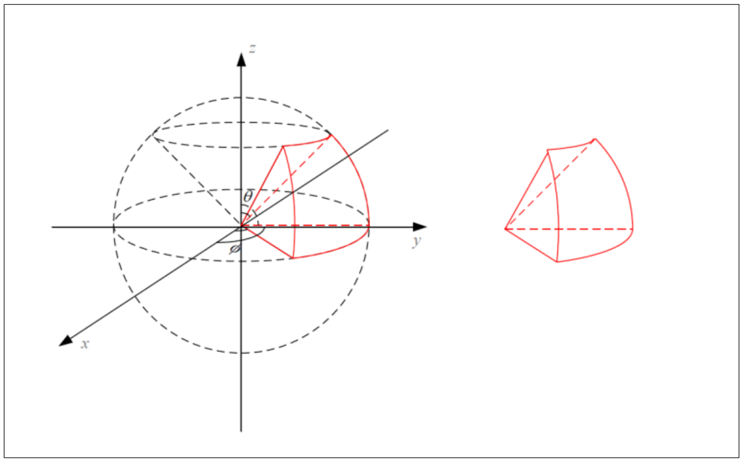

Figure 1: The surface of the question

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a) To find the volume *v* of a closed surface we first find out *dv*, the volume element. In spherical coordinates, *dv* is given by $dv = r^2 \sin\theta \, d\varphi dr d\theta$ **.** Once we get the expression of *dv*, we integrate *dv* over the entire volume.

$$dv = r^2 \sin\theta \, d\theta dr d\varphi$$

$$v = \iiint dv$$

$$v = \iiint r^2 \sin\theta \, d\theta dr d\varphi$$

$$v = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_0^2 r^2 dr \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta \, d\theta$$

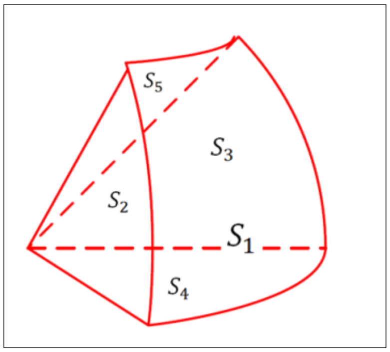
$$v = (\,\varphi|_{\frac{\pi}{4}}^{\frac{\pi}{2}})\,(\frac{r^3}{3}|_0^2)(-\cos\theta\,|_{\frac{\pi}{4}}^{\frac{\pi}{2}})$$

$$v = \frac{\sqrt{2}\,\pi}{3} = 1.48096\, m^3$$

b) The area of the closed surface is given by

$$S_{enclosed} = S_1 + S_2 + S_3 + S_4 + S_5$$

We need to find dS_1, dS_2, dS_3, dS_4, dS_5. Then we need to integrate them over their boundary. It is obvious that $S_2 = S_3$.



Now we have,

$$dS_1 = r^2 \sin\theta \, d\theta d\varphi \, \hat{a}_r$$

$$dS_2 = dS_3 = r d\theta dr \, \hat{a}_\varphi$$

$$dS_4 = dS_5 = r \sin\theta \, dr d\varphi \, \hat{a}_\theta$$

So,

$$\int dS_1 = r^2 \iint \sin\theta \, d\theta d\varphi \;\; \hat{a}_r$$

$$\int dS_2 = \int dS_3 = \iint r d\theta dr \;\; \hat{a}_\varphi$$

$$\int dS_4 = \sin\theta_1 \iint r dr d\varphi \;\; \hat{a}_\theta$$

$$\int dS_5 = \sin\theta_2 \iint r dr d\varphi \;\; \hat{a}_\theta$$

Calculating the integrals will give

1- $S_1 = 2^2 \int_{\pi/4}^{\pi/2} \sin\theta \, d\theta \int_{\pi/4}^{\pi/2} d\varphi$

$$S_1 = 2.2211 \, m^2$$

2- $S_2 = S_3 = \int_0^2 r dr \int_{\pi/4}^{\pi/2} d\theta$

$$S_2 = S_3 = \frac{\pi}{2} \, m^2 = 1.571 \, m^2$$

3- $S_4 = \sin 90° \int_0^2 r dr \int_{\pi/4}^{\pi/2} d\varphi$

$$S_4 = \frac{\pi}{2} \, m^2 = 1.571 \, m^2$$

4- $S_5 = \sin 45° \int_0^2 r dr \int_{\pi/4}^{\pi/2} d\varphi$

$$S_5 = \frac{\sqrt{2}\pi}{2} \, m^2 = 1.1107 \, m^2$$

$$S_{enclosed} = 8.0451 \, m^2$$

a) The following MATLAB code verify the previous answers:

```
V  = 0;          %initialize volume of the closed surface to 0
S1 = 0;          %initialize the area of S1 to 0
S2 = 0;          %initialize the area of S2 to 0
S3 = 0;          %initialize the area of S3 to 0
S4 = 0;          %initialize the area of S4 to 0
S5 = 0;          %initialize the area of S5 to 0

r     = 0;         %initialize r to the its lower boundary
theta = pi/4;      %initialize theta to the its lower boundary
phi   = pi/4;      %initialize phi to the its lower boundary

Number_of_r_Steps     = 1000;        %initialize the r discretization
Number_of_theta_Steps = 1000;        %initialize the theta
discretization
Number_of_phi_Steps   = 1000;        %initialize the phi discretization

dr     = (2-0)/Number_of_r_Steps;                %The r increment
dtheta = (pi/2-pi/4)/Number_of_theta_Steps;      %The theta increment
dphi   = (pi/2-pi/4)/Number_of_phi_Steps;        %The phi increment

%%the following routine calculates the volume of the enclosed surface
for k = 1: Number_of_phi_Steps
    for j = 1: Number_of_theta_Steps
        for i = 1: Number_of_r_Steps
            V = V + r^2 * sin(theta) * dphi * dr * dtheta;
        end
        r = r + dr;     %The first loop increment the r
    end
    r = 0;
    theta = theta + dtheta; %The second loop increment theta
end

Volume = V   %the volume of the shape

%%To calculate the circular area
r = 2 ; theta = pi/4;
for j = 1: Number_of_theta_Steps
    for i = 1: Number_of_phi_Steps
        S1 = S1 + r^2 * sin(theta) * dphi * dtheta;
    end
    theta = theta + dtheta;
end
```

```
%To calculate the left and right areas which have the same area
r = 0;                          %re-enter the initial values
for k = 1: Number_of_theta_Steps
    for j = 1: Number_of_r_Steps
        S2 = S2 + r* dtheta * dr;
    end
    r = r + dr;
end

if dphi ~= 0
    S3 = S2;
end

%%To calculate the lower area on the xy plane
r = 0; theta = pi/2;        % re-enter the the initial values
for k = 1: Number_of_r_Steps
    for i = 1: Number_of_phi_Steps
        S4 = S4 + r * sin(theta) * dphi * dr;
    end
    r = r + dr;
end

%%To calculate the upper area
r = 0; theta = pi/4;      % re-enter the the initial values
for k = 1: Number_of_r_Steps
    for i = 1: Number_of_phi_Steps
        S5 = S5 + r * sin(theta) * dphi * dr;
    end
    r = r + dr;
end

Surface_area = S1+S4+S3+S4+S5    %the area of the enclosed surface
```

Answers:

```
Volume = 1.4785

Surface_area = 8.0384
```