TUTORIAL ON DIGITAL MODULATIONS

Part 15: m-QAM

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m-QAM: characteristics

1. Band-pass modulation

2. 2D signal set

3. Basis signals $p(t)cos(2\pi f_0 t)$ e $p(t)sin(2\pi f_0 t)$

4. Constellation = m points on the plane (typically on a grid)

5. Information associated to both the amplitude and the carrier phase

m-QAM: *constellation*

- SIGNAL SET

- **NOT CONSTANT ENVELOPE**

$$M = \{ \, s_i(t) = A_i \, p(t) \cos(2\pi f_0 t - \varphi_i) \, \}_{i=1}^{m}$$

- Information associated to both the **amplitude** and the **carrier phase**

m-QAM: constellation

$$M = \{ \ s_i(t) = A_i \, p(t) \cos(2\pi f_0 t - \varphi_i) \ \}_{i=1}^{m}$$

- We can write

$$s_i(t) = (A_i \cos\varphi_i)\, p(t) \cos(2\pi f_0 t) + (A_i \sin\varphi_i)\, p(t) \sin(2\pi f_0 t)$$

- Clearly, we have two versors

$$b_1(t) = p(t)\cos(2\pi f_0 t)$$

$$b_2(t) = p(t)\sin(2\pi f_0 t)$$

m-QAM: constellation

- SIGNAL SET

$$M = \{ s_i(t) = A_i\, p(t) \cos(2\pi f_0 t - \varphi_i) \}_{i=1}^{m}$$

- VERSORS

$$b_1(t) = p(t)\cos(2\pi f_0 t)$$

$$b_2(t) = p(t)\sin(2\pi f_0 t)$$

- VECTOR SET

$$M = \{ \underline{s}_i = (\alpha_i, \beta_i) \}_{i=1}^{m} \subseteq R^2$$

$$\alpha_i = A_i \cos\varphi_i$$

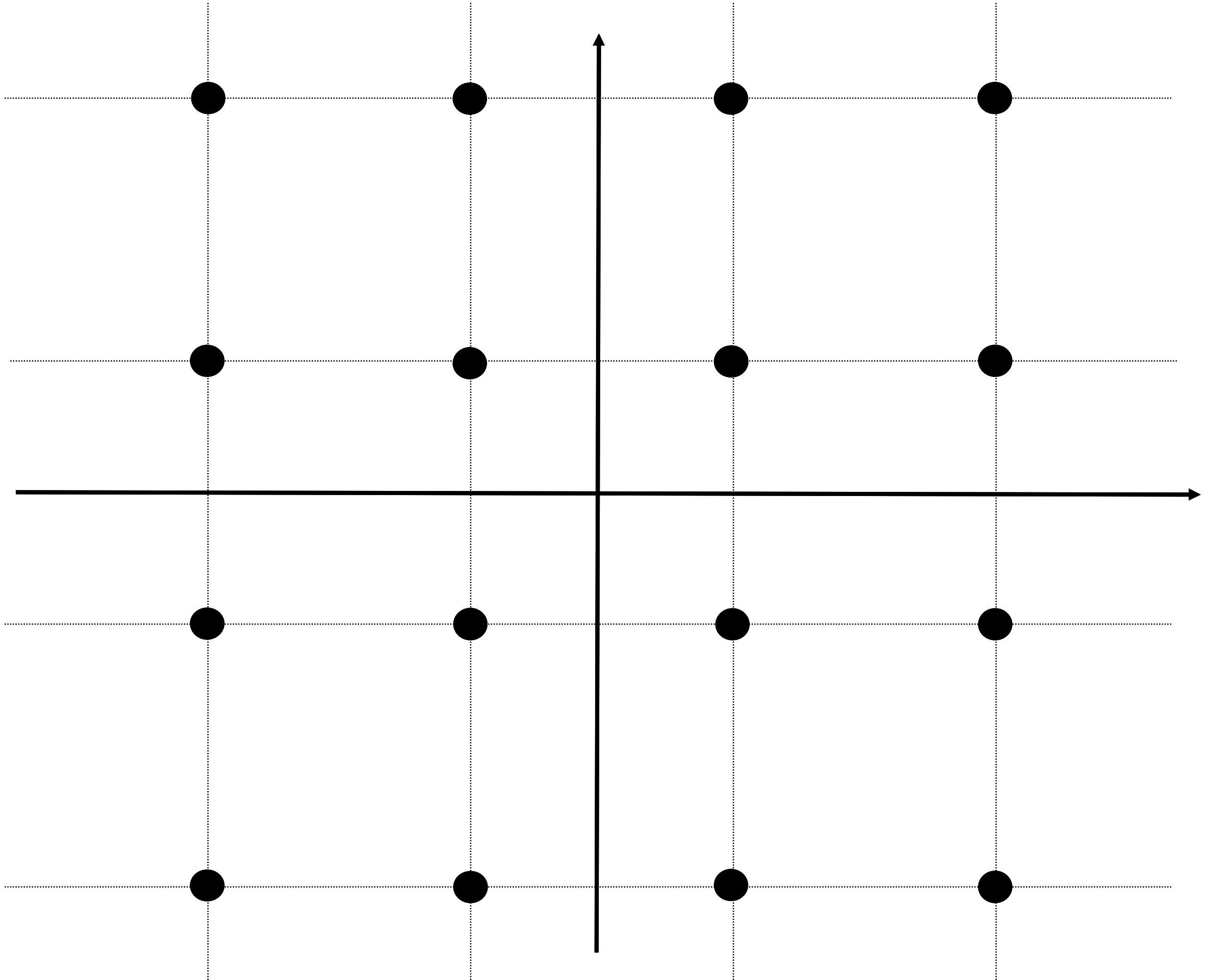
$$\beta_i = A_i \sin\varphi_i$$

- We focus on **grid QAM**

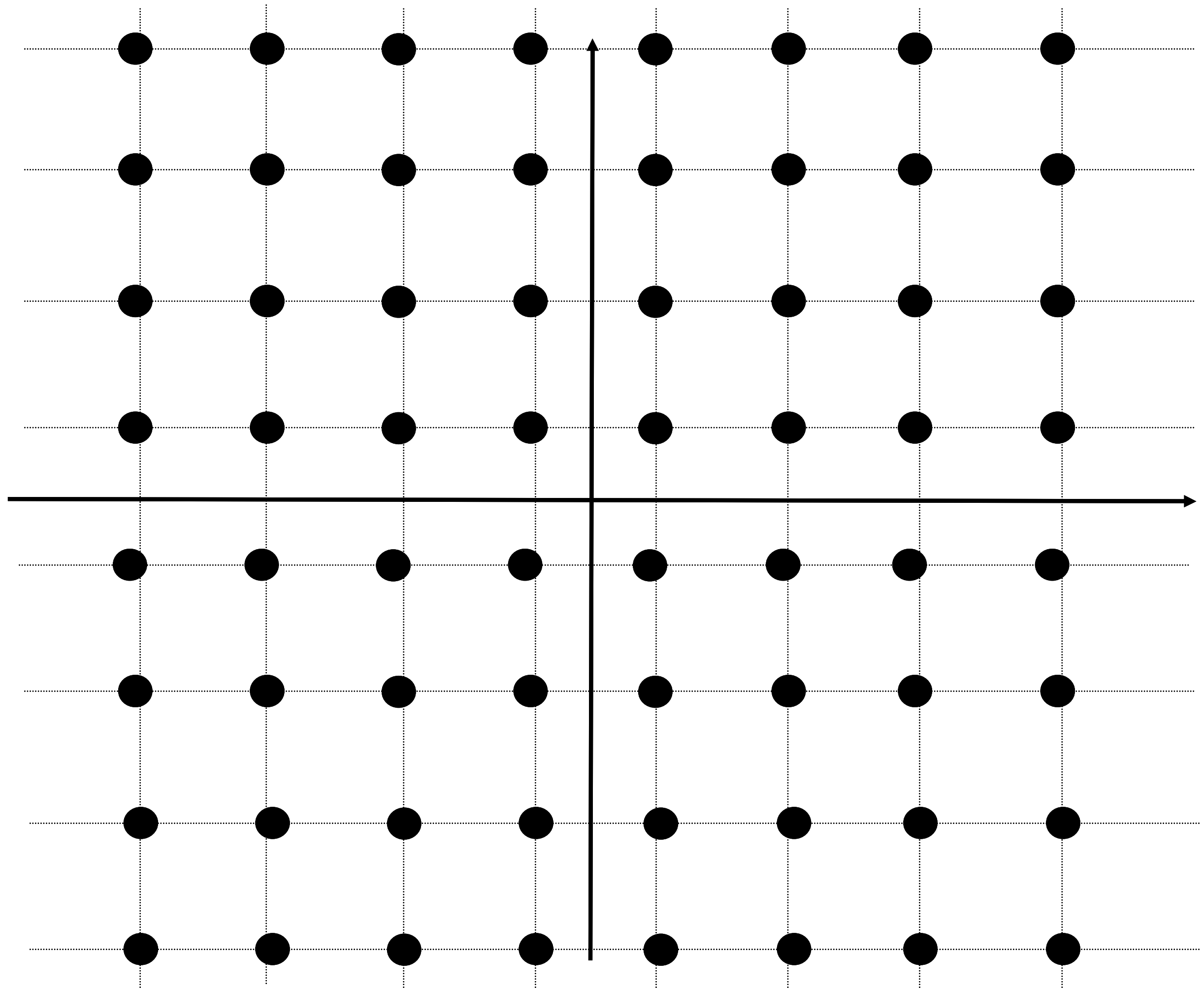
- If m is a square ($m=q^2$)
  - we use square grid QAM with q points on each face
    - [16-QAM = 4 x 4]
    - [64-QAM = 8 x 8]
    - [256-QAM = 16 x 16]

- If m is a not a square ($m \neq q^2$)
- we use a subset of the following square grid QAM
    - [8-QAM $\subseteq$ 16-QAM]
    - [32-QAM $\subseteq$ 64-QAM]
    - [128-QAM $\subseteq$ 256-QAM]

Example: 16-QAM

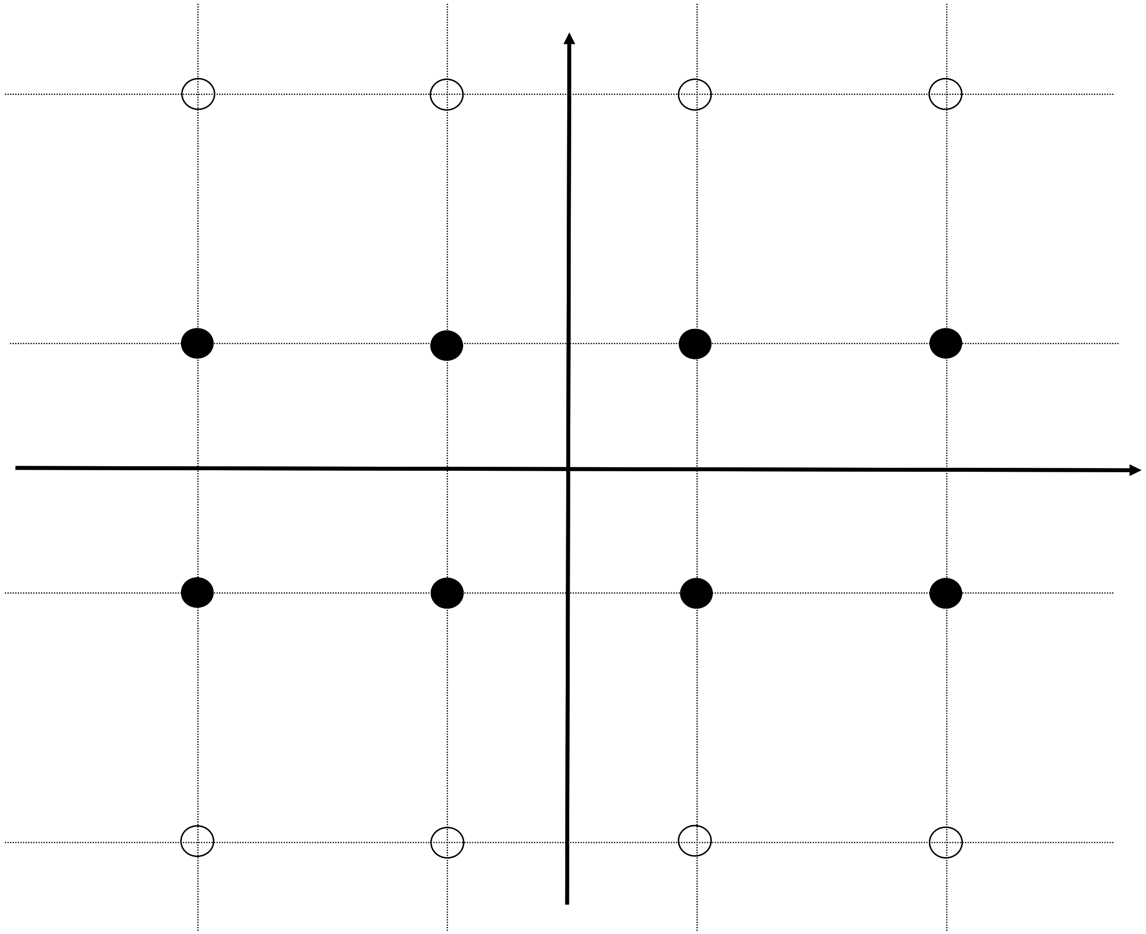


Example: 64-QAM



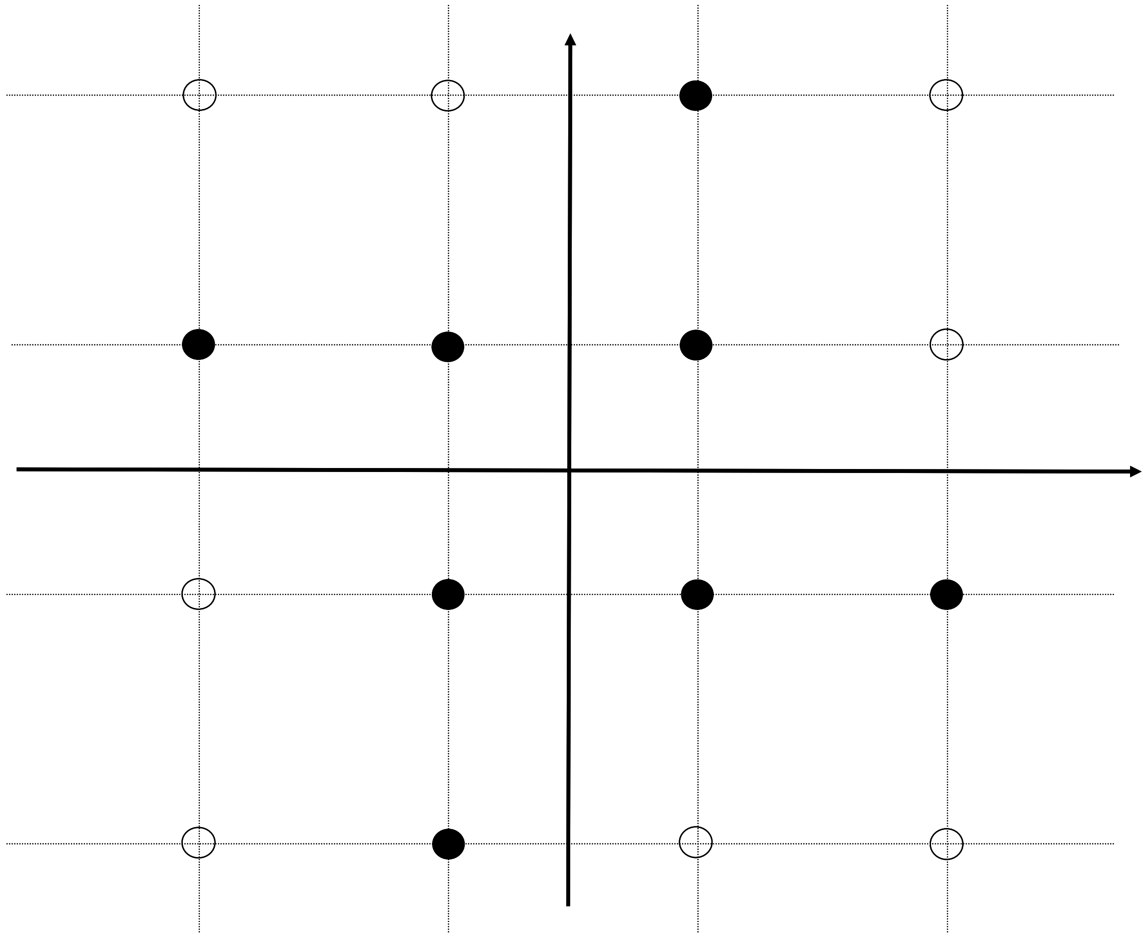
Example 8-QAM

- 8-QAM $\subseteq$ 16-QAM



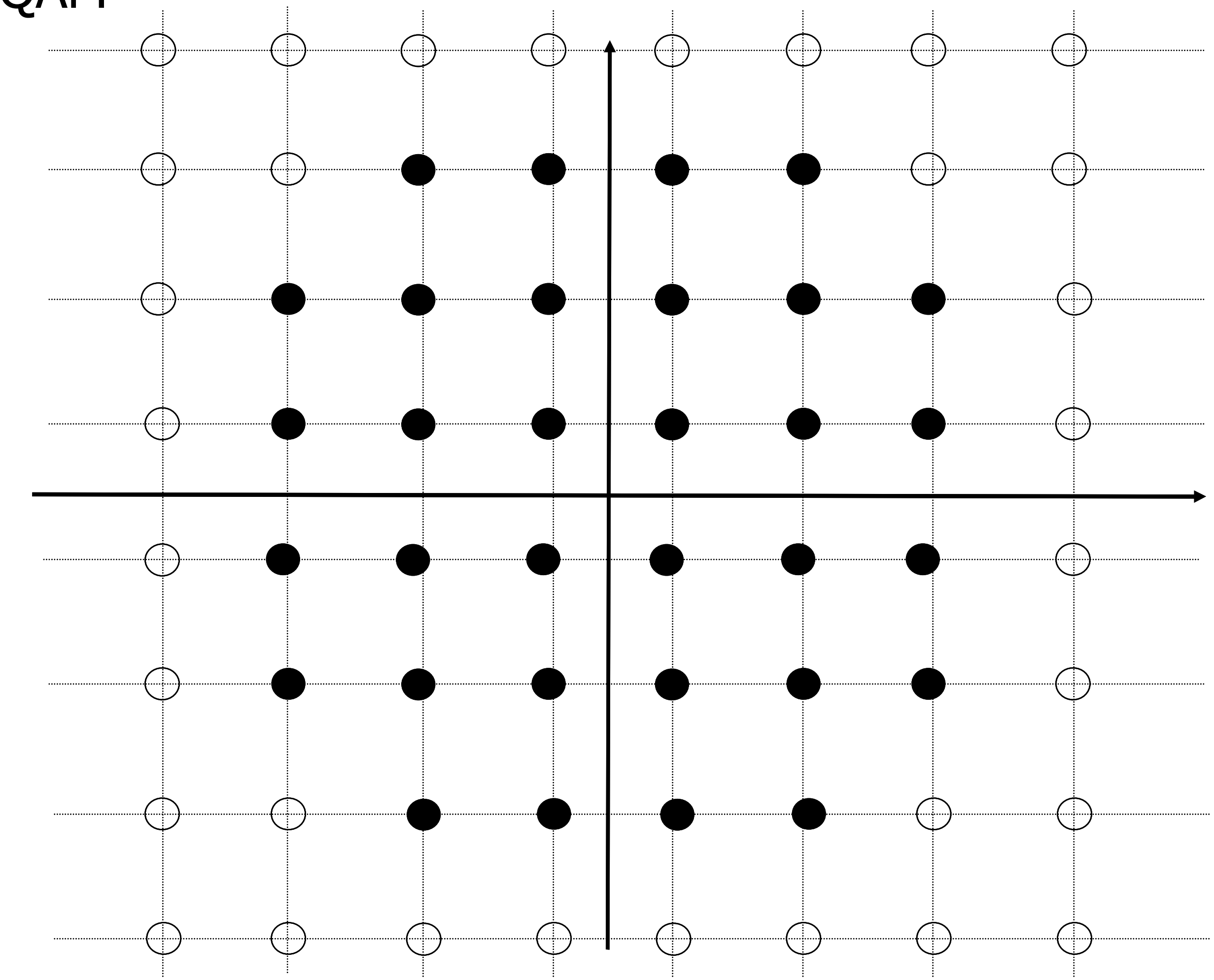
Example 8-QAM

- 8-QAM $\subseteq$ 16-QAM (not unique choice)



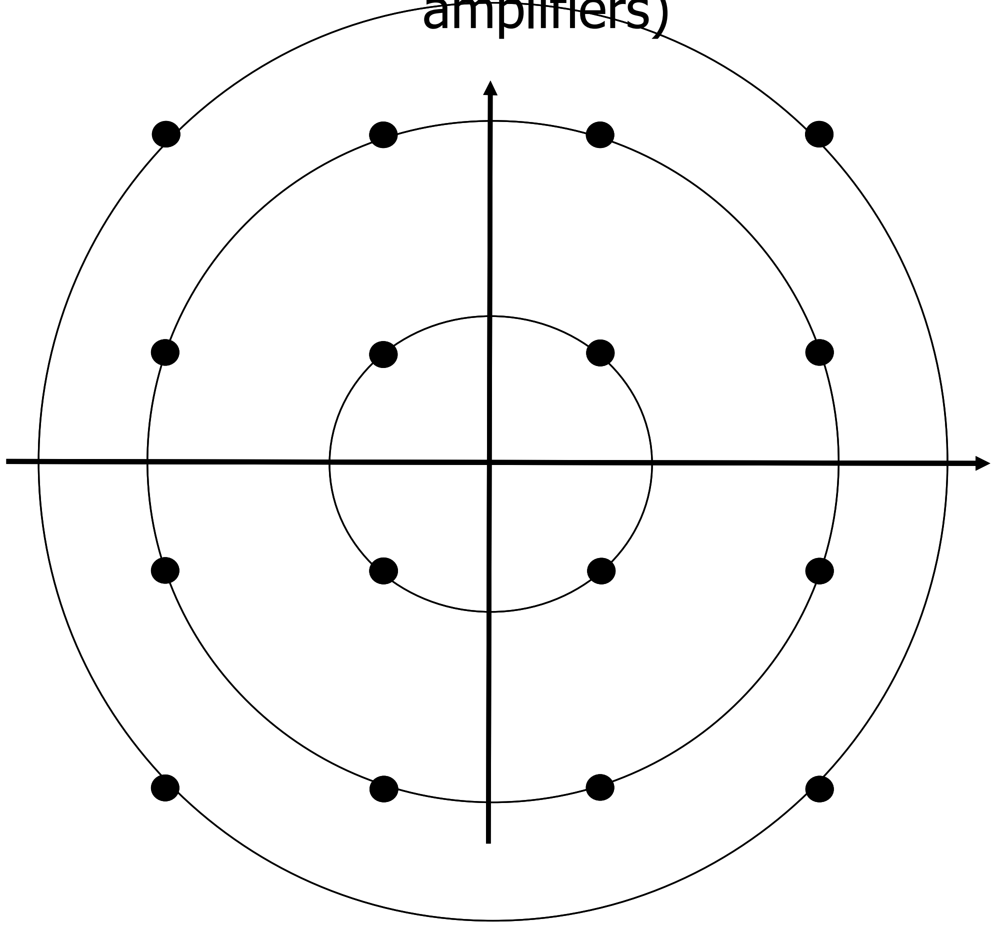
Example 32-QAM

- 32-QAM $\subseteq$ 64-QAM



***m-QAM**: constellation*

- The constellation envelope is not constant (problems with power amplifiers)

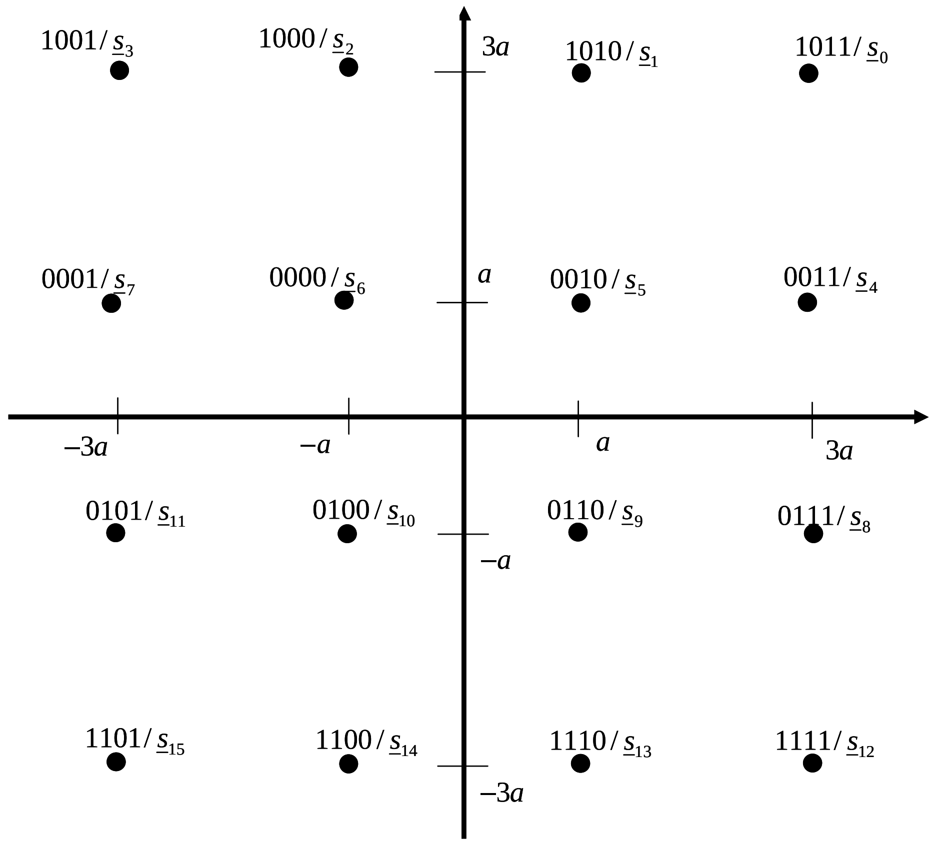


***m-QAM**: binary labeling*

❑

$e : H_k \leftrightarrow M$

❑For grid QAM it is always possible to build Gray labelings



m-QAM: transmitted waveform

$k = \log_2 m$ $\boxed{T = kT_b}$ $\boxed{R = \frac{R_b}{k}}$

- **Each symbol has duration T**
- **Each symbol component (α and β) lasts for T second**

- Transmitted waveform

$$s(t) = \underbrace{\left[\sum_n \alpha[n]p(t-nT)\right]}_{i(t)}\cos(2\pi f_0 t) + \underbrace{\left[\sum_n \beta[n]p(t-nT)\right]}_{q(t)}\sin(2\pi f_0 t)$$

- I component (in phase)
- Q component (in quadrature)

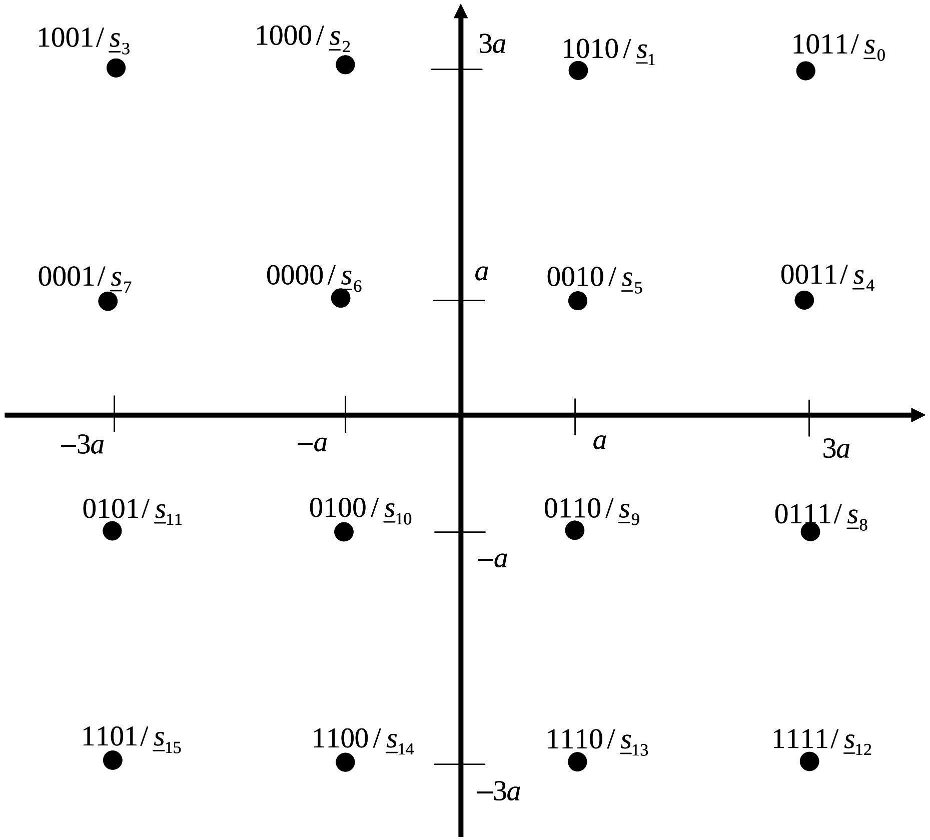
Example: 16-QAM transmitted waveform

- 16- QAM

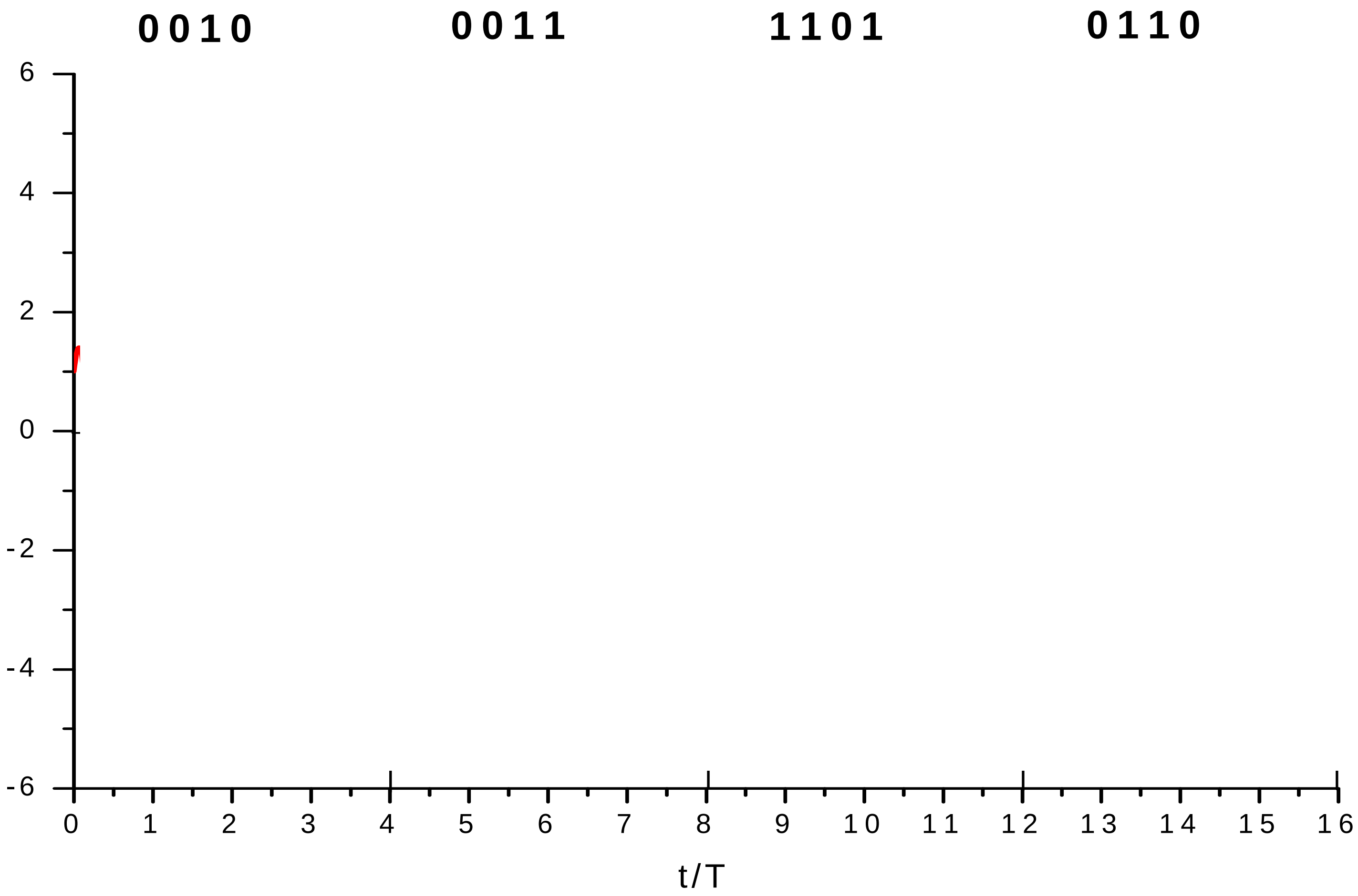
$$p(t) = \frac{1}{\sqrt{T}} P_T(t)$$

$$f_0 = 2R_b$$

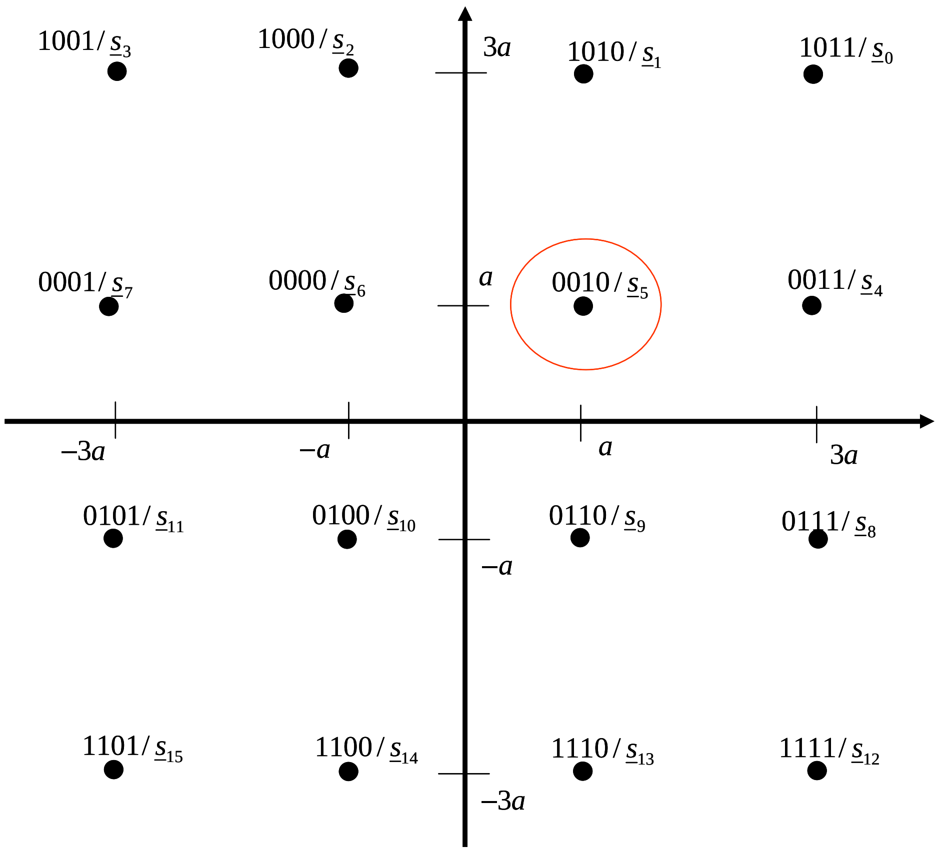
$$\underline{u}_T = 0010001111010110$$



Example

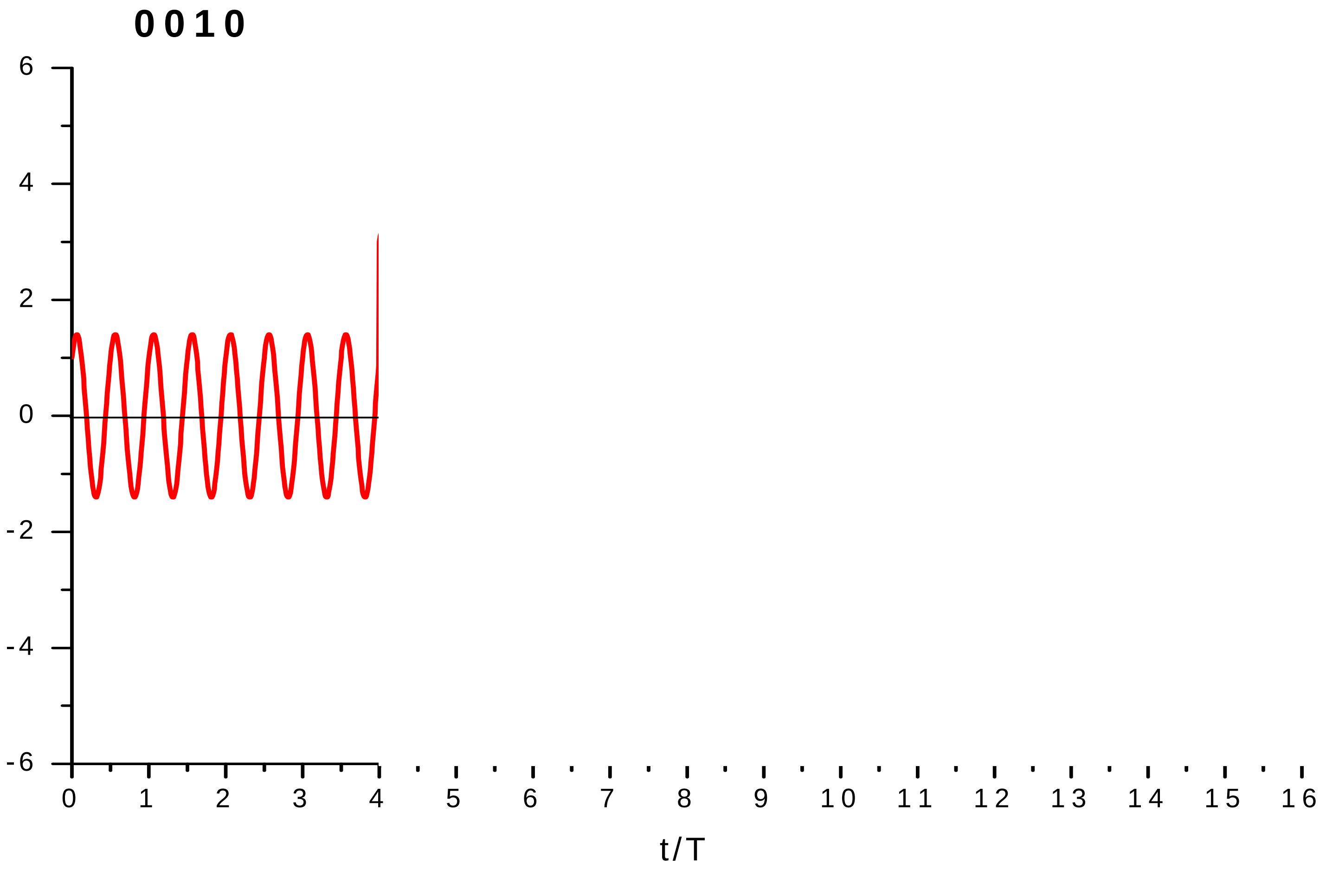


Example

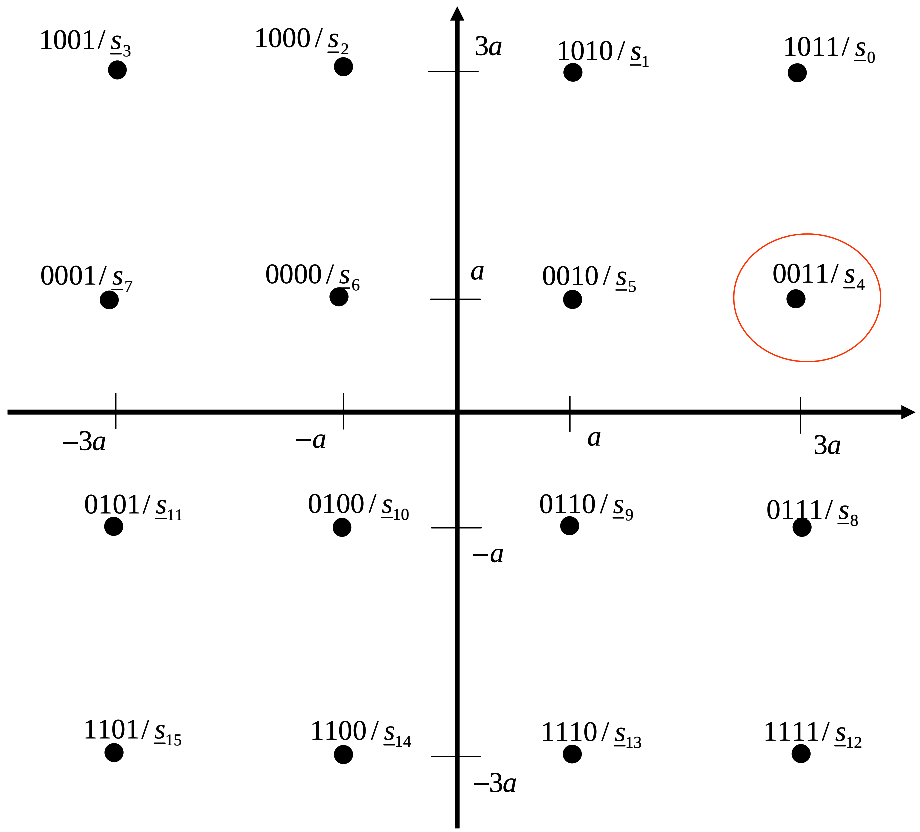


$$\underline{v}_T[0] = 0010 \longrightarrow s_T[0] = a\cos(2\pi f_0 t) + a\sin(2\pi f_0 t) = \sqrt{2}a\cos\left(2\pi f_0 t - \frac{\pi}{4}\right)$$

Example

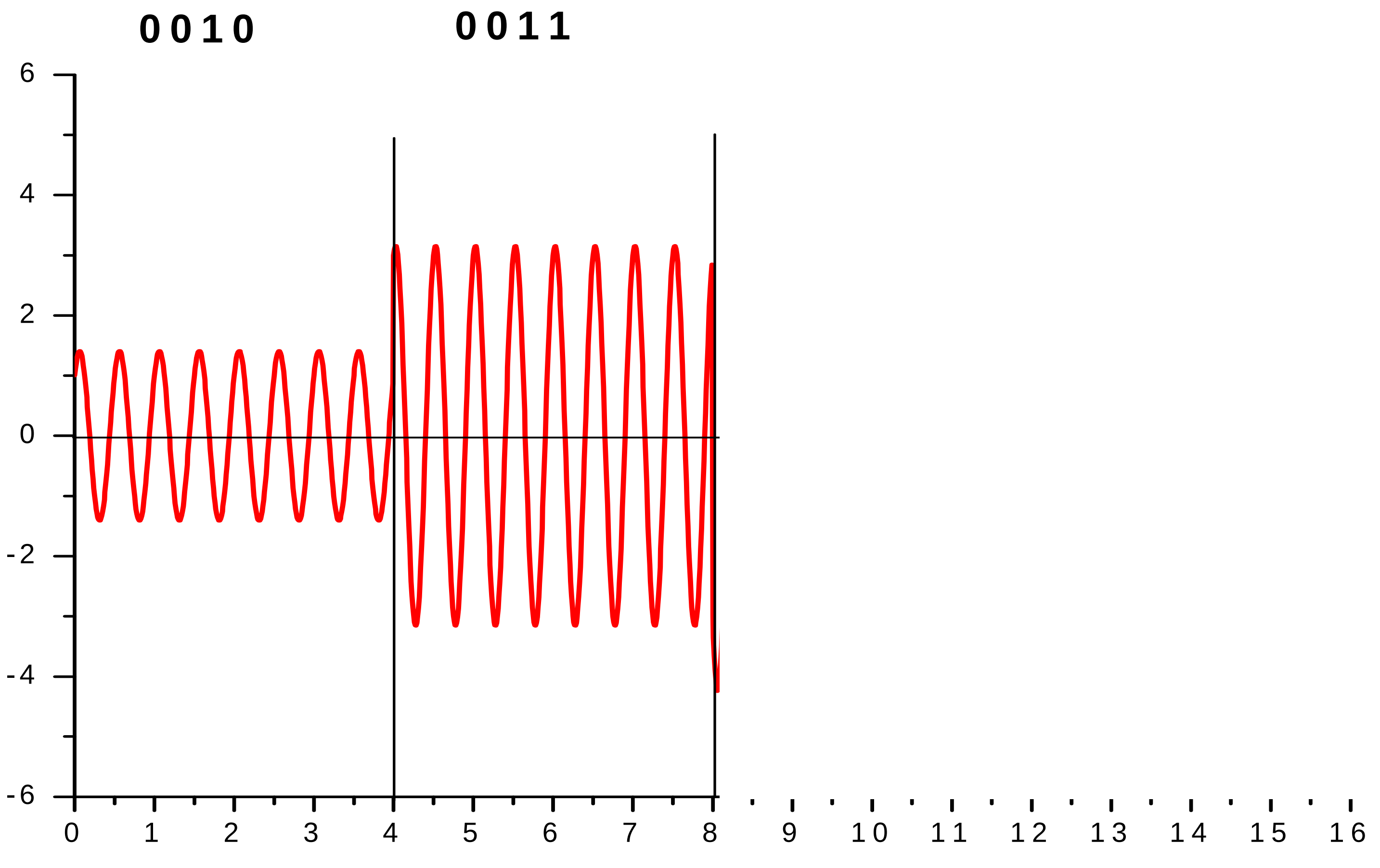


Example

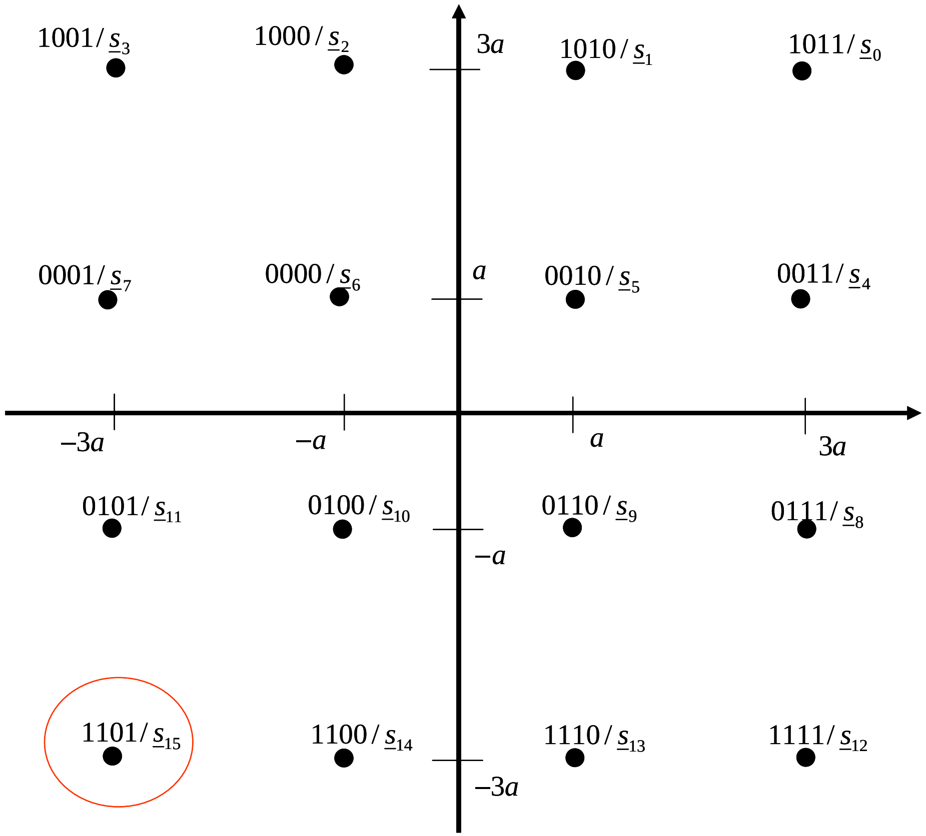


$$\underline{v}_T[0] = 0011 \longrightarrow s_T[0] = 3a\cos(2\pi f_0 t) + a\sin(2\pi f_0 t) = \sqrt{10}a\cos\left(2\pi f_0 t - \frac{\pi}{10}\right)$$

Example

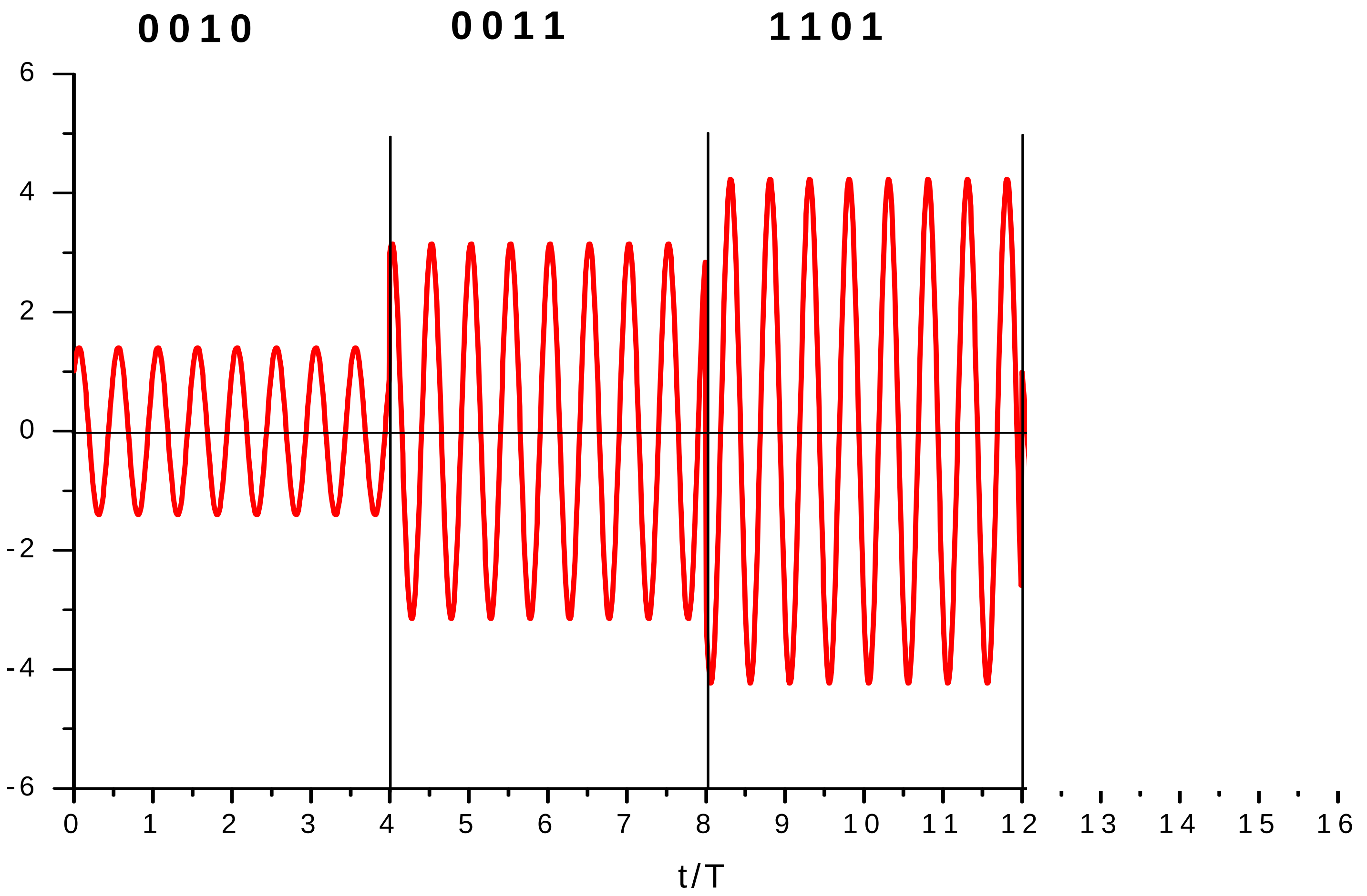


Example

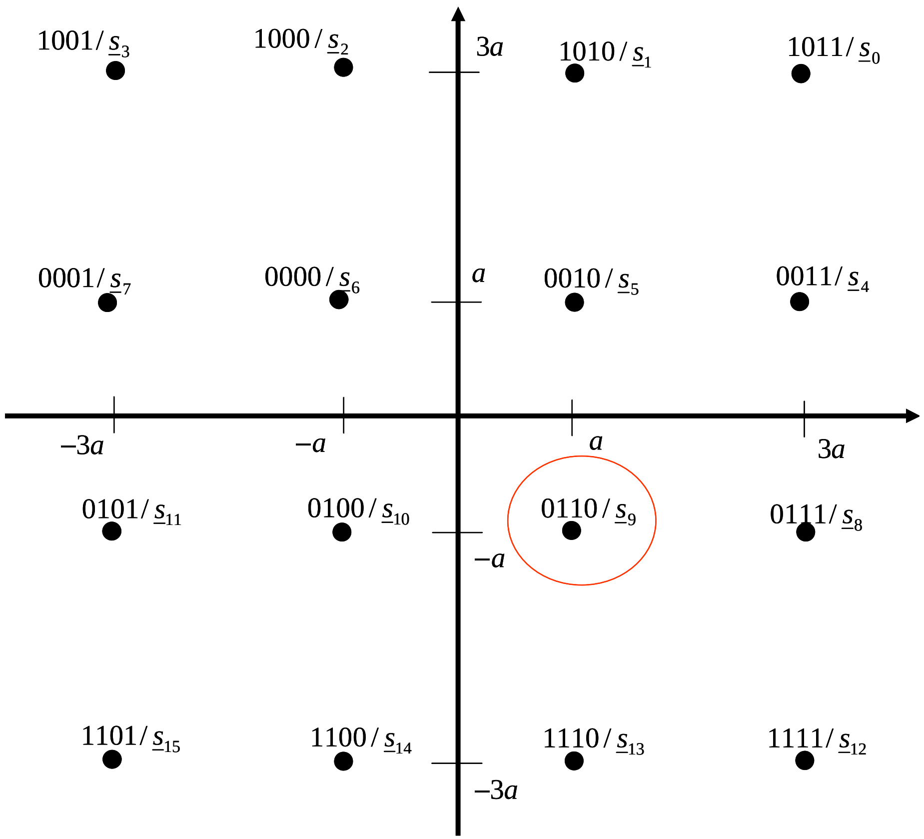


$$\underline{v}_T[0] = 1101 \longrightarrow s_T[0] = -3a\cos(2\pi f_0 t) - 3a\sin(2\pi f_0 t) = 3\sqrt{2}a\cos\left(2\pi f_0 t + 3\frac{\pi}{4}\right)$$

Example

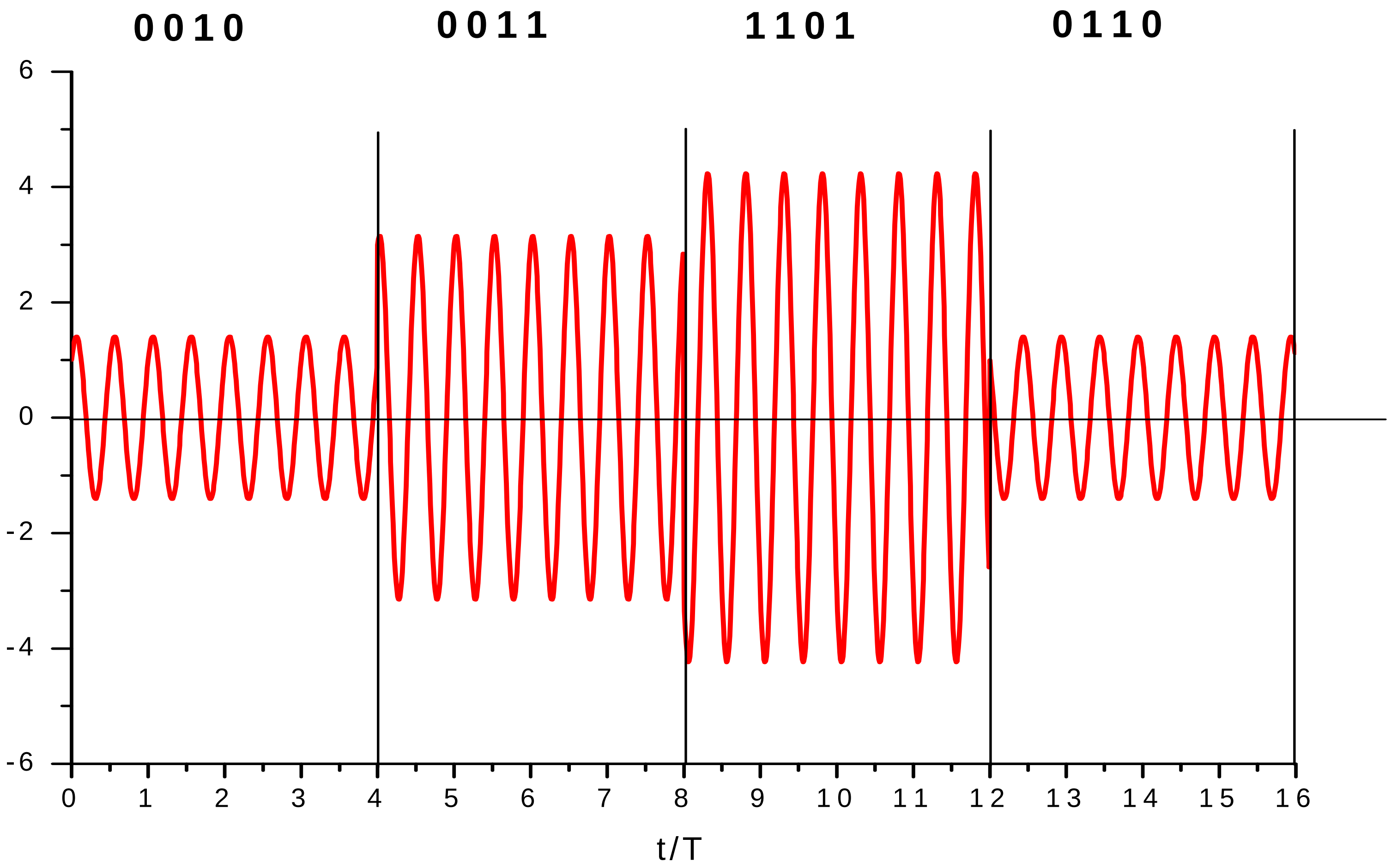


Example



$$\underline{v}_T[0] = 0110 \longrightarrow s_T[0] = a\cos(2\pi f_0 t) + -a\sin(2\pi f_0 t) = \sqrt{2}a\cos\left(2\pi f_0 t + \frac{\pi}{4}\right)$$

Example



m-QAM: *analytic signal*

$$s(t) = \underbrace{\left[\sum_n \alpha[n]p(t-nT)\right]}_{i(t)}\cos(2\pi f_0 t) + \underbrace{\left[\sum_n \beta[n]p(t-nT)\right]}_{q(t)}\sin(2\pi f_0 t)$$

$$s(t) = \mathrm{Re}\left[\dot{s}(t)\right] = \mathrm{Re}\left[\tilde{s}(t)e^{j2\pi f_0 t}\right]$$

$$\tilde{s}(t) = i(t) - jq(t) = \sum_n \gamma[n]p(t-nT) \qquad \gamma[n] = \alpha[n] - j\beta[n]$$

***m-QAM**: bandwidth and spectral efficiency*

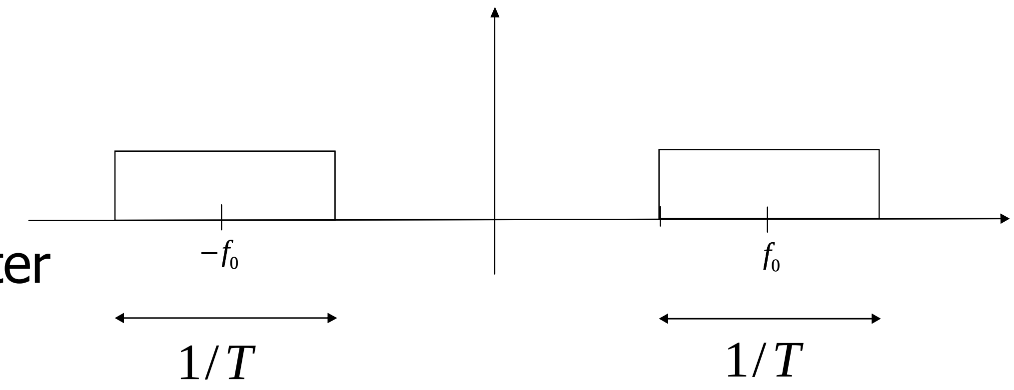
- Transmitted waveform

$$s(t) = \left[\sum_n \alpha[n] p(t - nT) \right] \cos(2\pi f_0 t) + \left[\sum_n \beta[n] p(t - nT) \right] \sin(2\pi f_0 t)$$

$$G_s(f) = z\left[\left|P(f - f_0)\right|^2 + \left|P(f + f_0)\right|^2 \right] \qquad z \in R$$

- Each symbol $\alpha[n]$ and $\beta[n]$ has time duration $T = kT_b$

m-QAM: *bandwidth and spectral efficiency*



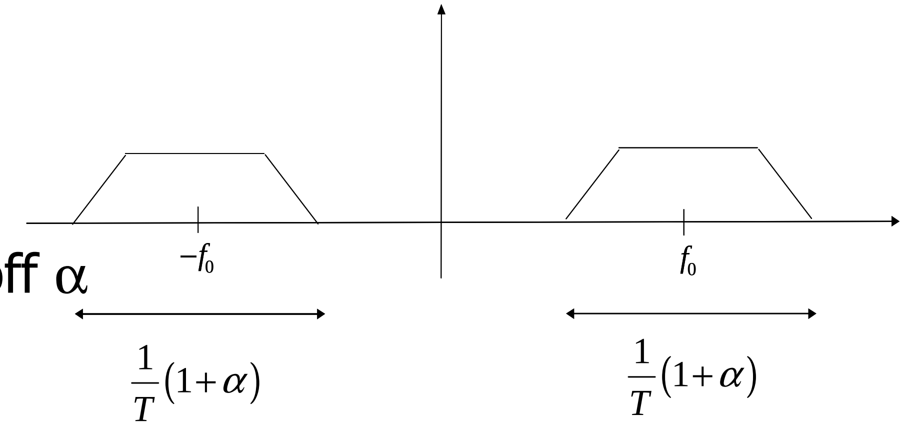
- Case 1: $p(t)$ = ideal low pass filter

$$B_{id} = R = \frac{R_b}{k}$$

- Total bandwidth
- (ideal case)

$$\eta_{id} = \frac{R_b}{B_{id}} = k \ bps/Hz$$

***m-QAM**: bandwidth and spectral efficiency*



- Case 2: $p(t)$ = RRC filter with roll off α

$$B = R(1+\alpha) = \frac{R_b}{k}(1+\alpha)$$

- Total bandwidth

$$\boxed{\eta = \frac{R_b}{B} = \frac{k}{(1+\alpha)} \; bps/Hz}$$

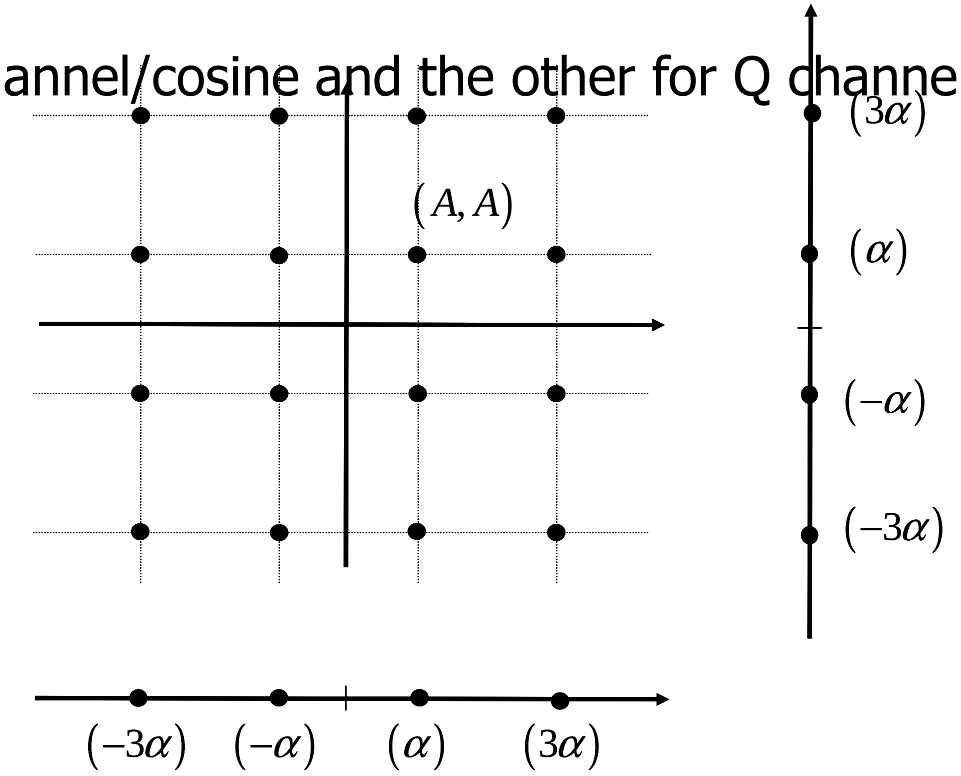
Exercize

- Given a bandpass channel with bandwidth B = 4000 Hz, centred around f_0=2 GHz, compute the maximum bit rate R_b we can transmit over it with an 16-QAM constellation or a 64-QAM constellation in the two cases:

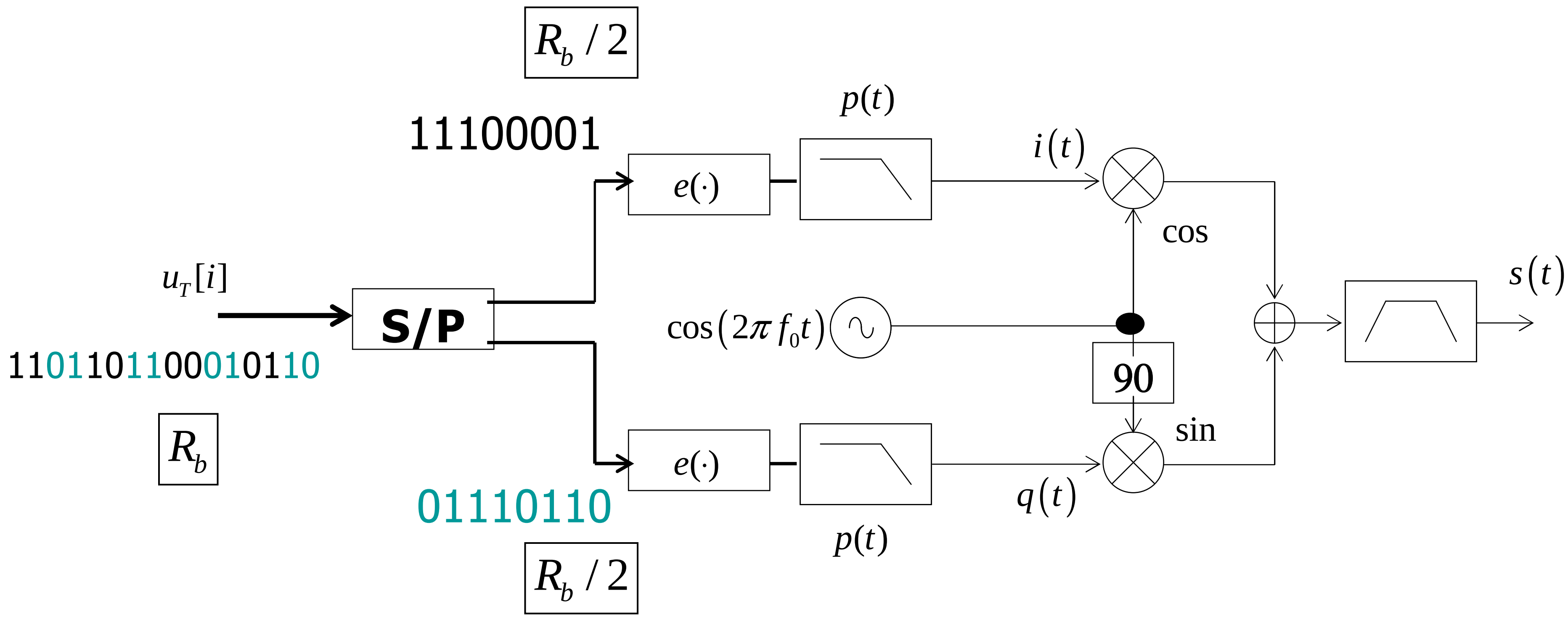
- Ideal low pass filter
- RRC filter with α=0.25

m-QAM: interpretation

- Square grid QAM constellations ($m=q^2$):
- it can be view as the Cartesian product of two independent q-ASK constellations
- (one for I channel/cosine and the other for Q channel/sine)

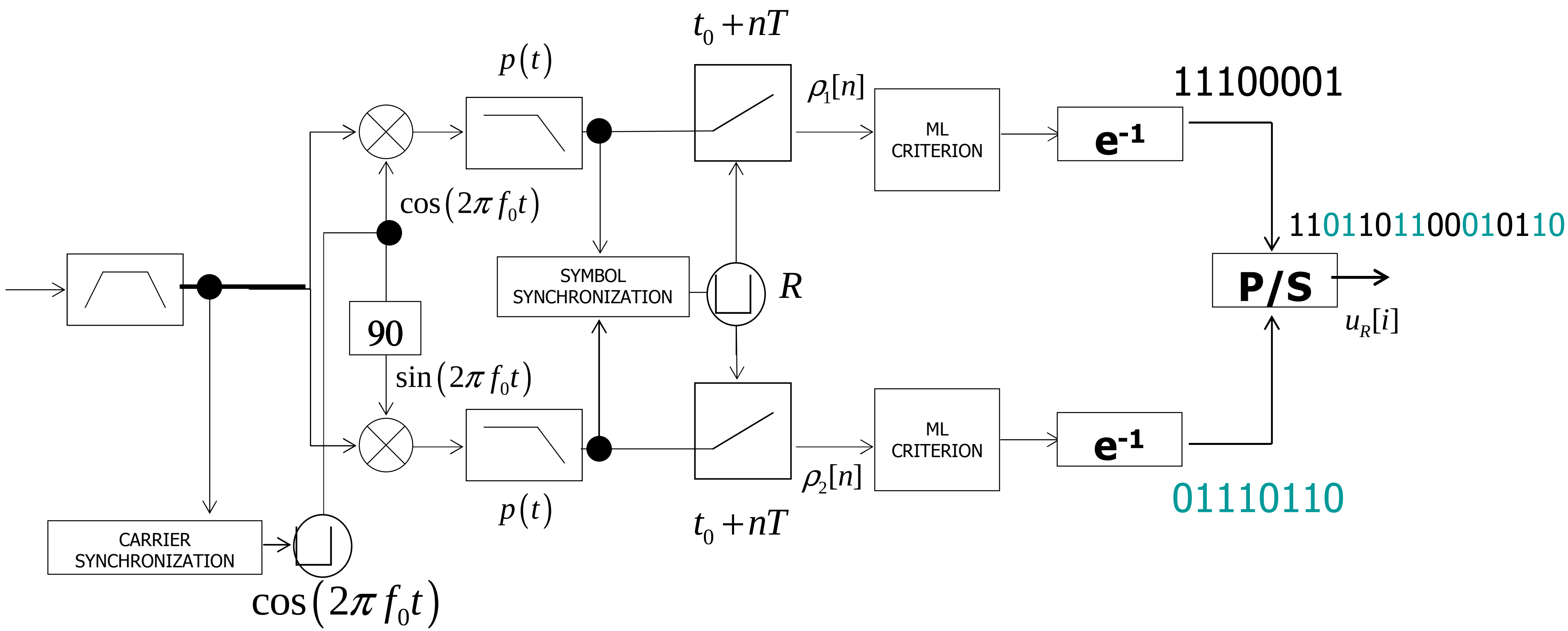


m-QAM: *modulator for* ***square*** *constellations*



- Example: 16-QAM = 4-ASK x 4-ASK
- One symbol = 4 bits, two on I channel and two on Q channel

m-QAM: *demodulator for* ***square*** *constellations*

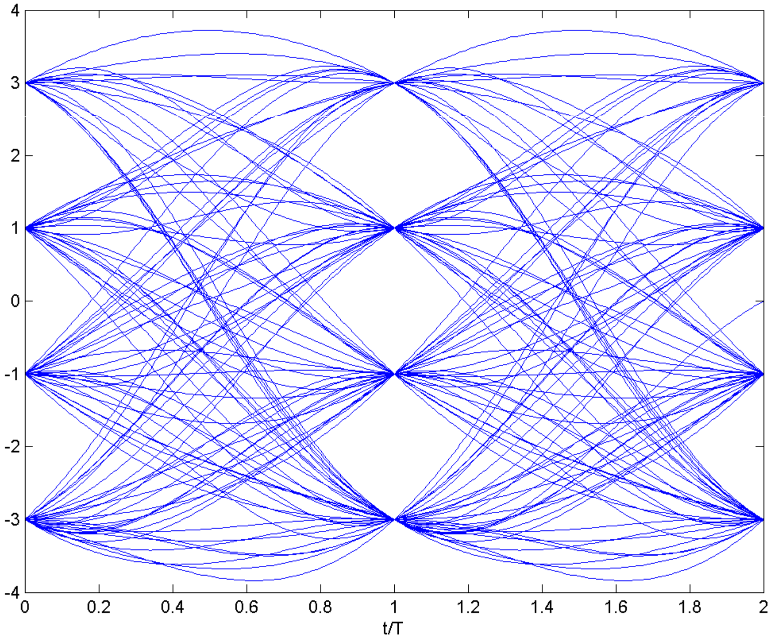


- Decision is taken separately on I channel (based on $\rho_1[n]$)
- and on Q channel (based on $\rho_2[n]$)

***m-QAM**: eye diagram*

- 16-QAM constellation with RRC filter (α=0.5)
  - [α and β components = -3 , -1 , +1 , +3]

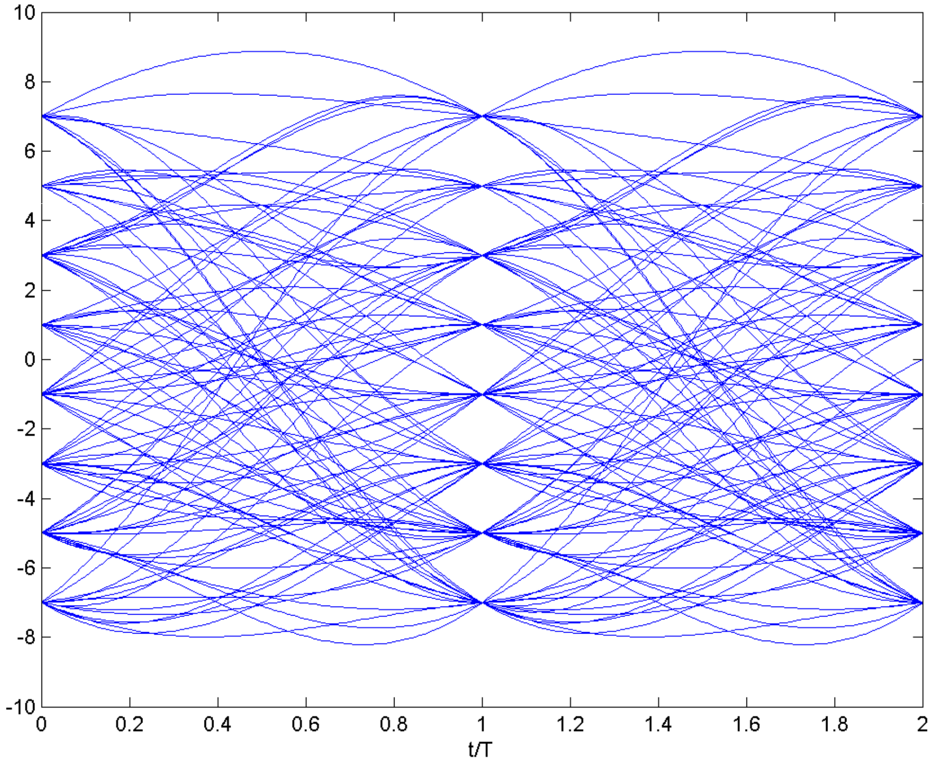
- I and Q channel



m-QAM: *eye diagram*

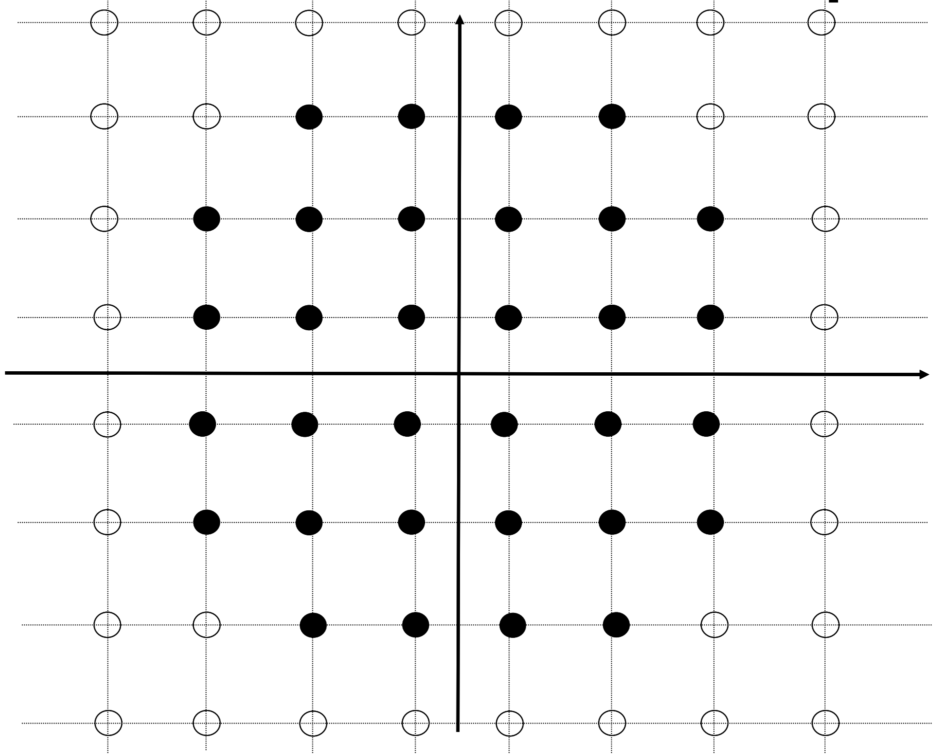
- 64-QAM constellation with RRC filter (α=0.5)
  - [α and β components = +7,+5,+3,+1,-1,-3,-5,-7]

- I and Q channel

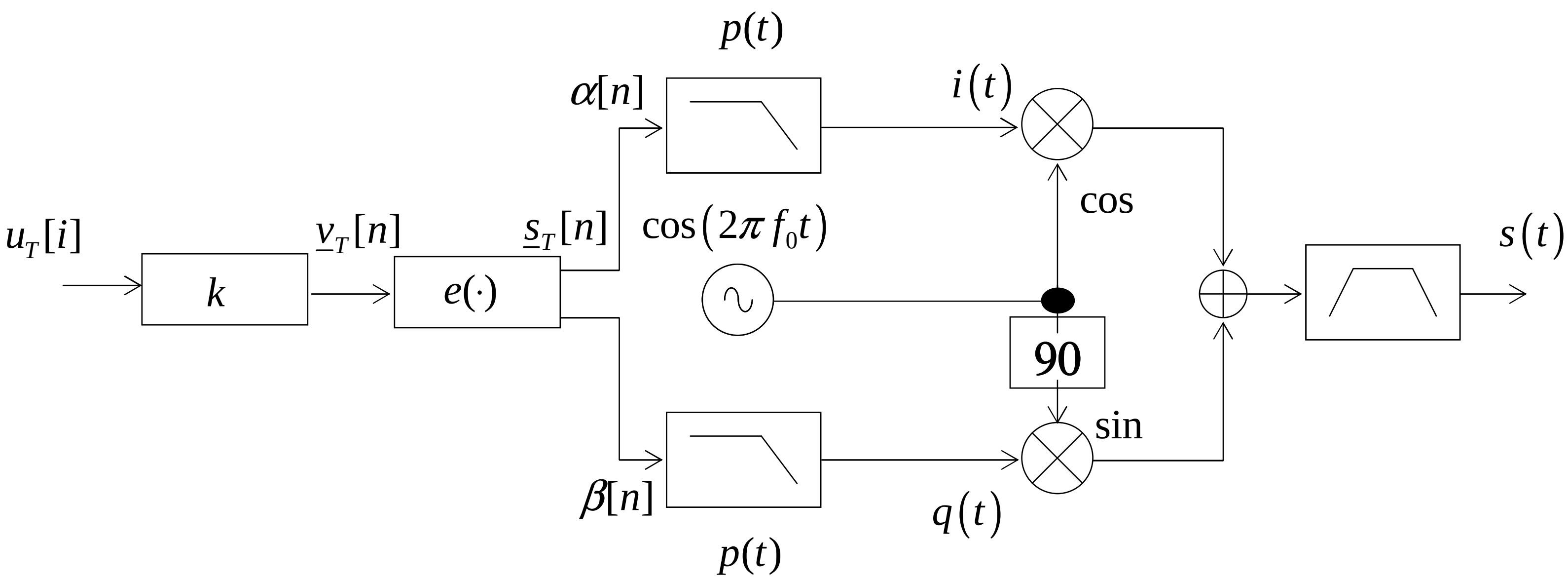


m-QAM: *modulator for non-square constellations*

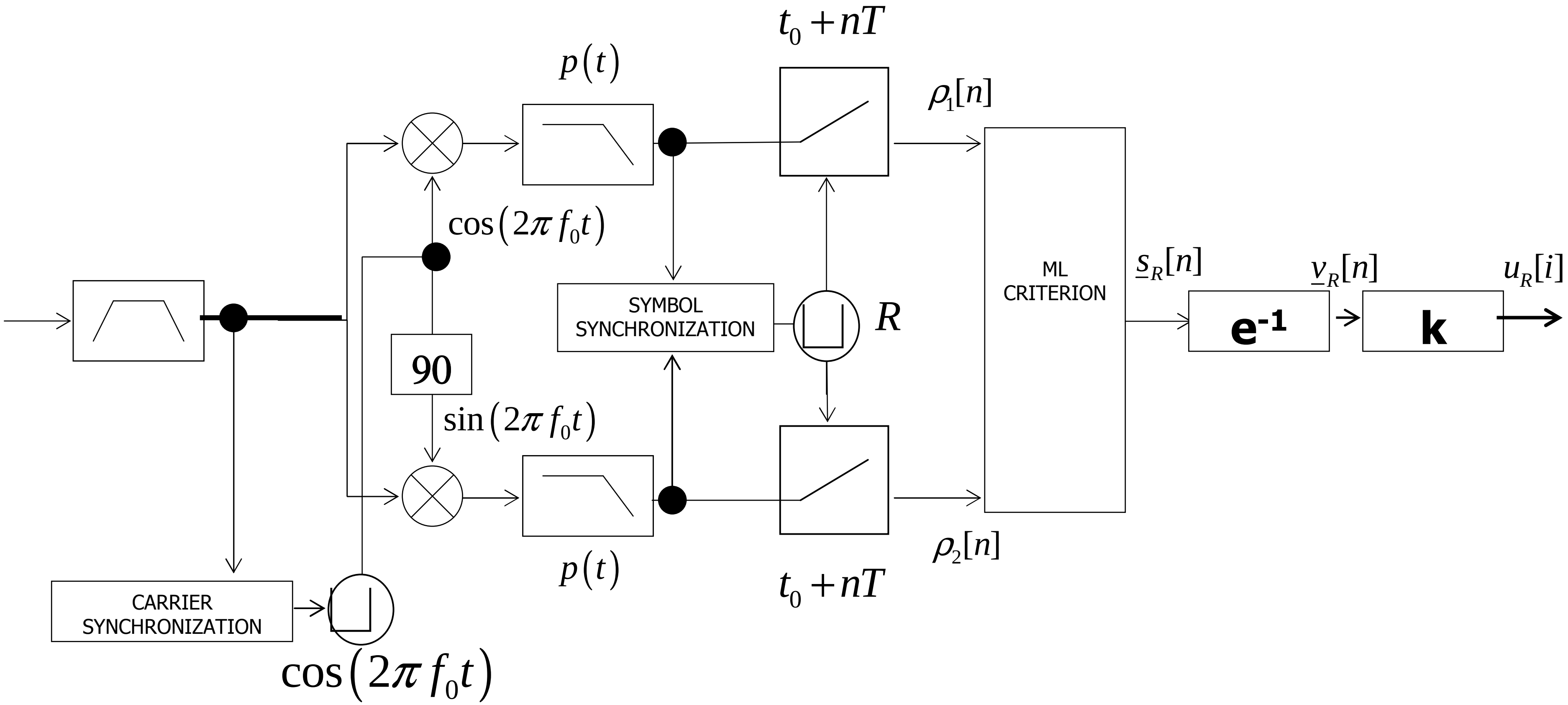
- Non-Square grid QAM constellations ($m \neq q^2$):
- cannot be view as the Cartesian product of two independent q-ASK:
- **modulator and demodulator do not work independently on I and Q channels.**



m-QAM: *modulator for* ***non-square*** *constellations*



m-QAM: *demodulator for* ***non-square*** *constellations*



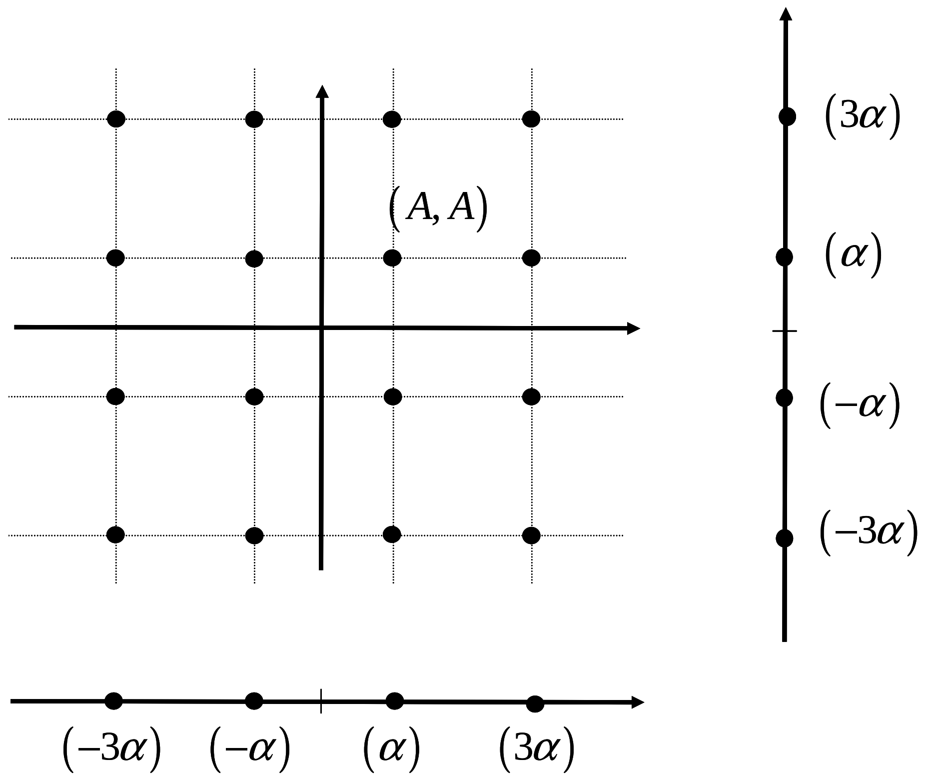
m-QAM: *error probability*

- Square grid $m=q^2$ -QAM

q^2-QAM = Cartesian product of

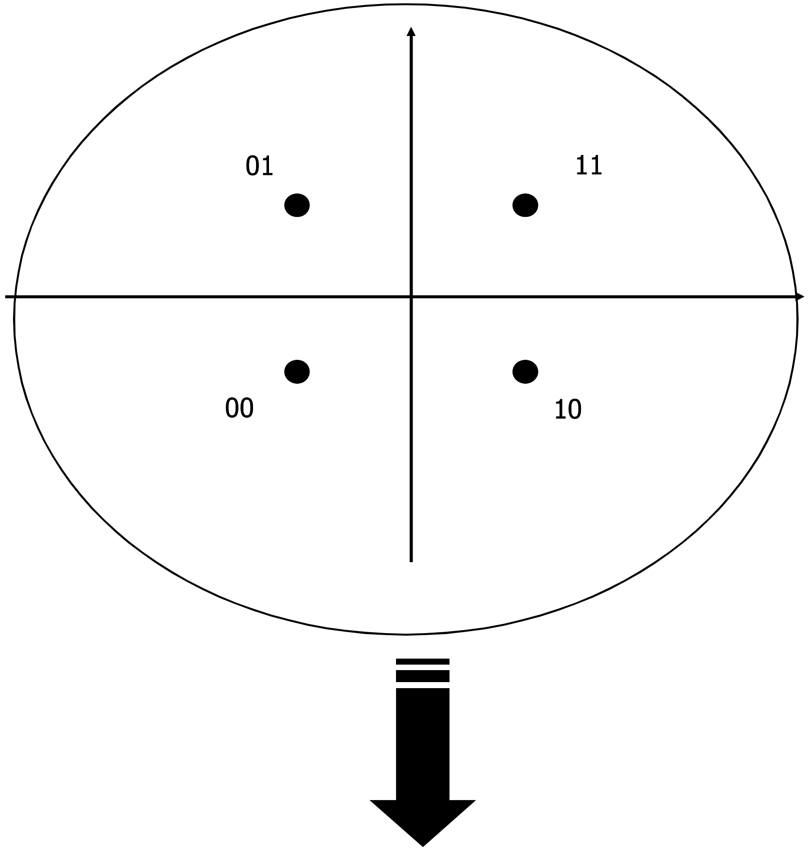
- two independent q-ASK (q-PAM)

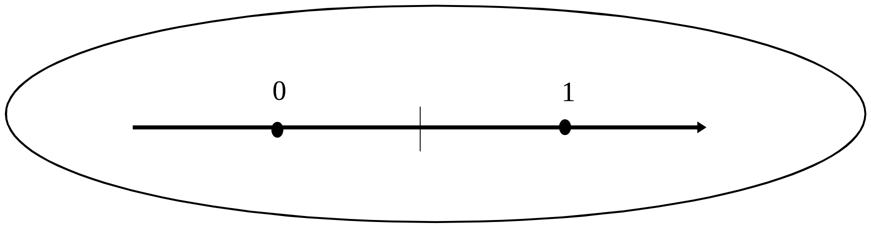
- q^2-QAM has the same error performance of q-PAM



m-QAM: *error probability*

- 4-QAM = 2-PAM

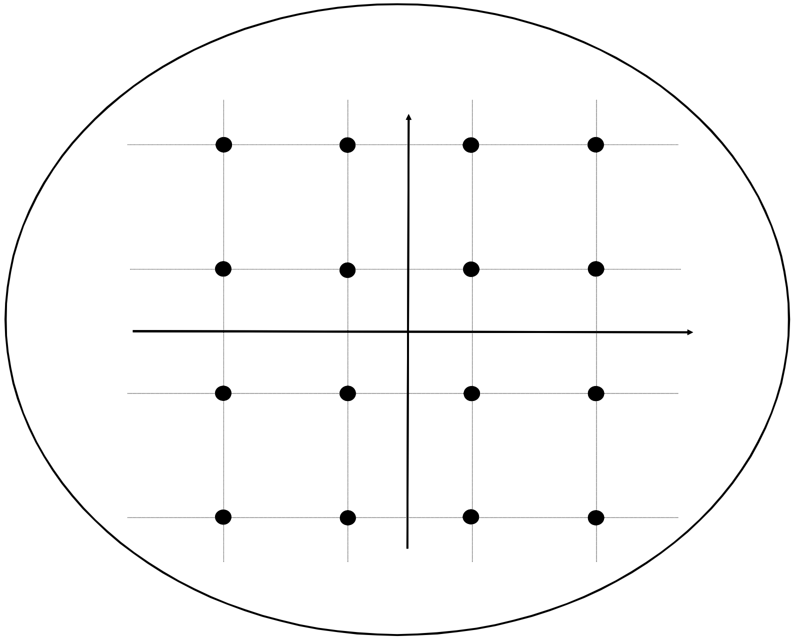


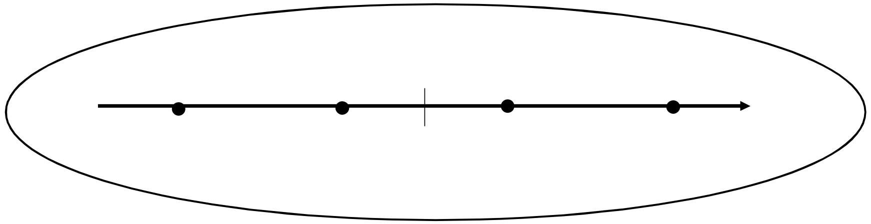


$$P_b(e) = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

m-QAM: *error probability*

- 16-QAM = 4-PAM

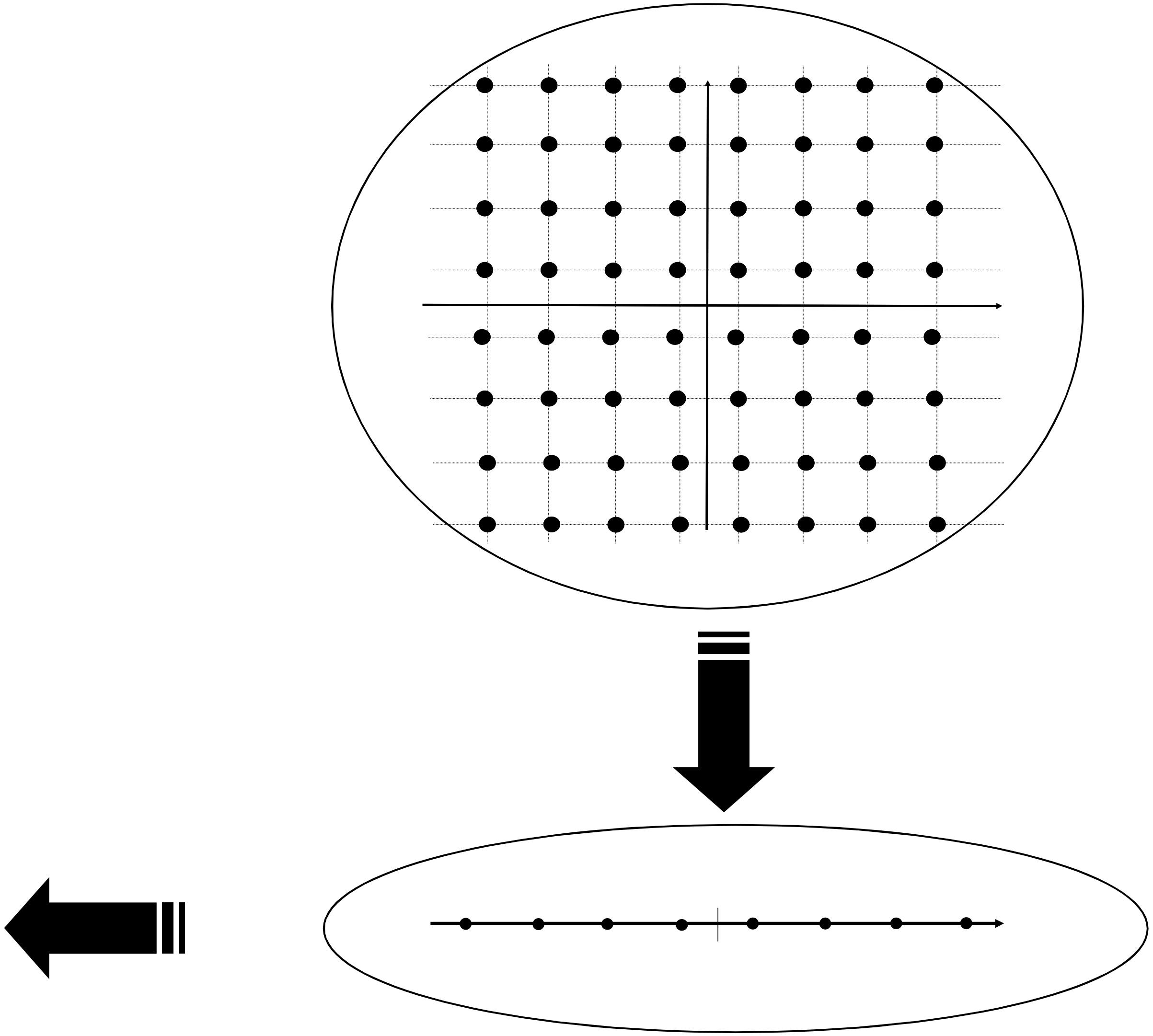




$$P_b(e) \approx \frac{3}{8} erfc\left(\sqrt{\frac{2}{5}\frac{E_b}{N_0}}\right)$$

m-QAM: *error probability*

- 64-QAM = 8-PAM



$$P_b(e) \approx \frac{7}{24}\, erfc\left(\sqrt{\frac{1}{7}\frac{E_b}{N_0}}\right)$$

m-QAM: error probability

- q^2-QAM = q-PAM

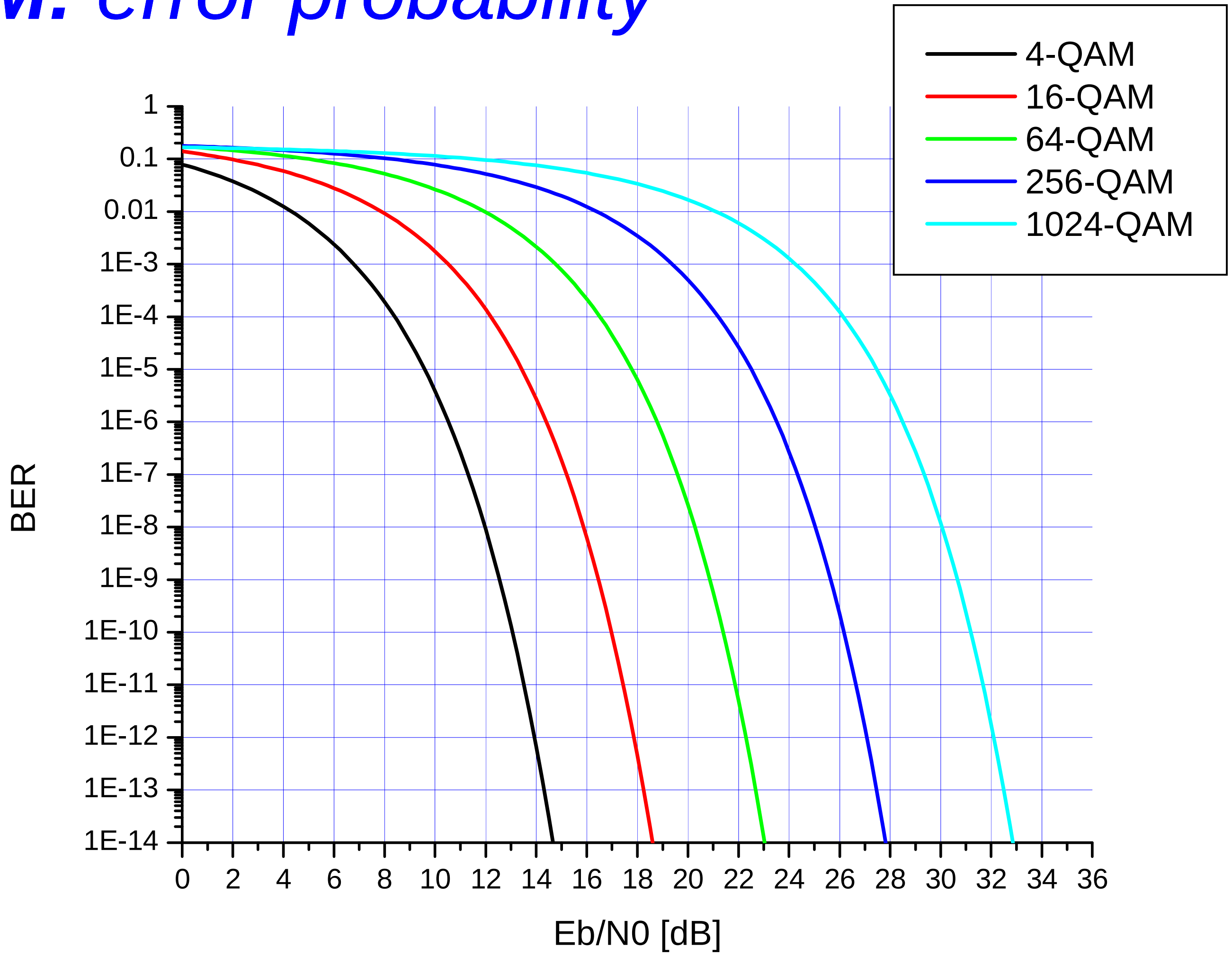
- q^2-QAM

- General expression
- derived for m-PAM constellations

- q-PAM

$$P_b(e) \approx 2\frac{\sqrt{m}-1}{\sqrt{mk}} erfc\left(\sqrt{\frac{3k}{2(m-1)}\frac{E_b}{N_0}}\right)$$

m-QAM: *error probability*



❑**For increasing *m*, the performance decrease**

m-QAM: *error probability (vs. ASK)*

- $m=q^2$-QAM

- **1. COMPARISON QAM / ASK**

- An m-QAM constellation and an m-ASK constellation have:

  - the same spectral efficiency

  - m-QAM has better performance, because equal to q-PAM (remember that, for increasing m, PAM performance decreases)

    - 16-QAM = 4-PAM | 16-ASK=16-PAM
    - 64-QAM = 8-PAM | 64-ASK=64-PAM

m-QAM: *error probability (vs. PSK)*

- $m=q^2$-QAM
- **2. COMPARISON QAM / PSK**

- An m-QAM constellation and an m-PSK constellation have:
  - the same spectral efficiency
  - m-QAM has better performance (better distribution of points on the plane, larger minimum distance)

m-QAM: *error probability (vs. PSK)*

- **COMPARISON QAM / PSK**

$m-\mathrm{PSK}$ $$P_b(e) \approx \frac{1}{k} erfc\left(\sqrt{k \frac{E_b}{N_0} \sin^2\left(\frac{\pi}{m}\right)}\right)$$

$m-\mathrm{QAM}$ $$P_b(e) \approx 2 \frac{\sqrt{m}-1}{\sqrt{mk}} erfc\left(\sqrt{\frac{3k}{2(m-1)} \frac{E_b}{N_0}}\right)$$

- Fixed a (sufficiently large) BER, PSK requires higher E_b/N_0

m-QAM: *error probability (vs. PSK)*

- Fixed a (sufficiently large) BER, PSK requires higher E_b/N_0

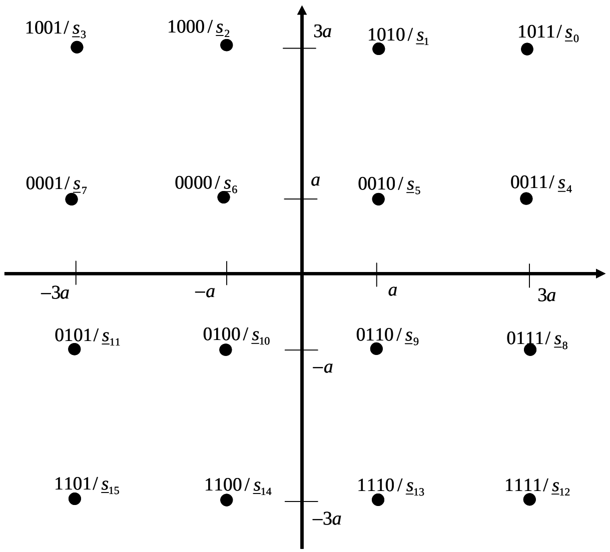
$$\left(\frac{E_b}{N_0}\right)_{\text{PSK}} \approx \left(\frac{E_b}{N_0}\right)_{\text{QAM}} \left(\frac{3}{2(m-1)\sin^2(\pi/m)}\right)$$

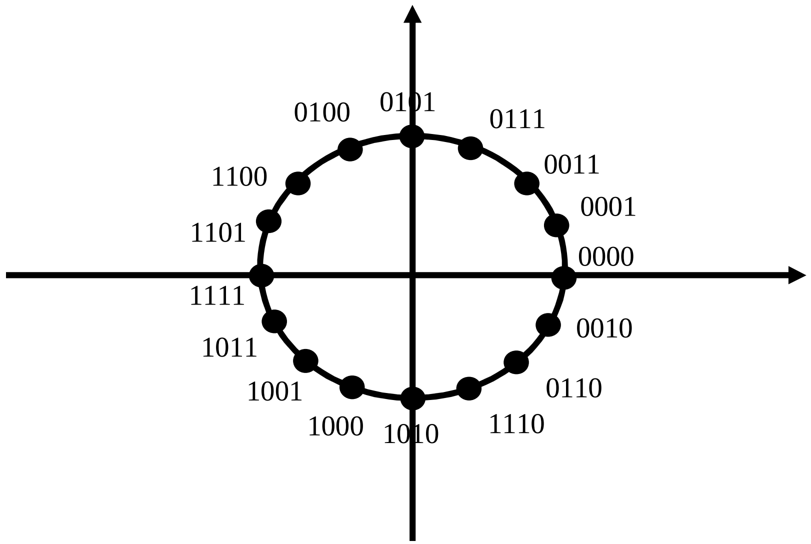
$$m=16 \quad \text{difference} = 4.20\ \text{dB}$$

$$m=64 \quad \text{difference} = 9.96\ \text{dB}$$

m-QAM: *error probability (vs. PSK)*

- Comparison 16-QAM vs. 16-PSK



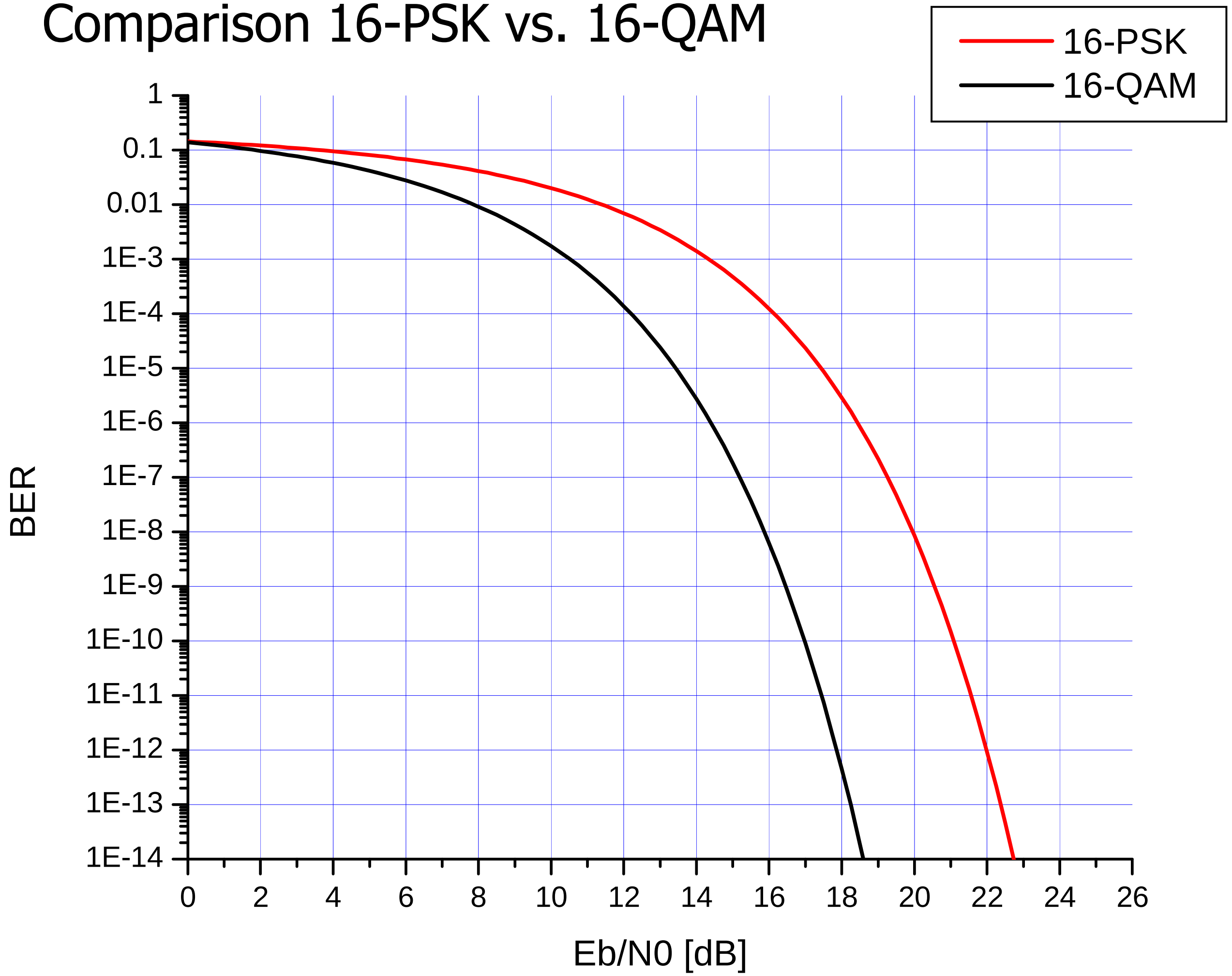


$$P_b(e) \approx \frac{3}{8} erfc\left(\sqrt{\frac{2}{5}\frac{E_b}{N_0}}\right) \qquad P_b(e) \approx \frac{1}{4} erfc\left(\sqrt{0.152\frac{E_b}{N_0}}\right)$$

$$\text{difference} = 4.20\ \text{dB}$$

m-QAM: *error probability (vs. PSK)*

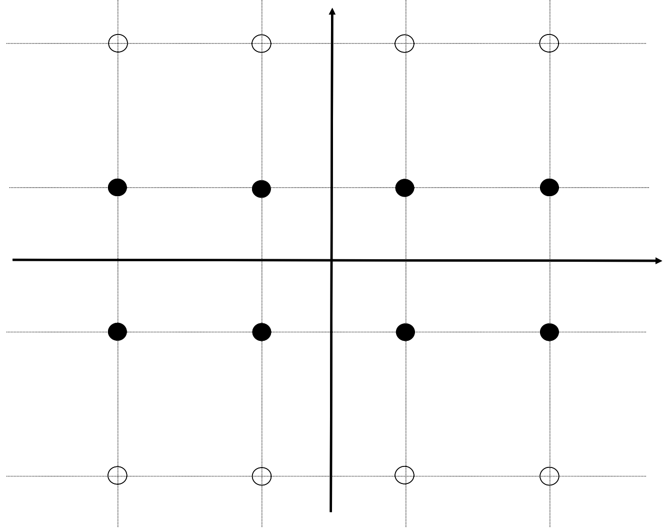
- Comparison 16-PSK vs. 16-QAM

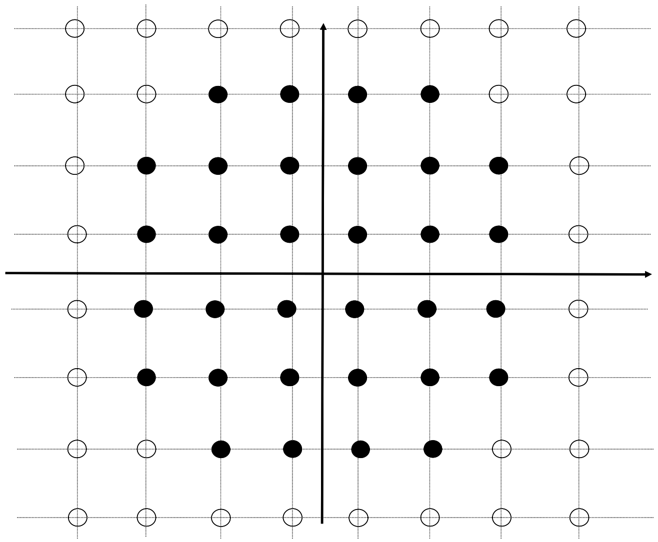


difference = 4.20 dB

m-QAM: *error probability*

- Non-square grid $m=q^2$ –QAM (8-QAM, 32-QAM, 128-QAM, 512-QAM,...)





Not yet a Cartesian product of

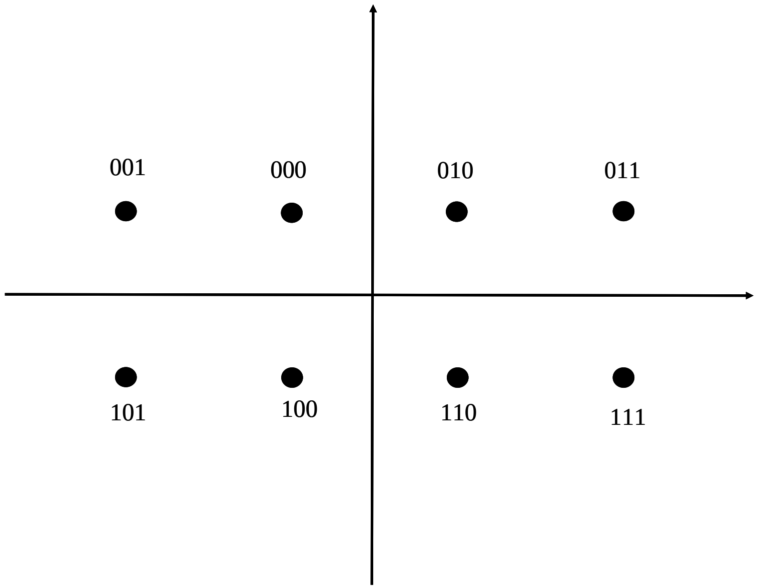
- two independent ASK

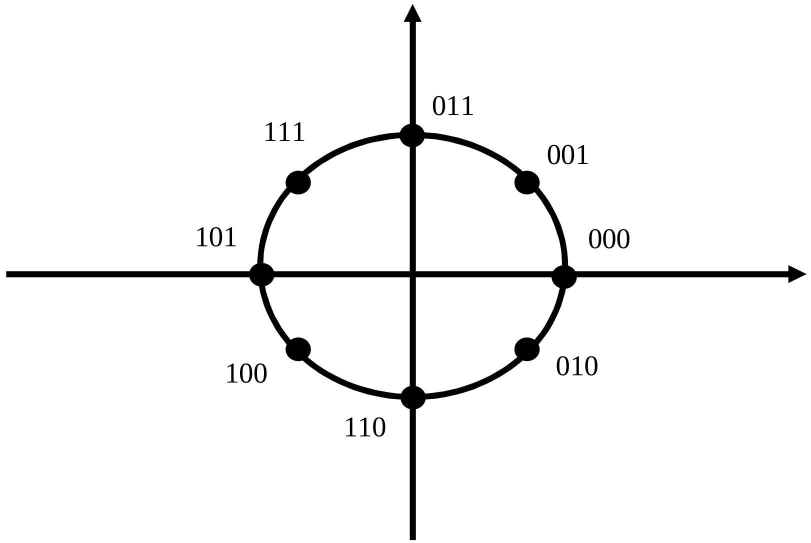
⇒

- The performance must be computed case by case

m-QAM: *error probability (vs. PSK)*

- Comparison 8-QAM vs. 8-PSK



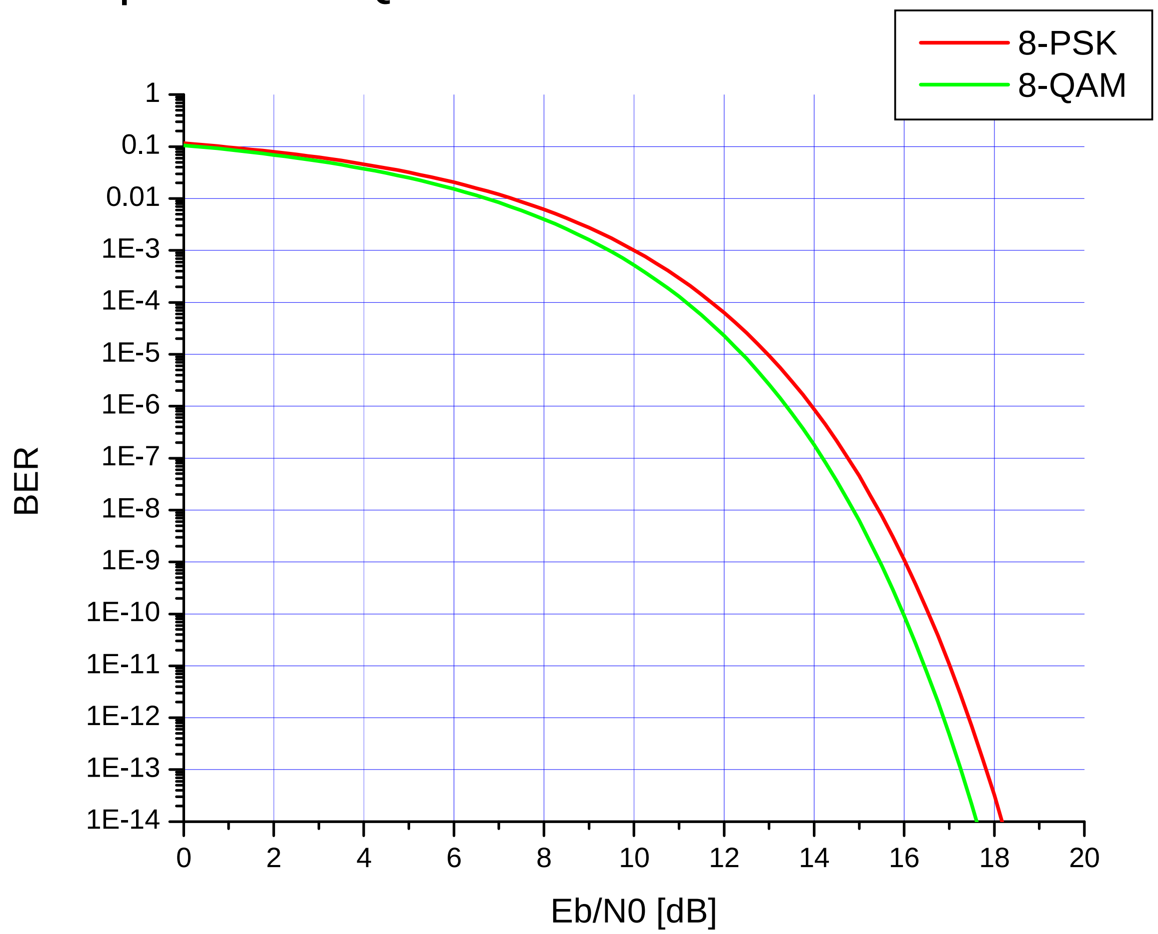


$$P_b(e) \approx \frac{5}{12} erfc\left(\sqrt{\frac{1}{2}\frac{E_b}{N_0}}\right)$$

$$P_b(e) \approx \frac{1}{3} erfc\left(\sqrt{0.439\frac{E_b}{N_0}}\right)$$

m-QAM: *error probability (vs. PSK)*

- Comparison 8-QAM vs. 8-PSK



difference = 0.56 dB

Exercize

- Given a baseband channel with bandwidth B = 4000 Hz, compute the maximum bit rate R_b we can transmit over it by:
  - a 4-PAM constellation
  - a 16-QAM constellation (carrier frequency f_0 = 2kHz)
- when ideal low pass TX filters are supposed in both cases.
- Which constellation has better performance?

***m-QAM**: applications*

- Digital radio links (Up to 128-QAM)
- Some satellite links (up to 16-QAM)
- Internet modems (V90: 33600 bps in uplink, 1024-QAM)
- ADSL modems (OFDM modulation, up to 256-QAM for each carrier)
- DVB-T, DAB (OFDM)
- …