Experiment 7
Pulse Code Modulation (PCM)
Part 2

Prepared by

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Elements of the PCM System

- The PCM system consists of three parts: Sampler, Quantizer, and Encoder.
- The **sampler** operates at a rate higher than the Nyquist rate. Its input is a continuous-time continuous-amplitude waveform and its output is a discrete-time continuous amplitude waveform. **The quantizer was studies in Experiment 5**.
- The **quantizer** transforms the output of the sampler into discrete-time discrete amplitude waveform. **Introduced in Experiment 6.**
- The **encoder** converts the discrete amplitudes into binary digits.
- In Experiment 6, we used a dc signal for the purpose of finding the various characteristics of the PCM system. Here, we use a time varying message
- In Experiment 6 and 7, we will consider the following topics:
  - characteristics of the linear and nonlinear quantizers
  - Characteristics of the compressor/expander
  - Resolution of the quantizer
  - Basics of the encoding part.
  - quantization noise
  - Differential Pulse Code Modulation.

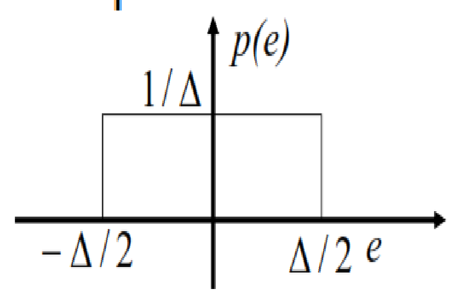
Quantization Error

- The quantization error per sample is the absolute difference between the input and output of the quantizer, i.e.

$$e = |x - \hat{x}|$$

- The maximum error (also referred to as the resolution) = Δ/2
- This error, , can be assumed to be uniform random variable over the interval $-\Delta|2 < e < \Delta|2$

$$\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$$

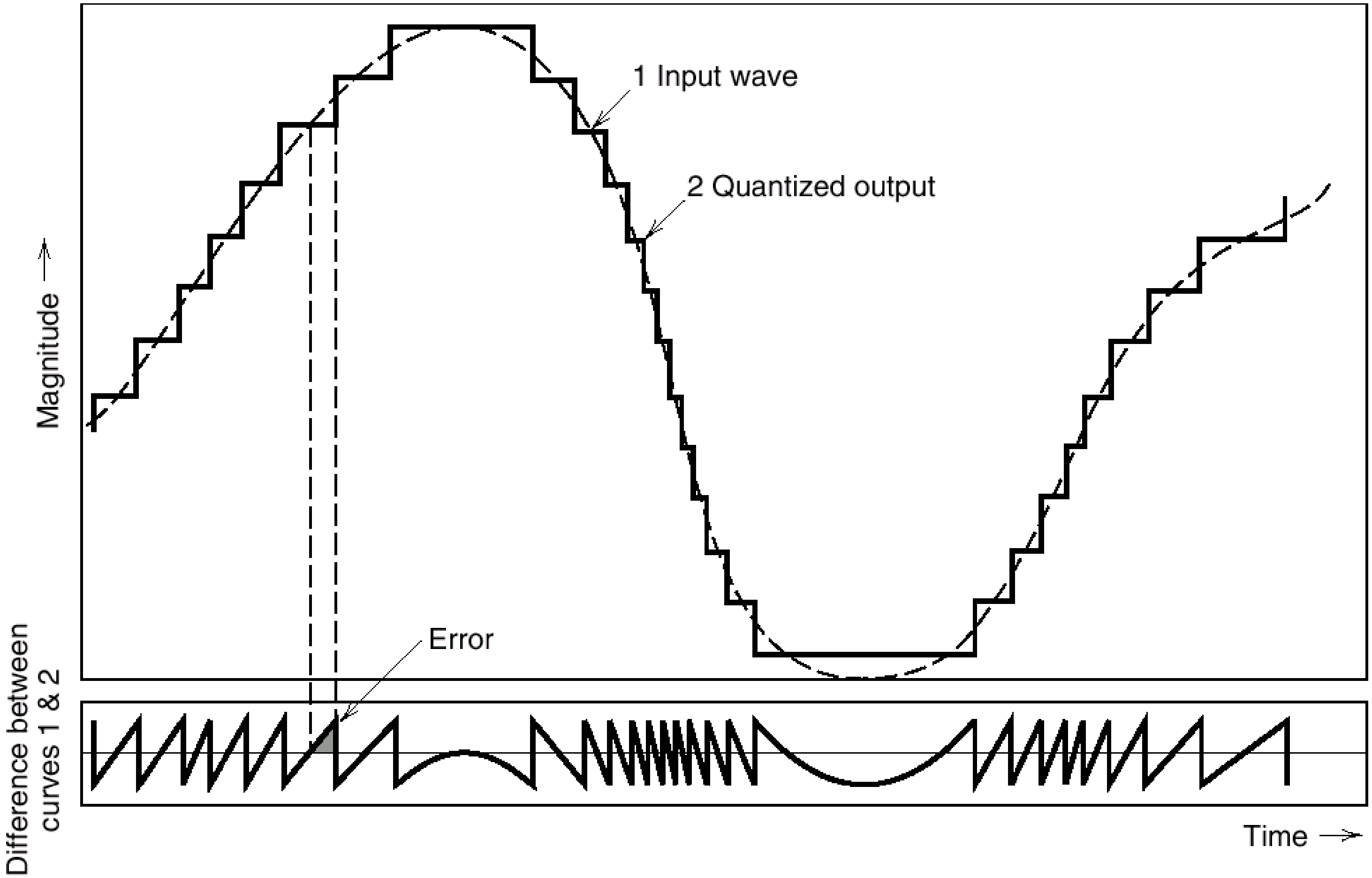


The average quantization error (distortion) over all samples of the signal is

$$D = E(x - \hat{x})^2 = E(e)^2$$

$$D = \frac{1}{\Delta}\int_{-\Delta/2}^{+\Delta/2} (e)^2\, de = \Delta^2|12$$

Quantization Noise



The quantization noise depends on Δ

Δ depends on the number of quantization levels L. As L increases, the error decreases.

Differential Pulse Code Modulation (DPCM)

- The quantizers that we studied so far are memoryless, in the sense that quantization is done on a sample by sample basis, where the correlation between successive samples is not utilized.
- A quantizer that utilizes the fact that successive samples are correlated is the *differential pulse-code modulation (DPCM) quantizer*.
- This quantizer, instead of quantizing successive samples, it quantizes the difference between a sample and a predicted value of that sample.
- The prediction is based, in general, on past m samples of the signal.
- If successive samples are highly correlated, the predictor will do a good job in predicting the next sample value and thus the quantized prediction error will be small.
- This means the error has a small variance, which further means that fewer bits will be needed to represent the error
- Since the range of differences in amplitudes between successive samples of the signal is less than the range of the actual sample amplitudes, fewer bits are required to represent the difference signals.
- At the receiver, a predictor similar to the one used at the transmitter is used to construct the original waveform

Differential Pulse Code Modulation (DPCM)

- Linear Prediction Filter

It is a discrete time, finite-duration impulse response filter (FIR), which consists of three blocks :

1. Set of p (p: prediction order) unit-delay elements (z^{-1})

2. Set of multipliers with coefficients $w_1, w_2, \ldots w_p$

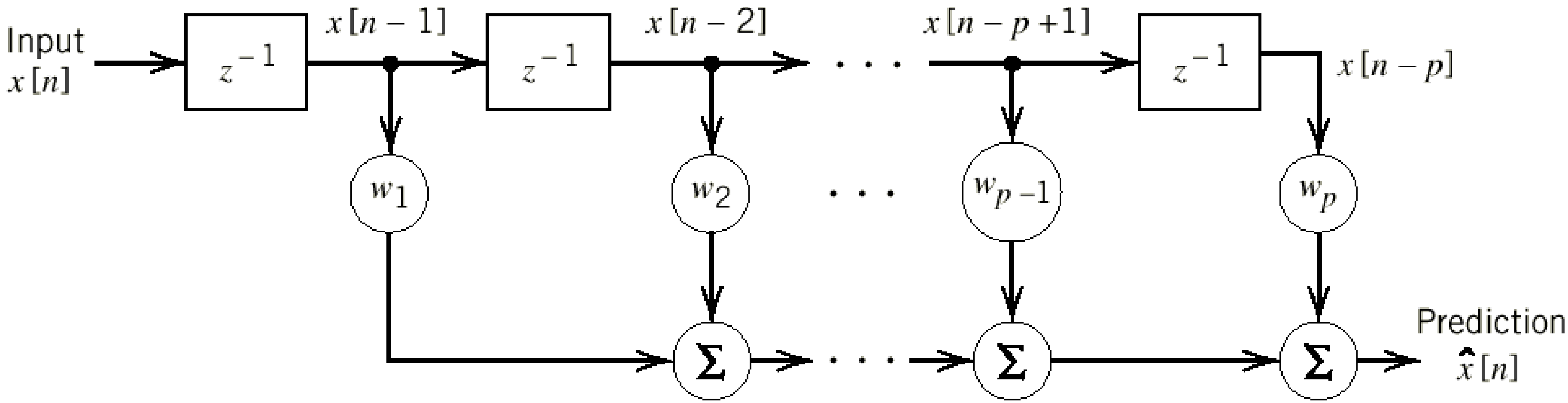
3. Set of adders (Σ)

This filter expresses the predicted value of the sample at time (nT_s) as a linear combination of the past p samples of the signal.

$$\hat{x}(n) = w_1 x(n-1) + w_2 x(n-2) + \cdots + w_p x(n-p)$$

The coefficients $w_1, w_2, \ldots, w_p$ are chosen so as to minimize the mean square error $E((\hat{x}(n) - x(n))^2$.

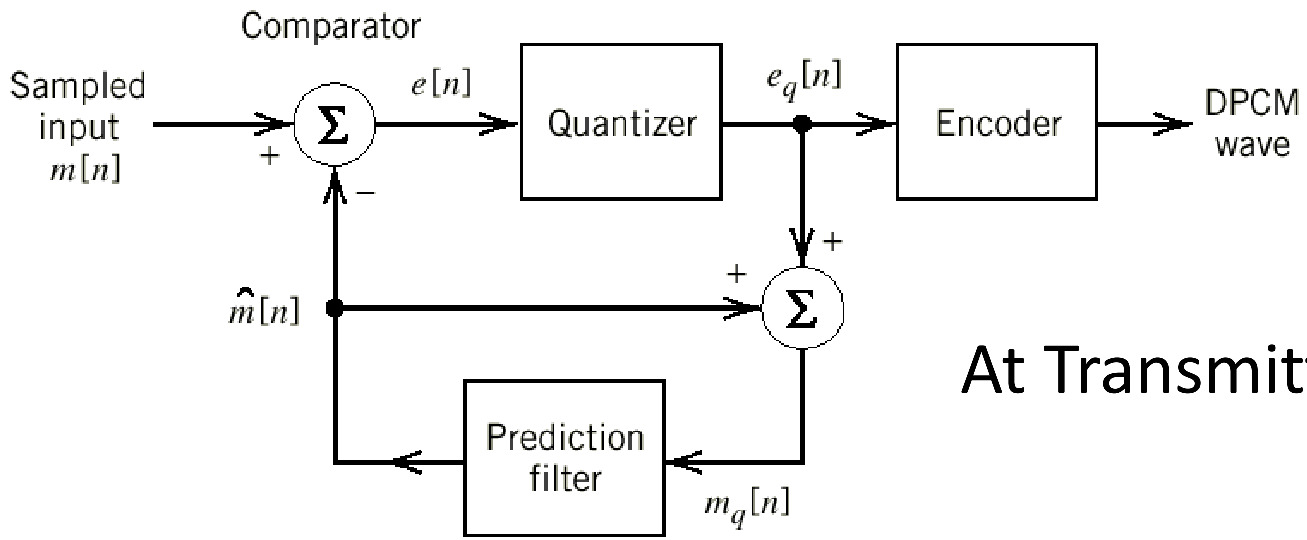
Differential Pulse Code Modulation (DPCM)



$$\hat{x}(n) = w_1 x(n-1) + w_2 x(n-2) + \cdots + w_p x(n-p)$$

The Prediction Equation

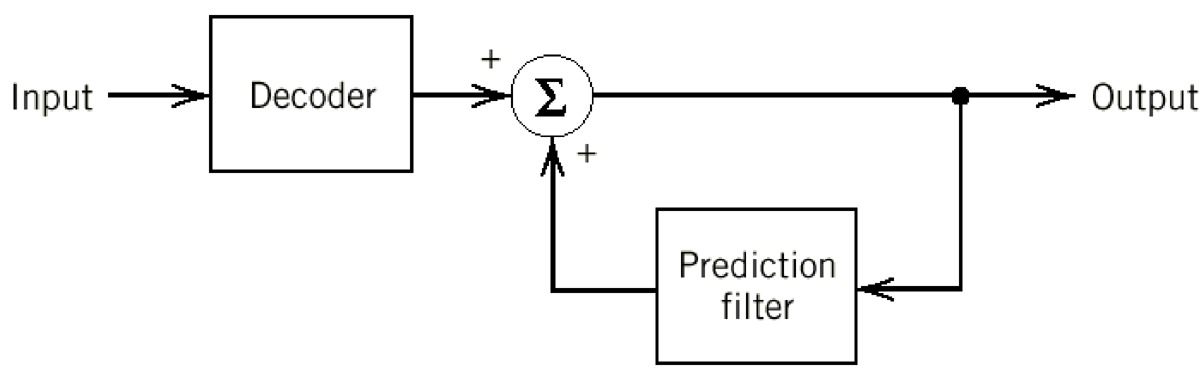
Differential Pulse Code Modulation (DPCM)



At Transmitter

(a)

At Receiver



(b)