Experiment 10
Noise in DSB and SSB

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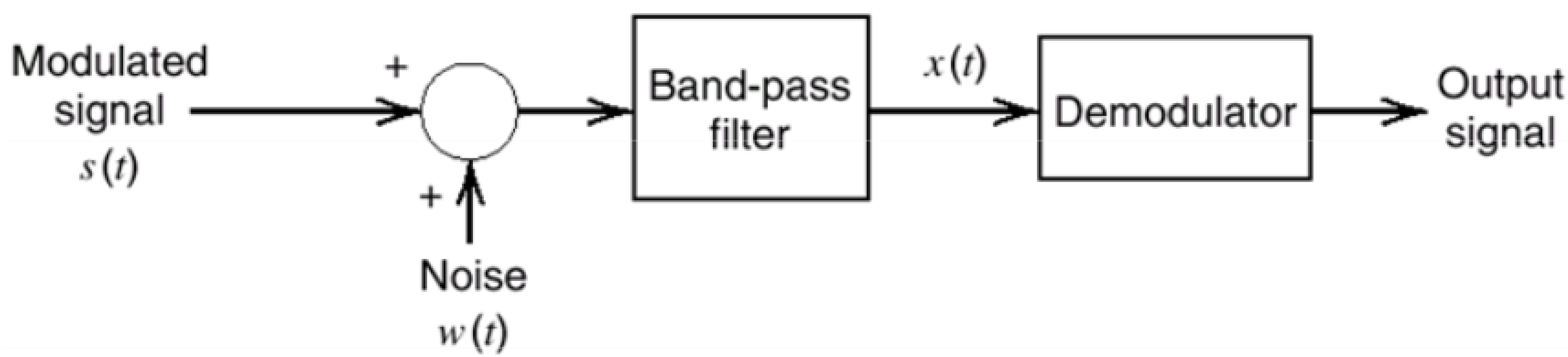
Additive White Gaussian Noise

- The type of noise that is normally assumed to impair the transmission of messages over a communication channel is what is referred to as *additive white Gaussian noise.*

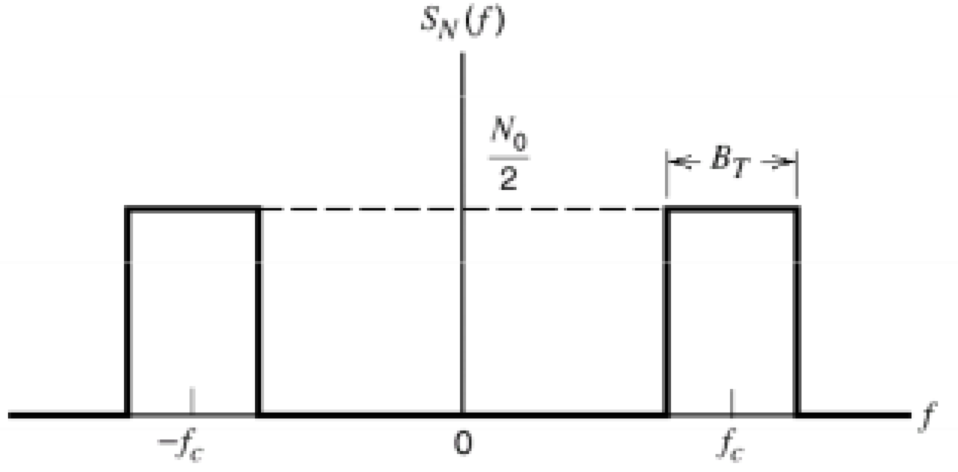
Receiver Model:

- *s(t) denotes the incoming modulated signal.*
- *w(t) denotes front-end receiver noise. The power spectral density of* the noise *w(t) is denoted by* $N_0/2$, defined for both positive and negative frequencies. $N_0/2$ is the average noise power per unit bandwidth measured at the front end of the receiver.
- The bandwidth of this band-pass filter is just wide enough to pass the modulated signal without distortion.



Noise Model

- Assume the band-pass filter is ideal, having a bandwidth equal to the transmission bandwidth B_T of the modulated signal *s(t)* , *and a mid*-band frequency equal to the carrier frequency fc.



Idealized characteristic of band-pass filtered noise.

The Noise Model

- The filtered noise *n(t) may be treated as a narrow band noise* represented in the canonical form

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$n_I(t)$, $n_Q(t)$ and $n(t)$ have the same variance

$$E\{n(t)^2\} = E\{n_I(t)^2\} = E\{n_Q(t)^2\} = \sigma^2$$

After band-pass filtering, the resulting signal is

$$x(t) = s(t) + n(t)$$

The average noise power at the demodulator input is equal to the total area under the curve of the power spectral density

$$P_N = \int_{-\infty}^{\infty} S_N(f)\,df = 2B_T \frac{N_0}{2} = B_T N_0 = \sigma^2$$

Noise in DSB-SC

- A Double-sideband suppressed-carrier (DSB-SC) modulated signal is represented as

$$s(t) = A_c m(t) \cos(2\pi f_c t)$$

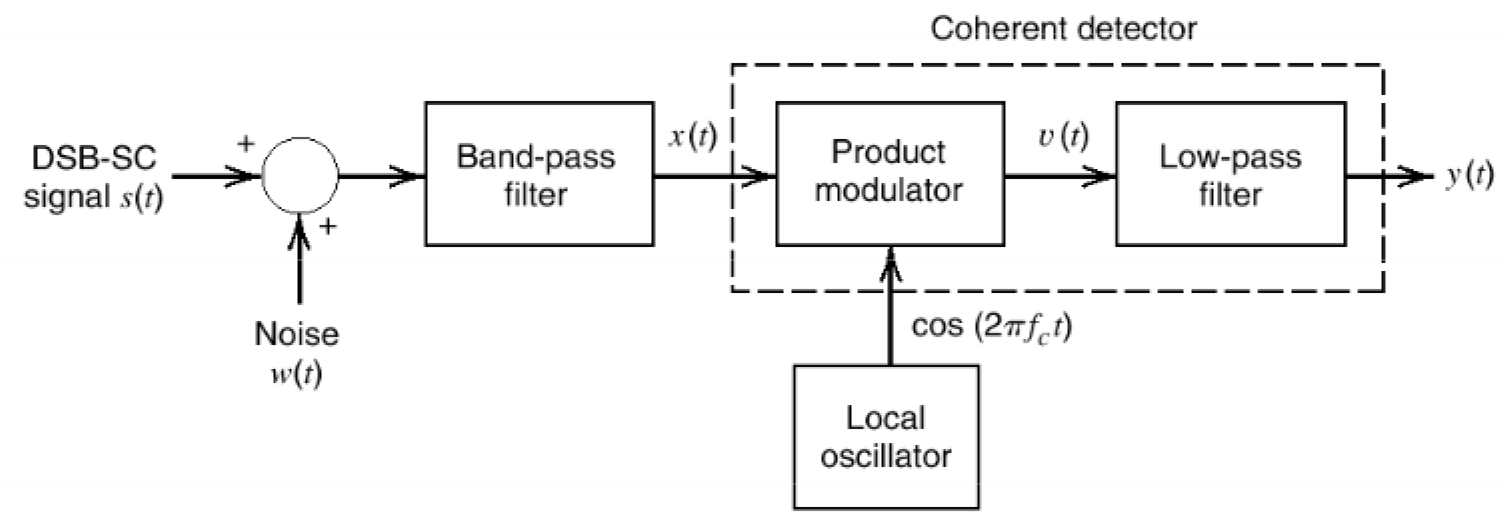
  - fc is the carrier frequency
  - m(t) is the message signal

- After band-pass filtering, the resulting signal is

$$x(t) = A_c m(t) \cos(2\pi f_c t) + n(t)$$

Noise in DSB-SC

- The figure below shows the model of a DSB-SC receiver using a coherent detector.
- For the demodulation scheme to operate satisfactorily, it is necessary that the local oscillator be synchronized both in phase and in frequency with the oscillator generating the carrier wave in the transmitter. We assume that this synchronization has been achieved



Noise in DSB-SC

The total signal after band – pass filtering, is

$$x(t) = s(t) + n(t)$$

$$x(t) = A_c m(t)\cos(2\pi f_c t) + n(t)$$

The signal power is:

$$P_s = E(s^2(t)) = (A_C^2/2)E(m^2(t)) = (A_C^2/2)P_m$$

$$P_s = (A_C^2/2)(A_m^2/2), \text{ when } m(t) = \cos(2\pi f_m t)$$

The noise power is:

$$P_N = B_T N_0 = (2W)N_0; \text{ W is the message bandwidth}$$

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{DSB}} = \frac{A_c^2 P}{2N_0 B_T}$$

Noise in DSB-SC

The signal at the input to the coherent detector :

$$x(t) = s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$v(t) = x(t)\cos(2\pi f_c t)$$

$$= \tfrac{1}{2}(A_c m(t) + n_I(t))$$

$$+ \tfrac{1}{2}(A_c m(t) + n_I(t))\cos(4\pi f_c t) - \tfrac{1}{2}n_Q(t)\sin(4\pi f_c t)$$

$$\cos A\cos A = \frac{1+\cos 2A}{2} \quad \text{and} \quad \sin A\cos A = \frac{\sin 2A}{2}$$

The high frequency components are removed with a LPF

$$y(t) = \tfrac{1}{2}(A_c m(t) + n_I(t))$$

Noise in DSB-SC

$$y(t) = \tfrac{1}{2}(A_c m(t) + n_I(t))$$

The message component is $A_c m(t)/2$

The post – detection signal power is $A_C^2 P_m /4$

The spectral density of $n_I(t)$ is:

$S_{NI} = N_0$ over $-B_T/2$ to $B_T/2$

the output noise power is (in a bandwidth W):

$$E[n_I^2(t)] = \int_{-W}^{W} N_0 df = 2N_0 W$$

$$\text{Post – detection}: \text{SNR}_{\text{post}}^{\text{DSB}} = \frac{\frac{1}{4}(A_c^2)P_m}{\frac{1}{4}(2N_0 W)} = \frac{A_c^2 P_m}{2N_0 W}$$

Figure of Merit in BSB-SC

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{DSB}} = \frac{A_c^2 P_m}{2N_0 B_T} = \frac{A_c^2 P_m}{4N_0 W} = \frac{A_c^2 P_m / 2}{2N_0 W}$$

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}} = \frac{A_c^2 P_m}{2N_0 \mathrm{W}} = \frac{A_c^2 P_m / 2}{N_0 \mathrm{W}}$$

$$\text{Post} - \text{detection SNR is twice pre} - \text{detection SNR.}$$

$$\text{Figure of merit} = \frac{\mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}}}{\mathrm{SNR}_{\mathrm{ref}}} = 1$$

$$\mathrm{SNR}_{\mathrm{ref}} = \frac{\text{average power of the modulated signal}}{\text{average power of noise mesured in the message bandwidth}}$$

Noise in Single Sideband Modulation

The modulated wave is:

$$s(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t)\sin(2\pi f_c t)$$

$m(t)$ and $\hat{m}(t)$ are uncorrelated with each other.

Therefore, their power spectral densities are additive

$\hat{m}(t)$ is obtained by passing $m(t)$ through a linear filter

with transfer function $-j\,\mathrm{sgn}(f)$

$|-j\,\mathrm{sgn}(f)|^2 = 1$ for all f

Hence, $m(t)$ and $\hat{m}(t)$ have the same average power P_m

Noise in Single Sideband Modulation

The pre – detection $\text{SNR}_{\text{pre}}^{\text{SSB}} = \dfrac{A_c^2 P}{4N_0 \text{W}}$

The band – pass signal after multiplication with $\cos(2\pi f_c t)$ is

The modulated wave is:

$$v(t) = x(t)\cos(2\pi f_c t)$$

$$= \tfrac{1}{2}\left(\frac{A_c}{2} m(t) + n_I(t)\right)$$

$$+ \tfrac{1}{2}\left(\frac{A_c}{2} m(t) + n_I(t)\right)\cos(4\pi f_c t) - \tfrac{1}{2}\left(\frac{A_c}{2} \hat{m}(t) + n_Q(t)\right)\sin(4\pi f_c t)$$

After low – pass filtering,

$$y(t) = \tfrac{1}{2}\left(\frac{A_c}{2} m(t) + n_I(t)\right)$$

Noise in Single Sideband Modulation

The spectrum of $n_I(t)$

$$s_{N_I}(f) = \begin{cases} \dfrac{N_0}{2}, & -\mathrm{W} \le f \le \mathrm{W} \\ 0, & \text{otherwise} \end{cases}$$

The post – detection signal – to – noise ratio

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{SSB}} = \frac{A_c^2 P}{4N_0 \mathrm{W}}$$

The figure of merit for the SSB system

$$\text{Figure of merit} = \frac{\mathrm{SNR}_{\mathrm{post}}^{\mathrm{SSB}}}{\mathrm{SNR}_{\mathrm{ref}}} = 1$$