



BERZIET UNIVERSITY

FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

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**ENEE 4113**

communication Laboratory.

**Experiment 5**

**Pulse Amplitude Modulation - Part1**

**Prepared by:** Anas Nimer 1180180

**Instructor:** Dr. mohammad jubran

**TA:** Eng.Ruba Eid

**Section # :** 3

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## **1. Abstract:**

In this experiment, the student will be introduced to how they can deal with Pulse Amplitude modulation using written python code in GitHub simulator, they will generate a periodic pulse train and a train of impulses in both time and frequency domain and then show some parameters that affect them, then they will be introduced to the ideal sampling in both frequency and time domain and know how the signal reconstruction work by LPF and its cut-off frequency to do that in a right way, Furthermore, the experiment will provide the student the possibility to thoroughly observe the difference between natural and flat-topped sampling in both domains, then they will emphasize the sampling theorem that talks about the relation between signal frequency and sampling frequency. Last but not least the experiment will talk about time division multiplexing and its power in communication.

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## 2. Procedure:

### 2.1 The Periodic Pulse Train in the Time and Frequency Domains:

A periodic square function is characterized by its amplitude, frequency  $f$ , and duty cycle. Here, we will generate a square wave function to see how changing the duty cycle affects the signal in the time and frequency domains.

Firstly, we will generate a pulse train with duty cycle (dc)=50% and frequency  $f=10$  then plot it in time domain as shown in Figure1 and frequency domain as shown in Figure2.

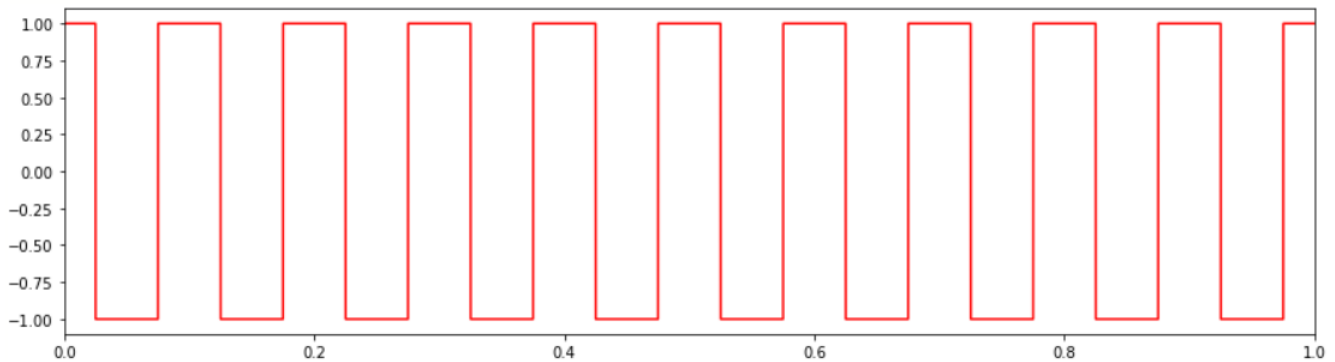


Figure 1: periodic square function with D.C=50% and  $f=10$  Hz in time domain

- **Note:** when  $dc=0.5$  we notice the period during which it is  $T_{on}=50\%$ . and the period of square function =  $1/10$ .

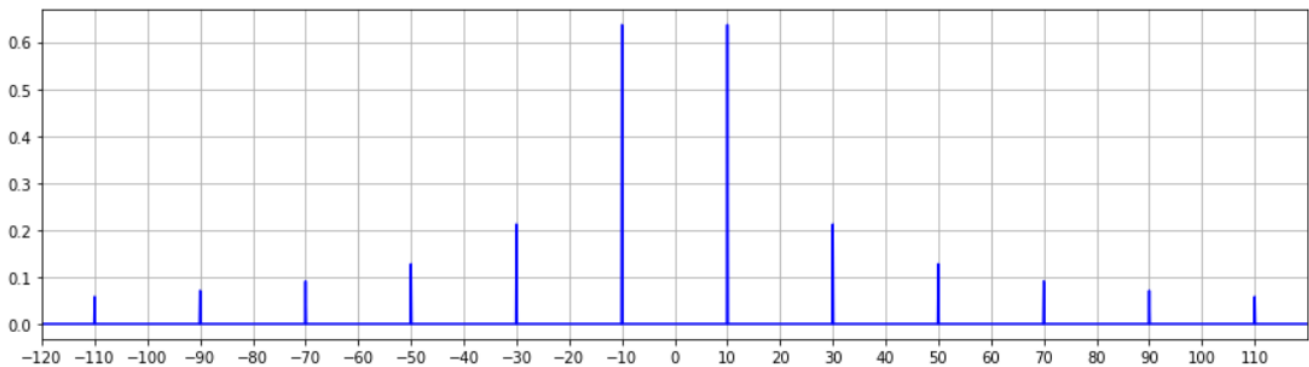


Figure 2: periodic square function with D.C=50% and  $f=10$  Hz in frequency domain

- **Note:** from the above figure we notice that there is no value at zero because there is no have dc value. Also the spectral components on  $2nf = 0$  because its null of sinc function.

### **Exercise:**

The following will show the changing of the dc to 0.2, 0.4 and 0.7 respectively and plot it in both time then frequency domains:

#### **1- When dc=0.2 in time domain:**

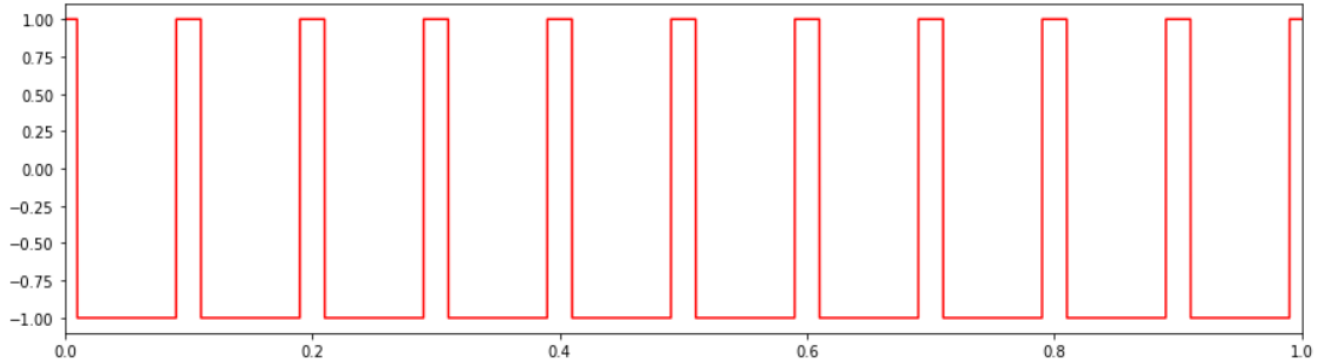


Figure 3: D.C=0.2 &  $f=10$  in time domain

#### **2- When dc=0.4 in time domain:**

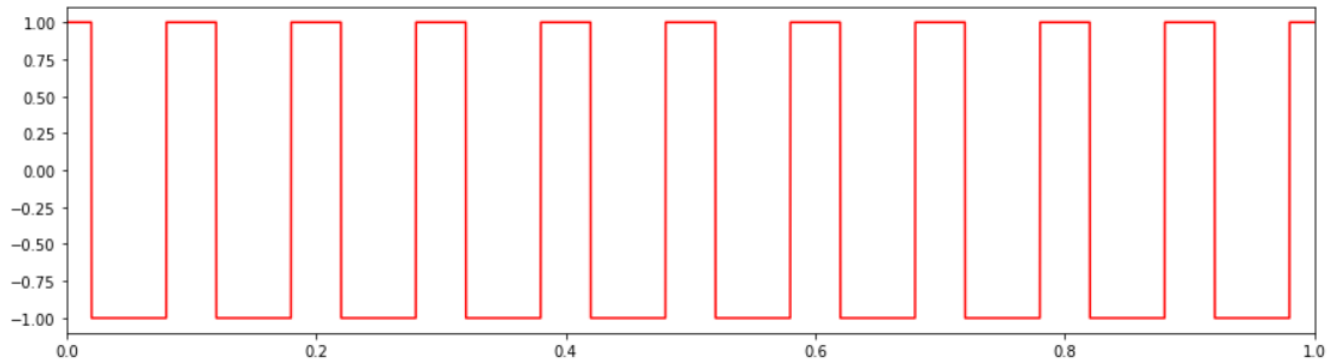


Figure 4: D.C=0.4 &  $f=10$  in time domain

#### **3- When dc=0.7 in time domain:**

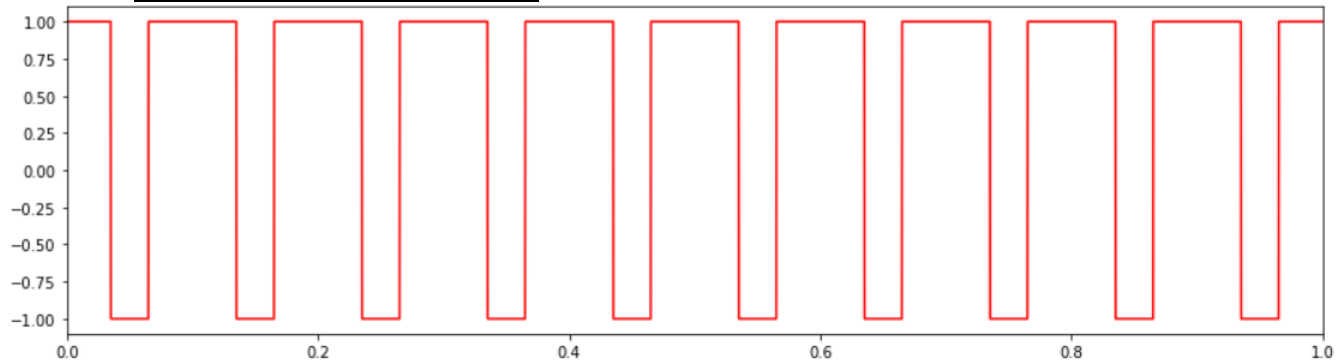


Figure 5: D.C=0.7 &  $f=10$  in time domain

- **Note:** we note from Figure 3&4&5 that show the pulse train in time domain, when we increase the duty cycle the range when the signal is on will increase and vice versa. Where  $T_{on}$  in fig3=20%, fig4=40% and fig5=70%. While changing the dc does not effect on period of square wave value.

#### 4- When dc=0.2 in frequency domain:

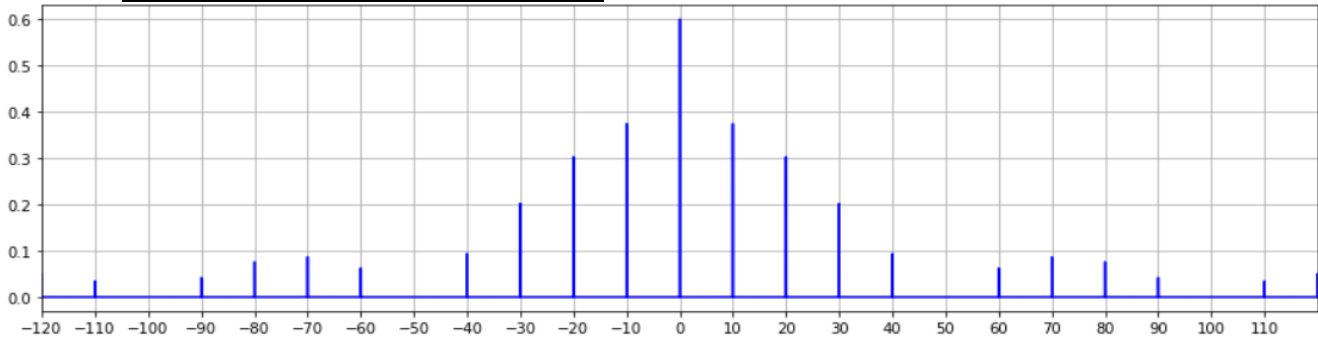


Figure 6: D.C=0.2 & f=10 in frequency domain

- **Note:** we notice when change the dc value the null position difference by  $f/dc$ , so in this case the spectral components will be zero every  $2nf$ , s.t  $n$  is multiple of 2.5. While the position of frequency does not effect because  $f$  doesn't change.

#### 5- When dc=0.4 in frequency domain:

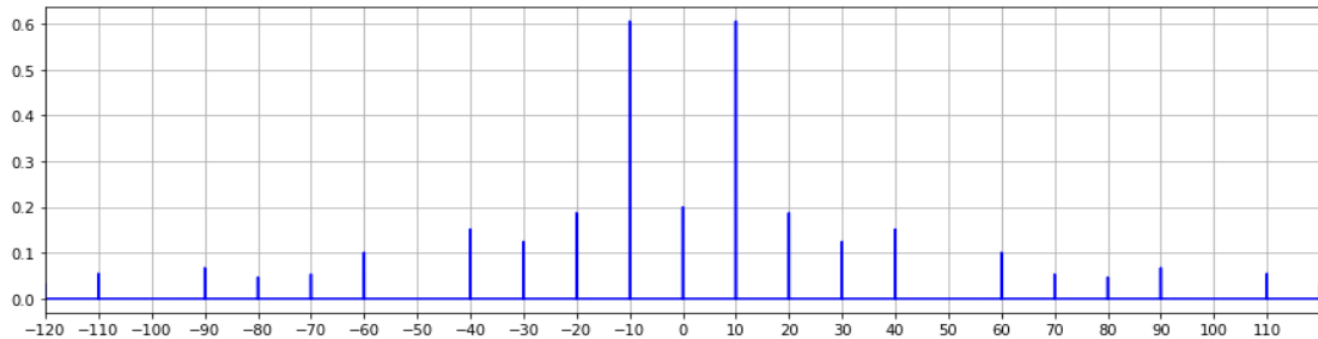


Figure 7: D.C=0.4 & f=10 in frequency domain

- **Note:** we notice when change the dc value the null position difference by  $f/dc$ , so in this case the spectral components will be zero every  $2nf$ , s.t  $n$  is multiple of 2.5. While the position of frequency does not effect because  $f$  doesn't change.

#### 6- When dc=0.7 in frequency domain:

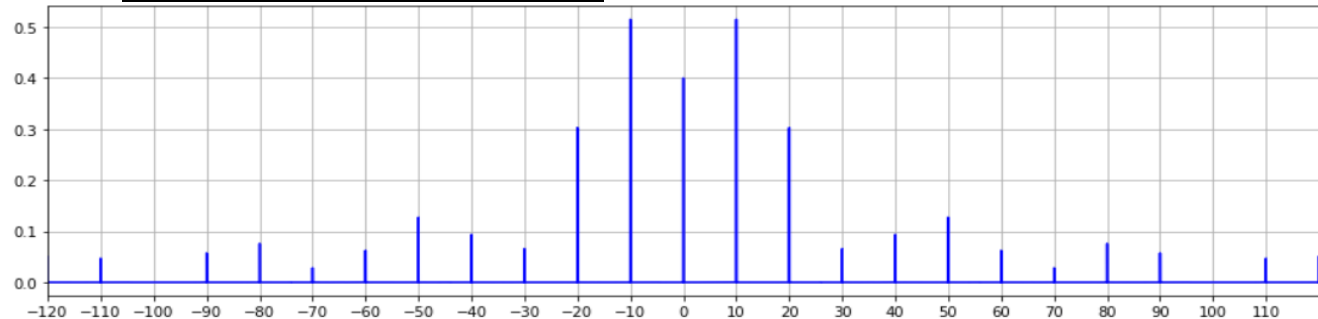


Figure 8: D.C=0.7 & f=10 in frequency domain

- **Note:** we notice when change the dc value the null position difference by  $f/dc$ , so in this case the spectral components will be zero every  $2nf$ , s.t  $n$  is multiple of 5. While the position of frequency does not effect because  $f$  doesn't change.



The following will show the changing of the frequency value to 20 Hz and plot it in both time then frequency domains:

**1- When f=20 Hz in time domain:**

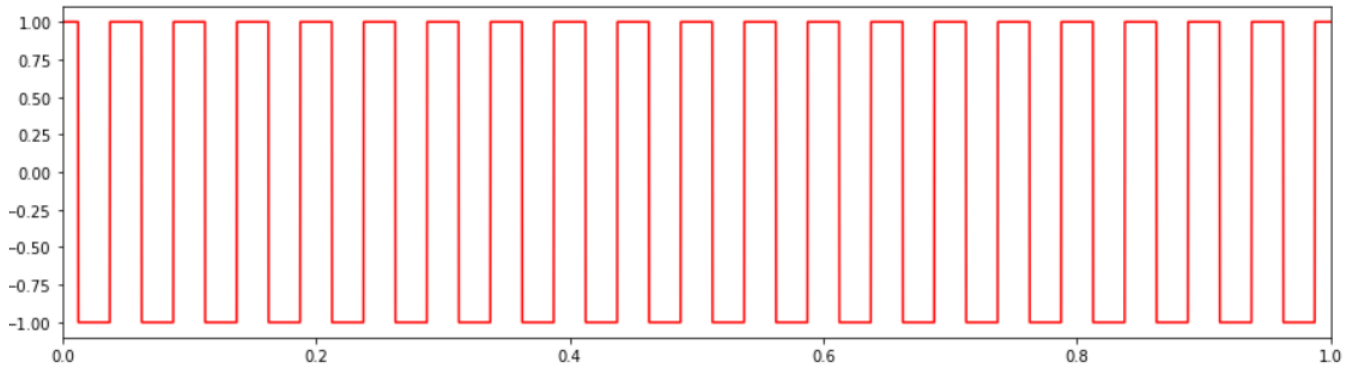


Figure 9: D.C=0.5 & f=20 in time domain

- **Note:** As we can see in the above figure, when we change the value of f, the value of  $T_{on}$  changes by  $dc/f$ . In addition to, the period value of square wave changed by  $1/f$ . In this case the values of  $T_{on}$  and period decreased because we have increased f.

**2- When f=20 Hz in frequency domain:**

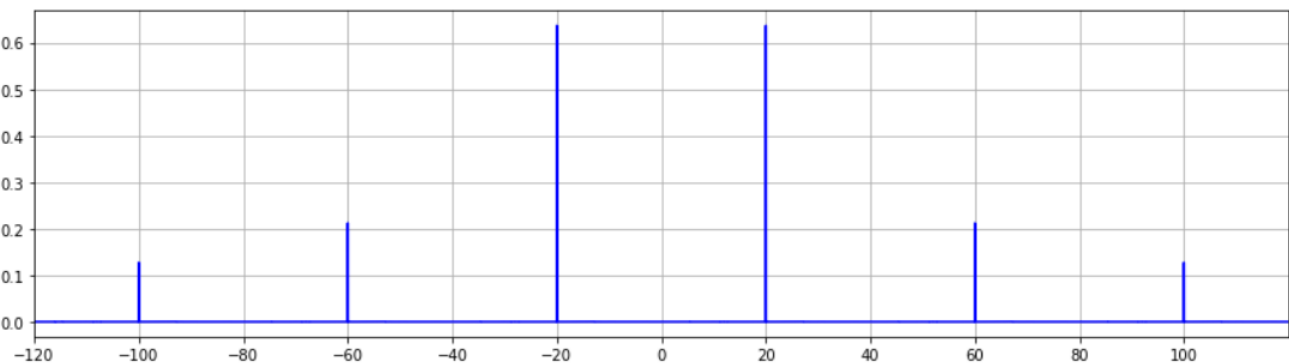


Figure 10: D.C=0.5 & f=20 in frequency domain

- **Note:** we notice when change f value the position of frequency changed by f value. In addition to, the null value for sinc changed by  $f/dc$ .

## 2.2 The Impulse Train in the Time and Frequency Domains:

An impulse train is characterized by its amplitude, and frequency  $f$ . Here, we will generate an impulse train to see how changing the frequency affects the signal in the time and frequency domains.

Firstly, we will generate a train of impulses with frequency  $f=10$ , and magnitude =1 then plot it in time domain as shown in Figure11 and frequency domain as shown in Figure12.

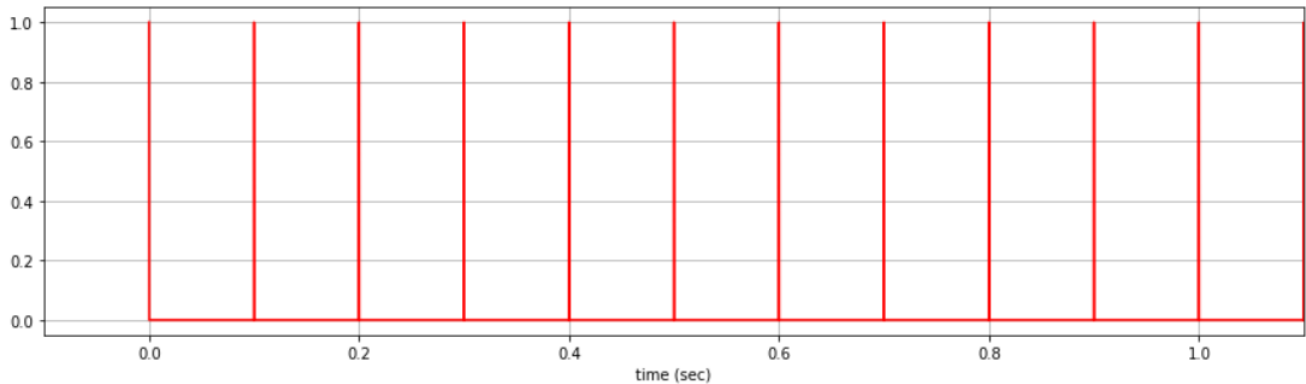


Figure 11: Amp=1 &  $f=10$  in time domain

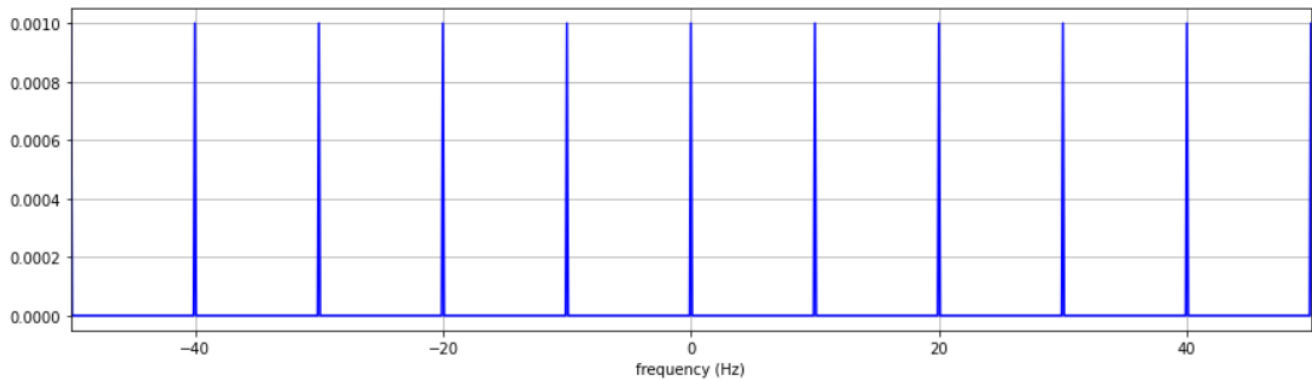


Figure 12: Amp=1 &  $f=10$  in frequency domain

- **Note:** we note from fig11 that shows the impulse train in time domain that the impulse is periodic every 0.1 sec according to this equation:  $T=1/f$ , we note that there is no effect for magnitude. While in fig12 the location of pulses periodic every  $nf$  Hz,  $n$  is integer. In addition to, the value of amplitude in frequency domain change if  $f$  change.

**Exercise:**

let us vary the frequency of the impulse train to  $f=20$  Hz to emphasize that equation.

**1- When  $f=20$  Hz in time domain:**

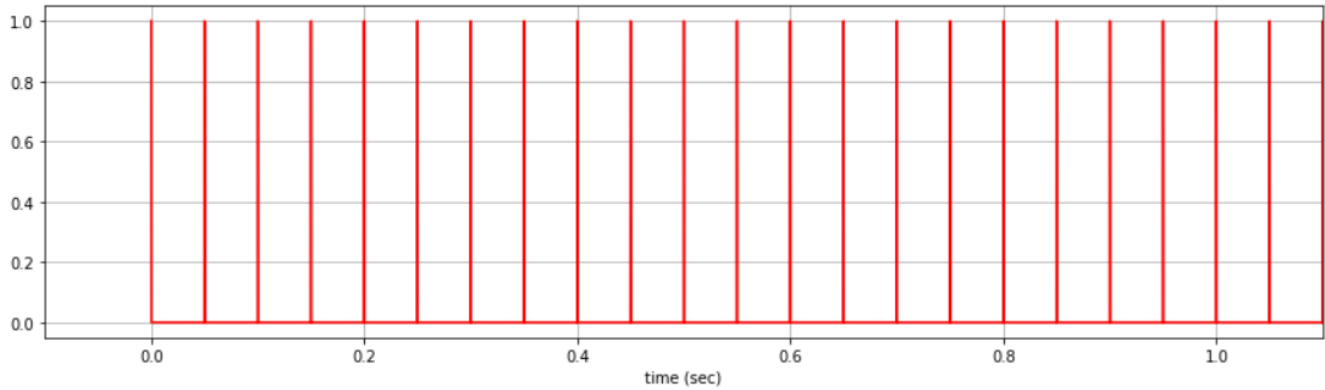


Figure 13:  $Amp=1$  &  $f=20$  in time domain

- **Note:** we notice when change  $f$  value the impulse is periodic every 0.05 sec. So in this case the value of impulse periodic decreased because we have increased  $f$  value.

**2- When  $f=20$  Hz in frequency domain:**

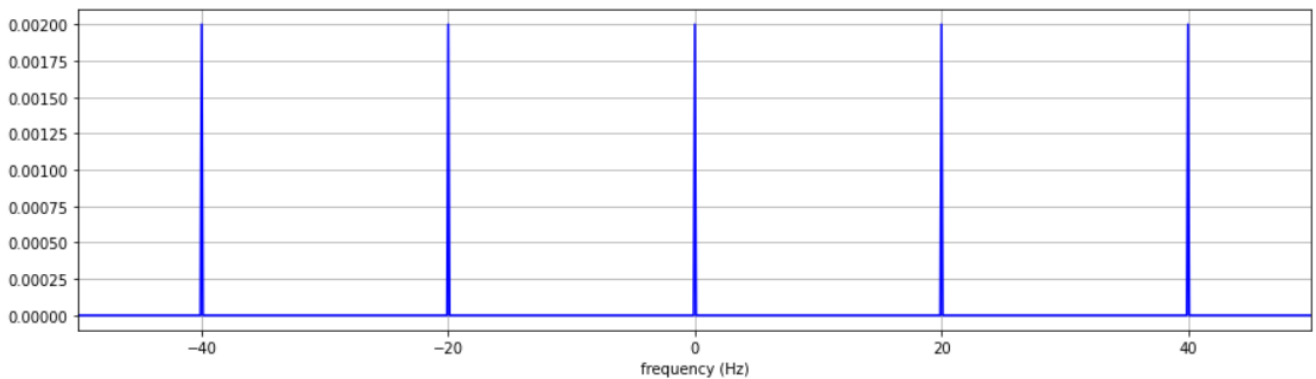


Figure 14:  $Amp=1$  &  $f=20$  in frequency domain

- **Note:** from above figure we show when change  $f$  the location of pulses changes as the value of  $f$  changes and periodic by  $nf$ ,  $n$  is integer. In addition to, the amplitude value for pulses change.

Now let us change the value of the magnitude to 4.

**1- When Amp=4 in time domain:**

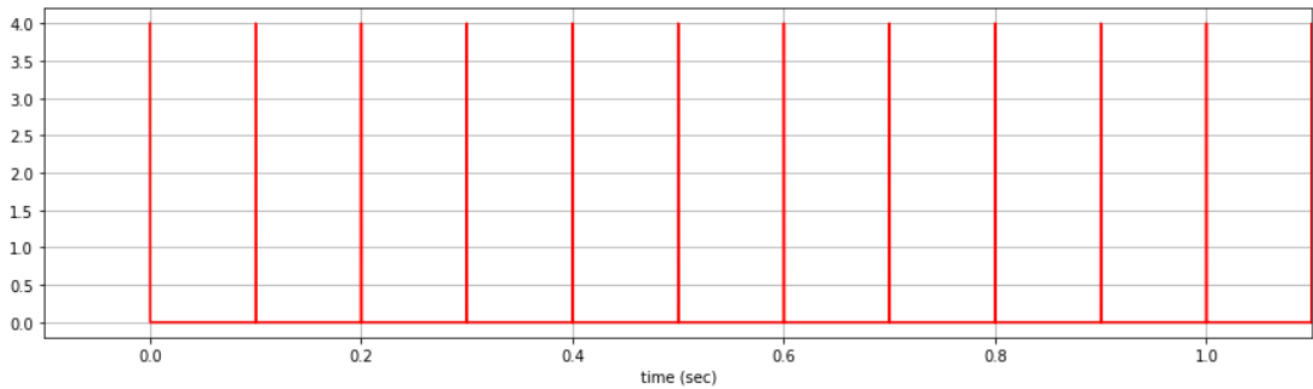


Figure 15: Amp=4 &  $f=10$  in time domain

- **Note:** we notice from above figure when change the amp, the period value doesn't affect.

**2- When Amp=4 in frequency domain:**

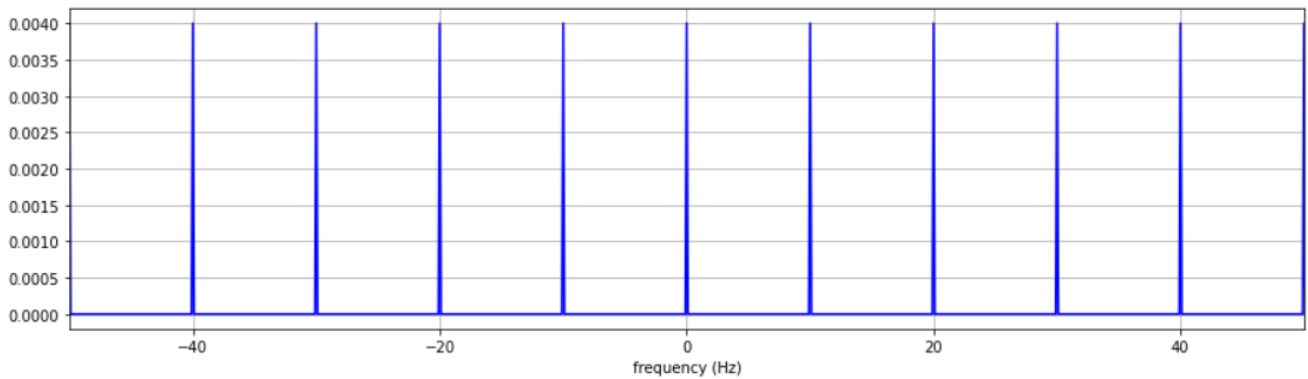


Figure 16: Amp=4 &  $f=10$  in frequency domain

- **Note:** we can show from the above figure, the change of the value of amp effect on amplitude of the impulses.

## 2.3 Ideal Sampling in the Time and Frequency Domains:

In this section we will use ideal sampling method to sample the signal  $m(t)=\cos(2\pi f_m t)$ . The sampled signal is generated by multiplying the message signal by an impulse train with period  $T_s$ .

### 2.3.1 Sampling in Time domain:

Firstly, we will sample a message of  $f_m=1$ ,  $A_m=1$  with train of impulses (ideal) of  $Amp=1$  and  $f_s=20$ .

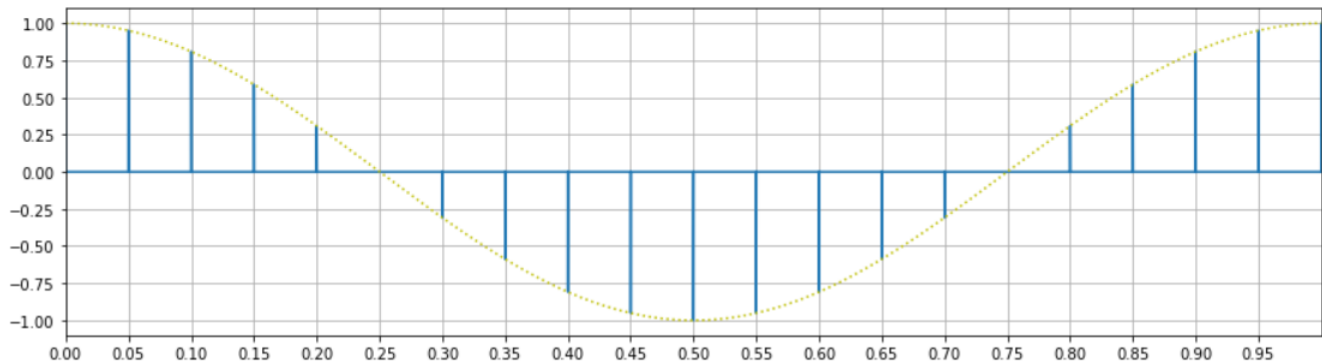


Figure 17:sampled signal in time domain with  $F_s=20$  Hz

- **Note:** we can show from above figure when multiplying the message signal by an impulse train the amplitude of impulses are taking the same amplitude of message.

## Exercise:

Let's change the  $f_s$  to 5,10,30 and plot it below to see what is the effect of  $f_s$  on sampling:

### 1- When $f_s=5$ Hz:

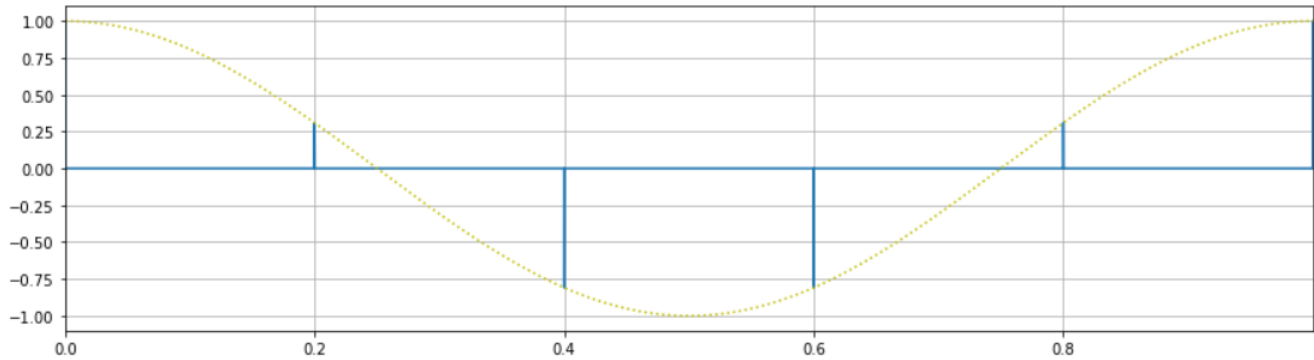


Figure 18:sampled signal in time domain with  $F_s=5$  Hz

### 2- When $f_s=10$ Hz:

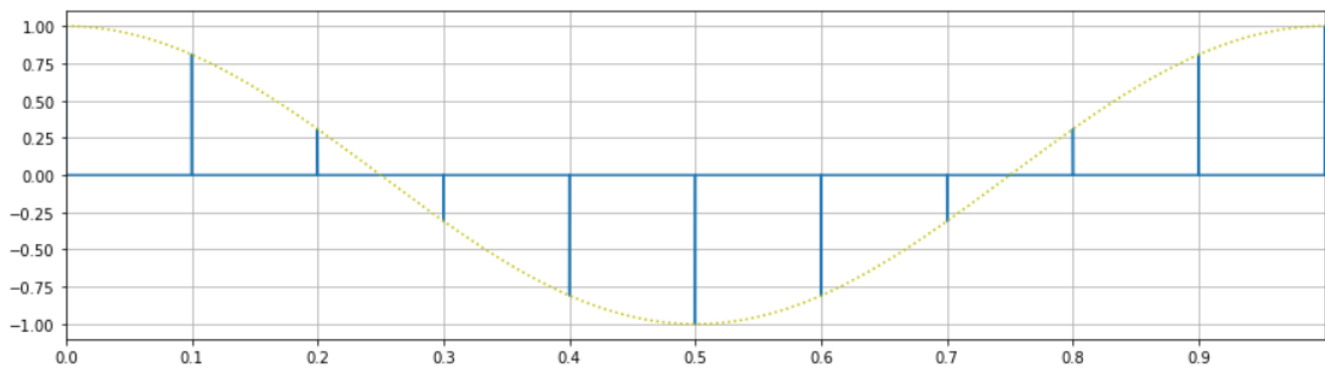


Figure 19:sampled signal in time domain with  $F_s=10$  Hz

### 3- When $f_s=30$ Hz:

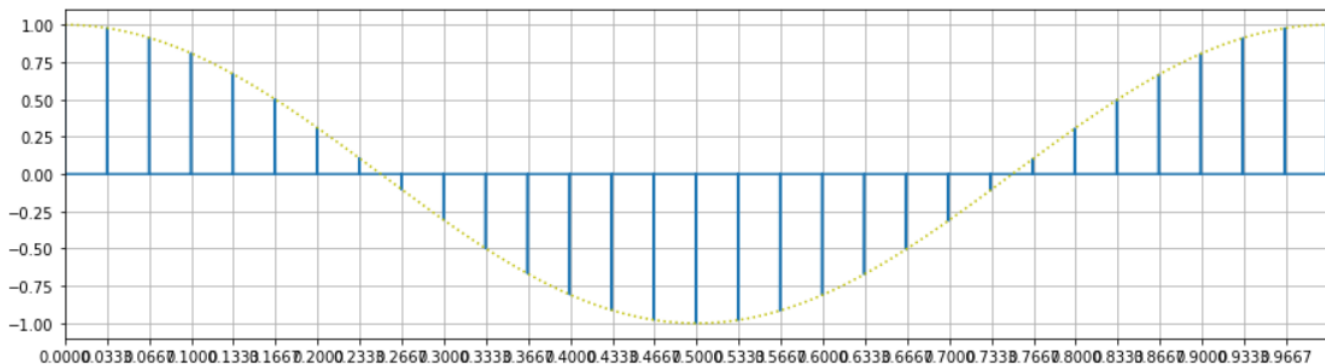


Figure 20:sampled signal in time domain with  $F_s=30$  Hz

- **Note:** We observe from three above figures that we take a sample of the message every  $T=1/f_s$ , so when  $f_s$  increases the  $T$  between samples is decreasing, so the quality of sampled message will increase.

#### 4- When $A_m=4$ :

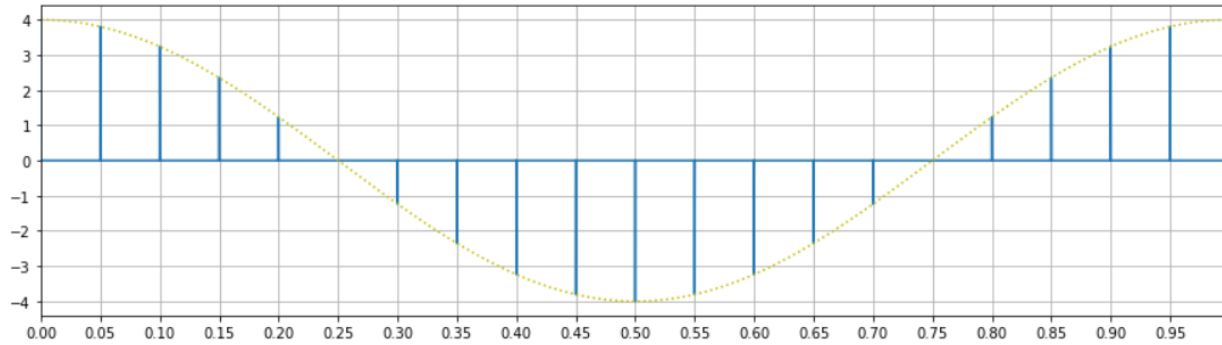


Figure 21:sampled signal in time domain with  $A_m=4$

- **Note:** We notice from the above figure that when we change the message amplitude ( $A_m$ ), the value of impulses amplitude will change by the same amount as  $A_m$  change.

#### 2.3.2 Sampling in frequency domain:

Now we will see the same previous example in frequency domain, you can see what each Figure shows under it (all the figures are in frequency domain).

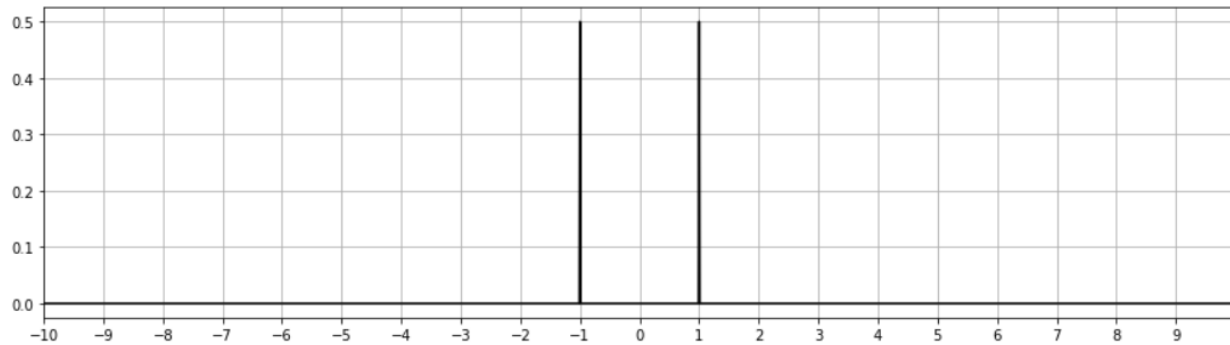


Figure 22:message signal  $F_m=1$  Hz

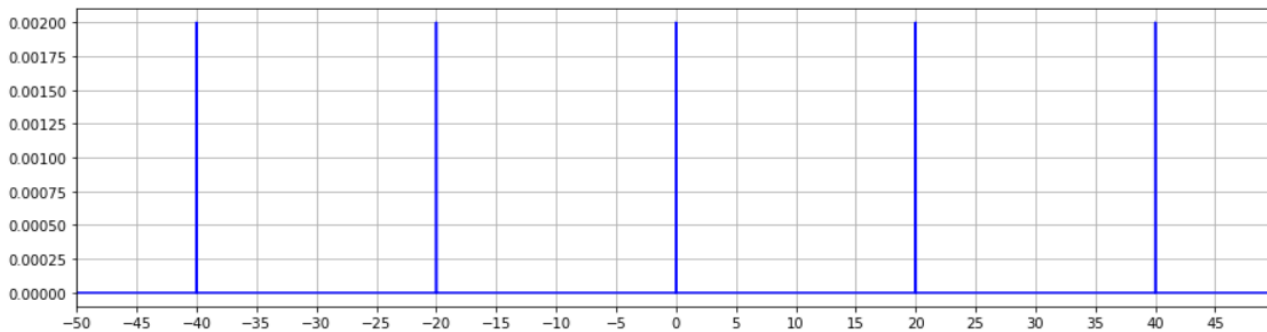


Figure 23:impulse with  $f=20$  Hz

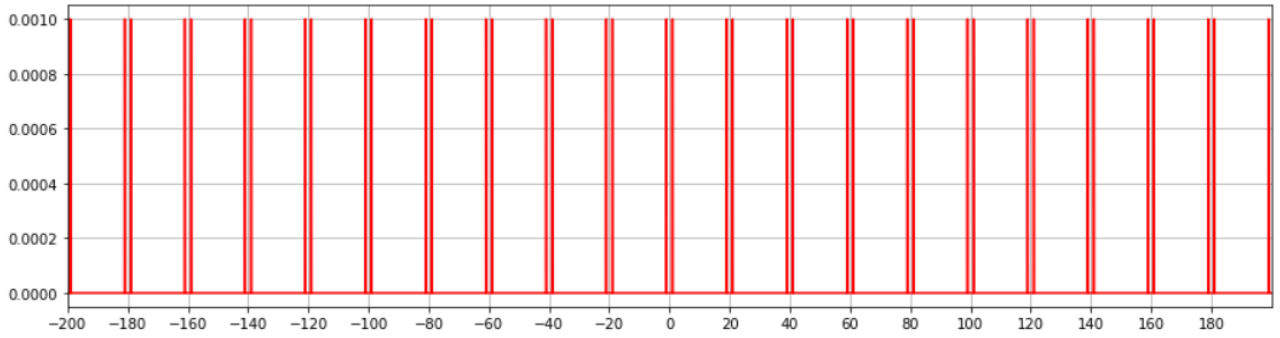


Figure 24:sampled signal in the frequency domain

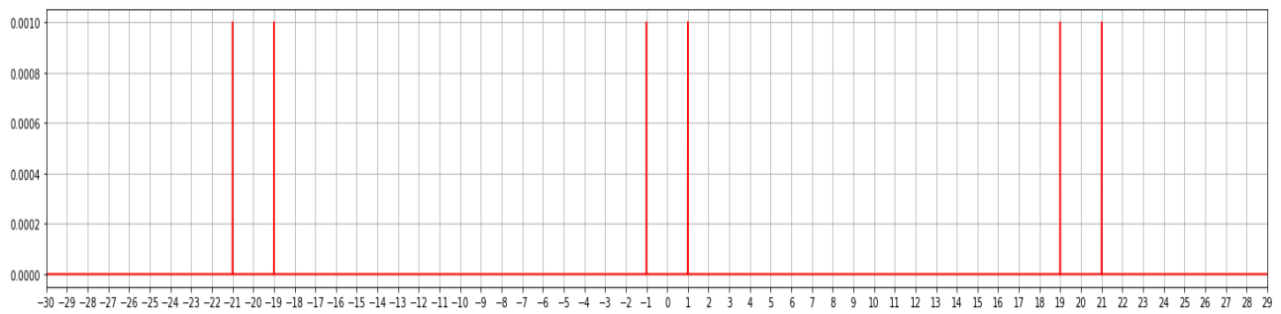


Figure 25:sampled signal zoomed in

- **Note:** The message signal is convoluted at every  $f_s$ , so we can see in Figure25 that there are two impulses centered on  $f_s$ , impulse on  $k(f_s+f_m)$  and the other on  $k(f_s-f_m)$ , where  $k$  is integer.



**Exercise:**

1- let's emphasize this by changing  $f_m=5$  and plot all figures below.

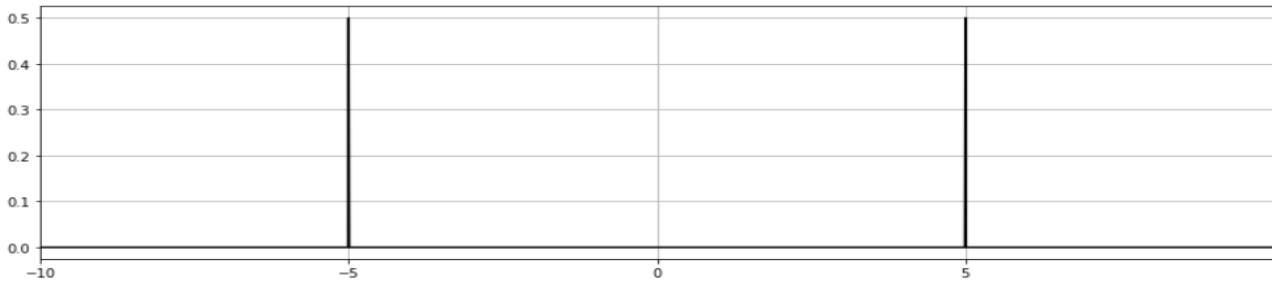


Figure 26: message signal with  $F_m=5$  Hz

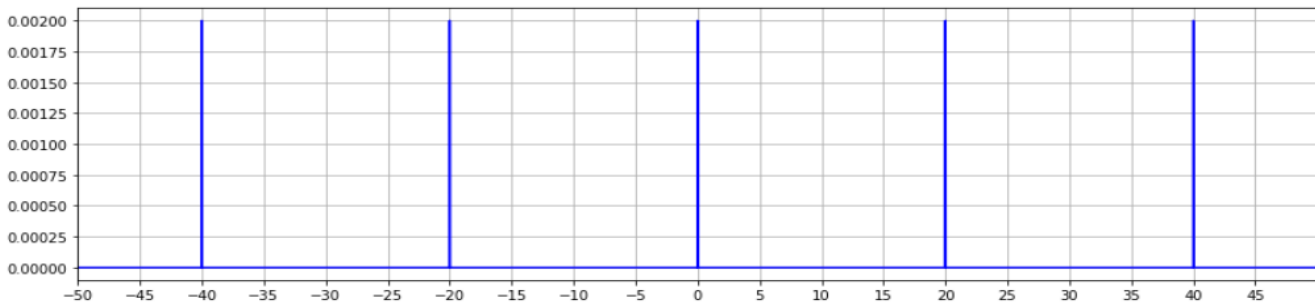


Figure 27: impulse train with  $F_s=20$  Hz

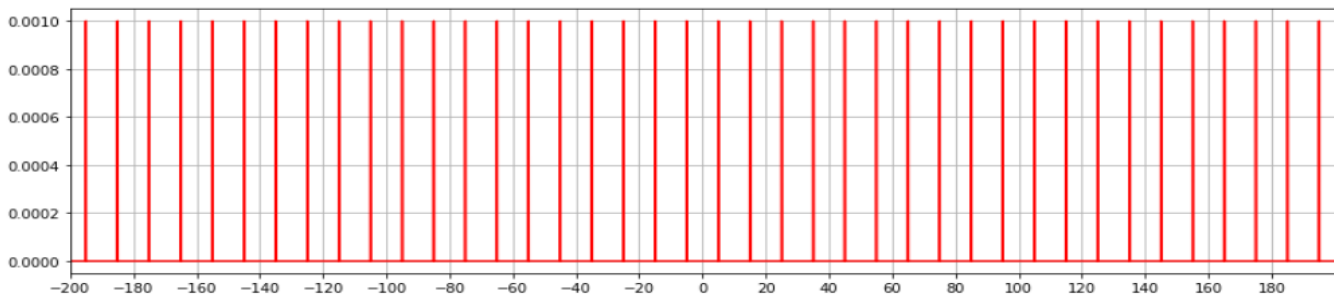


Figure 28: sampled signal

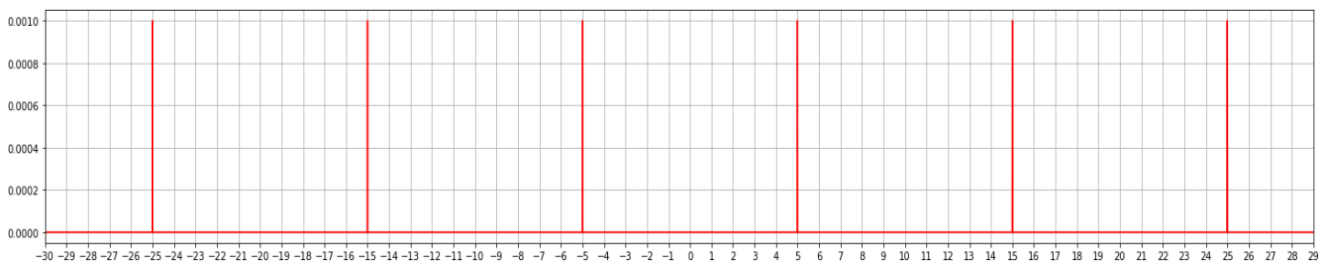


Figure 29: sampled signal zoomed in

- **Note:** We also note from the above figures that changing the value of  $f_m$  affects the location of the message frequency and affects the site of frequency when performing the conflation process, but it does not affect the site of frequency for the impulses and does not affect the amplitude for frequency in all of the above cases.

2- Now let's change  $f_m=5$  and plot all figures below:

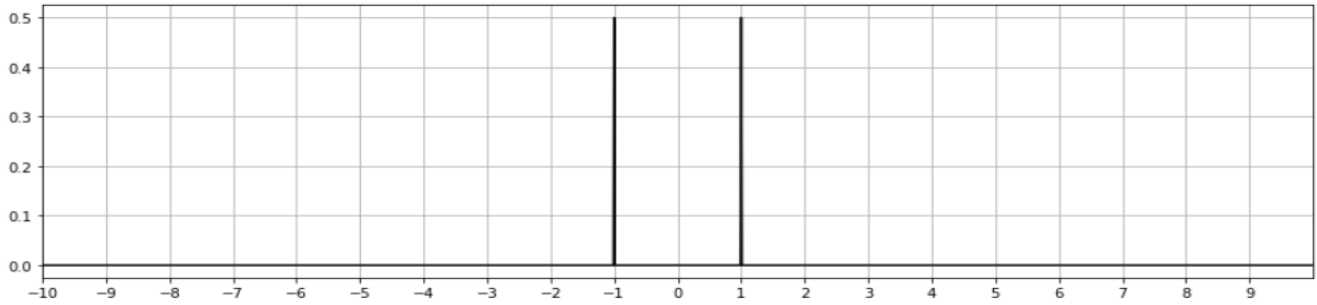


Figure 30::message signal with  $F_m=1$  Hz

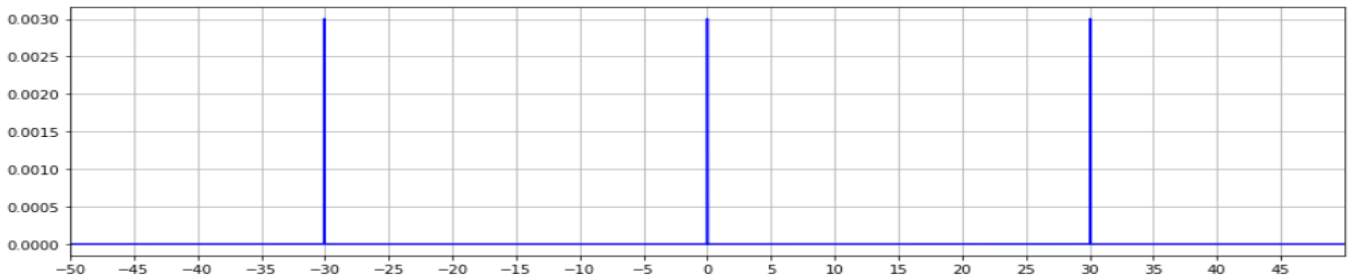


Figure 31:impulse train with  $F_s=30$  Hz

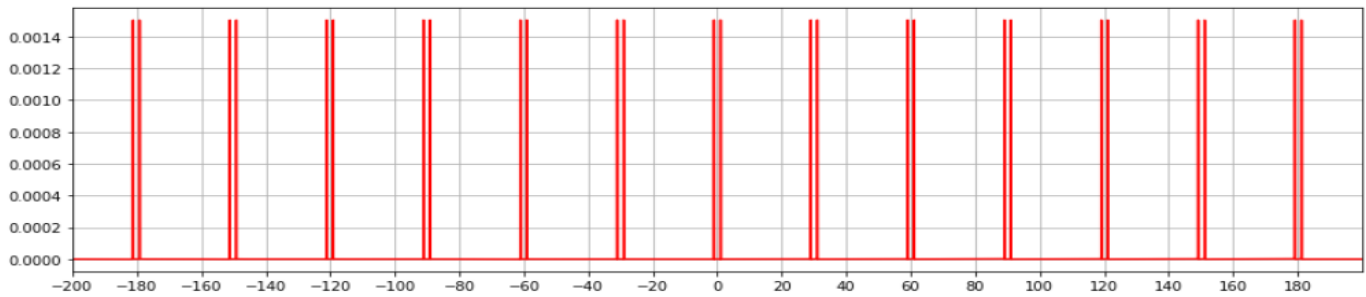


Figure 32:sampled signal

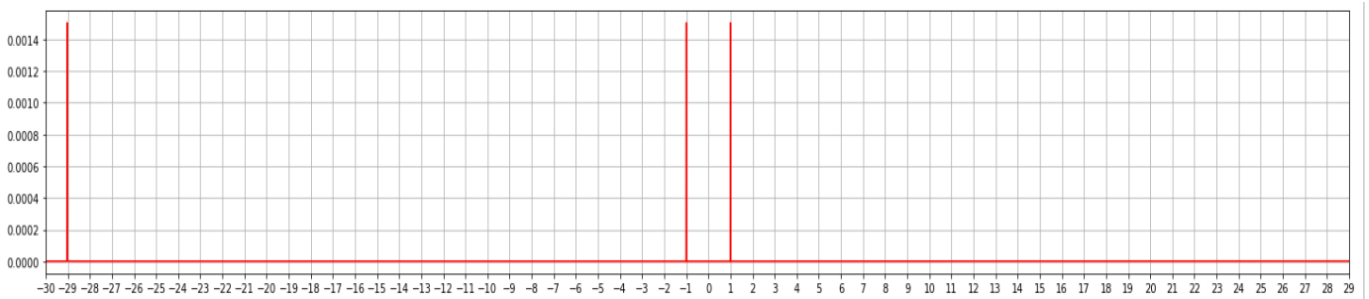


Figure 33:sampled signal zoomed in

- **Note:** We notice from the above figures that changing the value of  $f_c$  doesn't affect the location of the message frequency. But it effects on the site of frequency when performing the conflation process, (but if we focus on this matter, then the change of  $f_s$  has affected the location of all pulses frequency except for frequency which at zero was not affected). In addition to, it effects on the site of frequency for the impulses. Also it effects on the amplitude for frequency in sample signal and impulse signal frequency.

### 2.3.3 Signal Reconstruction (LPF):

In this section we will use Low Pass filter to reconstruct the message signal from the samples. So we have to define a Low Pass filter with Bandwidth equal to 3dB.

We define a LPF with cut-off frequency =1000Hz , forder=3 and plot it

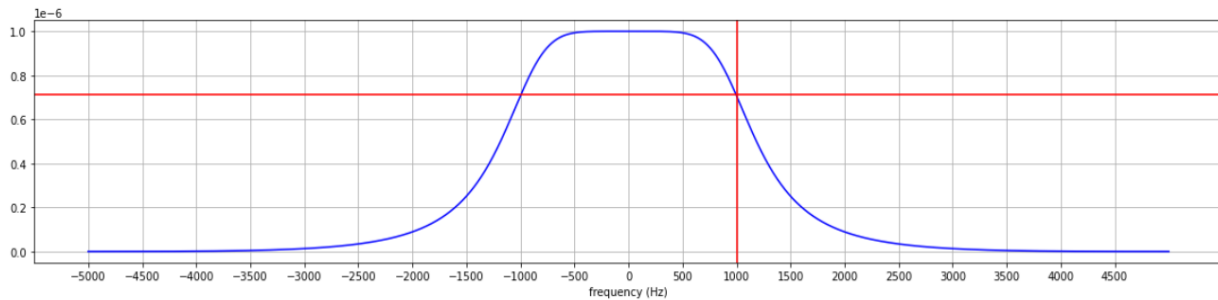


Figure 34:LPF cut-off = 1000 Hz

- **Note:** we can show from above figure the magnitude of the filter will be 0.73 at  $f=1000$  Hz which is the cut-off frequency.

#### Exercise:

Change the cut-off frequency and Let it  $f=2000$  Hz.

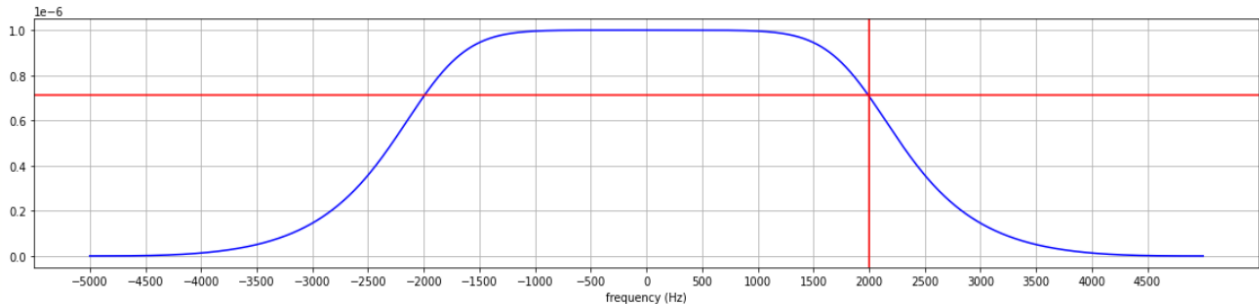


Figure 35:LPF cut-off = 2000 Hz

- **Note:** we can show from above figure If we change the cut-off frequency of the LPF to 2000, the magnitude of the filter will be 0.73 at  $f=2000$ .

Now let's apply the LPF to the sampled signal trying to reconstruct the message that sampled with train of impulses of  $F_s=40$ , we will show it in Figure 32 in time domain and frequency domain.

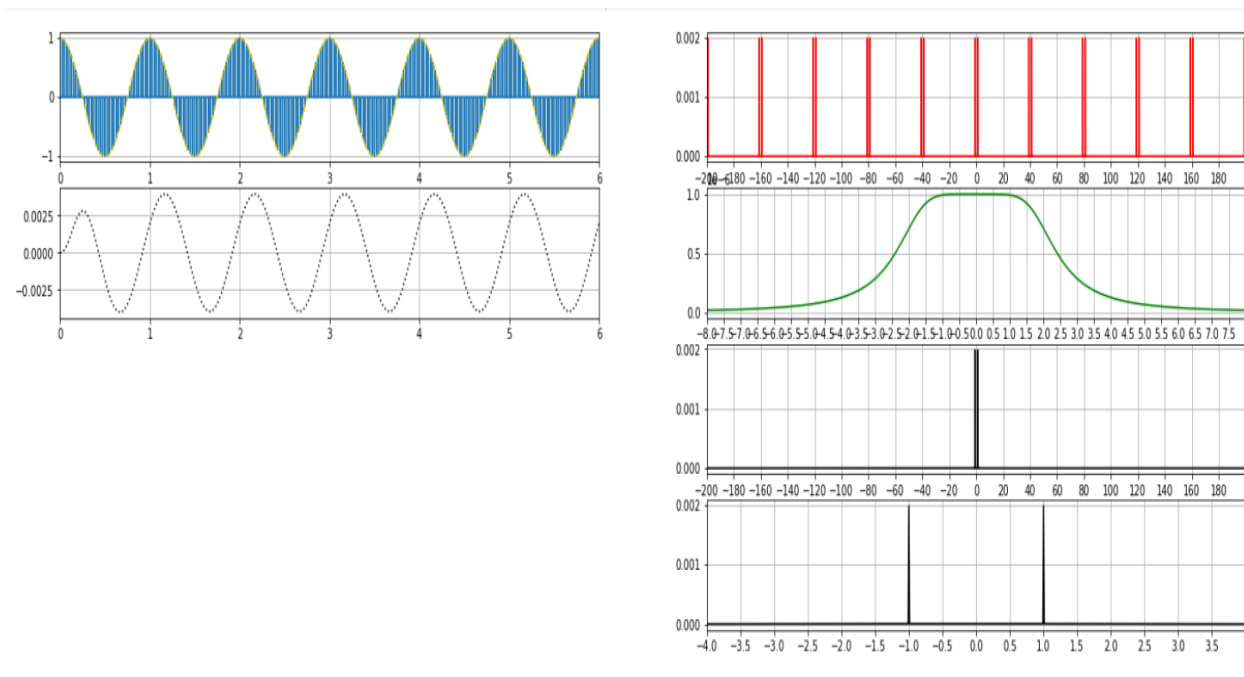


Figure 36:sampled and reconstructed signals in time and frequency domain

- **Note:** In the above figure, we see two parts, the left part represents the sampled signal in the top then the reconstructed signal in the bottom and we can see at the beginning of recovered message some of distortion that because the type of filter but this has no effect. while the left part represents the sampled signal 1<sup>st</sup> from the top, then the LPF with cut-off =2, then the reconstructed signal, then the reconstructed signal zoomed in. We could reconstruct the message signal as it was before sampling because of that the LPF is with cut-off frequency less than  $f_s - f_m$  and bigger than  $f_m$ .

**Exercise:**

Now we need change the cut off frequency and let it cut-off =4

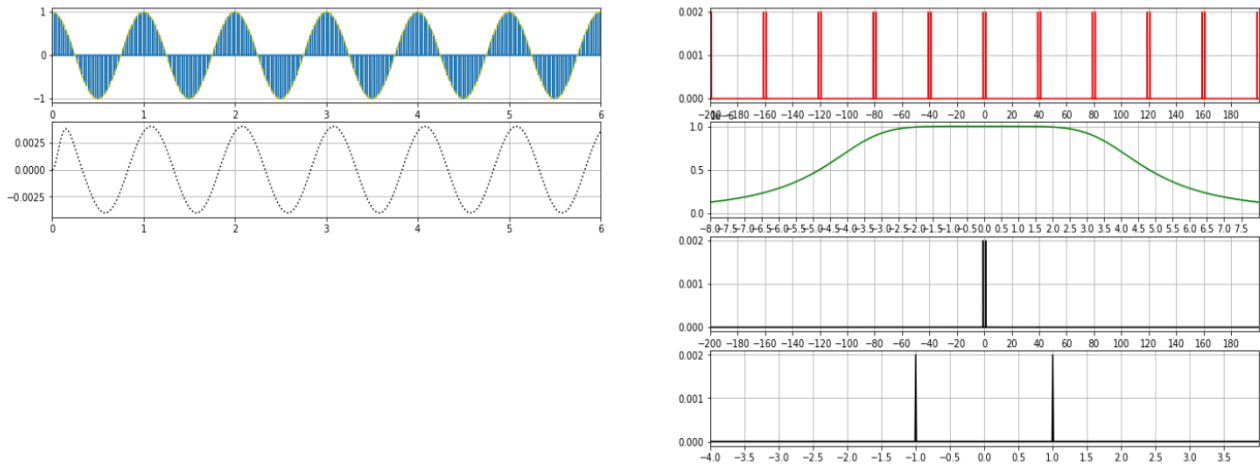


Figure 37:sampled and reconstructed signals in time and frequency domain, when cut-off=4

- **Note:** we can notice we are still able reconstruct the message signal as it was before sampling because of that the LPF is with cut-off frequency less than  $F_s - F_m$  and bigger than  $F_m$ . In the end, as we note after the introduction of the sampled signal to the LPF, we were able to recovered the original message, but there is some difference in amplitude and this is something that can be treatment through the use of amplifier.

Let us consider a new message signal  $m(t) = A_m1 \cos(2\pi f_{m1}t) + A_m2 \cos(2\pi f_{m2}t)$

With  $f_{m1} = 1$ ,  $A_{m1} = 1$ ,  $f_{m2} = 2$ ,  $A_{m2} = 1$ , sampled with  $f_s = 4$ , as we did before, I will show all of this in time and frequency domain.

We made the reconstruction by LPF of cut-off frequency = 8

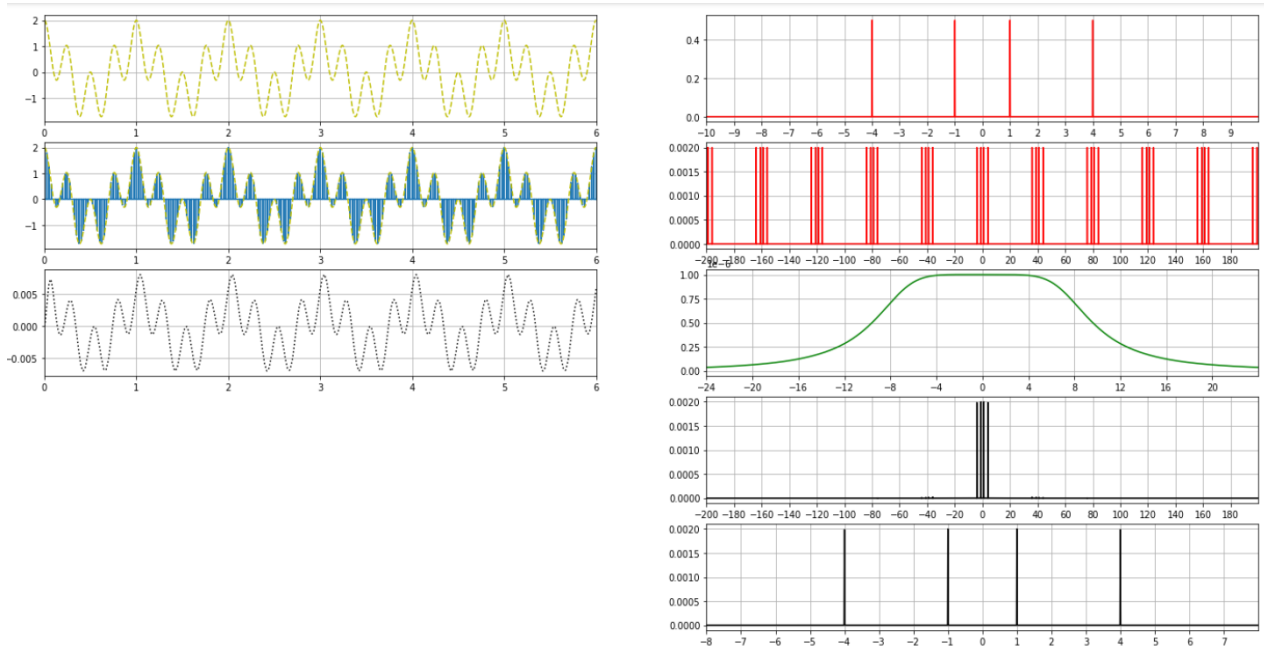


Figure 38: sampled and reconstructed signals in time and frequency domain

- Note:** In the above figure, we see two parts, the left part represents message signal in 1<sup>st</sup> plot from the top then the sampled signal and then the reconstructed signal. While the right part represents the message signal 1<sup>st</sup> from the top, then sampled signal, then the LPF with cut-off = 8, then the reconstructed signal, then the reconstructed signal zoomed in. We can see that we could reconstruct the signal because the cut-off frequency of the LPF is bigger than  $\max\{F_{m1}, F_{m2}\}$  and less than  $F_s - \max\{F_{m1}, F_{m2}\}$ . In the end, as we note after the introduction of the sampled signal to the LPF, we were able to recover the original message. Also, we can notice that an amplitude for each frequency are equal, where in this case amplitude for each frequency are equal as shown in reconstructed signal zoomed figure because  $A_{m1} = A_{m2} = 1$ .

In order to clarify this matter further, we will change the value of  $A_{m1}$  and  $A_{m2}$  as shown in the exercise below.

**Exercise:**

**1- Let us change the value of Am1 and Am2 and make them Am1=2, Am2=4**

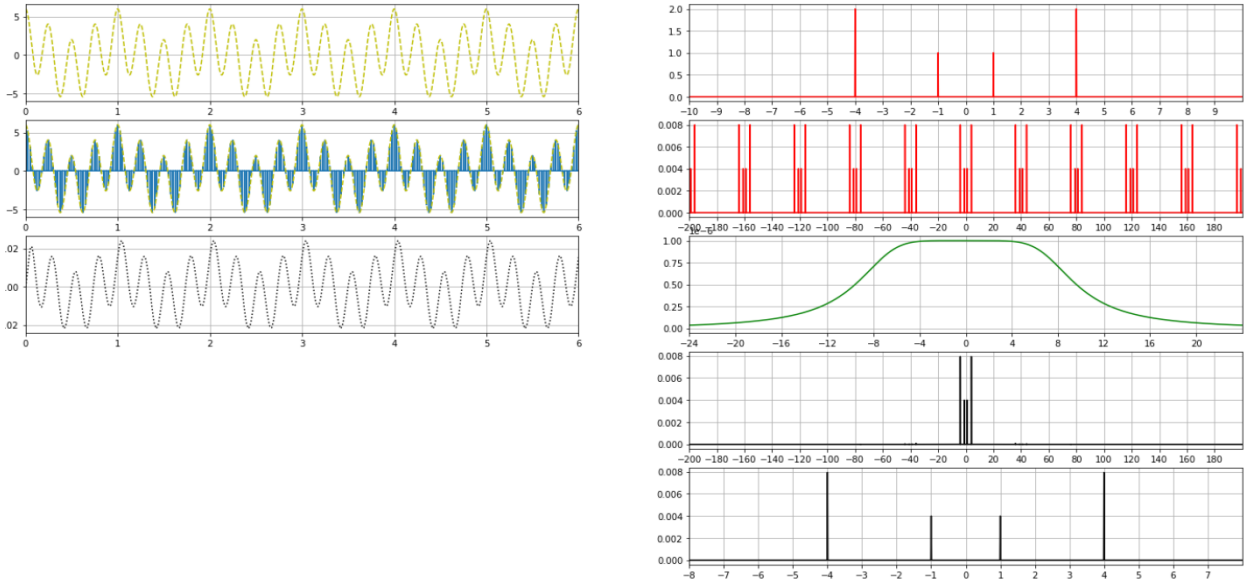


Figure 39:sampled and reconstructed signals in time and frequency domain when Am1=2,Am2=4

- **Note:** In this case, when we change the value of Am1 and Am2, we notice in the reconstructed signal zoomed figure the amplitude of two pulses of m1(t) are equal and the amplitude of two pulses of m2(t) are equal.

**2- Now let us try to make the LPF cut-off = 2 and see the output in**

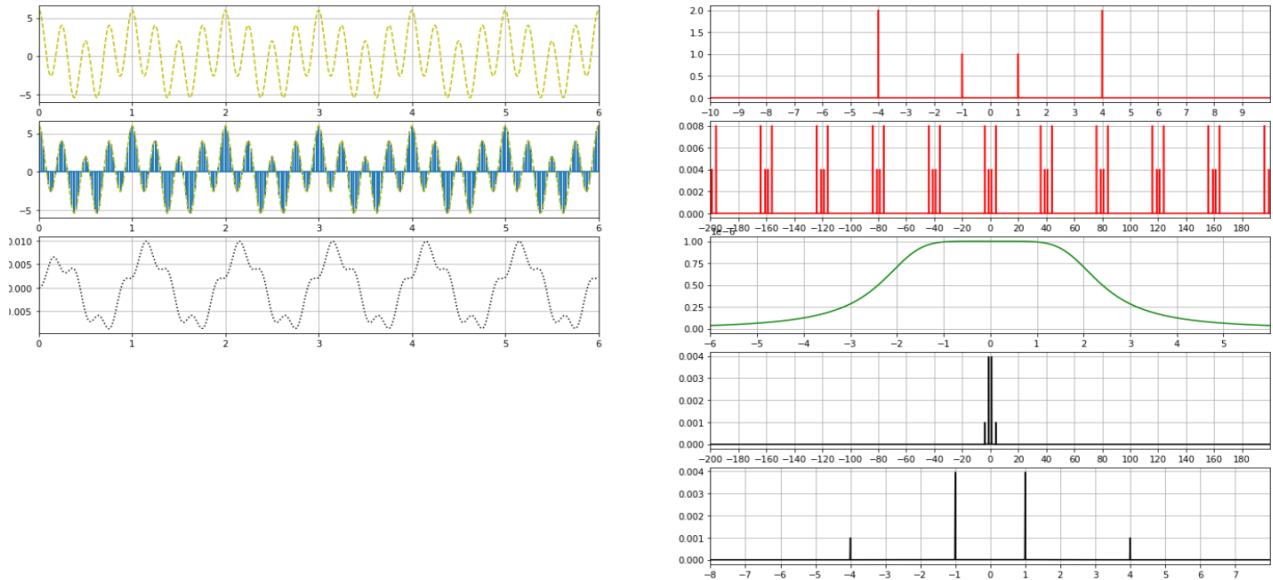


Figure 40:sampled and reconstructed signals in time and frequency domain when cutoff=2

- **Note:** We see from figure above that we couldn't reconstruct the message signal because that the cut-off frequency of LPF doesn't meet the condition -(the cut-off frequency of the LPF must be bigger than  $\max \{F_{m1}, F_{m2}\}$ )-.

### 3- Now let's change $f_m=8$

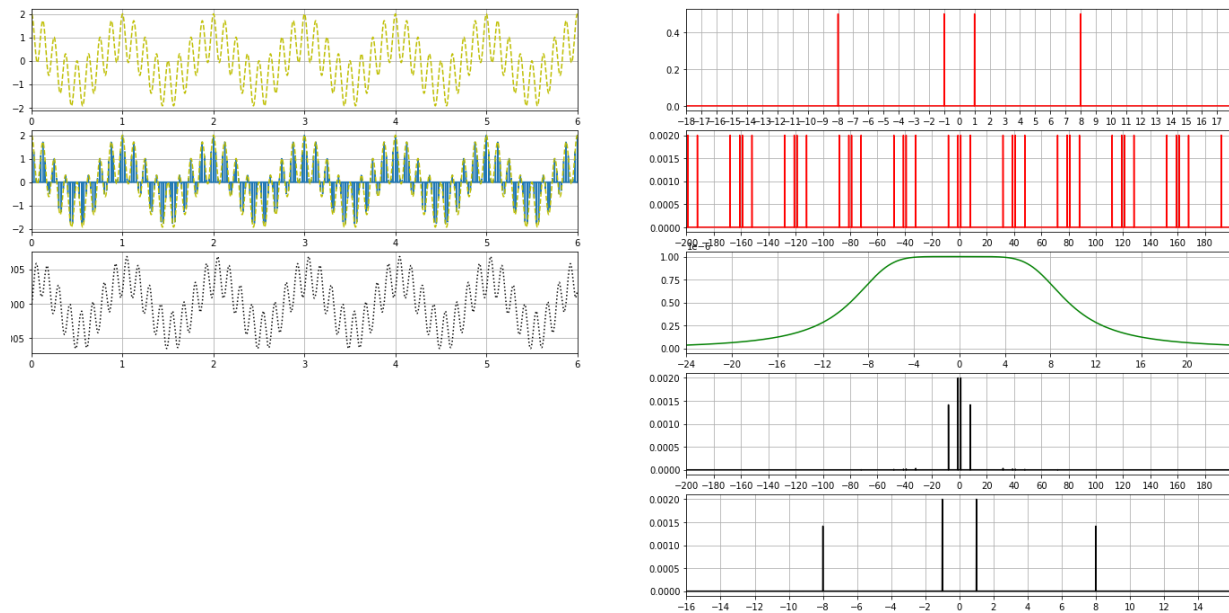


Figure 41: sampled and reconstructed signals in time and frequency domain with  $f_m=8$

- **Note:** in this case when increased the value for  $f_m$  we notice the amplitude for 4 components which are located around  $f_s=0$  they don't have the same value because the cut-off frequency is small, but we still can recover the message signal with some of different in amplitude (distortion).



**4- Now let's change  $fm2=8$  , and change cut-off frequency= $16$**

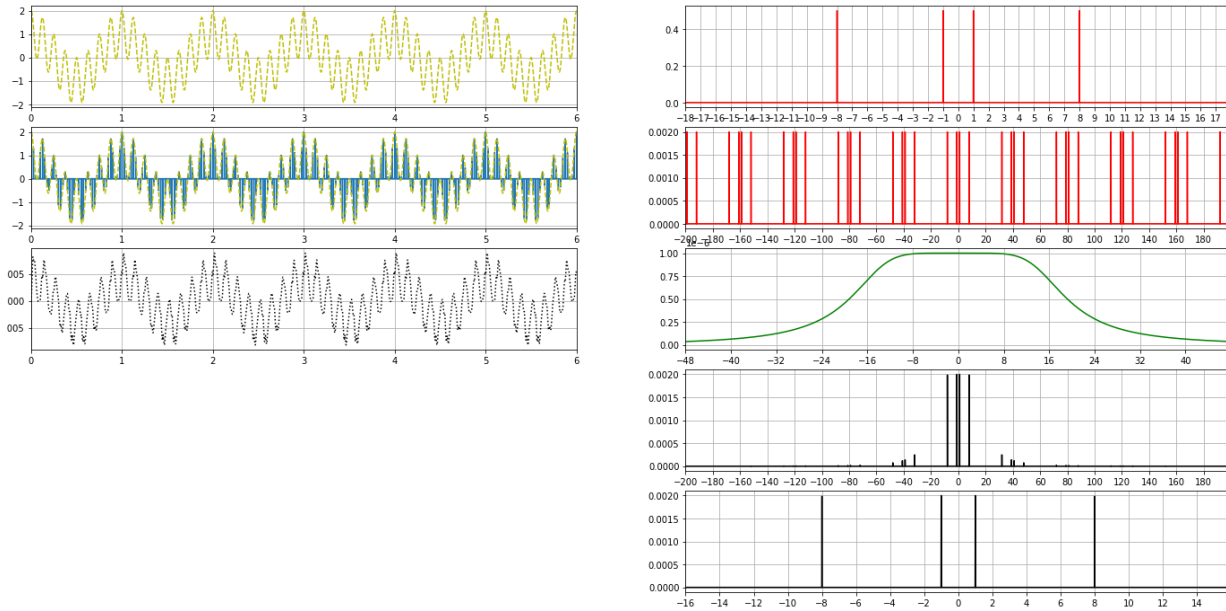


Figure 42:sampled and reconstructed signals in time and frequency domain with  $fm2=8$  and cut-off= $16$

- **Note:** in this case when increase the value of  $fm2$  and change the value of cut-off frequency for filter we notice the presence of a component of the Signal at around  $fs=40$  and  $fs=-40$  that passed through the filter because of the order of filter, so it was not able to make rejection of this component.

**5- Now let's see what will happen when we change filter order= $5$**

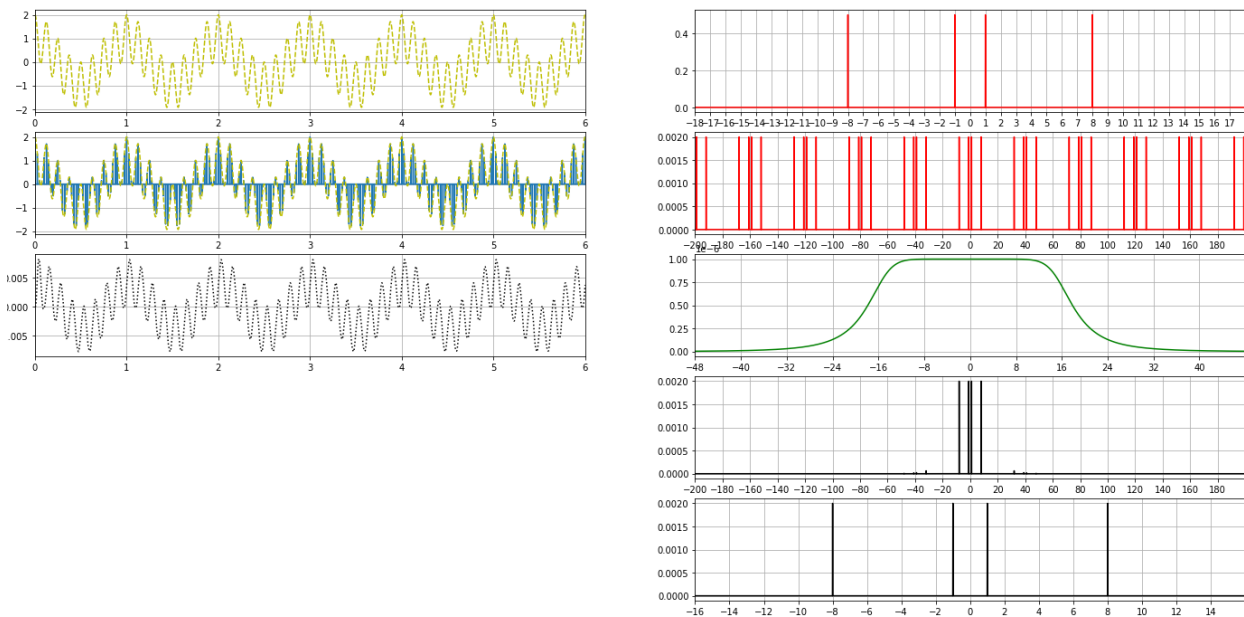


Figure 43:sampled and reconstructed signals in time and frequency domain with order= $5$

- **Note:** in this case when increase the value of filter order the sharpens of filter increased and The filter did not pass any component except the component that is included in the ring of the filter.

## 2.4 Natural Sampling in the Time and Frequency Domains:

In this section we will use natural sampling method to sample the signal  $m(t)=\cos(2\pi f_m t)$ . The sampled signal is generated by multiplying the message signal by a periodic pulse train.

### 2.4.1 Sampling in Time domain:

Firstly, we will sample a message of  $f_m=1$ ,  $A_m=1$  with train of pulses with  $dc=0.5$  and  $f_s=20$ .

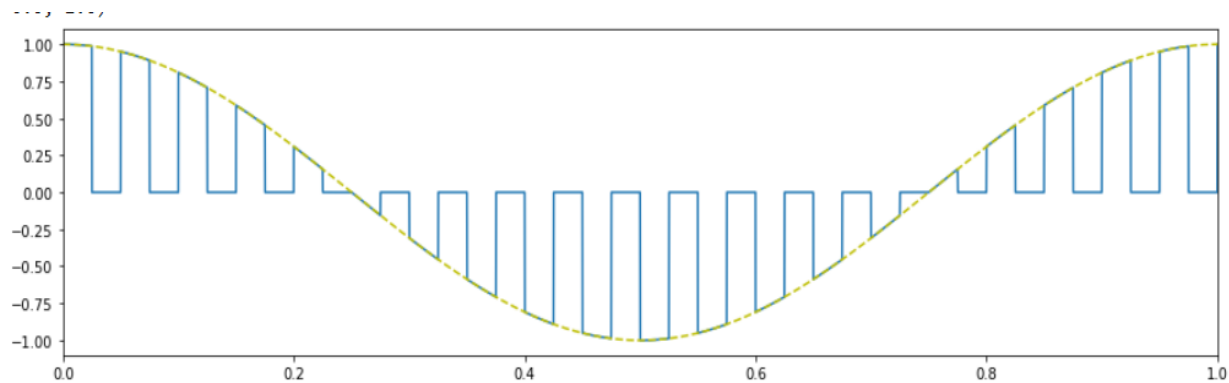


Figure 44:sampled signal  $f_s=20$  Hz

- **Note:** we can show from above figure when multiplying the message signal by an pulse train the amplitude of impulses are taking the same amplitude of message, and in this type of sampling the pulses have width(dc).

### Exercise:

Let's change the  $f_s$  to 5,10,30 and plot it below to see what is the effect of  $f_s$  on sampling:

#### 1- When $f_s=5$ Hz:

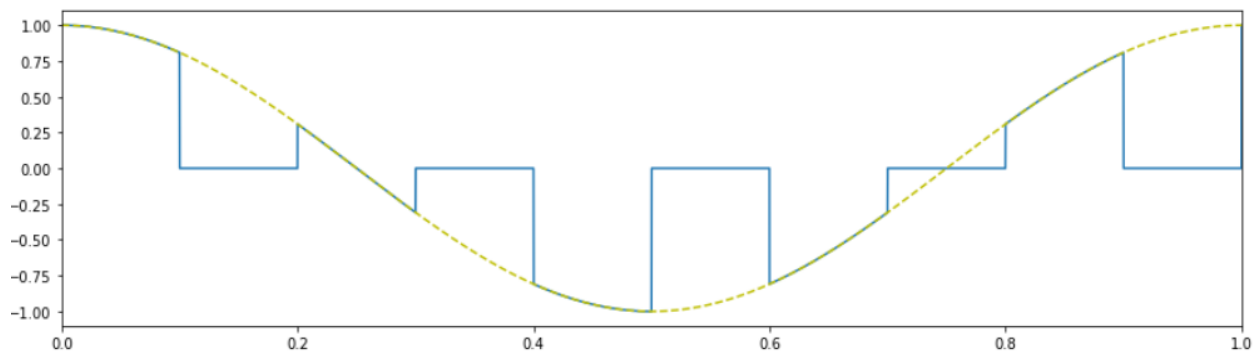


Figure 45:sampled signal  $f_s=5$  Hz

## 2- When $f_s=10$ Hz:

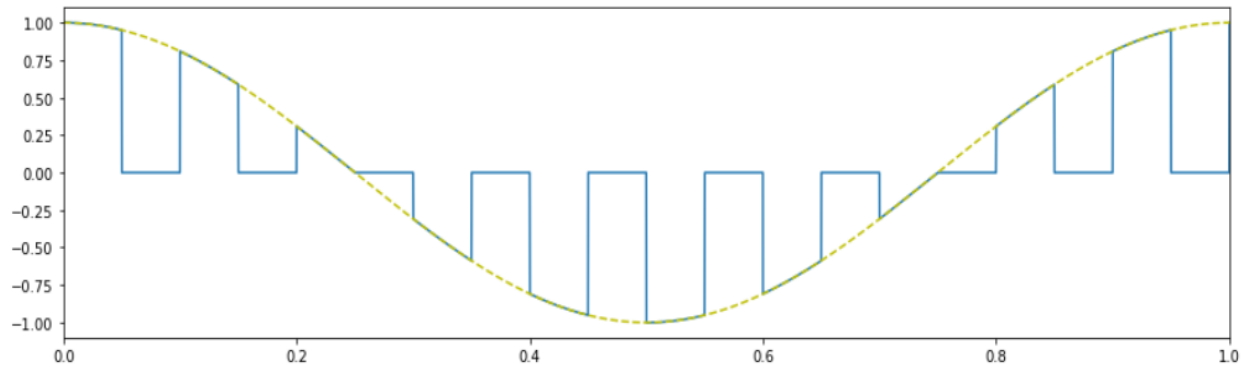


Figure 46:sampled signal  $f_s=10$  Hz

## 3- When $f_s=30$ Hz:

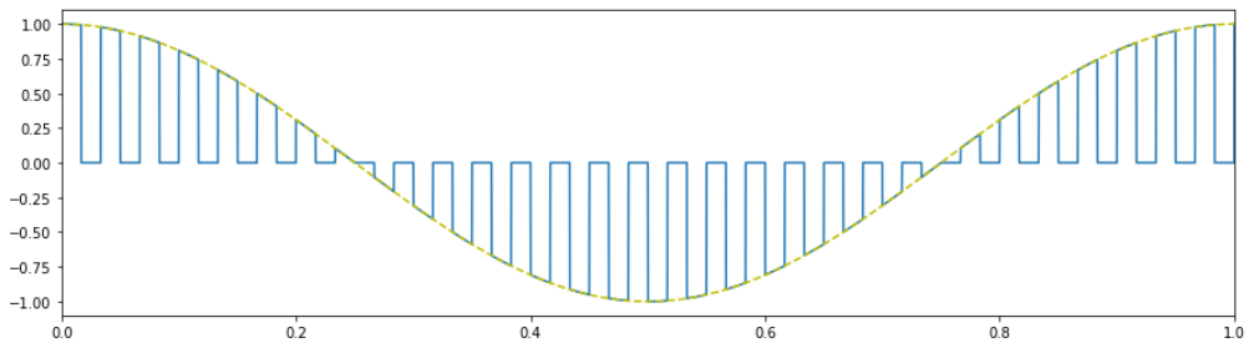


Figure 47:sampled signal  $f_s=30$  Hz

- **Note:** We observe from figures above that when we increase  $f_s$ , the number of samples that are taken in specific period will increase, but the range of sample that touch the message will decrease.

Now with  $f_s=10$  let's change the dc to 0.2, 0.7 and plot it below to see what is the effect of  $f_s$  on sampling:

**1- When  $dc=0.2$ :**

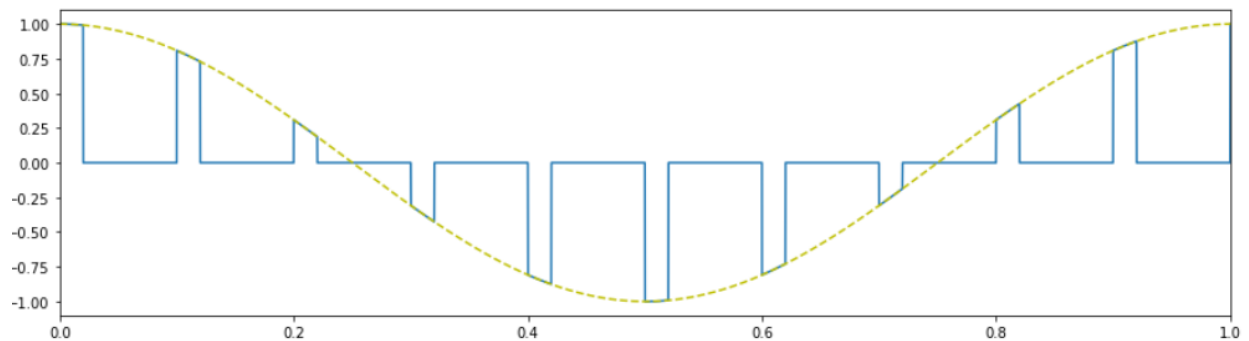


Figure 48:sampled signal  $dc=0.2$

**2- When  $dc=0.7$ :**

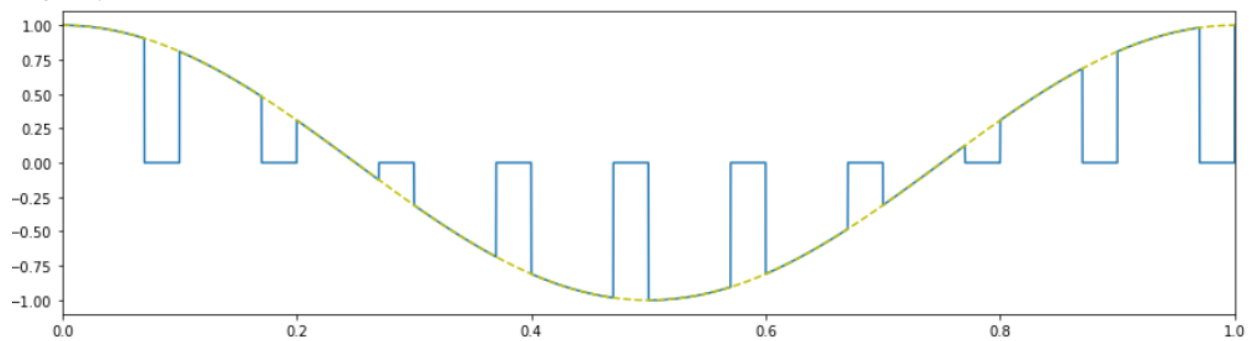


Figure 49:sampled signal  $dc=0.7$

- **Note:** We can observe from figures shown above that when we increase the duty cycle the range which the sample will be on will increase, so the touched period between the sample and the message will increase and vice versa.

## 2.4.2 Sampling in frequency domain:

Now we will see the same previous example in frequency domain, you can see what each Figure shows under it (all the figures are in frequency domain).

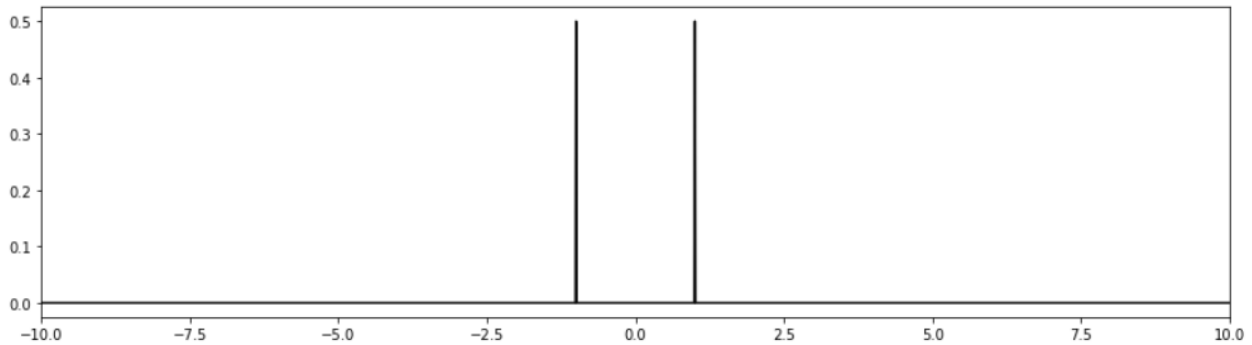


Figure 50: message signal

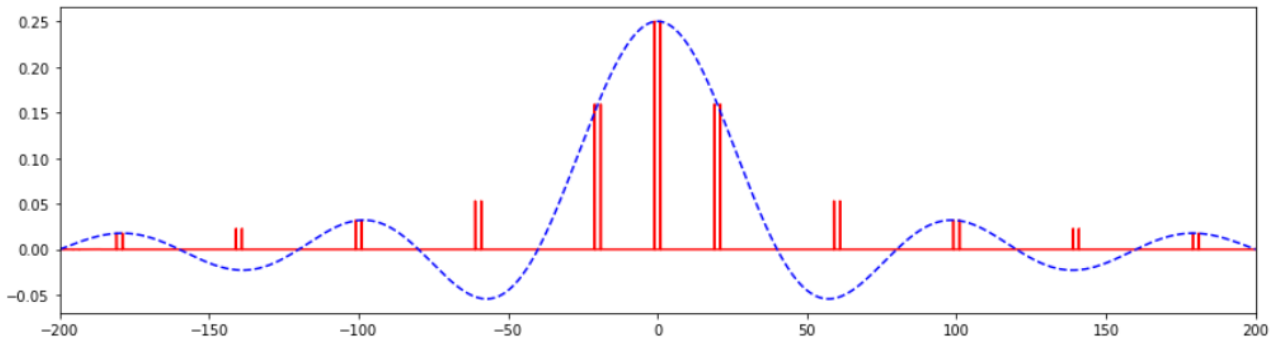


Figure 51: sampled signal

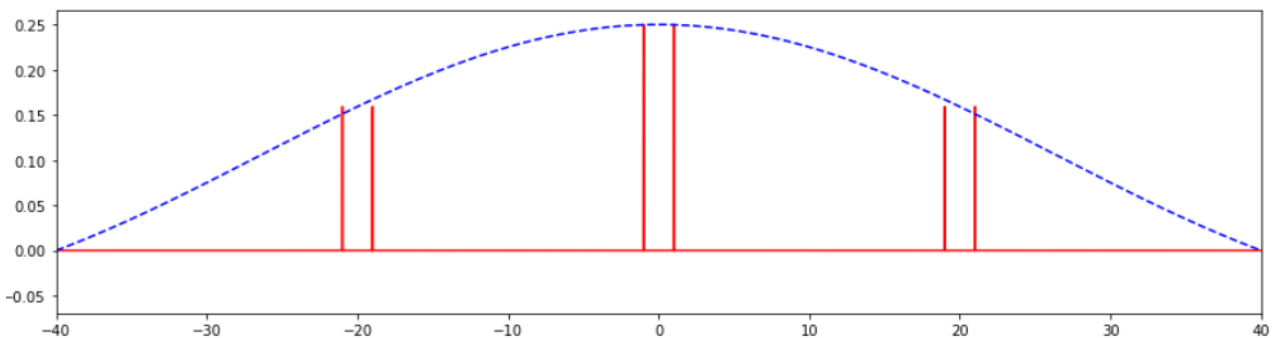


Figure 52: sampled signal zoomed in

- **Note:** We can observe the difference between the sampled signal in natural sampling and ideal sampling that all the pulses are equal in ideal sample, but in natural sampling the pulses follow the sinc function and every couple of pulses that centered on  $kFs$  where  $k$  is integer is equal. Also, if we notice in the case of natural sampling the amplitude for all pairs of  $\Delta$  that coming from the same Signal they have the same value of amplitude, but if we notice the frequency when the value 21 Hz and -21 Hz it does not take the same shape of sinc function.

### 2.4.3 Signal Reconstruction:

In this section we will use Low Pass filter to reconstruct the message signal from the samples. So we have to define a Low Pass filter with Bandwidth equal to 3dB.

Now let's apply the LPF to the sampled signal trying to reconstruct the message that sampled with natural sample method of  $f_s=40$ ,  $d_c=0.5$ , we will show it in time and frequency domain.

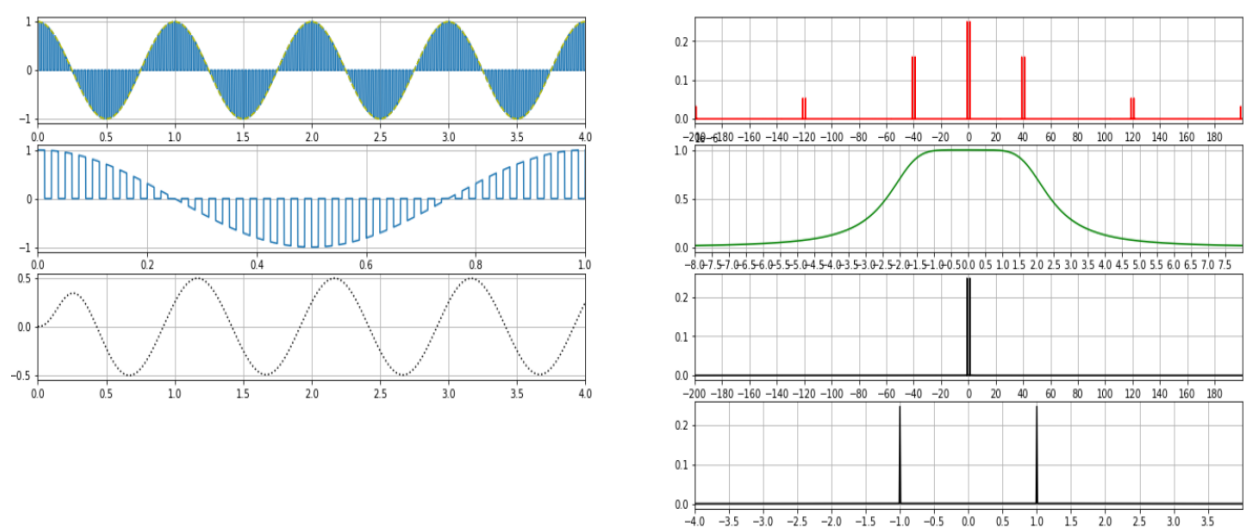


Figure 53:sampled and reconstructed signals in time and frequency domain

- Note:** In the above figure, we see two parts, the left part represents the sampled signal in the top then then zoom-in of sampled signal, then the reconstructed signal in the bottom and we can see at the beginning of recovered message some of distortion that because the type of filter but this has no effect. while the right part represents the sampled signal 1<sup>st</sup> from the top, then the LPF with cut-off =2, then the reconstructed signal, then the reconstructed signal zoomed in. We could reconstruct the message signal as it was before sampling because of that the LPF is with cut-off frequency less than  $f_s/2$  and bigger than  $f_m$ . In the end, as we note after the introduction of the sampled signal to the LPF, we were able to recover the original message, but there is some difference in amplitude and this is something that can be treated through the use of amplifier.

Now Let use consider a new message signal  $m(t)=A_{m1}\cos(2\pi f_{m1}t)+A_{m2}\cos(2\pi f_{m2}t)$ .  
 With  $f_{m1} = 1$ ,  $A_{m1} = 1$ ,  $f_{m2} = 2$ ,  $A_{m2} = 1$ , sampled with  $f_s=40$  and  $dc=0.5$ , as we did before, I will show all of this in time and frequency domain.

We made the reconstruction by LPF of cut-off frequency = 8

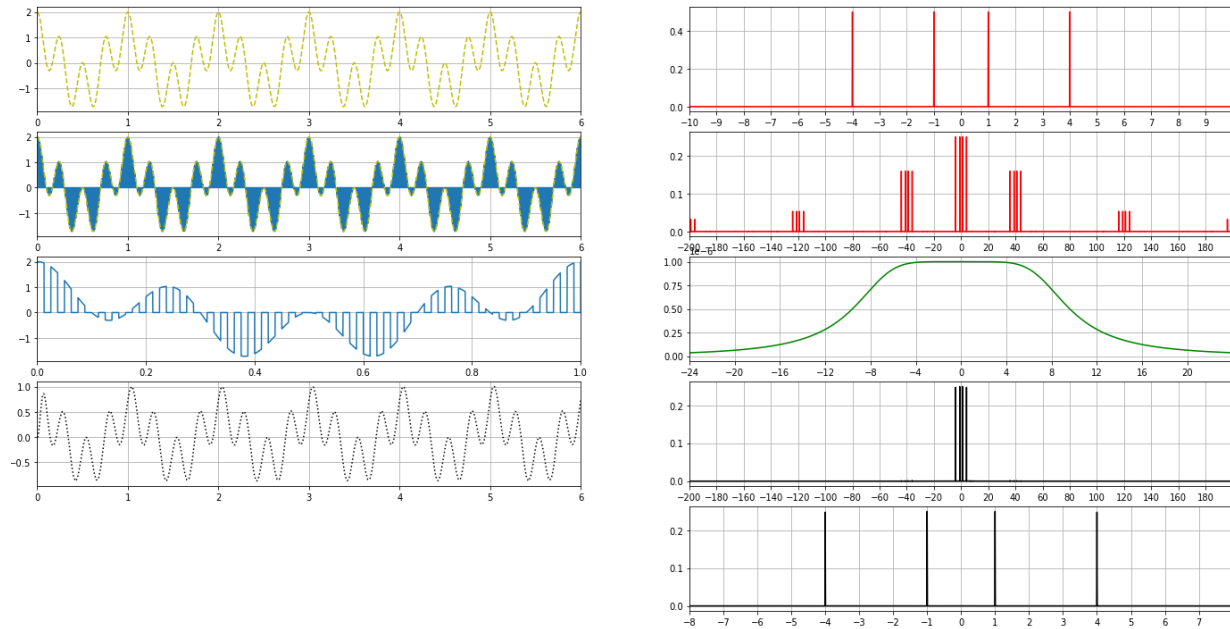


Figure 54:sampled and reconstructed signals in time and frequency domain

- Note:** In the above figure, we see two parts, the left part represents message signal in 1<sup>st</sup> plot from the top then the sampled signal then zoomed-in sampled signal, then the reconstructed signal. While the left part represents the message signal 1<sup>st</sup> from the top, then sampled signal, then the LPF with cut-off =8, then the reconstructed signal, then the reconstructed signal zoomed in. we can see that we could reconstruct the signal because the cut-off frequency of the LPF is bigger than  $\max\{F_{m1},F_{m2}\}$  and less than  $F_s-\max\{F_{m1},F_{m2}\}$ .In the end, as we note after the introduction of the sampled signal to the LPF, we were able to recovered the original message, Also, we can notice that an amplitude for each frequency are equal, where in this case amplitude for each frequency are equal as shown in reconstructed signal zoomed figure because  $A_{m1}=A_{m2}=1$ .  
 In order to clarify this matter further, we will change the value of  $A_{m1}$  and  $A_{m2}$  as shown in the exercise below.

**Exercise:**

**1- Let us change the value of Am1 and Am2 and make them Am1=2, Am2=4**

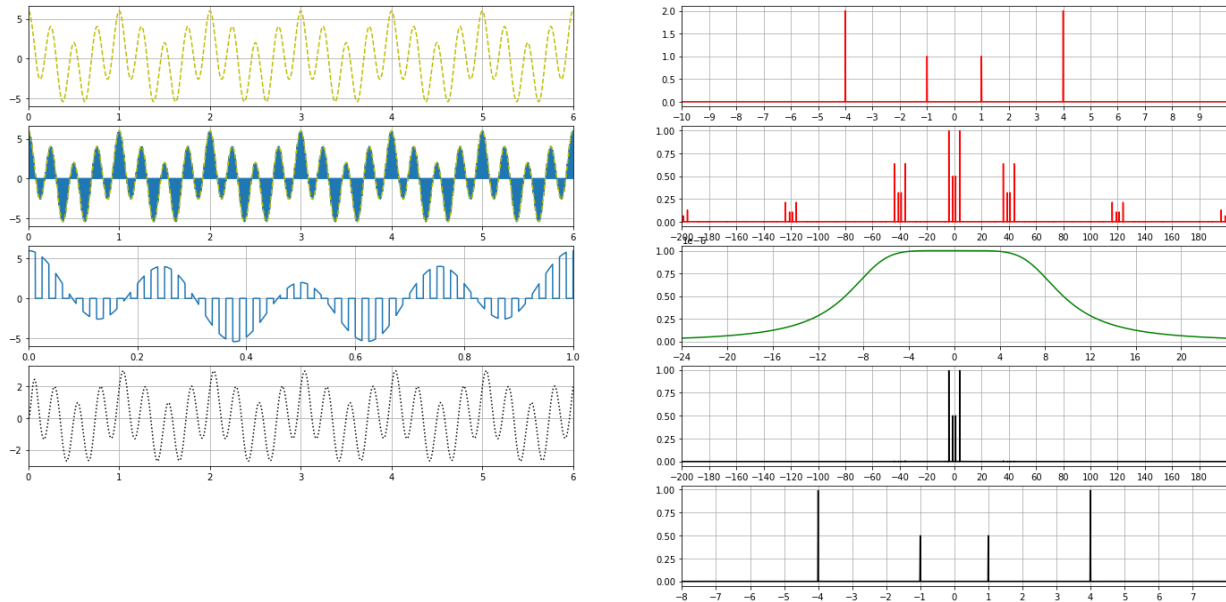


Figure 55:sampled and reconstructed signals in time and frequency domain when Am1=2,Am2=4

- **Note:** In this case, when we change the value of Am1 and Am2, we notice in the reconstructed signal zoomed figure the amplitude of two pulses of m1(t) are equals and the amplitude of two pulses of m2(t) are equal.

**2- Let us transgress this by making the LPF cut-off = 3 and see the output:**

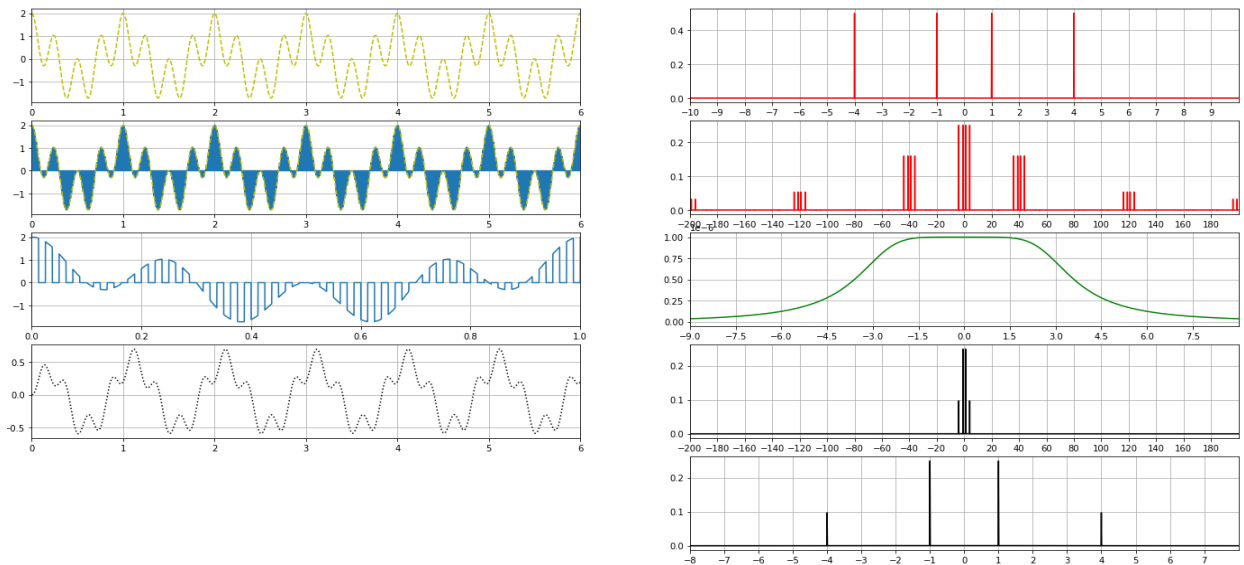


Figure 56:sampled and reconstructed signals in time and frequency domain when cut-off=3

- **Note:** We see from figure above that we couldn't reconstruct the message signal because that the cut-off frequency of LPF doesn't meet the condition -(the cut-off frequency of the LPF must be bigger than max {Fm1, Fm2})-.



### 3- Now let's change $fm2=8$

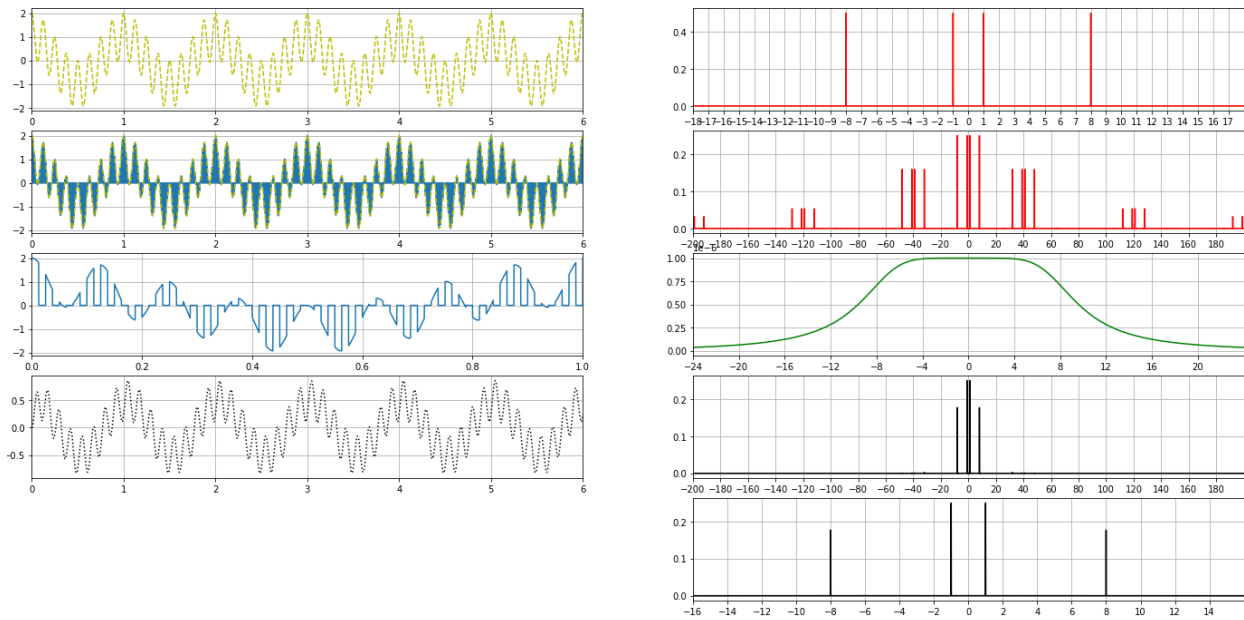


Figure 57: sampled and reconstructed signals in time and frequency domain with  $fm2=8$

- **Note:** in this case when increased the value for  $fm$  we notice the amplitude for 4 components which are located around  $fs=0$  they don't have the same value because the cut-off frequency is small, but we still can recover the message signal with some of different in amplitude (distortion).

**4- Now let's change  $f_m=8$  , and change cut-off frequency= 16**

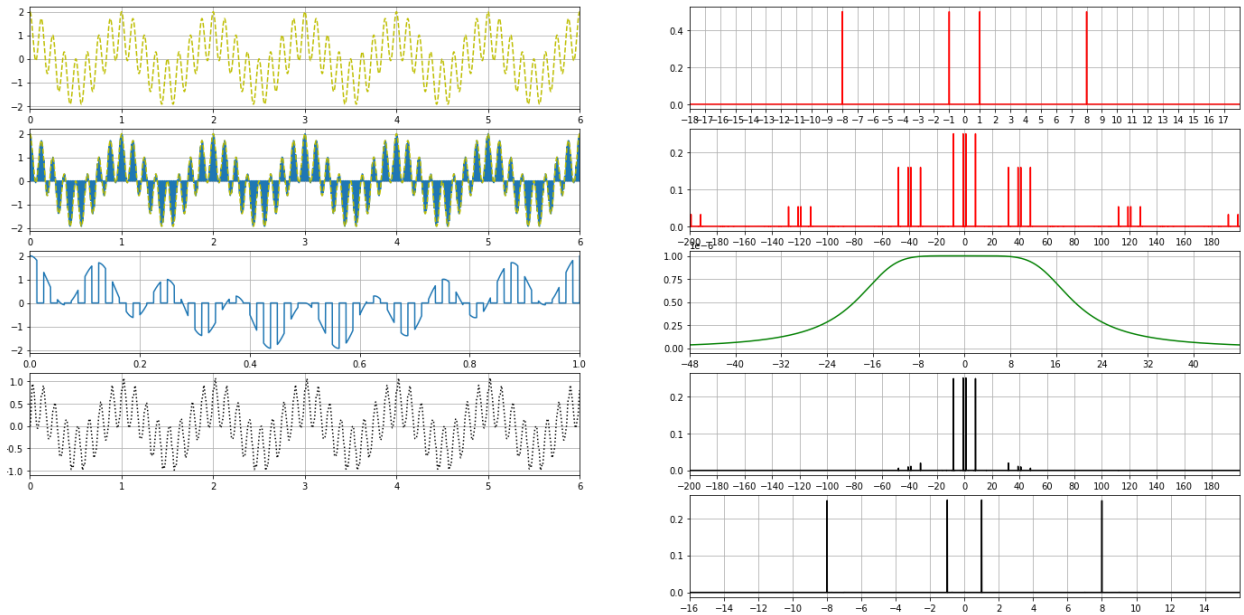


Figure 58:sampled and reconstructed signals in time and frequency domain with  $f_m=8$  and cut-off=16

- **Note:** in this case when increase the value of  $f_m$  and change the value of cut-off frequency for filter we notice the presence of a component of the Signal at around  $f_s=40$  and  $f_s=-40$  that passed through the filter because of the order of filter, so it was not able to make rejection of this component.

**5- Now let's see what will happen when we change filter order=5**

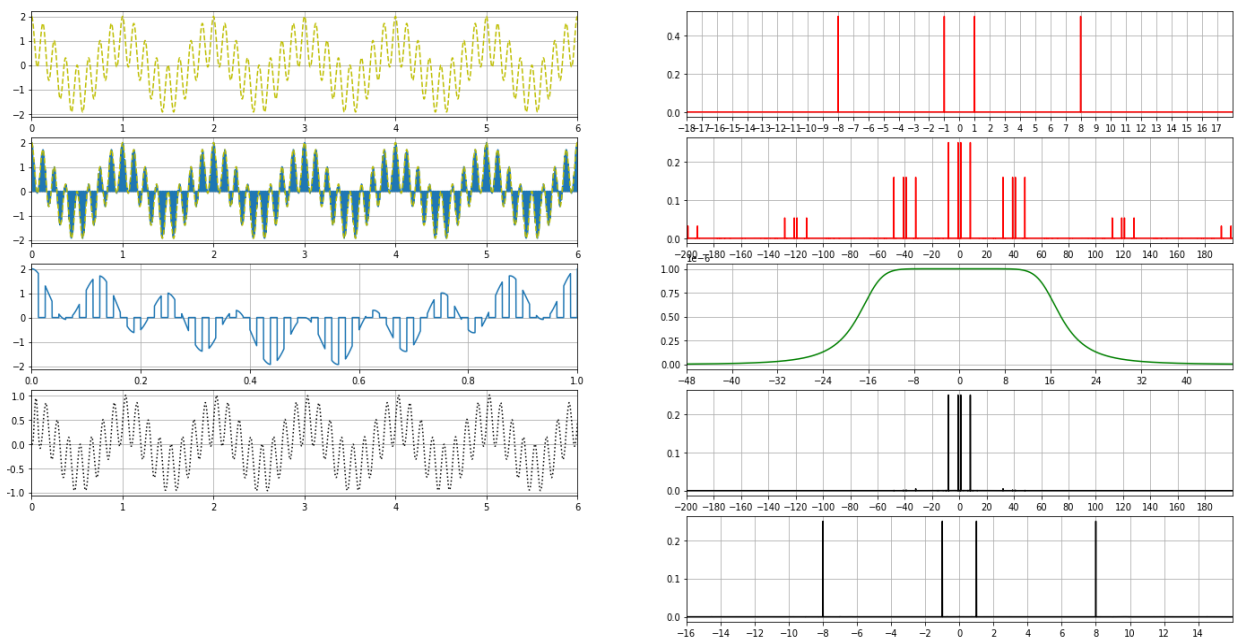


Figure 59:sampled and reconstructed signals in time and frequency domain with order=5

- **Note:** in this case when increase the value of filter order the sharpens of filter increased and The filter did not pass any component except the component that is included in the ring of the filter.

## 2.5 Flat-Topped Sampling in the Time and Frequency Domains:

In this section we will use flat-topped sampling method to sample the signal  $m(t)=\cos(2\pi f_m t)$ , the sampled signal is generated using the sample and hold method, which means that the message signal is sampled at time  $T_s$ , and the value of the signal is held for a fraction of the sampling period that can be controlled using the duty cycle.

### 2.5.1 Sampling in Time domain:

Firstly, we will sample a message of  $f_m=1$ ,  $A_m=1$  with train of pulses with  $dc=0.5$  and  $f_s=20$ .

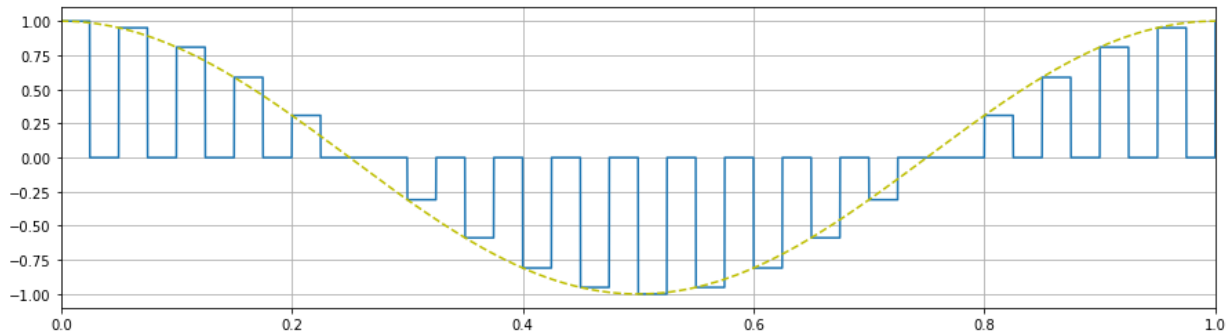


Figure 60:sampled signal

- **Note:** we can show from above figure when multiplying the message signal by pulse train the amplitude of the impulses Don't take the same amplitude of message, and in this type of sampling the pulses have width( $dc$ ).

### Exercise:

Let's change the  $f_s$  to 5,10,30 and plot it below to see what is the effect of  $f_s$  on sampling:

#### 1- When $f_s=5$ Hz:

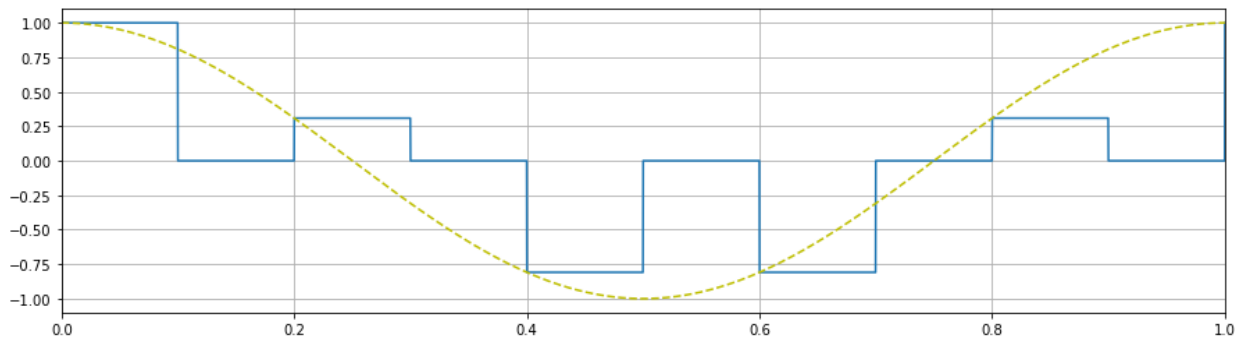


Figure 61:sampled signal with  $f_s=5$  Hz

## 2- When $f_s=10$ Hz:

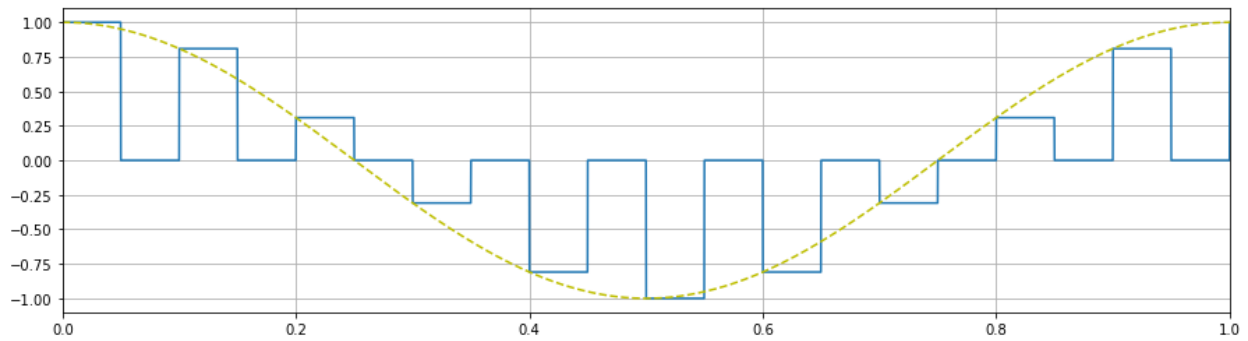


Figure 62:sampled signal with  $f_s=10$  Hz

## 3- When $f_s=30$ Hz:

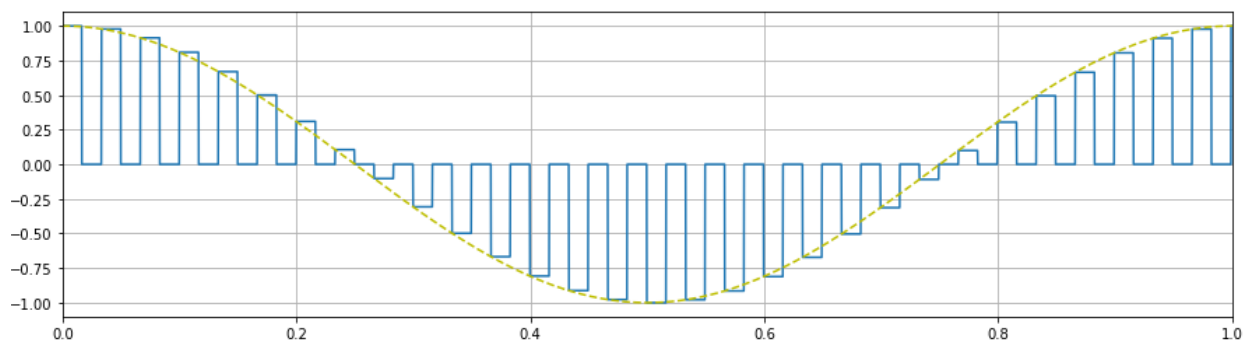


Figure 63:sampled signal with  $f_s=30$  Hz

- **Note:** We observe from figures above that when we increase  $f_s$ , the number of samples that are taken in specific period will increase, but the range of sample that touch the message will decrease.

Now with  $F_s=10$  let's change the D.C to 0.2, 0.7 and plot it below to see what is the effect of  $F_s$  on sampling:

**1- When  $dc=0.2$ :**

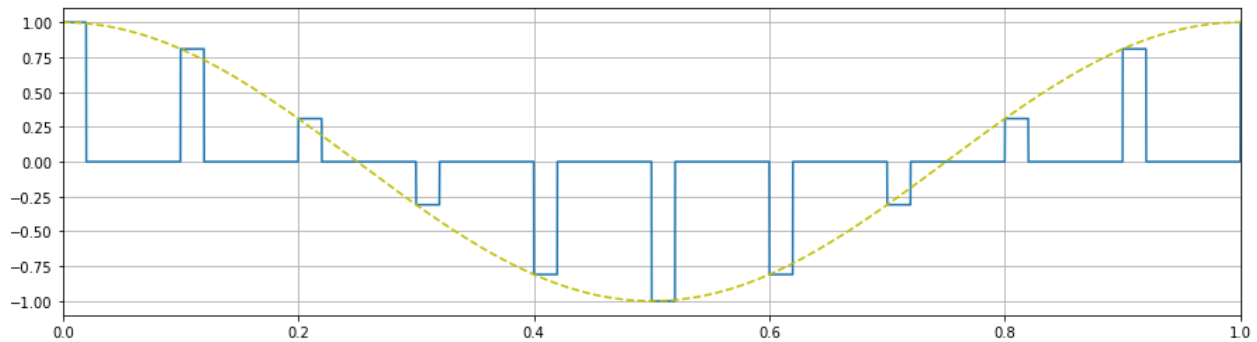


Figure 64:sampled signal with  $dc=0.2$

**2- When  $dc=0.7$ :**

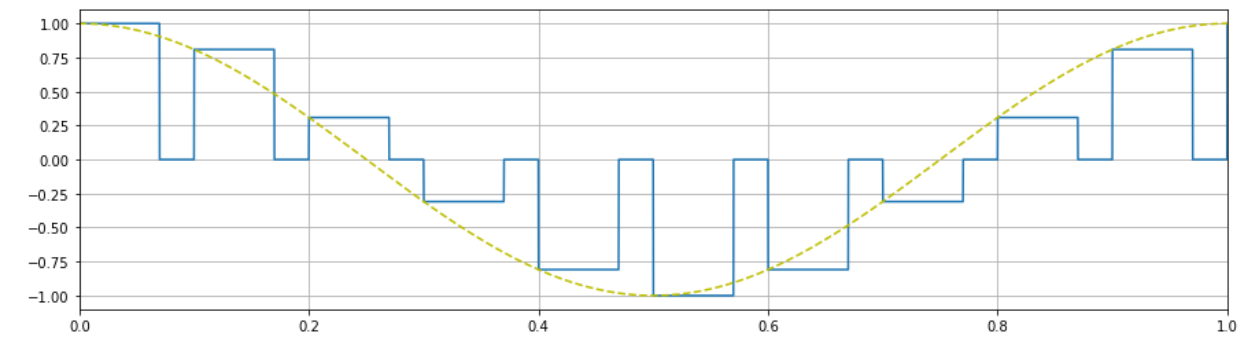


Figure 65:sampled signal with  $dc=0.7$

- **Note:** We can observe from figures shown above that when we increase the duty cycle the range which the sample will be on will increase, so the touched period between the sample and the message will increase and vice versa.

## 2.4.2 Sampling in frequency domain:

Now we will see the same previous example ( $F_s=10$ ,  $dc=0.5$ ) in frequency domain, you can see what each Figure shows under it (all the figures are in frequency domain).

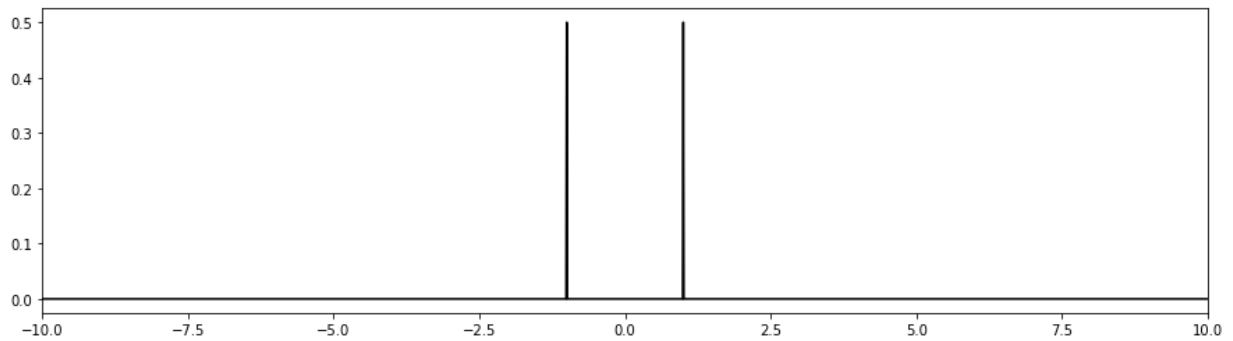


Figure 66: message signal

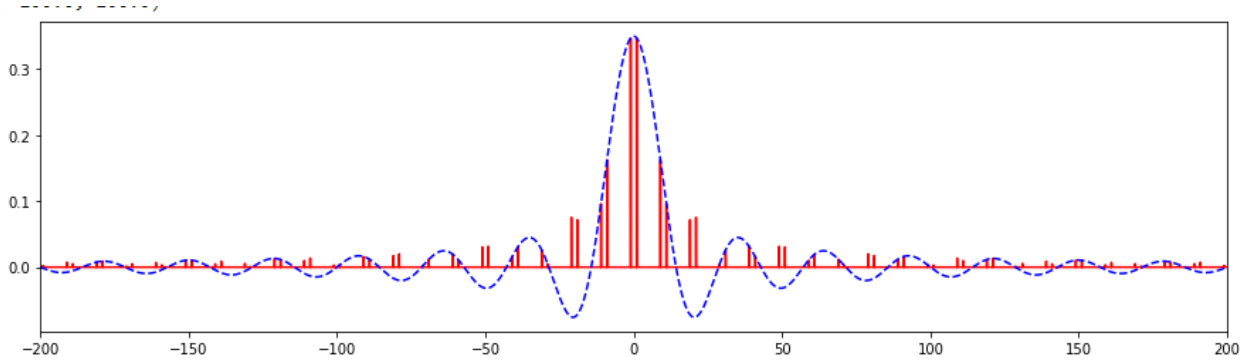


Figure 67: sampled signal in flat-topped method

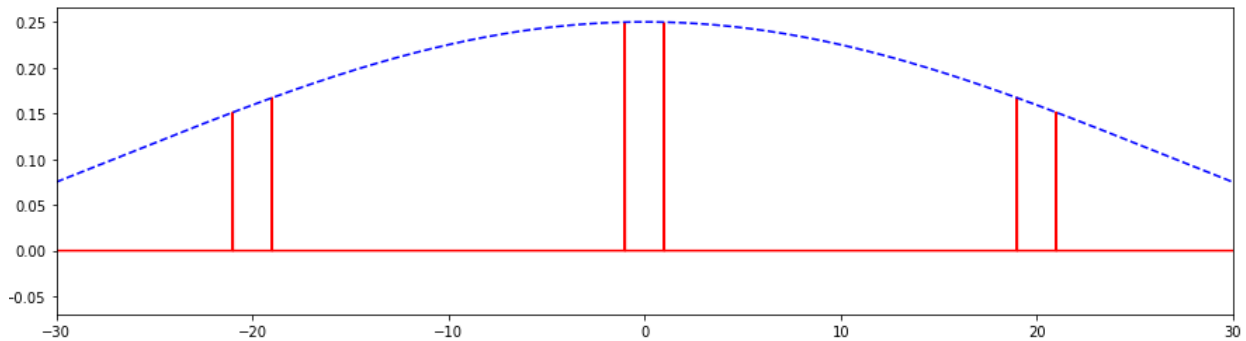


Figure 68: sampled signal zoomed-in

- **Note:** We can observe the difference between the sampled signal in flat-topped sampling and ideal sampling, that all the pulses are equal in ideal sampling, but in flat-topped sampling the pulses follow the sinc function and every couple of pulses that are centered on  $kF_s$  where  $k$  is an integer is not equal and follows the sinc function. Except when  $F_s$  equals zero, we can see that the two impulses are equal, so we can recover the signal without any extra thing just by LPF, we can see the reconstruction next section.

### 2.4.3 Signal Reconstruction:

In this section we will use Low Pass filter to reconstruct the message signal from the samples. So we have to define a Low Pass filter with Bandwidth equal to 3dB.

Now let's apply the LPF to the sampled signal trying to reconstruct the message that sampled with natural sample method of  $f_s=40$ ,  $d_c=0.5$ , we will show it in time domain and frequency domain.

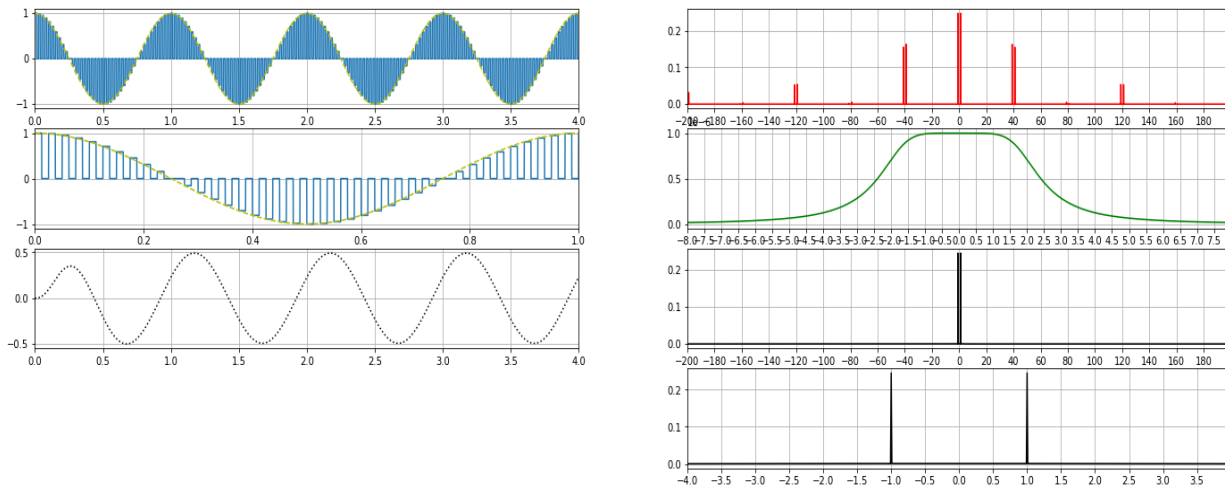


Figure 69:sampled and reconstructed signals in time and frequency domain

- Note:** In the above figure, we see two parts, the left part represents the sampled signal in the top then zoom-in of sampled signal, then the reconstructed signal in the bottom and we can see at the beginning of recovered message some of distortion that because the type of filter but this has no effect. while the right part represents the sampled signal 1<sup>st</sup> from the top, then the LPF with cut-off =2, then the reconstructed signal, then the reconstructed signal zoomed in. We could reconstruct the message signal as it was before sampling because of that the LPF is with cut-off frequency less than  $f_s/2$  and bigger than  $f_m$ . In the end, as we note after the introduction of the sampled signal to the LPF, we were able to recover the original message, but there is some difference in amplitude and this is something that can be treated through the use of amplifier.

Now Let use consider a new message signal  $m(t)=Am_1\cos(2\pi f_{m1}t)+Am_2\cos(2\pi f_{m2}t)$ .

With  $F_{m1} = 1$ ,  $A_{m1} = 1$ ,  $F_{m2} = 2$ ,  $A_{m2} = 1$ , sampled with  $F_s=40$  and D.C=0.5, as we did before, I will show all of this in time and frequency domain.

We made the reconstruction by LPF of cut-off frequency = 8

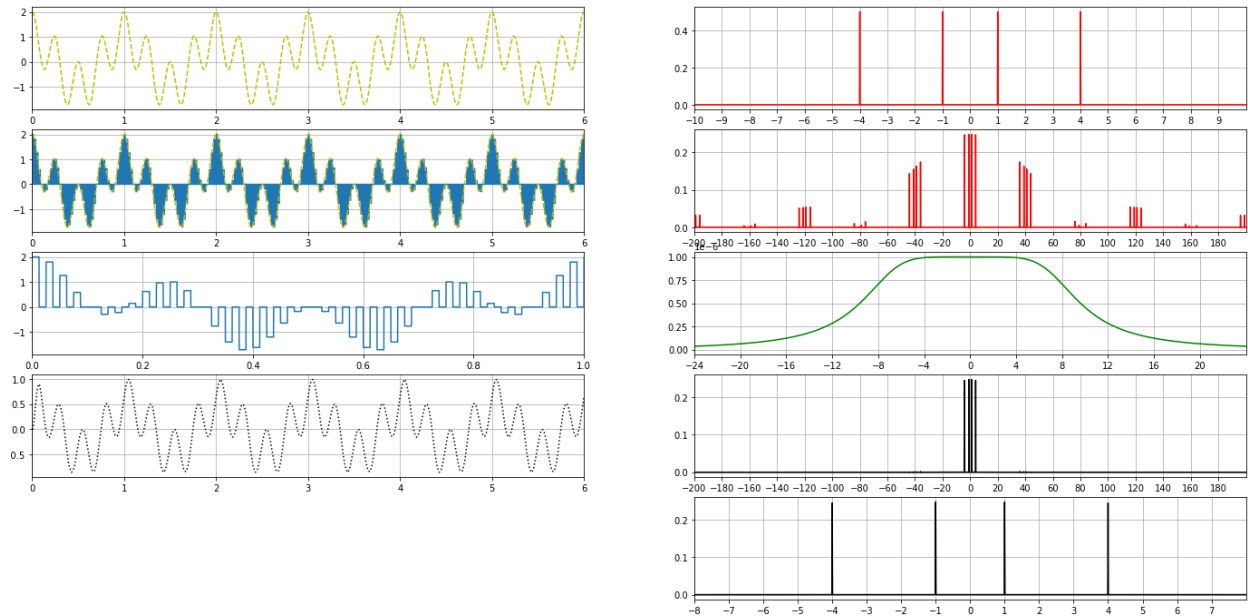


Figure 70:sampled and reconstructed signals in time and frequency domain

• **Note:** In the above figure, we see two parts, the left part represents message signal in 1<sup>st</sup> plot from the top then the sampled signal then zoomed-in sampled signal, then the reconstructed signal. While the right part represents the message signal 1<sup>st</sup> from the top, then sampled signal, then the LPF with cut-off =8, then the reconstructed signal, then the reconstructed signal zoomed in. we can see that we could reconstruct the signal because the cut-off frequency of the LPF is bigger than  $\max\{F_{m1},F_{m2}\}$  and less than  $F_s-\max\{F_{m1},F_{m2}\}$ . In the end, as we note after the introduction of the sampled signal to the LPF, we were able to recovered the original message, but there is some difference in amplitude and this is something that can be treatment through the use of amplifier. Also, we can notice from sampled signal figure in frequency domain that an amplitude for each frequency aren't equal, an example of this the amplitude for 4 components which is located around  $f_s=40$  are have different from each other, but have the same amplitude for the sinc function, as well as other pulses that are in the other frequency. In addition to, we can notice the amplitude for 4 components which are located around  $f_s=0$  they have almost the same value for each other because the dc value small and the dc value effects on null value for sinc function.

In order to clarify this matter further, we will increase the value of dc as shown in the exercise below.



**Exercise:**

**6- Let increase dc value to  $dc=1.5$**

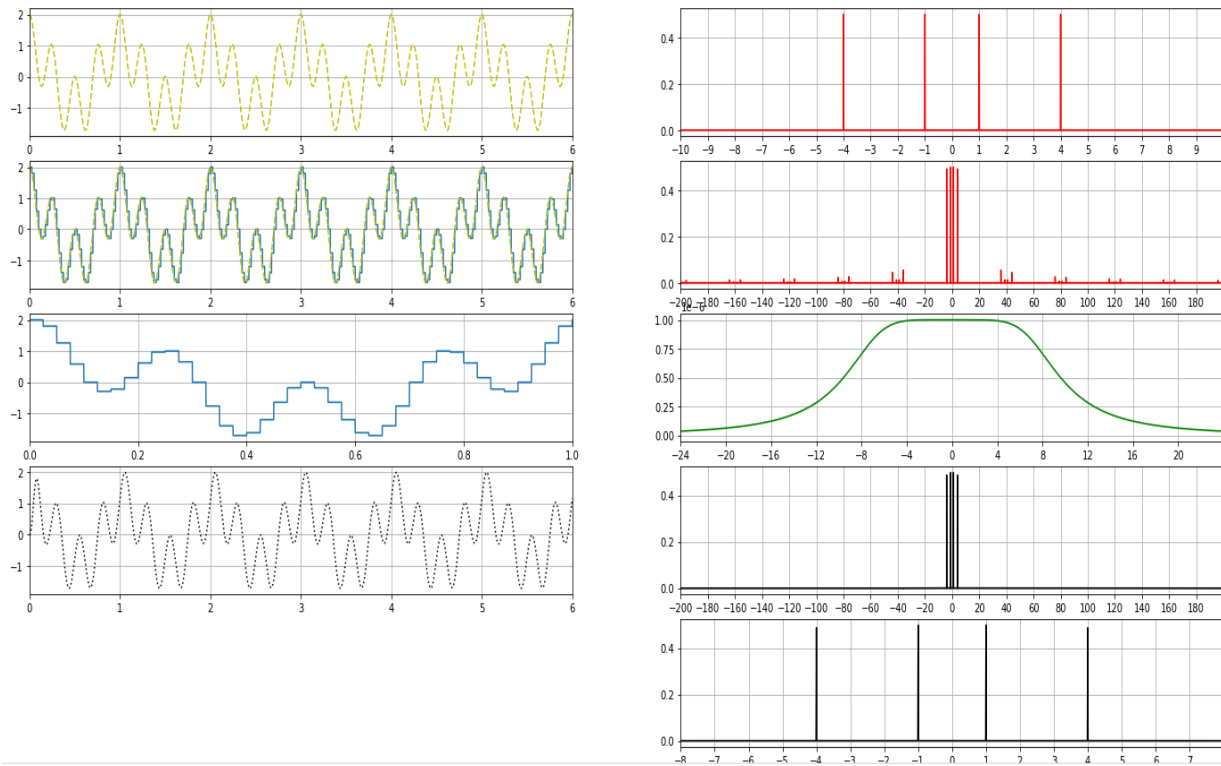
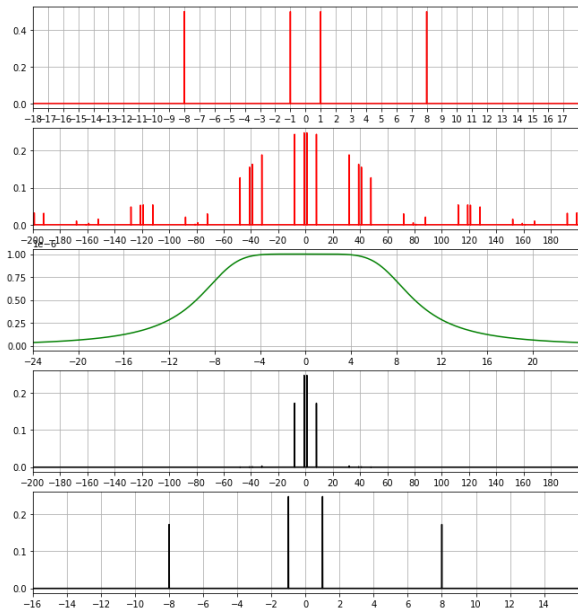
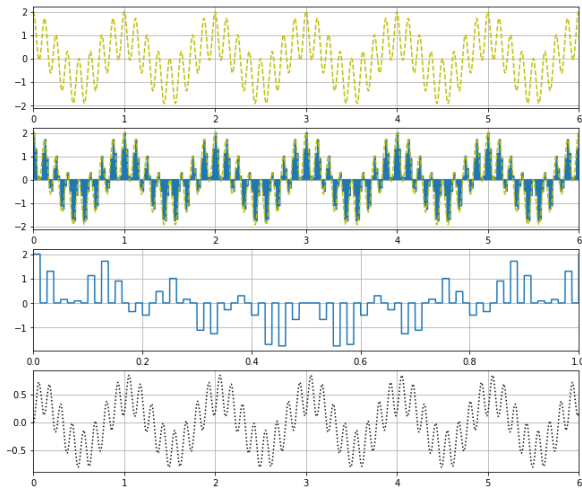


Figure 71:sampled and reconstructed signals in time and frequency domain with  $dc=1.5$

- **Note:** in this case when increased the value for  $dc$  we notice the amplitude for 4 components which are located around  $f_s=0$  they don't have the same value because the  $dc$  value big and the  $dc$  value effects on null value for sinc function.

### 7- Now let's change $f_m=8$



- **Note:** in this case when increased the value for  $f_m$  we notice the amplitude for 4 components which are located around  $f_s=0$  they don't have the same value because the cut-off frequency is small, but we still can recover the message signal with some of different in amplitude (distortion).

**8- Now let's change  $fm2=8$  , and change cut-off frequency= 16**

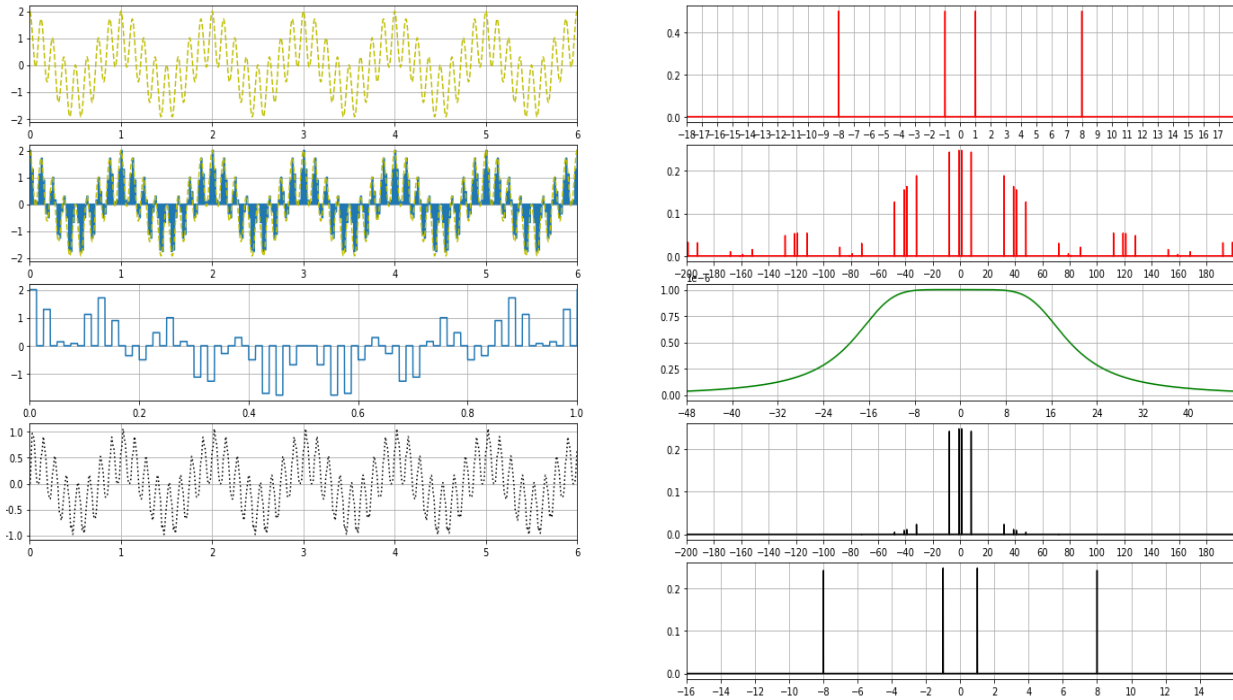


Figure 72:sampled and reconstructed signals in time and frequency domain with  $fm=8$  and  $cut-off=16$

- **Note:** in this case when increase the value of  $fm2$  and change the value of cut-off frequency for filter we notice the presence of a component of the Signal at around  $fs=40$  and  $fs=-40$  that passed through the filter because of the order of filter, so it was not able to make rejection of this component.

**9- Now let's see what will happen when we change filter order=5**

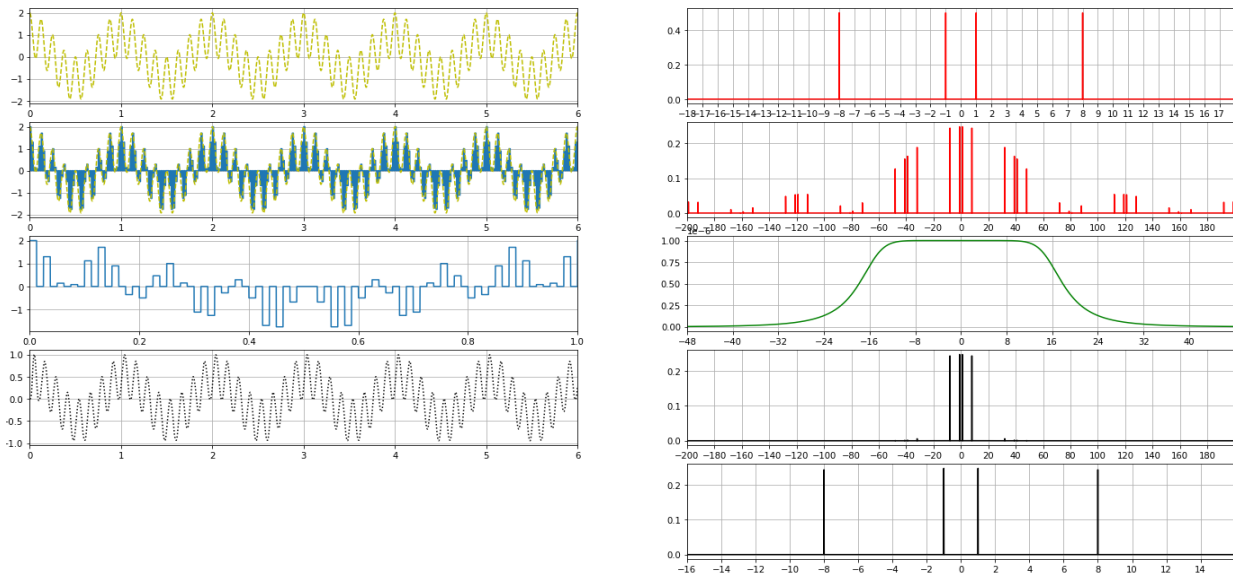


Figure 73:sampled and reconstructed signals in time and frequency domain with  $order=5$

- **Note:** in this case when increase the value of filter order the sharpens of filter increased and The filter did not pass any component except the component that is included in the ring of the filter.

### **3 Conclusion:**

In conclusion, we were able to understand the Working mechanism of Pulse Amplitude Modulation and understand the different between another type of sampling (ideal sampling, natural sampling, flat topped sampling). Also, we were able to understand the effect of changing the parameters on the recovered signal. We were able to understand the purpose of using different sampling on the type of the signal. Finally, the experiment ran smoothly using the Colab and our results were logical and convincing.