Engineering Electromagnetics

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Electromagnetics

Electromagnetics theory is a discipline concerned with the study of <u>CHARGES</u>, at <u>REST</u> and <u>MOTION</u>, that produce <u>CURRENT</u>, <u>ELICTRICAL</u>, and <u>MAGNATIC</u> fields.

Electromagnetics

James Clerk Maxwell
1831-1879

- **The study of EM includes:**
  - ❑ **Theoretical and applied concepts.**

- **The theoretical concepts are described by a set of:**
  - ❑ **Basic laws formulated through experiments.**
  - ❑ **These laws known as**

Maxwell Equations

Maxwell's Equations

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Ampere's Circuital Law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday's Law of Induction

$$\nabla \cdot \mathbf{D} = \rho_v$$

Gauss' Law for the electric field

$$\nabla \cdot \mathbf{B} = 0$$

Gauss's Law for the magnetic field

where

D the electric flux density Coulombs per meter squared

B the magnetic flux density Weber per meter squared

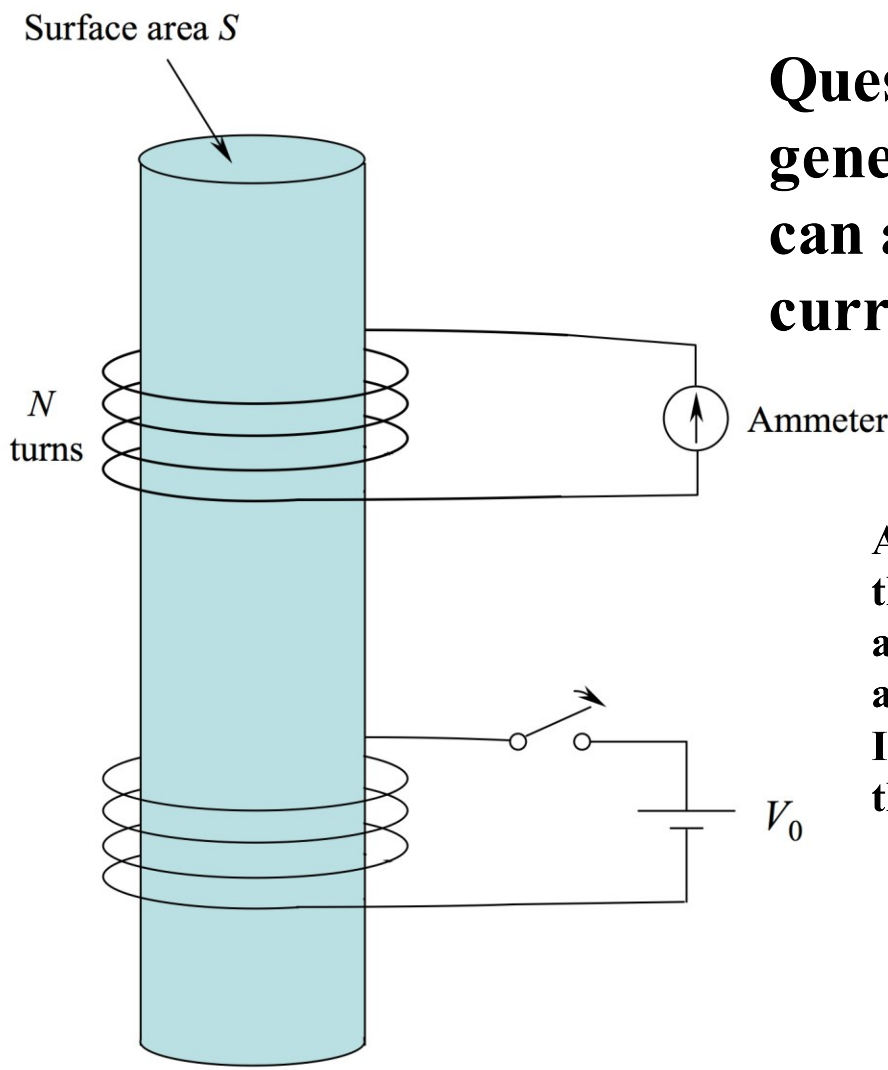
E the electric field intensity Volts per meter

H the magnetic field intensity Amperes per meter

ρ_v the volume charge density Quantity of charge per cubic meter

J the current density Ampere per meter squared

Faraday's Experiment



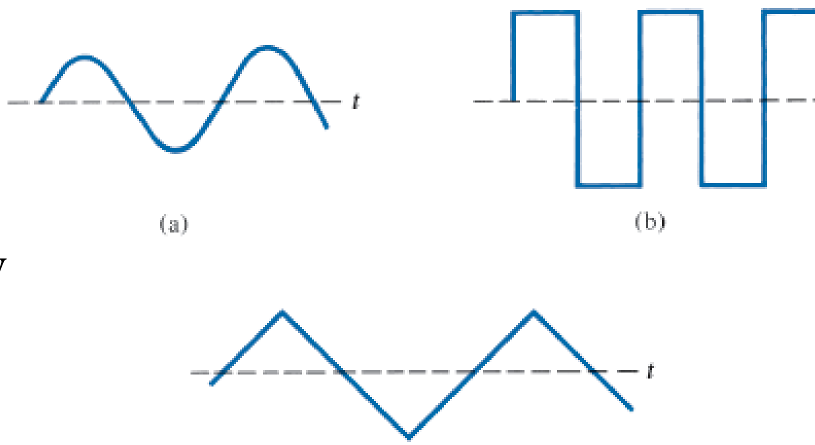
Question: If a current can generate a magnetic field, then can a magnetic field generate a current?

An experiment similar to that conducted to answer that question is shown here. Two sets of windings are placed on a shared iron core. In the lower set, a current is generated by closing the switch as shown. In the upper set, any induced current is registered by the ammeter.

Some insights about EM fields

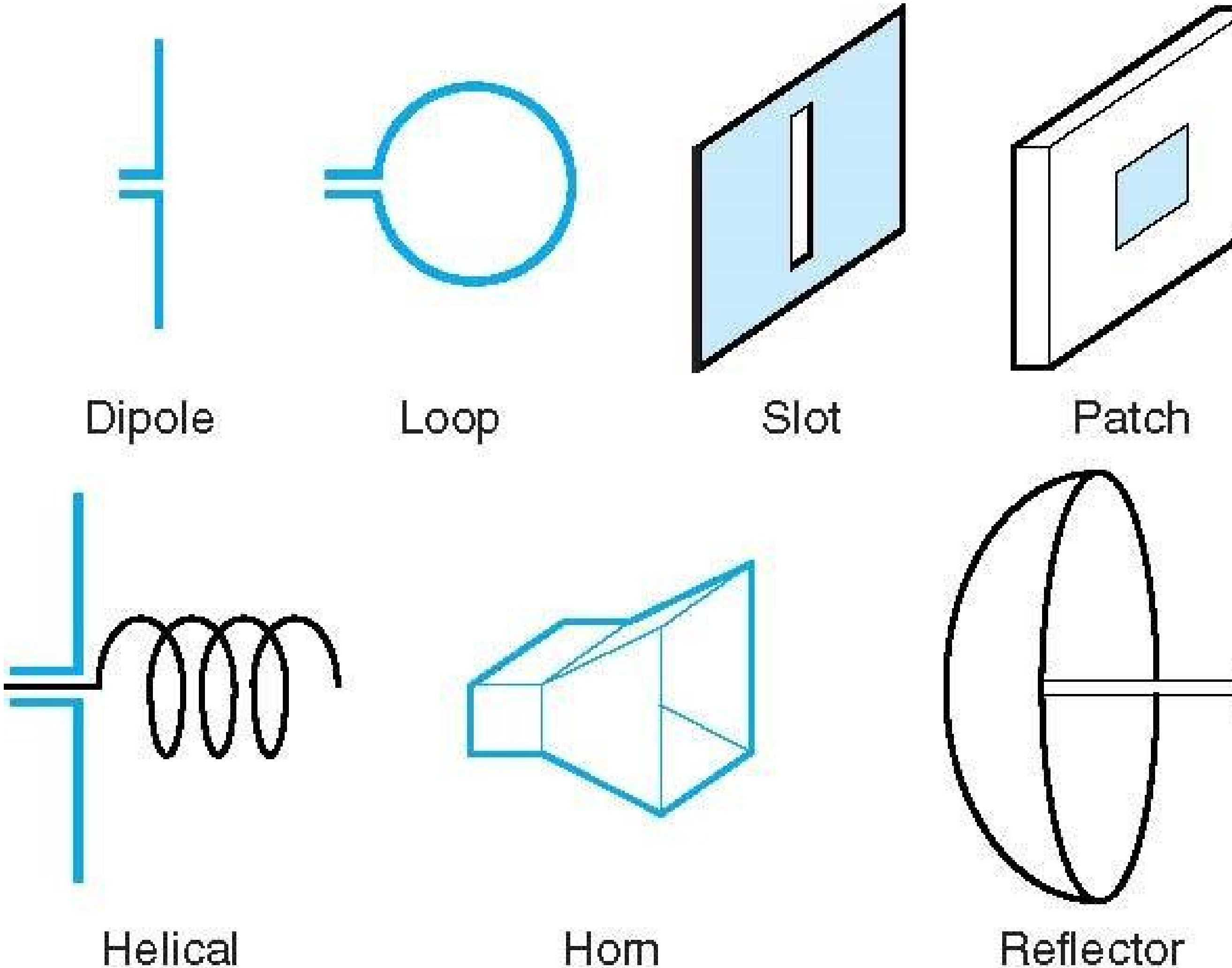
- **In static EM fields, electric and magnetic fields are independent of each other, whereas in dynamic EM fields, the two fields are interdependent.**

- **Electrostatic fields are usually produced by static electric charges, whereas Magnetostatic fields are due to motion of electric charges with uniform velocity (direct current) or static magnetic charges (magnetic poles)**



(a) (b) (c)

stationary charges	$\rightarrow$	electrostatic fields
steady currents	$\rightarrow$	magnetostatic fields
time-varying currents	$\rightarrow$	electromagnetic fields



Common single-element antennas.

Vectors Analysis

What is a Scaler quantity?

- The term *scalar* refers to a quantity whose value may be represented by a single (positive or negative) **real number**.
- **Examples:**

  Distance, temperature, mass, density, pressure, volume, volume resistivity, and voltage.

What is a Vector quantity

- A ***vector*** **quantity** has both a <u>magnitude</u> and a <u>direction</u> in space.
- **Examples**
  - Force, velocity, acceleration,

What is the field?

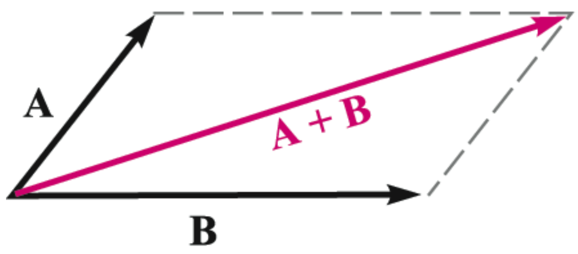
- **A field (scalar or vector) is a function that connects an arbitrary origin to a general point in space.**
- **The value of a field varies in general with both position and time.**

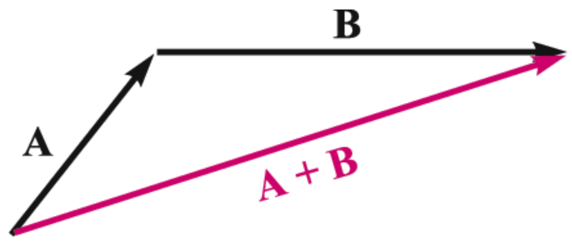
- Both ***scalar*** *fields* and ***vector*** *fields* exist.
  - The **temperature** and the **density** are examples of scalar fields.
  - The **gravitational** and **magnetic** fields of the earth, **voltage gradient**, and the **temperature gradient** are examples of vector fields.

Vectors characteristics

- Vectors may be multiplied by scalars.
- When the scalar is positive, the magnitude of the vector changes, but its direction does not.
- It **reverses direction** when multiplied by a **negative scalar**.
- Multiplication of a vector by a scalar also obeys the **associative** and **distributive** laws of algebra.

Vector Addition





Associative Law: $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$

Distributive Law: $(r + s)(\mathbf{A} + \mathbf{B}) = r(\mathbf{A} + \mathbf{B}) + s(\mathbf{A} + \mathbf{B})$

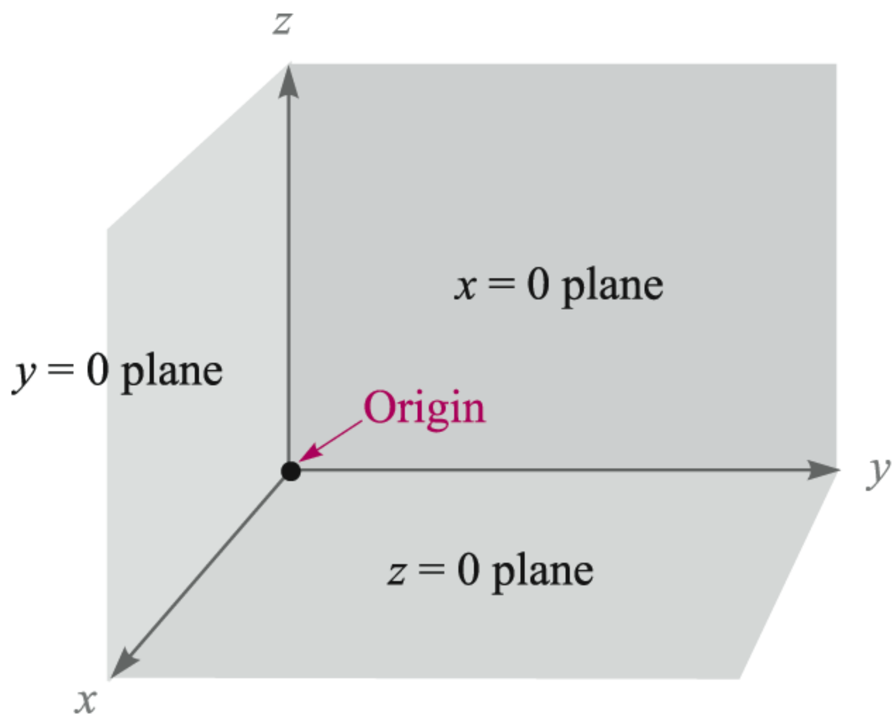
Describe a vector

To describe a vector accurately, some specific lengths, directions, angles, projections, or components must be given.

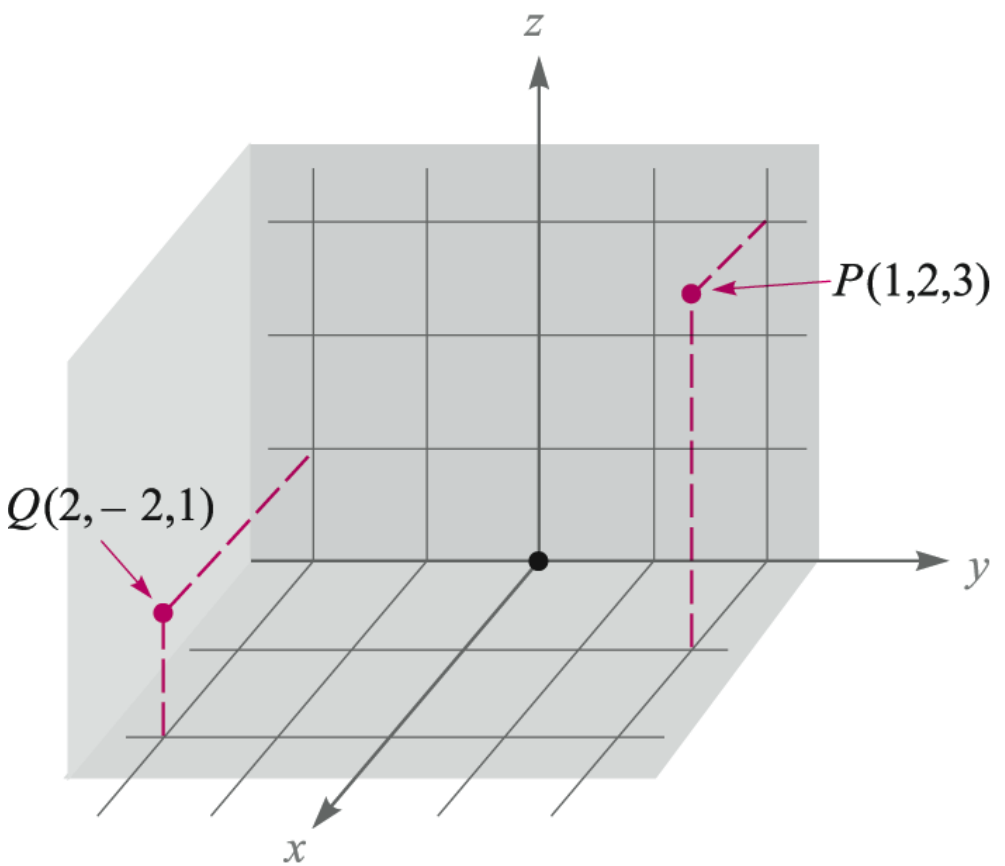
There are three simple methods of doing this,

- **Rectangular Cartesian coordinate system.**
- **cylindrical coordinate system and**
- **spherical coordinate system**

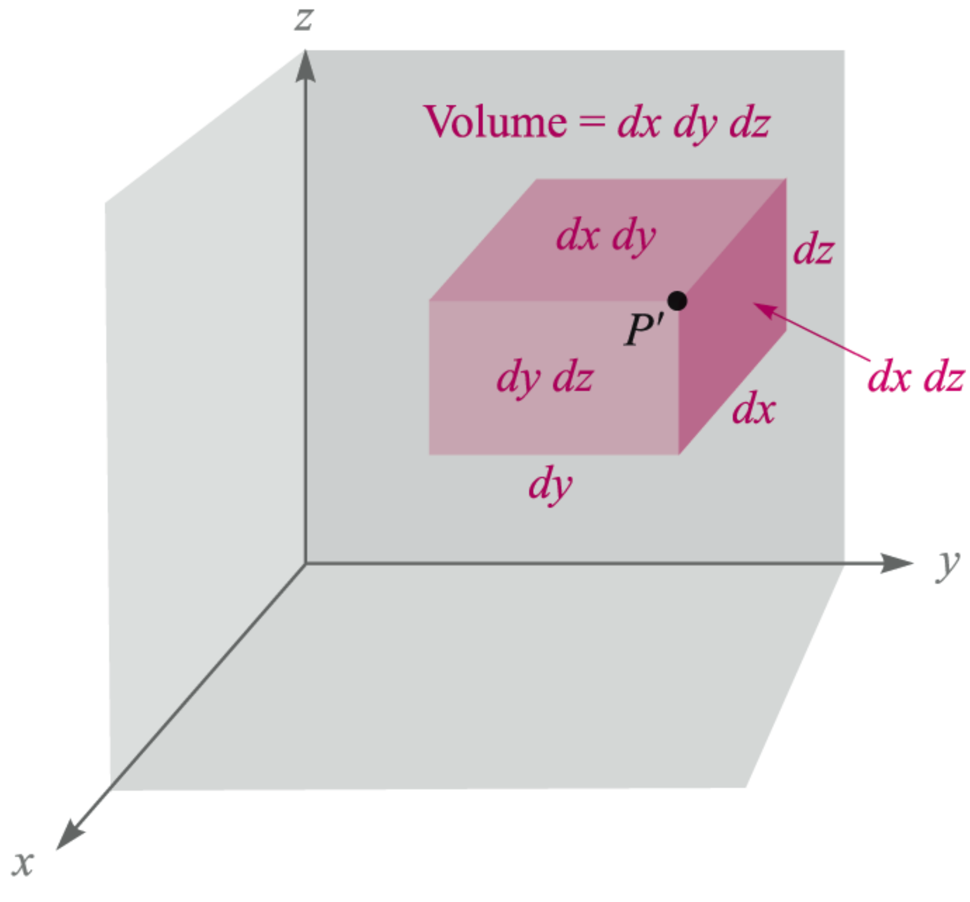
Rectangular Coordinate System



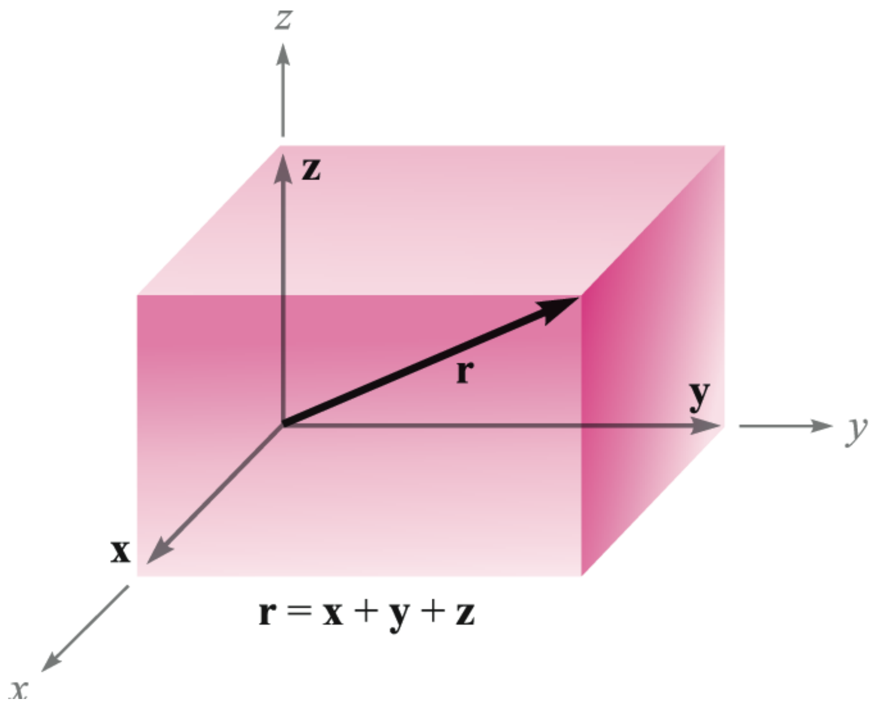
Point Locations in Rectangular Coordinates



Differential Volume Element



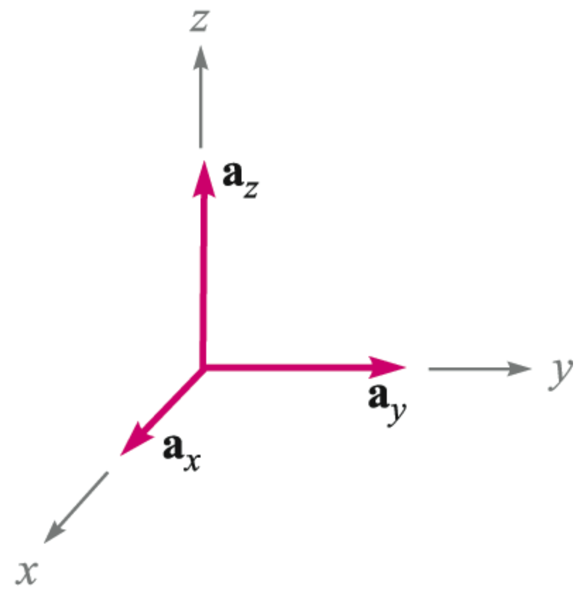
Orthogonal Vector Components



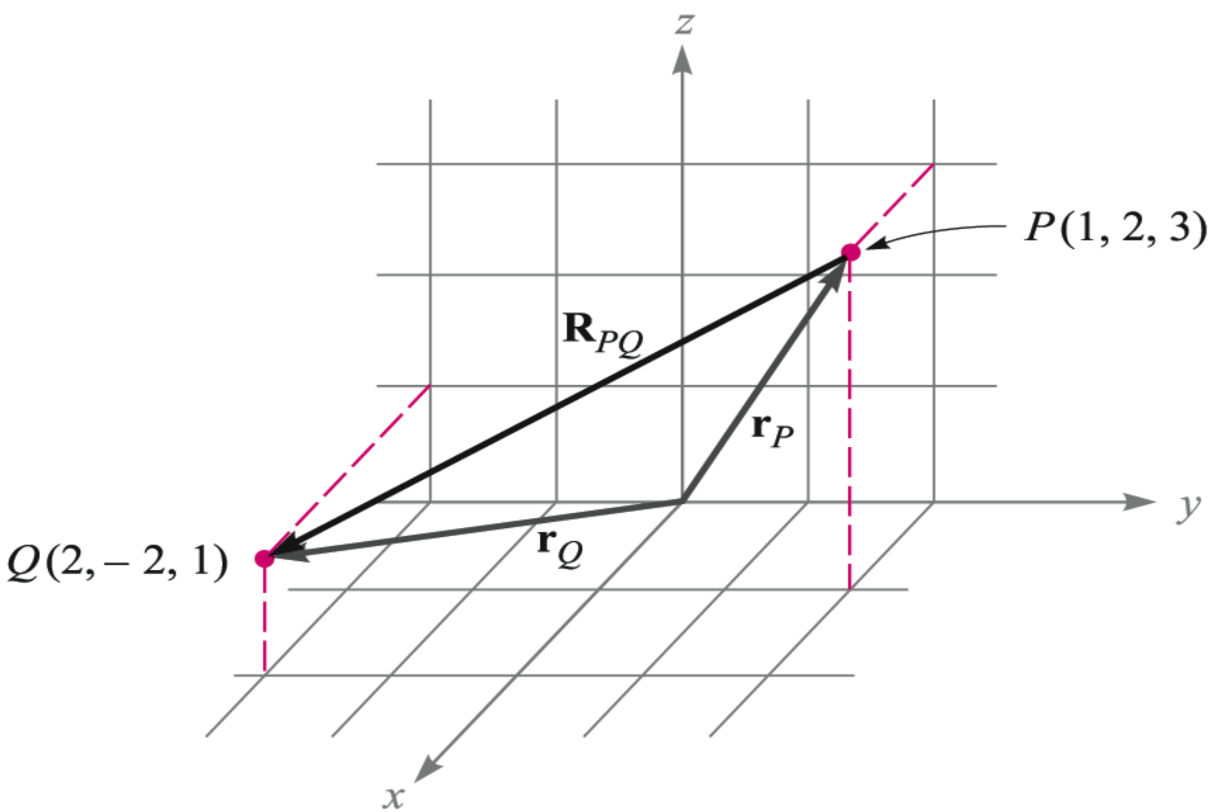
$\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$

Orthogonal Unit Vectors

unit
vectors having unit magnitude by definition



Vector Representation in Terms of Orthogonal Rectangular Components



$$\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P = (2-1)\mathbf{a}_x + (-2-2)\mathbf{a}_y + (1-3)\mathbf{a}_z$$
$$= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$$

Vector Expressions in Rectangular Coordinates

General Vector, **B**:

$$\mathbf{B} = B_x \mathbf{a}_x + B_y \mathbf{a}_y + B_z \mathbf{a}_z$$

Magnitude of **B**:

$$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

Unit Vector in the Direction of **B:**

$$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}} = \frac{\mathbf{B}}{|\mathbf{B}|}$$

Example

Specify the unit vector extending from the origin toward the point $G(2, -2, -1)$

$$\mathbf{G} = 2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z$$

$$|\mathbf{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = 3$$

$$\mathbf{a}_G = \frac{\mathbf{G}}{|\mathbf{G}|} = \tfrac{2}{3}\mathbf{a}_x - \tfrac{2}{3}\mathbf{a}_y - \tfrac{1}{3}\mathbf{a}_z = \underline{0.667\mathbf{a}_x - 0.667\mathbf{a}_y - 0.333\mathbf{a}_z}$$

Vector Field

We are accustomed to thinking of a specific vector:

$$\mathbf{v} = v_x \mathbf{a}_x + v_y \mathbf{a}_y + v_z \mathbf{a}_z$$

A vector field is a *function* defined in space that has magnitude and direction at all points:

$$\mathbf{v}(\mathbf{r}) = v_x(\mathbf{r})\mathbf{a}_x + v_y(\mathbf{r})\mathbf{a}_y + v_z(\mathbf{r})\mathbf{a}_z$$

where $\mathbf{r} = (x,y,z)$

The Dot Product

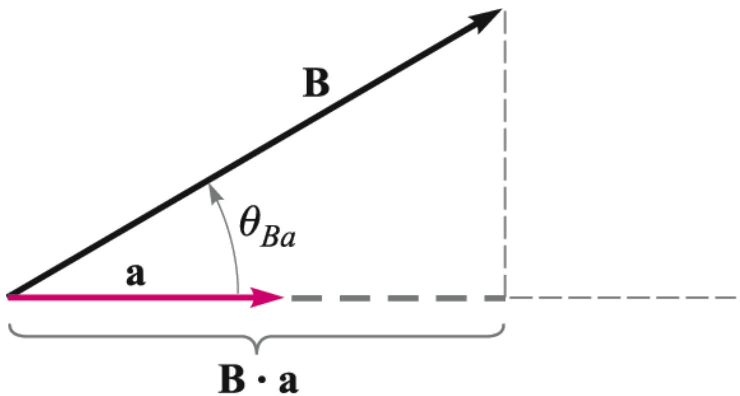
Given two vectors **A** and **B**, the *dot product*, or *scalar product*, is defined as the product of the magnitude of **A**, the magnitude of **B**, and the cosine of the smaller angle between them,

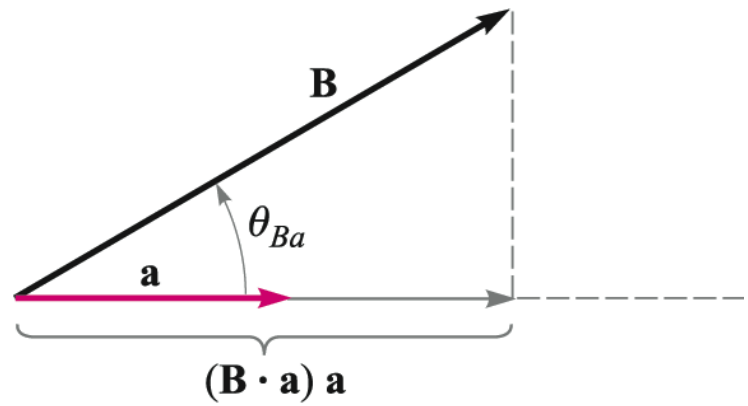
$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos \theta_{AB}$$

Commutative Law: $$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

Vector Projections Using the Dot Product

One of the most important applications of the dot product is that of finding the component of a vector in a given direction





B • a gives the component of **B** in the horizontal direction

(**B • a**) **a** gives the *vector* component of **B** in the horizontal direction

B · a is the projection of B in the a direction.

Operational Use of the Dot Product

Given

$$\begin{cases} \mathbf{A} = A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z \\ \mathbf{B} = B_x \mathbf{a}_x + B_y \mathbf{a}_y + B_z \mathbf{a}_z \end{cases}$$

Find

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

where we have used:

$$\begin{cases} \mathbf{a}_x \cdot \mathbf{a}_y = \mathbf{a}_y \cdot \mathbf{a}_z = \mathbf{a}_x \cdot \mathbf{a}_z = 0 \\ \mathbf{a}_x \cdot \mathbf{a}_x = \mathbf{a}_y \cdot \mathbf{a}_y = \mathbf{a}_z \cdot \mathbf{a}_z = 1 \end{cases}$$

Note also:

$$\mathbf{A} \cdot \mathbf{A} = A^2 = |\mathbf{A}|^2$$

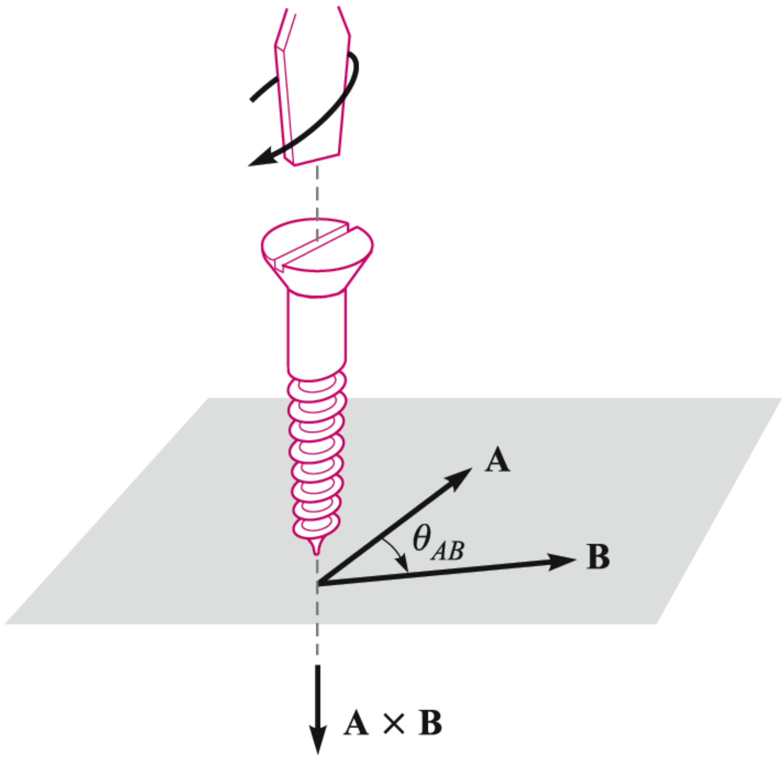
Cross Product

The cross product **A** × **B** is a vector; the magnitude of **A** × **B** is equal to the product of the magnitudes of **A**, **B**, and the sine of the smaller angle between **A** and **B**; the direction of **A** × **B** is perpendicular to the plane containing **A** and **B** and is along that one of the two possible perpendiculars which is in the direction of advance of a right-handed screw as **A** is turned into **B**.

$$\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}$$

Reversing the order of the vectors A and B results in a unit vector in the opposite direction, and we see that the cross product is not commutative, for

$$\mathbf{B} \times \mathbf{A} = -(\mathbf{A} \times \mathbf{B}).$$



Operational Definition of the Cross Product in Rectangular Coordinates

Begin with:
$$\mathbf{A}\times\mathbf{B} = A_x B_x \mathbf{a}_x \times \mathbf{a}_x + A_x B_y \mathbf{a}_x \times \mathbf{a}_y + A_x B_z \mathbf{a}_x \times \mathbf{a}_z + A_y B_x \mathbf{a}_y \times \mathbf{a}_x + A_y B_y \mathbf{a}_y \times \mathbf{a}_y + A_y B_z \mathbf{a}_y \times \mathbf{a}_z + A_z B_x \mathbf{a}_z \times \mathbf{a}_x + A_z B_y \mathbf{a}_z \times \mathbf{a}_y + A_z B_z \mathbf{a}_z \times \mathbf{a}_z$$

where
$$\begin{cases} \mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z \\ \mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x \\ \mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y \end{cases}$$

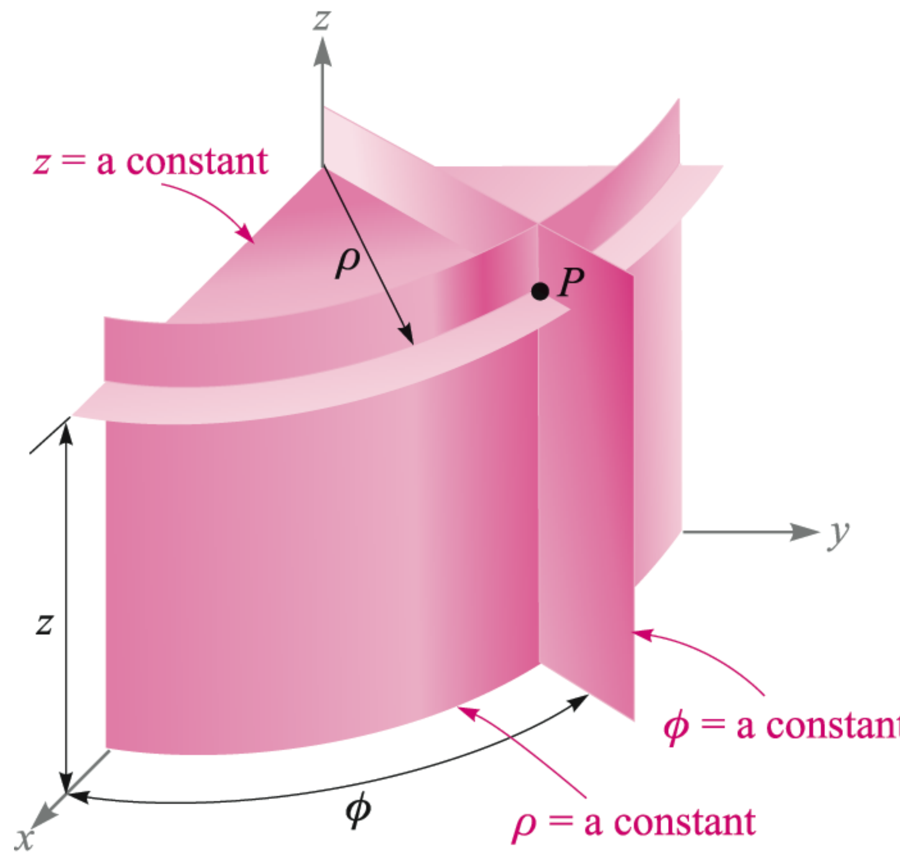
Therefore:

$$\mathbf{A}\times\mathbf{B} = (A_y B_z - A_z B_y)\mathbf{a}_x + (A_z B_x - A_x B_z)\mathbf{a}_y + (A_x B_y - A_y B_x)\mathbf{a}_z$$

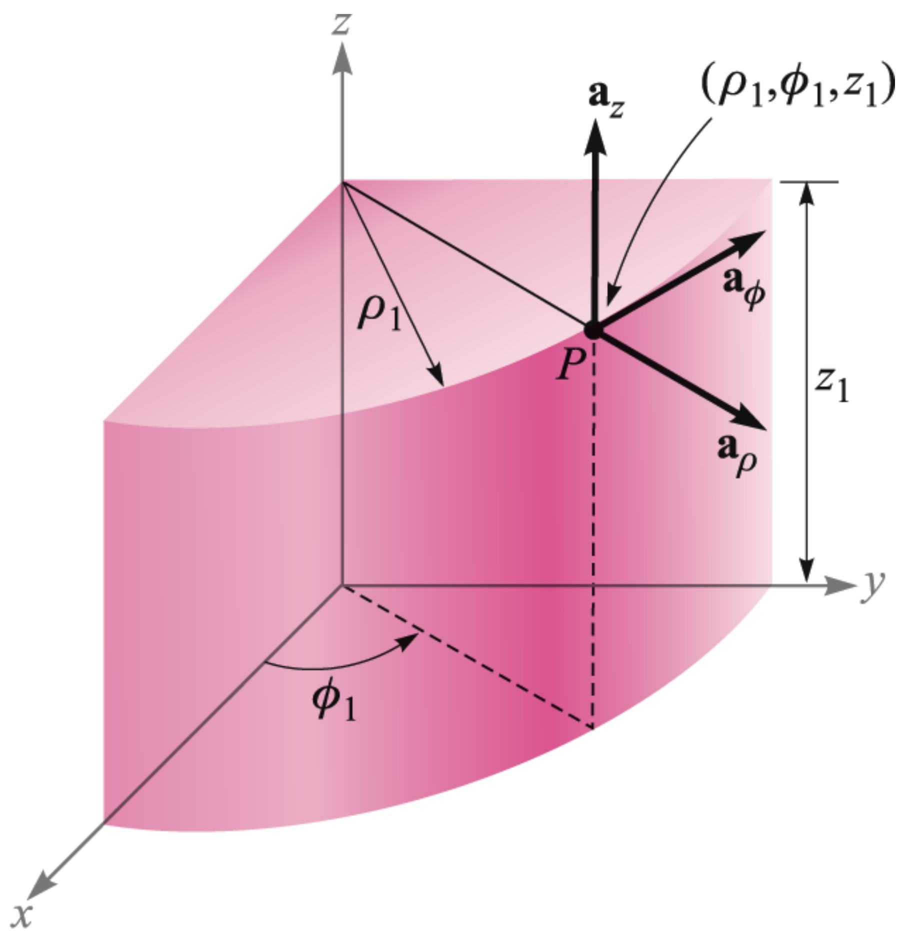
Or…
$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Circular Cylindrical Coordinates

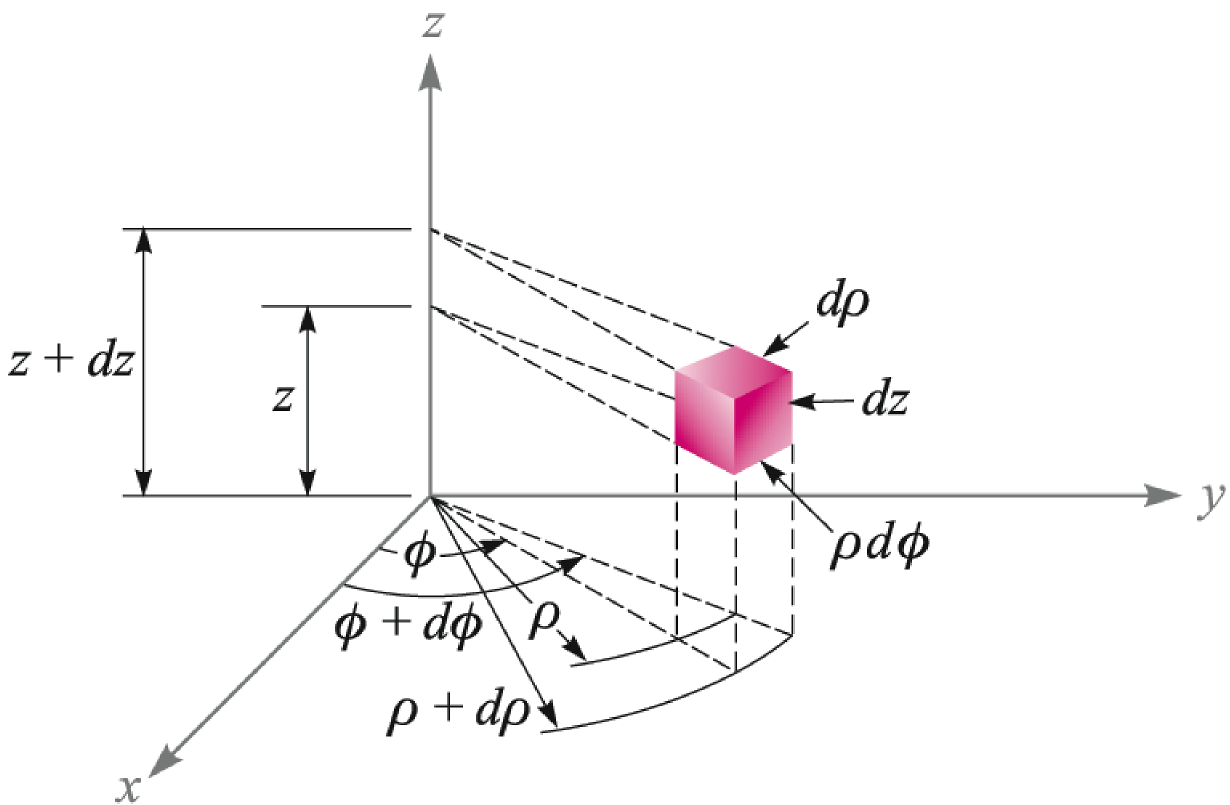
Point P has coordinates
Specified by $P(\rho, \phi, z)$



Orthogonal Unit Vectors in Cylindrical Coordinates



Differential Volume in Cylindrical Coordinates



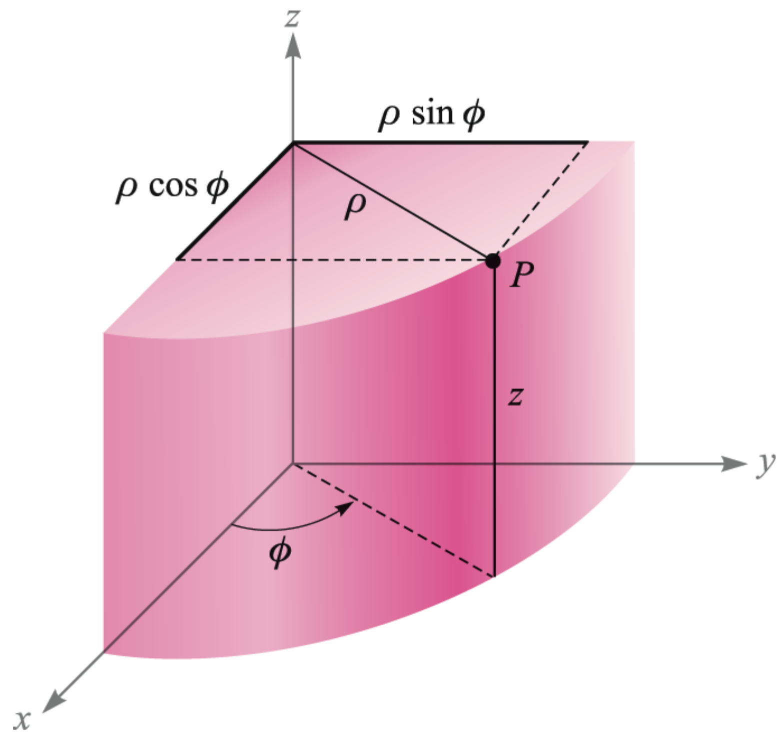
$$dV = \rho\, d\rho\, d\phi\, dz$$

Point Transformations in Cylindrical Coordinates

$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$z = z$$



$$x = \rho \cos\phi$$

$$y = \rho \sin\phi$$

$$z = z$$

Dot Products of Unit Vectors in Cylindrical and Rectangular Coordinate Systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

Example

Transform the vector,

$$\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$$

into cylindrical coordinates:

Use these:

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

Transform the vector,

$$\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$$

into cylindrical coordinates:

Start with:

$$B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho)$$

$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi)$$

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

Transform the vector,

$$\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$$

into cylindrical coordinates:

Then:

$$B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho)$$

$$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$

$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi)$$

$$= -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$$

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$z = z$

Transform the vector,

$$\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$$

into cylindrical coordinates:

Finally:

$$B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho)$$

$$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$

$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi)$$

$$= -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$$

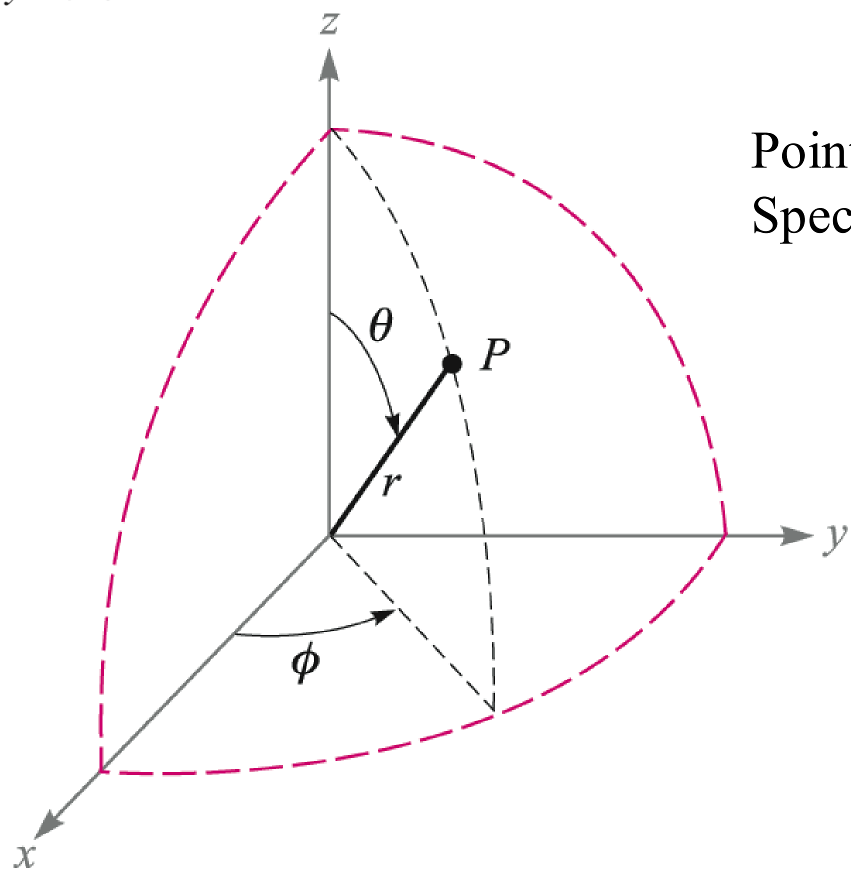
$$\mathbf{B} = -\rho\mathbf{a}_\phi + z\mathbf{a}_z$$

Spherical Coordinates

$$r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$$

$$\phi = \tan^{-1} \frac{y}{x}$$



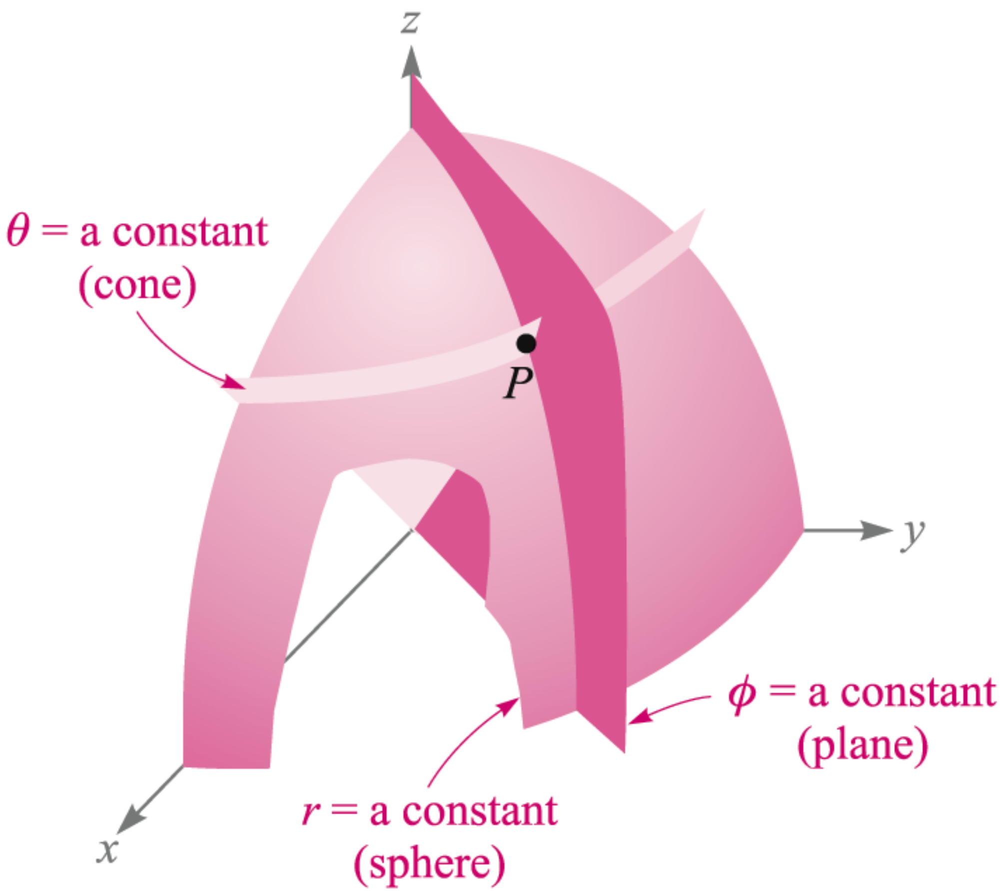
Point P has coordinates
Specified by $P(r, \theta, \phi)$

$$x = r \sin\theta \cos\phi$$

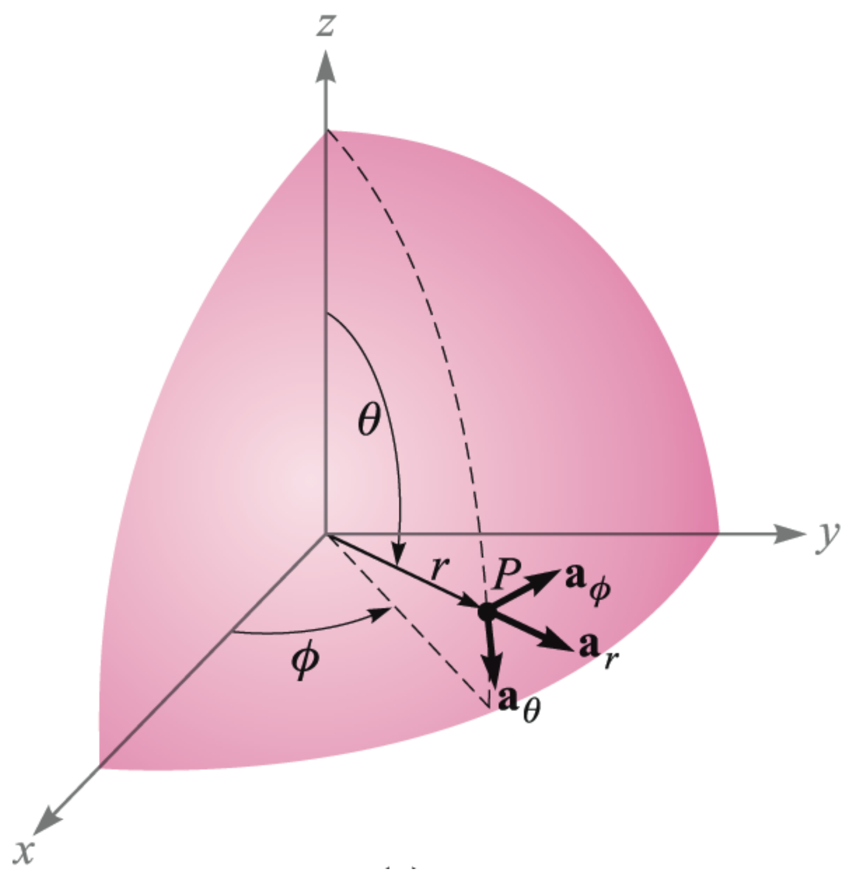
$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

Constant Coordinate Surfaces in Spherical Coordinates

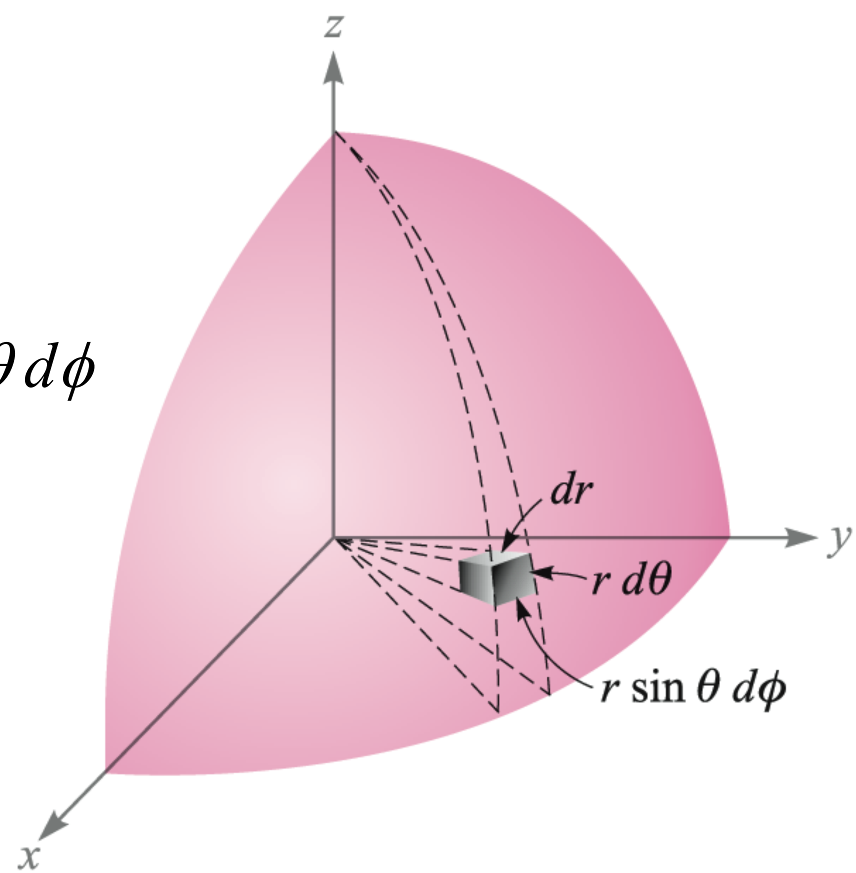


Unit Vector Components in Spherical Coordinates



Differential Volume in Spherical Coordinates

$$dV = r^2 \sin\theta \; dr\, d\theta\, d\phi$$



Dot Products of Unit Vectors in the Spherical and Rectangular Coordinate Systems

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

Example: Vector Component Transformation

Transform the field, $\mathbf{G} = (xz/y)\mathbf{a}_x$, into spherical coordinates and components

$$G_r = \mathbf{G} \cdot \mathbf{a}_r = \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_r = \frac{xz}{y} \sin\theta \cos\phi$$

$$= r \sin\theta \cos\theta \frac{\cos^2\phi}{\sin\phi}$$

$$G_\theta = \mathbf{G} \cdot \mathbf{a}_\theta = \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_\theta = \frac{xz}{y} \cos\theta \cos\phi$$

$$= r \cos^2\theta \frac{\cos^2\phi}{\sin\phi}$$

$$G\phi = \mathbf{G} \cdot \mathbf{a}_\phi = \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_\phi = \frac{xz}{y}(-\sin\phi)$$

$$= -r \cos\theta \cos\phi$$

$$\mathbf{G} = r \cos\theta \cos\phi \,(\sin\theta \cot\phi \,\mathbf{a}_r + \cos\theta \cot\phi \,\mathbf{a}_\theta - \mathbf{a}_\phi)$$

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta \cos\phi$	$\cos\theta \cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta \sin\phi$	$\cos\theta \sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$