



BIRZEIT UNIVERSITY

FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ENEE 4113

communication Laboratory.

Experiment 6

Pulse Amplitude Modulation – Part2

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Section # : 3

Date: 4.2.2021

1. Abstract:

In this experiment, the student will be introduced to how they can deal with Pulse Amplitude modulation using written python code in GitHub simulator, they will emphasize the sampling theorem and talks about the relation between signal frequency and sampling frequency. After that we will talk about time division multiplexing and its power in communication.

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2. Procedure:

2.1 The Sampling Theorem:

In this section we will sample three cosine signals with different bandwidths (f_m) to see the relation between the signal frequency f_m and the sampling frequency f_s . We will start with $f_s > 2W$, then $f_s = 2W$, and finally $f_s < 2W$.

2.1.1 Case 1: $f_s > 2W$:

We will plot the message signal $m(t) = 1 * \cos(2\pi(10)t)$.

$f_m = 10$ while $f_s = 100$ which means that $f_s > 2f_m$, the plots shown below.

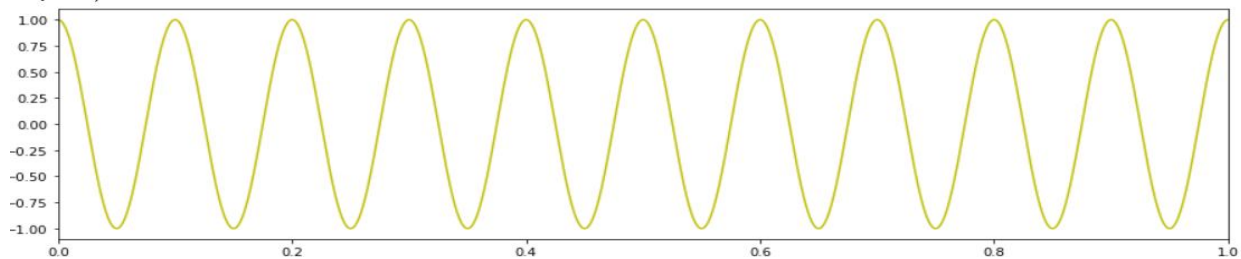


Figure 1: message signal with $f_m = 10$

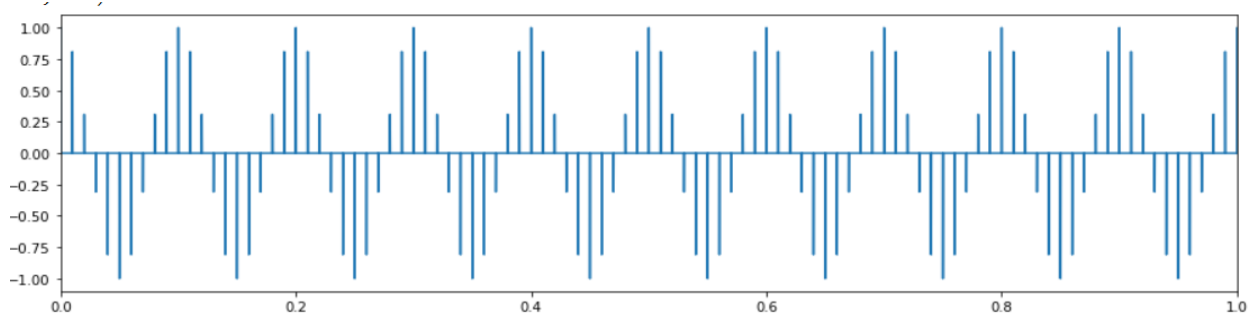


Figure 2: sampled signal with $f_s = 100$

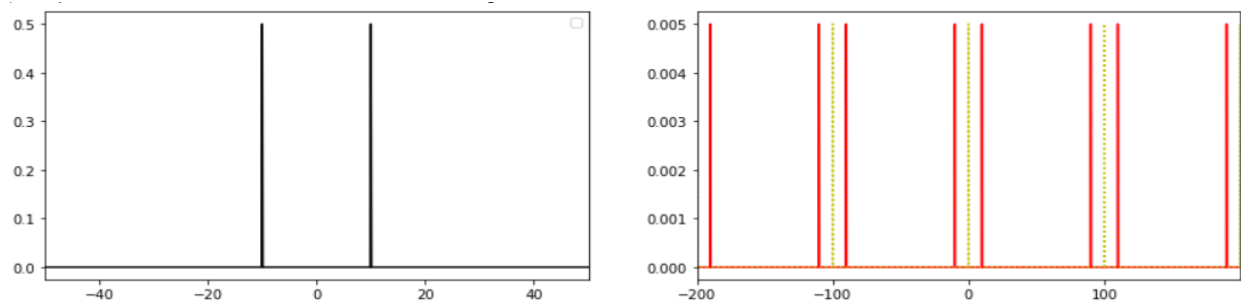


Figure 3: message signal and convolution signal in frequency domain

- **Note:** In figure1 shows the message signal, figure2 shows the train of impulses signal, figure3 in left part show the message signal in frequency domain but in right part show the sampled signal in red and the train of impulses that sampled on it in yellow color.

Now let's try to reconstruct the signal using LPF with cut-off frequency = $f_s/2$, the results in the following:

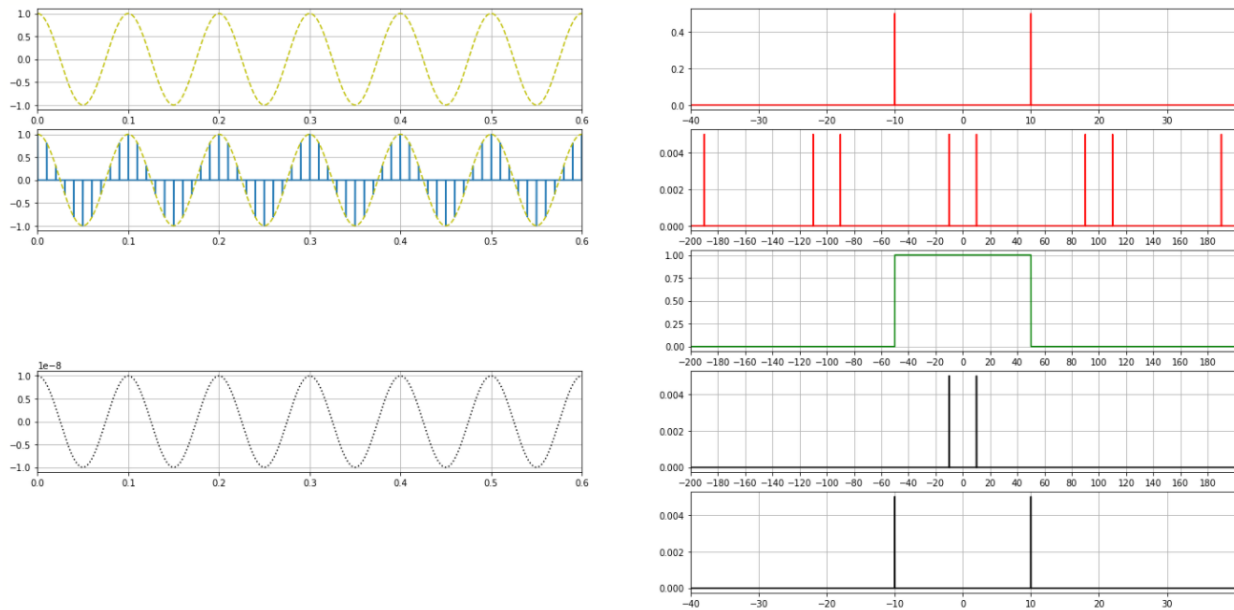


Figure 4: message, sampling and recovered message signal

- Note:** from above figure we show two part, left part shows all the signals in time domain, the 1st plot from the top shows the message signal, then the sampled signal, then the reconstructed signal. And in the right part we show all the signal in frequency domain, the 1st plot from the top shows the message signal, then the sampled signal, then the LPF with cut-off frequency used for reconstruct the signal, then the reconstructed signal so in this case we can recovered the message signal, the last plot shows the reconstructed signal zoomed-in.

Exercise:

1- let change $f_m=30$:

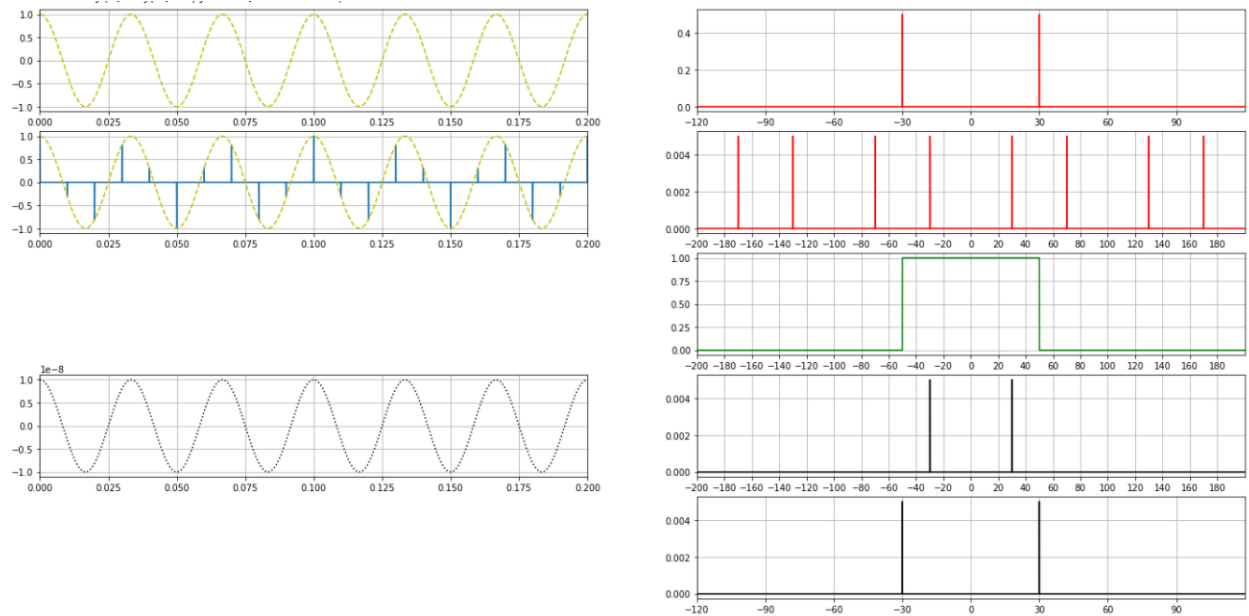


Figure 5: message, sampling and recovered message signal when $f_m=30$

- **Note:** in this case when change $f_m=30$ (f_m increased) we notice:
 - 1- The frequency of message signal change and moved away from each other and the BW change because the frequency change.
 - 2- The frequency for sampled signal change by $(f_s - n f_m, f_s + n f_m)$, n : integer number, also we notice when increase the value of f_m the upper side band for the left signal (f_s small) getting close to lower side band for the right signal (f_s large). For example: the upper side band for $f_s=0$ [$0+1*30=30$] is getting close to lower side band for $f_s=100$ [$100-1*50=50$]. And this idea will be cleared when changed $f_m=45$ in next part of exercise.
 - 3- When the f_m value is changed, this does not effect on cut-off value for LPF and the filter is still able to pass the Signal because $f_m < \text{cut-off}$.
 - 4- Finally, we notice when the signal pass from LPF we still can recovered the message signal.

2- let change $f_m=45$:

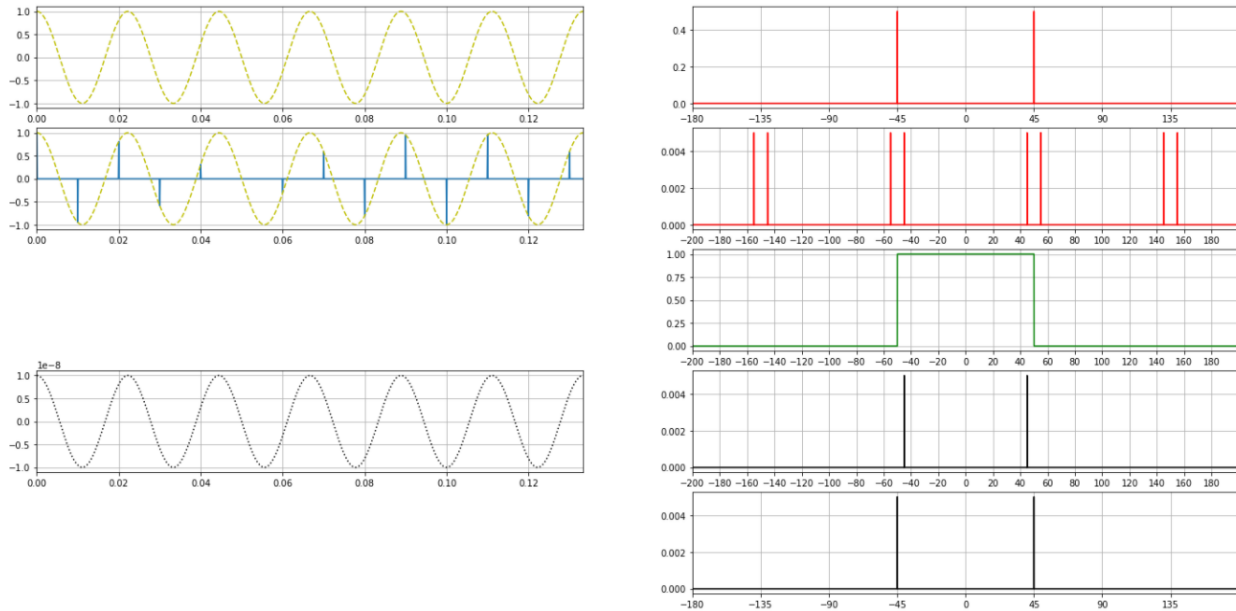


Figure 6: message, sampling and recovered message signal when $f_m=45$

- Note:** in this case when change $f_m=45$ (f_m increased) we notice:
 - 1- The frequency of message signal change and moved away from each other and the BW change because the frequency change.
 - 2- The frequency for sampled signal change by $(f_s - n f_m, f_s + n f_m)$, n : integer number, also we notice when increase the value of f_m the upper side band for the left signal (f_s small) getting close to lower side band for right signal (f_s large). For example: the upper side band for $f_s=0$ [$0+0*45=45$] is getting close to lower side band for $f_s=100$ [$100-45=55$].
 - 3- When the f_m value is changed, this does not effect on cut-off value for LPF and the filter is still able to pass the Signal because $f_m < \text{cut-off}$.
 - 4- Finally, we notice when the signal pass from LPF we still can recovered the message signal.

3- let change $A_m=3$:

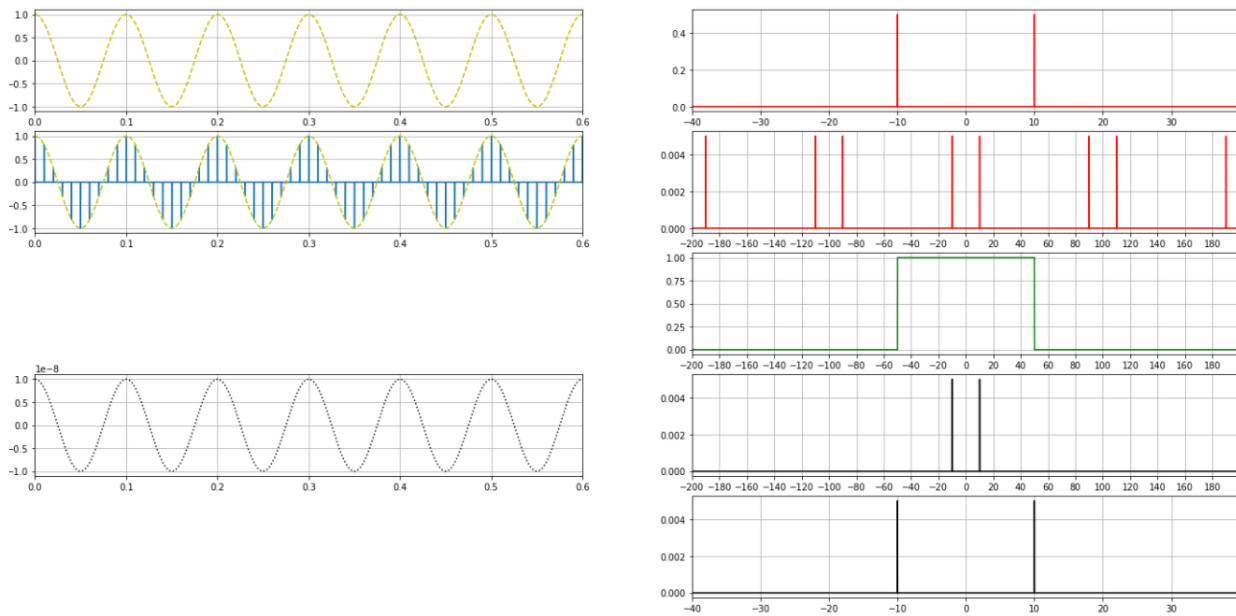
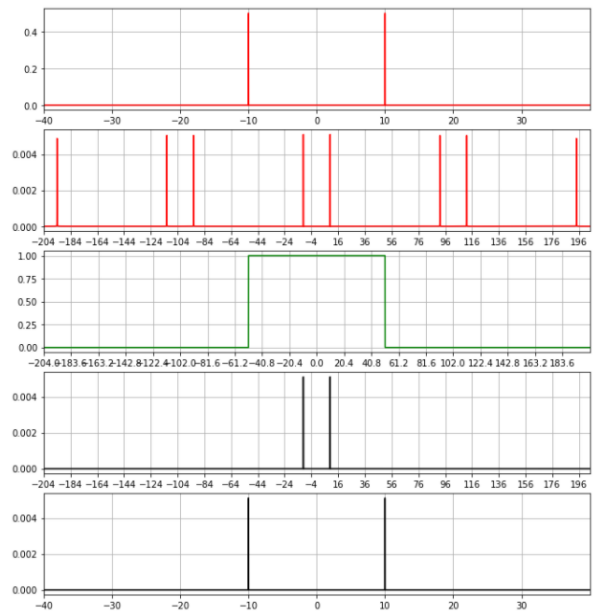
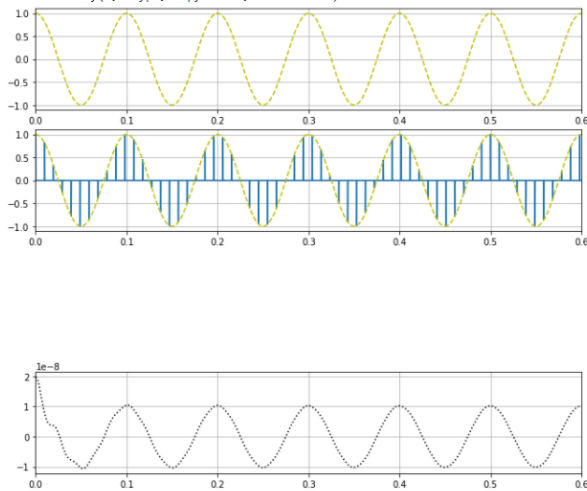


Figure 7: message, sampling and recovered message signal when $A_m=3$

- **Note:** in this case when change $A_m=3$ (A_m increased) we notice:
 - 1- The amplitude for message frequency changed by $A_m/2$ but the BW doesn't change because the frequency has not change.
 - 2- The amplitude for sampled frequency change because it is due to the product of the message signal by train of impulses so when the amplitude for message frequency change the amplitude for the sampled signal will be change.
 - 3- When the A_m value is changed, this does not effect on cut-off value for LPF and the filter is still able to pass the Signal because $f_m < \text{cut-off}$.
 - 4- Finally, we notice when the signal pass from LPF we still can recovered the message signal but the amplitude for this message is different from original message but we can use amplifier to get the same amplitude as the amplitude of the original message.

4- let change $f_s=102$:



- **Note:** in this case when change $f_s=102$ (still $f_s < 2f_m$) we notice:
 - 1- The frequency of message signal doesn't change and the BW doesn't change because the frequency doesn't change.
 - 2- The frequency for sampled signal change by $(f_s - n f_m, f_s + n f_m)$, n : integer number.
 - 3- When the f_s value is changed, this effect on cut-off value for LPF by $f_s/2$ and the filter is still able to pass the Signal because $f_m < \text{cut-off}$.
 - 4- Finally, we notice when the signal pass from LPF we still can recovered the message signal, but we notice we have some of distortion but this does not affect that much.

2.1.2 Case 2: $f_c = 2W$ (Nyquist rate):

We will plot the message signal $m(t) = 1 * \cos(2\pi(50)t)$.

$f_m = 50$ while $f_s = 100$ which means that $f_s = 2f_m$, the plots shown below.

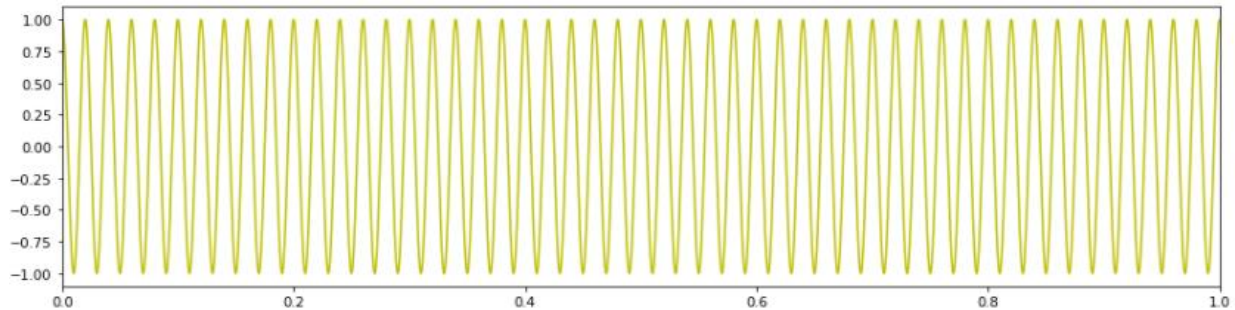


Figure 8: message signal with $f_m = 50$

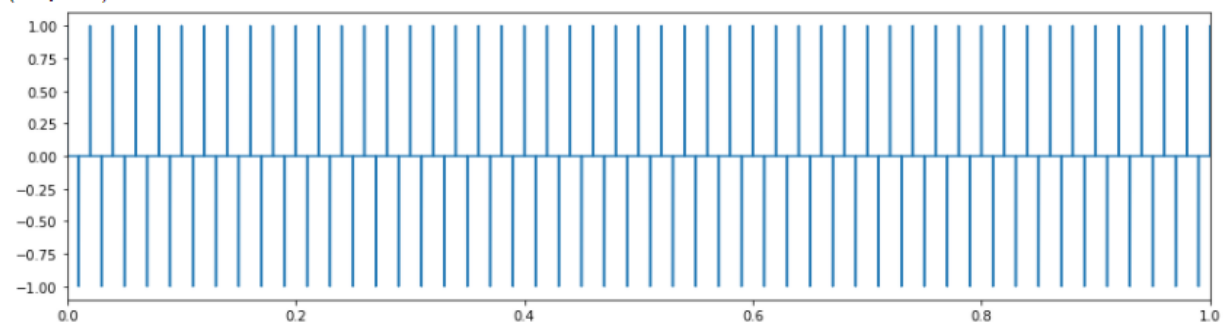


Figure 9: sampled signal with $f_s = 100$

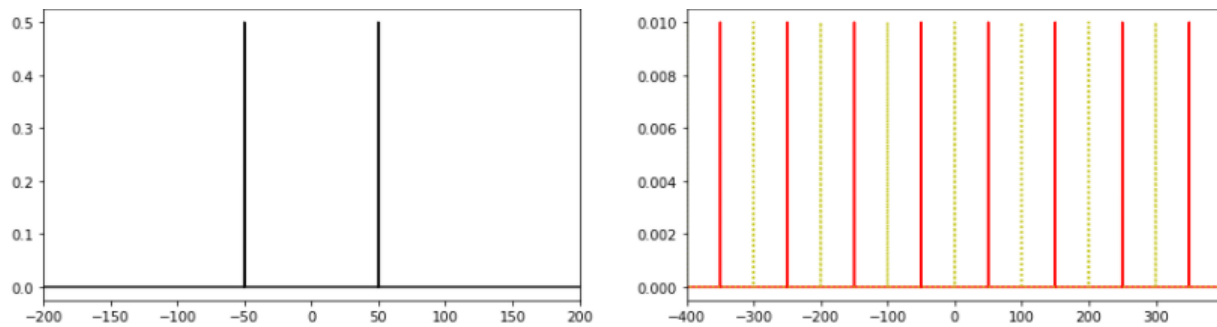


Figure 10: message signal and convolution signal in frequency domain

- **Note:** In figure 8 shows the message signal, figure 9 shows the train of impulses signal, figure 10 in left part show the message signal in frequency domain but in right part show the sampled signal in red and the train of impulses that sampled it in yellow color.

Now let's try to reconstruct the signal using LPF with cut-off frequency = $F_s/2$, the results in the following:

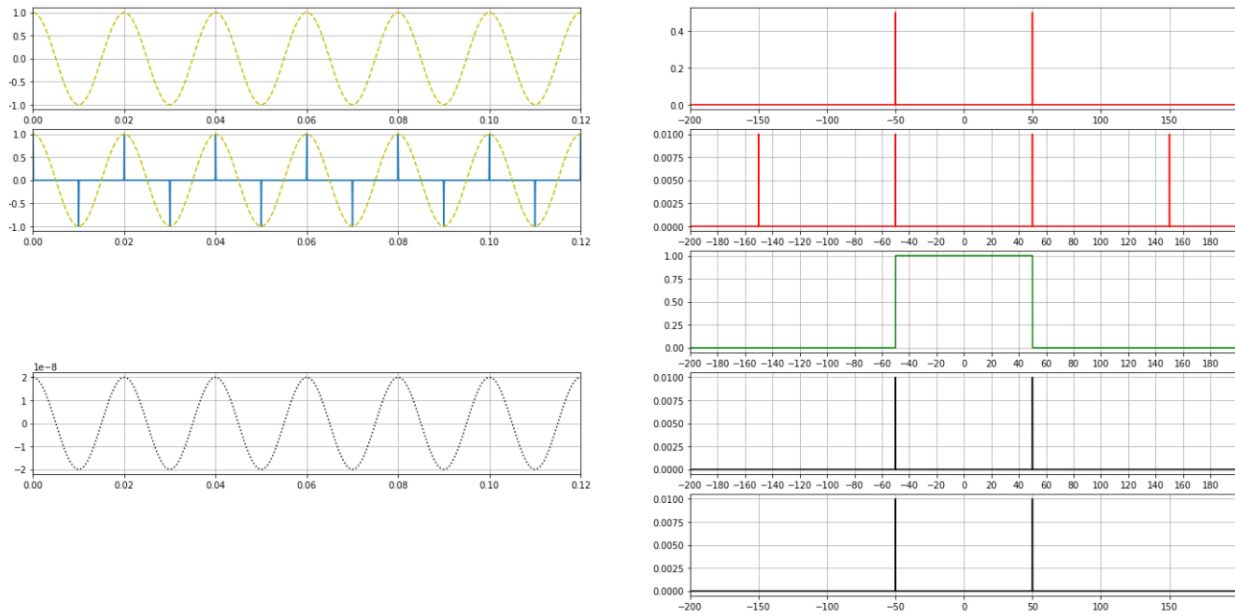


Figure 11: message, sampling and recovered message signal

- Note:** from above figure we show two part, left part shows all the signals in time domain, the 1st plot from the top shows the message signal, then the sampled signal, then the reconstructed signal. And in the right part we show all the signal in frequency domain, the 1st plot from the top shows the message signal, then the sampled signal and in this case we notice the upper side band for the left signal (f_s small) matched with lower side band for right signal (f_s large), then the LPF with cut-off frequency used for reconstruct the signal so in this case we still can recovered the message signal, then the reconstructed signal, the last plot shows the reconstructed signal zoomed-in.

Exercise:

In this case We don't want to change the value of f_m and A_m because if we decrease f_m we will come back to previous case ($f_s > 2f_m$) and we have discussed it earlier also if we increase it we will have aliasing ($f_s < 2f_m$), and this situation will be discussed later also if we change the value of A_m we will have discussed it and its effect will not differ in this case from the previous case. But in this case we need change the f_s and show what his change affect.

1- let change $f_s=102$:

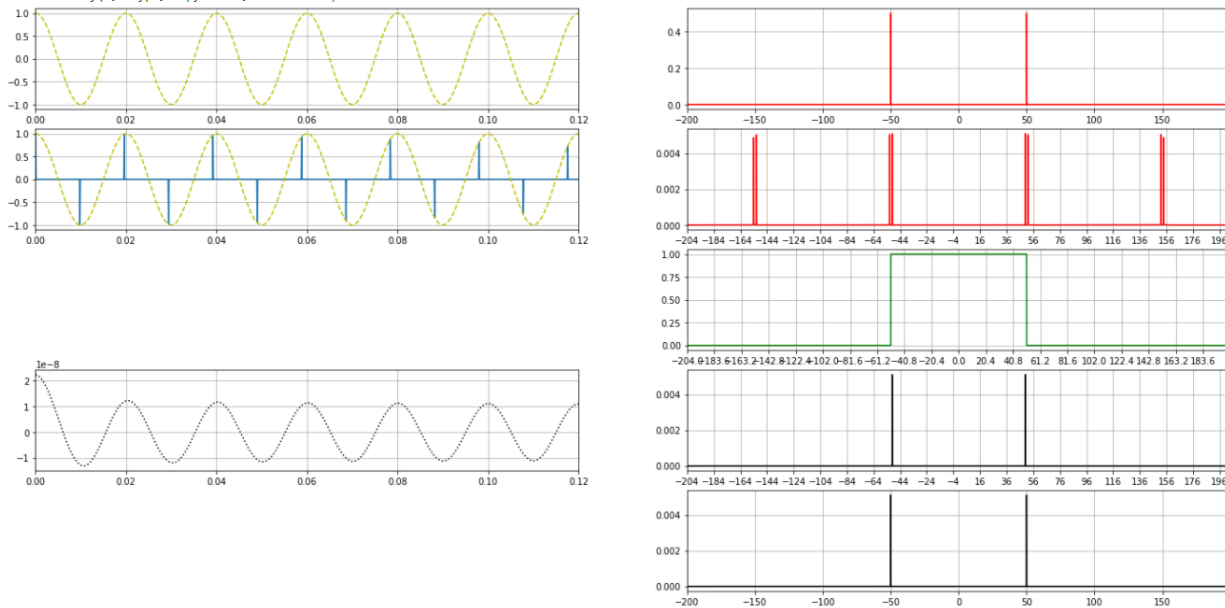


Figure 12: message, sampling and recovered message signal when $f_s=102$

- **Note:** in this case when change $f_s=102$ (f_s increased) we notice:
 - 1- The frequency of message signal doesn't change and the BW doesn't change because the frequency doesn't change.
 - 2- The frequency for sampled signal change by $(f_s - n f_m, f_s + n f_m)$, n : integer number, also we notice when increase the value of f_s the upper side band for the left signal (f_s small) doesn't match with lower side band for the right signal (f_s large). For example: the upper side band for $f_s=0[0+1*50=50]$ doesn't match with lower side band for $f_s=102[102-1*50=52]$ but we have some space between them.
 - 3- When the f_s value is changed, this effect on cut-off value for LPF by $f_s/2$ and the filter is still able to pass the Signal because $f_m < \text{cut-off}$.
 - 4- Finally, we notice when the signal pass from LPF we still can recovered the message signal.

2.1.3 Case 3: $f_c < 2W$ (aliasing):

We will plot the message signal $m(t)=1*\cos(2\pi(80)t)$.

$f_m = 80$ while $f_s=100$ which means that $f_s > 2f_m$, the plots shown below.

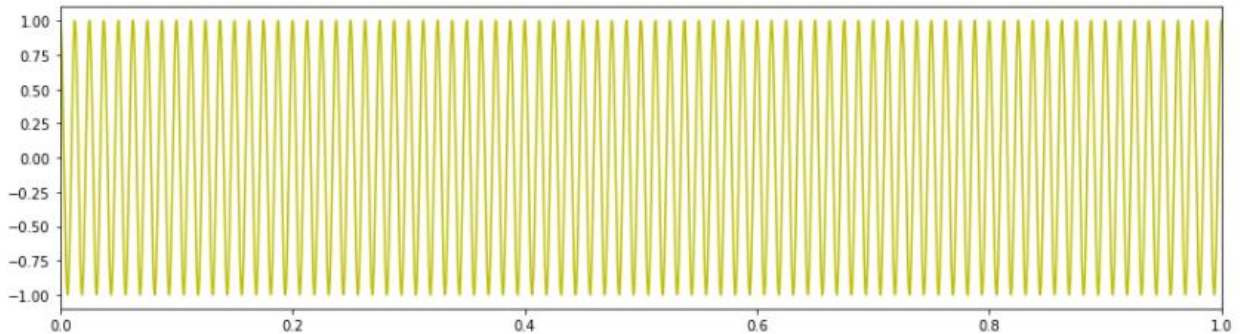


Figure 13: message signal with $F_m=80$

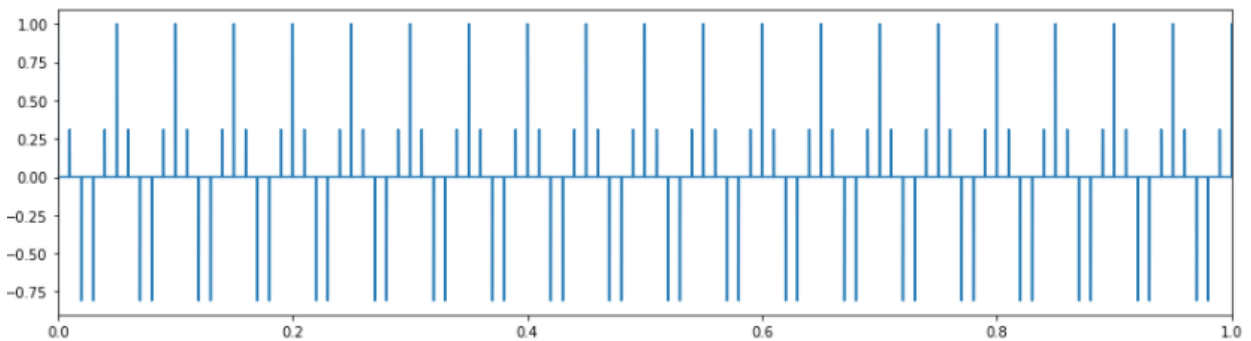


Figure 14: sampled signal with $F_s=100$

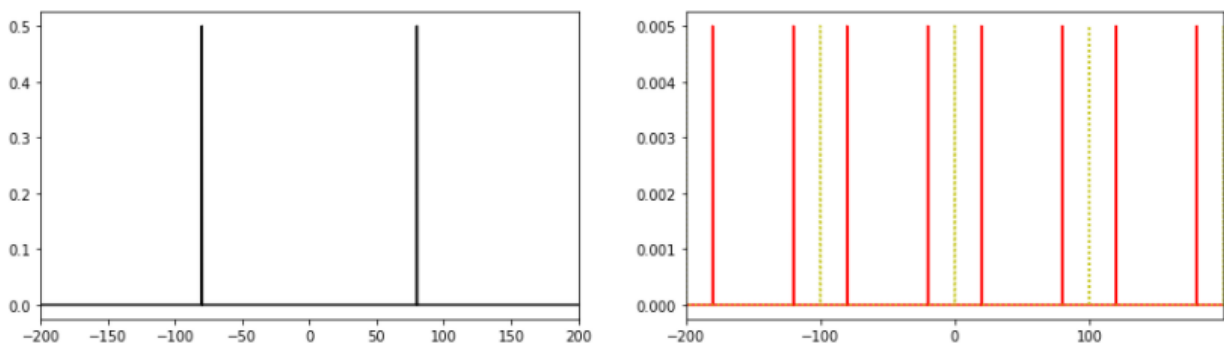


Figure 15: message signal and convolution signal in frequency domain

- **Note:** In figure 13 shows the message signal, figure 14 shows the train of impulses signal, figure 15 in left part show the message signal in frequency domain but in right part show the sampled signal in red and the train of impulses that sampled on it in yellow color.

Now let's try to reconstruct the signal using LPF with cut-off frequency = $F_s/2$, the results in the following:

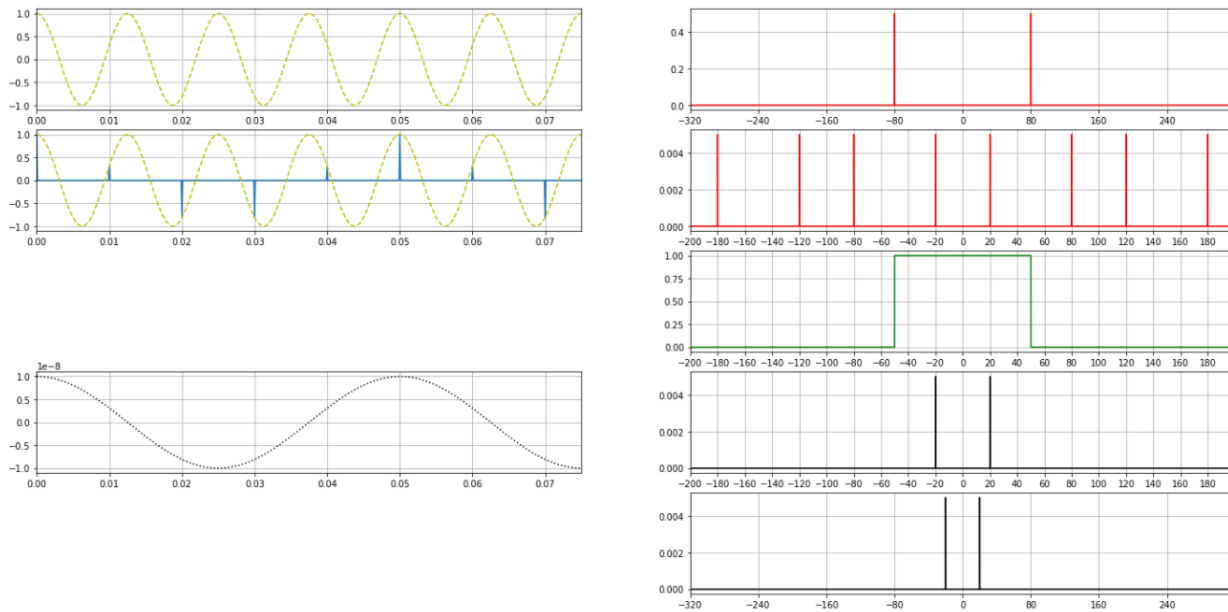


Figure 16: message, sampling and recovered message signal

- Note:** from above figure we show two part, left part shows all the signals in time domain, the 1st plot from the top shows the message signal, then the sampled signal, then the reconstructed signal. And in the right part we show all the signal in frequency domain, the 1st plot from the top shows the message signal, then the sampled signal and in this case we notice there is an overlap between the upper side band for the left signal (f_s small) with lower side band for right signal (f_s large), then the LPF with cut-off frequency used for reconstruct the signal, then the reconstructed signal also in this case we can't recovered the message signal, the last plot shows the reconstructed signal zoomed-in.

Exercise:

In this case We don't want to change the value of f_s and A_m because if we increase f_s we will come back to previous case ($f_s > 2f_m$) and we have discussed it earlier or case ($f_s = 2f_m$) and we have discussed it earlier also if we change the value of A_m we will have discussed it and its effect will not differ in this case from the previous case. But in this case we need change the f_m and show what his change affect.

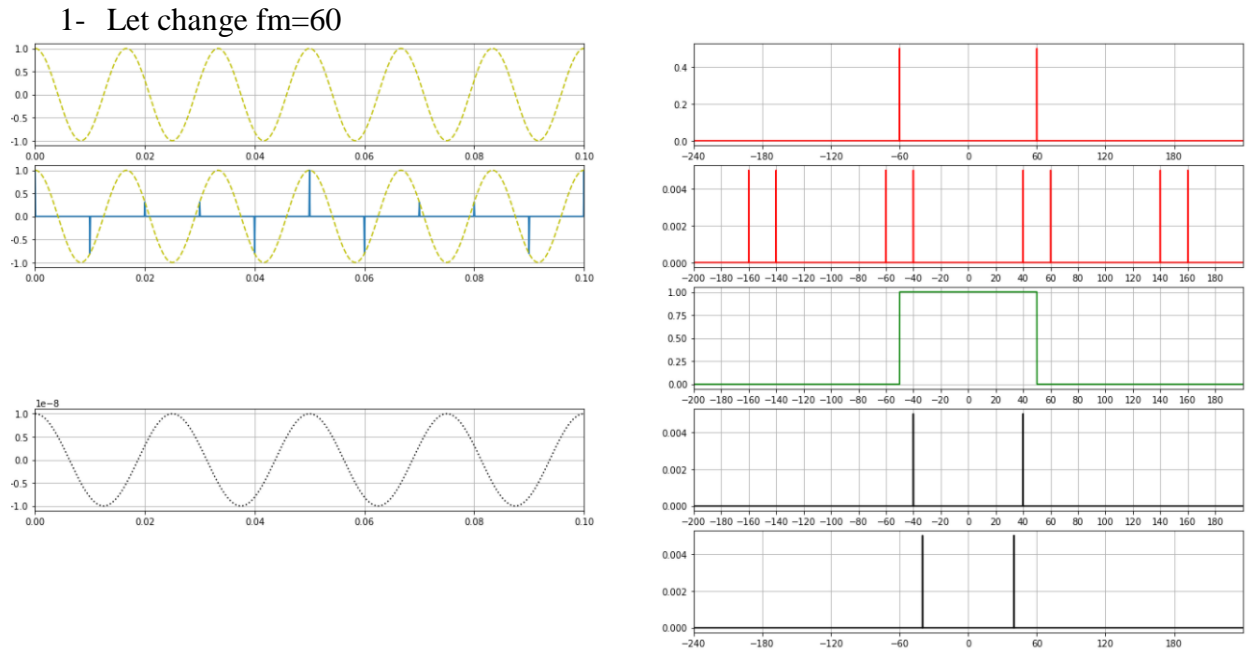


Figure 17: message, sampling and recovered message signal when $f_m=60$

- **Note:** in this case when change $f_m=80$ (f_m increased) we notice:
 - 1- The frequency of message signal change and moved away from each other and the BW change because the frequency change.
 - 2- The frequency for sampled signal change by $(f_s - n f_m, f_s + n f_m)$, n : integer number, also we notice when increase the value of f_m the upper sideband of the left signal (small f_s) overlapped the lower sideband of the right signal (large f_s). For example: the upper sideband for $f_s=0$ [$0+0*80=80$] and the lower sideband for $f_s=100$ [$100-1*60=40$] so we notice there is an overlap between them.
 - 3- When the f_m value is changed, this does not effect on cut-off value for LPF but in this case the filter passes the impulses have frequency $(-40,40)$ and these impulses followed to $f_s=100$ not $f_s=0$.
 - 4- Finally, we notice when the signal pass from LPF we can't recovered the message signal. Where we send message that has a $f_m=60$, but the signal that came out of the filter has a frequency=40.

2.2 Sampling of a Multitone Message Signal:

Let us apply the sampling theorem to the multitone message signal $m(t)=A_m1\cos(2\pi f_{m1}t)+A_m2\cos(2\pi f_{m2}t)+A_m3\cos(2\pi f_{m3}t)$

In this part when we want to know what case we are, we make a comparison between f_s with the large value for f_m .

2.2.1 Case 1: $f_s > 2W$:

Let $f_{m1}=20$, $f_{m2}=40$, $f_{m3}=60$, $f_s=200$

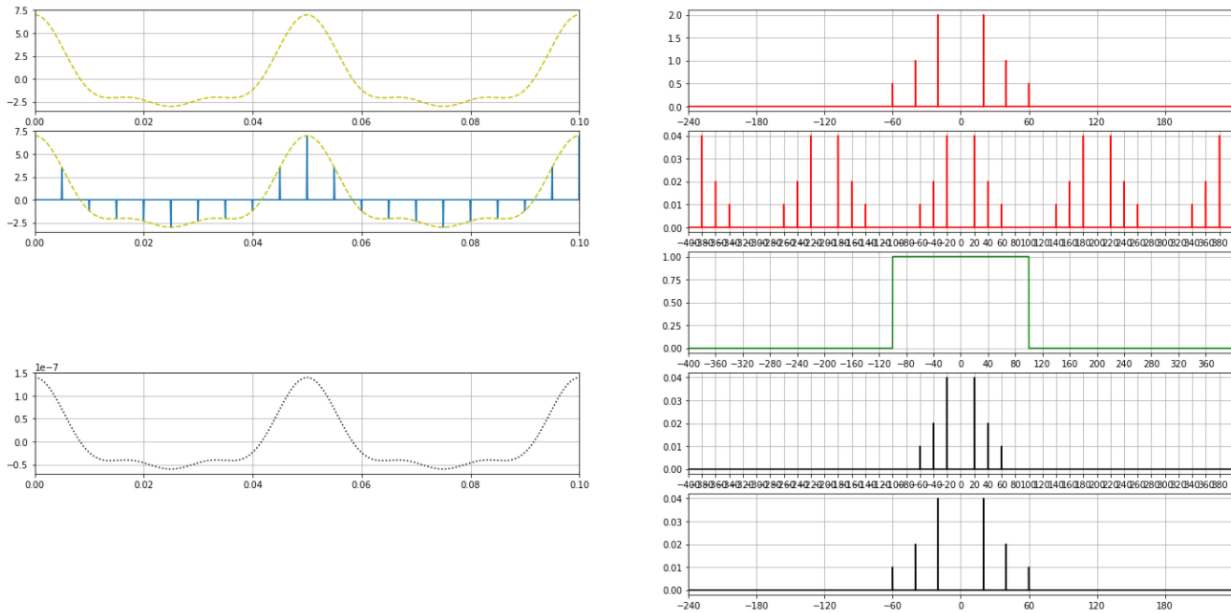


Figure 18: message, sampling and recovered message signal when $f_{m1}=20$, $f_{m2}=40$, $f_{m3}=60$, $f_s=200$

- Note:** in this case $f_s > 2f_m$ [$200 > 2 \cdot 60$] It will happen as it did in the previous part We have 6 impulses for message frequency $[(-f_{m1}, f_{m1}), (-f_{m2}, f_{m2}), (-f_{m3}, f_{m3})]$ also the frequency for sampling changed by $[(-f_s + n f_{mk}, f_s + n f_{mk})]$ where n : integer number and $k=1,2,3$. Also in this case we can notice we have space enough space between the impulses were followed $f_s=0$ and impulses were followed $f_s=200$ and so on. In addition to the LPF has cut-off $=f_s/2$ [$200/2=100$] where $f_m < \text{cut-off}$ so we can have recovered the message signal. finally in this case we can recovered the message signal.

2.2.2 Case 1: $f_s = 2W$:

Let $f_{m1}=20$, $f_{m2}=40$, $f_{m3}=60$, $f_s=120$

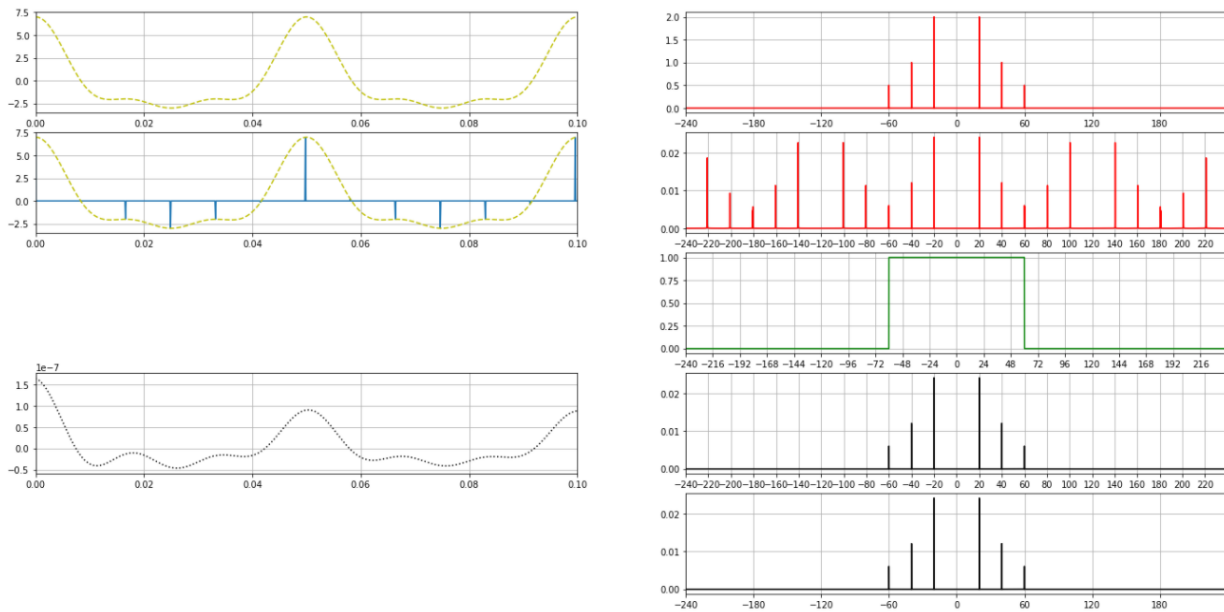


Figure 19: message, sampling and recovered message signal when $f_{m1}=20$, $f_{m2}=40$, $f_{m3}=60$, $f_s=120$

- Note:** in this case $f_s=2f_m$ [$120=2*60$] It will happen as it did in the previous part We have 6 impulses for message frequency $[(-f_{m1}, f_{m1}), (-f_{m2}, f_{m2}), (-f_{m3}, f_{m3})]$ also the frequency for sampling changed by $[(-f_s-nf_{mk}, f_s+nf_{mk})]$ where n : integer number and $k=1,2,3$. But in this case we can notice the third upper side band impulse for $f_s=0$ is match with the third lower side band impulse for $f_m=120$ [$(f_{s0}+f_{m3})=(0+60)=60$ also $(f_{s1}-f_{m3})=(120-60)=60$] so are will matches because of that the amplitude of this impulse (impulse at $f=60$) being doubled because it come from two f_s first one come from $f_s=0$ and second one come from $f_s=120$. In addition to the signal can traverse through LPF because $\text{cut-off}=f_s/2$ [$120/2=60$] where $f_m \leq \text{cut-off}$, but when this signal traverse through LPF the LPF passed the two matches impulses who are located at $f=60$. consequently, we were able to return the message signal, but there is some attenuation that come from doubling the value the amplitude for the impulse, which is located at $f=60$. finally in this case we can recovered the message signal.

2.2.3 Case 1: $f_s = 2W$:

Let $f_{m1}=20$, $f_{m2}=40$, $f_{m3}=60$, $f_s=100$

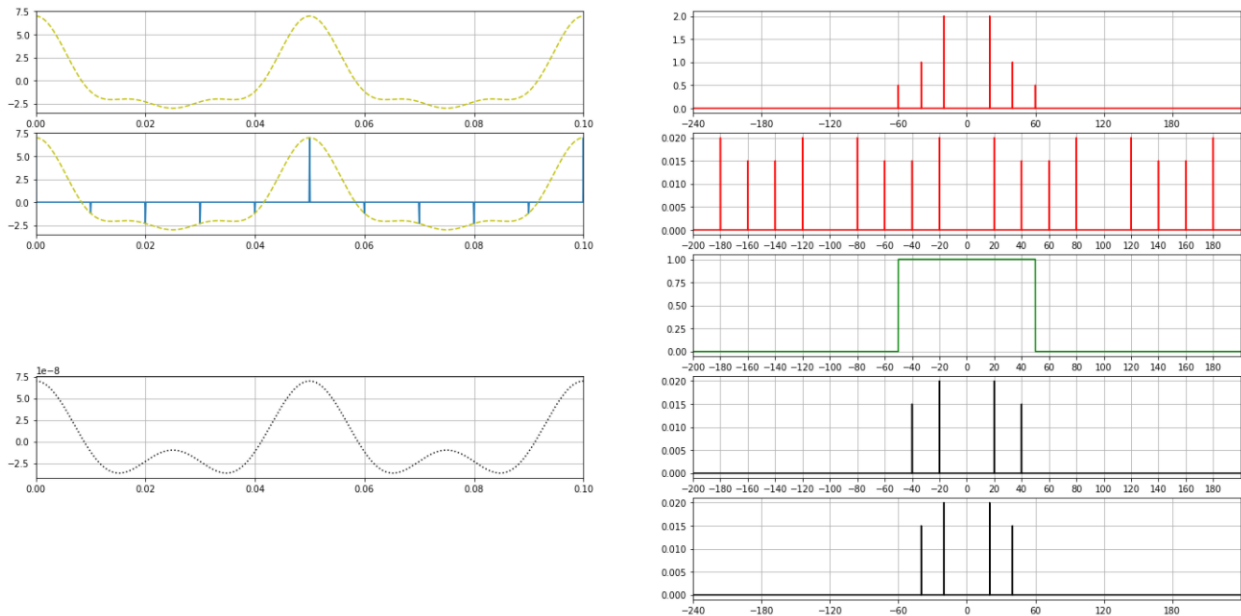


Figure 20: message, sampling and recovered message signal when $f_{m1}=20$, $f_{m2}=40$, $f_{m3}=60$, $f_s=100$

- Note:** in this case $f_s < 2f_m$ [$100 < 2 \cdot 60$] It will happen as it did in the previous part. We have 6 impulses for message frequency $[(-f_{m1}, f_{m1}), (-f_{m2}, f_{m2}), (-f_{m3}, f_{m3})]$ also the frequency for sampling changed by $[(-f_s - n f_{mk}, f_s + n f_{mk})]$ where n : integer number and $k=1,2,3$. But in this case we can notice the upper side band of the $f_s=0$ overlapped the lower side band of $f_s=100$. Consequently, when we pass the signal on the filter, the filter could not return the message signal, but it did return the impulses that fall within the range of its $[(-f_s/2) \text{ and } (f_s/2)]$ and these impulses followed $f_s=0$ and $f_s=100$. Finally, we notice when the signal pass from LPF we can't recover the message signal. Where we send message that has a $f_{m1}=20$, $f_{m2}=40$, $f_{m3}=60$, but the message that came out of the filter has a frequency = $[(-20,20) \text{ and } (-40,40)]$.

2.3 Time Division Multiplexing:

Time-division multiplexing (TDM) is a method of transmitting and receiving independent signals over a common signal path by means of synchronized switches at each end of the transmission line so that each signal appears on the line only a fraction of time in an alternating pattern, here we divide the time.

First of all, let's multiplex two signals, one of them is cosine signal with amp=1 and frequency=1, and the other is constant with amp=1 all sampled at $F_s=100$. The plots below show this.

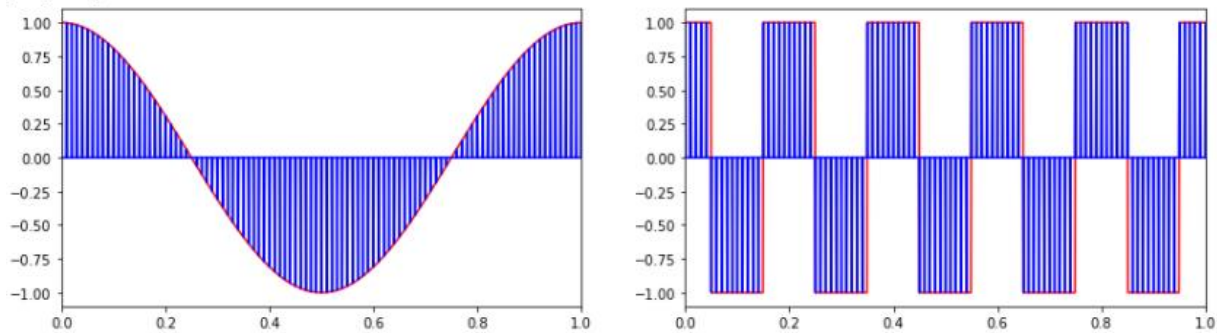


Figure 21: $m_1(t)=\cos(2\pi(1)t)$ and $m_2(t)=1$

- **Note:** from above figure we show two signals first one is cos signal in (red colour) and his impulses in (blue colour) and the second one is square signal in (red colour) and his impulses in (blue colour).

Let us try to multiplex the two above signals and plot the results below:

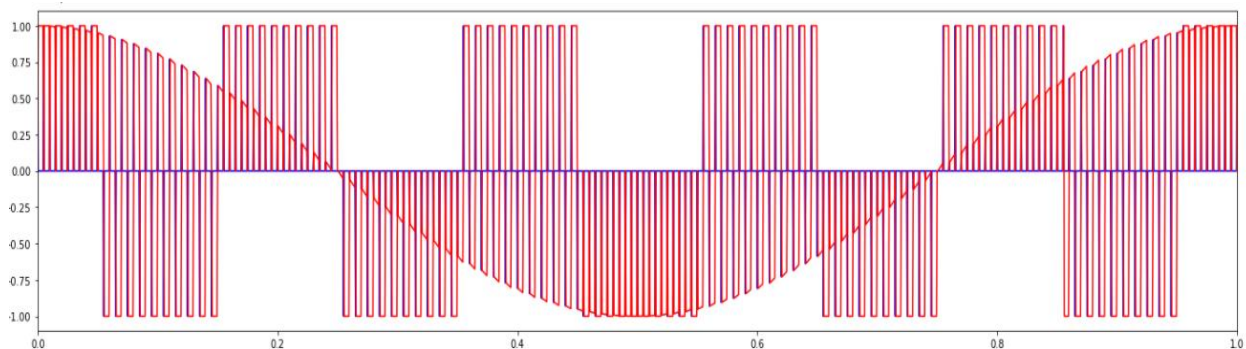


Figure 22: multiplexed the two signals

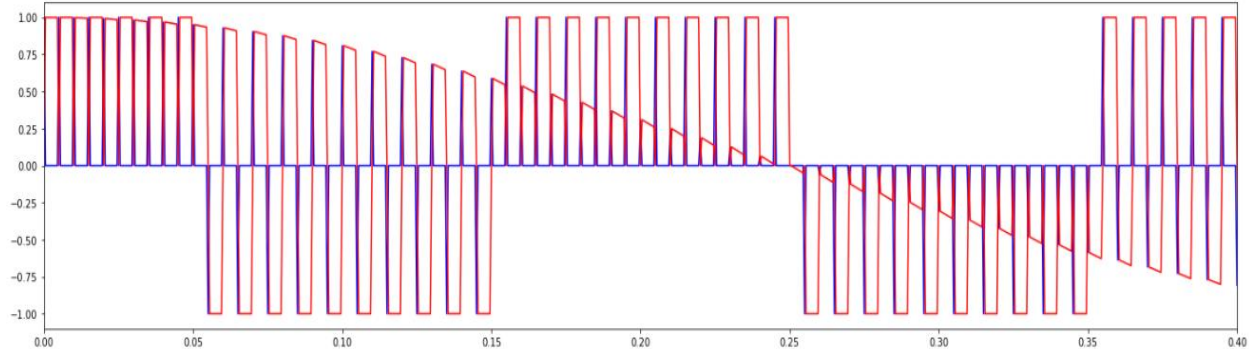


Figure 23: multiplexed the two signals zoomed-in

- **Note:** We can see from figure 22 and figure 23 that each independent signal appears in fraction of time while the other is off, where the signals are sent respectively once Cos signal and once Square signal. In order to recovered the original message of each signal, we must separate the pulses for two signal from each other and then insert them on the Low Pass Filter.

Exercise:

1- Let change $f_m=3$:

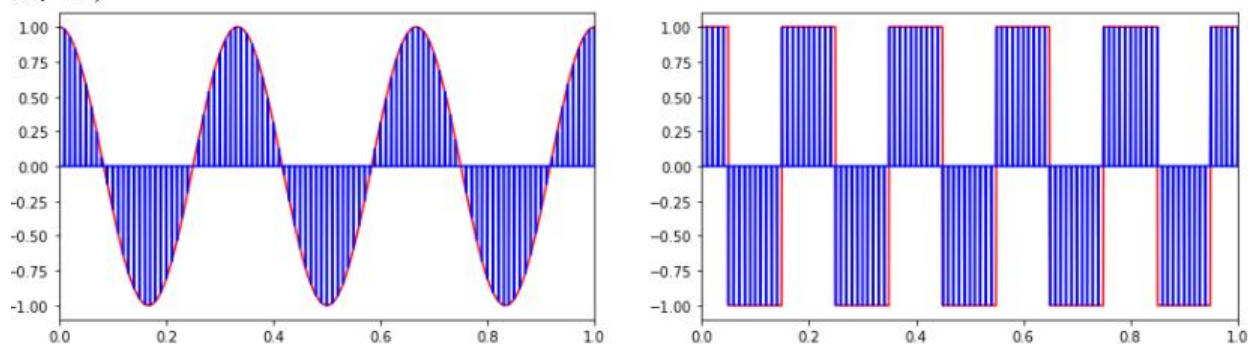


Figure 24: $m_1(t)=\cos(2\pi(1)t)$ and $m_2(t)=1$ when $f_m=3$

- **Note:** we can show when increase f_m this effects on the value of the period of the cos signal and affects the value of the BW, where when the value of f_m change the period of the signal decreased by $1/f_m$ and the pluses are close to each other, while change f_m it causes a difference in the period of the multiplexed signal and this different due to the difference of the cos signal periodic. But multiplexing process it will stay each indication appears independent at a fraction of the time while the other is off and that appears in the figure below.

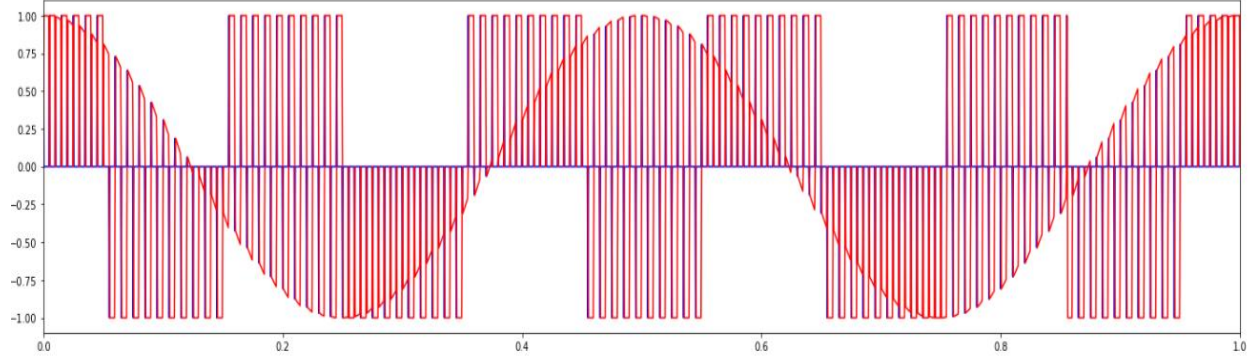


Figure 25: multiplexed the two signals when $f_m=3$

2- Let change $du=0.3$:

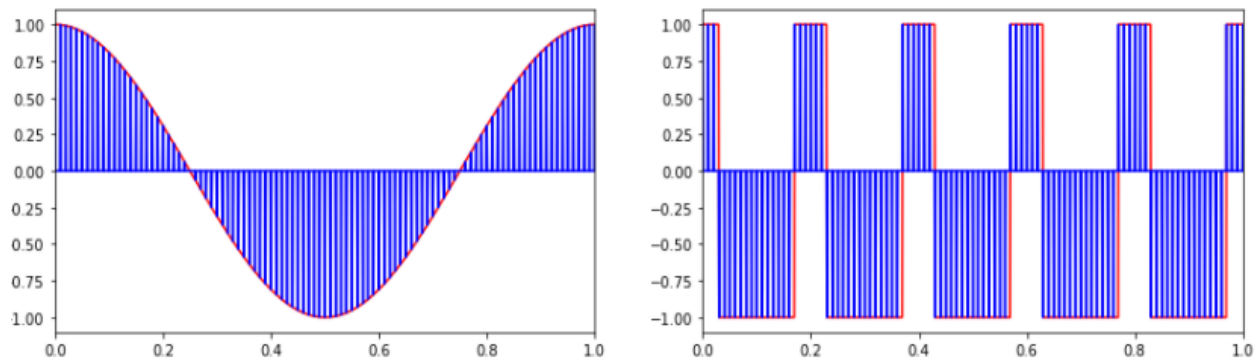


Figure 26: $m_1(t)=\cos(2\pi(1)t)$ and $m_2(t)=1$ when $du=0.3$

- **Note:** we can show when decreased du , this effects on period of (ON and OFF) for square signal, so in this case the allocated period when the square signal ON will be decreased. While change du does not effect on multiplexed process, where it will stay each indication appears independent at a fraction of the time while the other is off and that appears in the figure below.

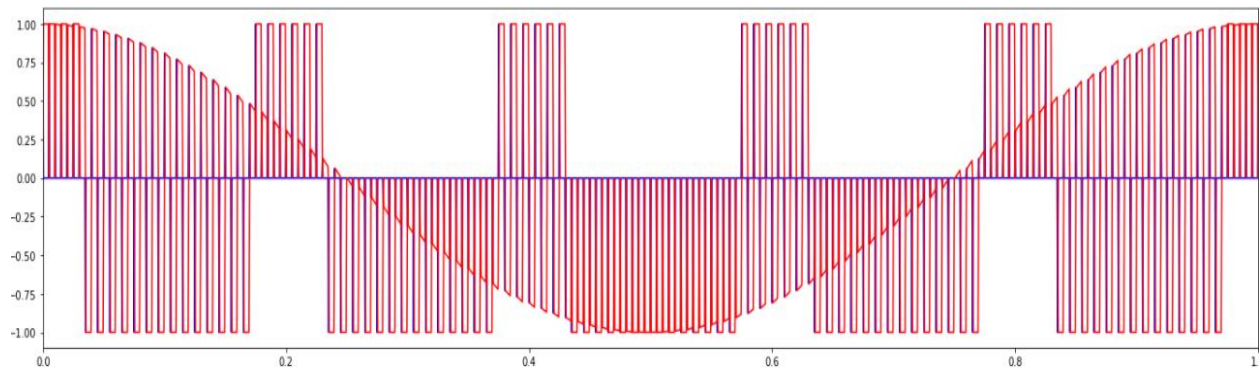


Figure 27: multiplexed the two signals when $du=0.3$

Now let take another example we will time-multiplex two single tone signals (cos and sin).

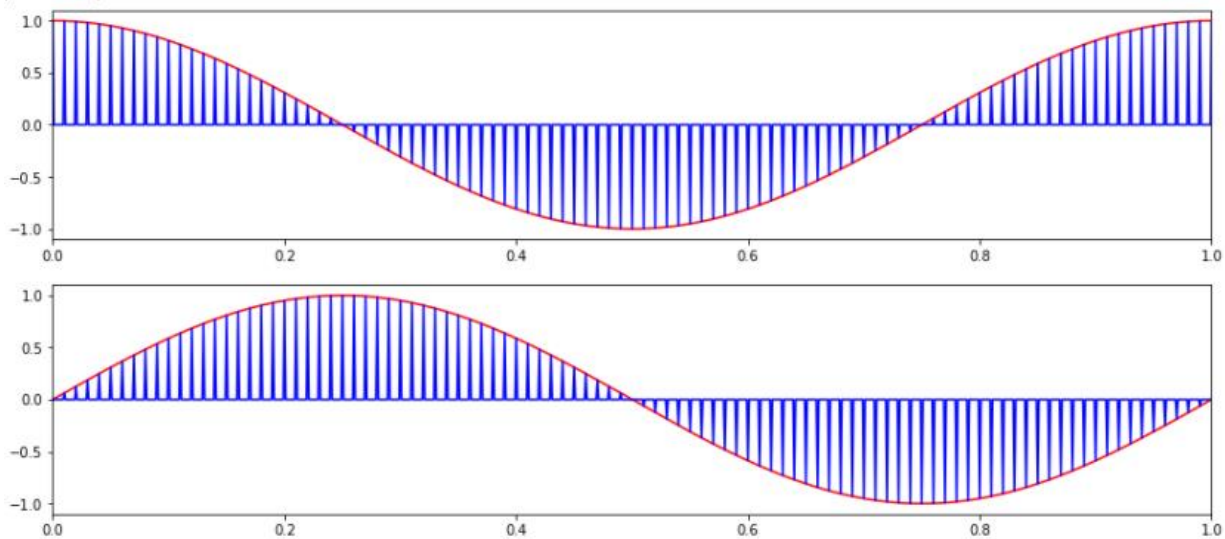


Figure 28: $m_1(t)=\cos(t)$ and $m_2(t)=\sin(t)$

- **Note:** from above figure we show two signals first one is cos signal in (red colure) and his impulses in (blue colure) and the second one is sin signal in (red colure) and his impulses in (blue colure).

Let us try to multiplex the two above signals and plot the results below:

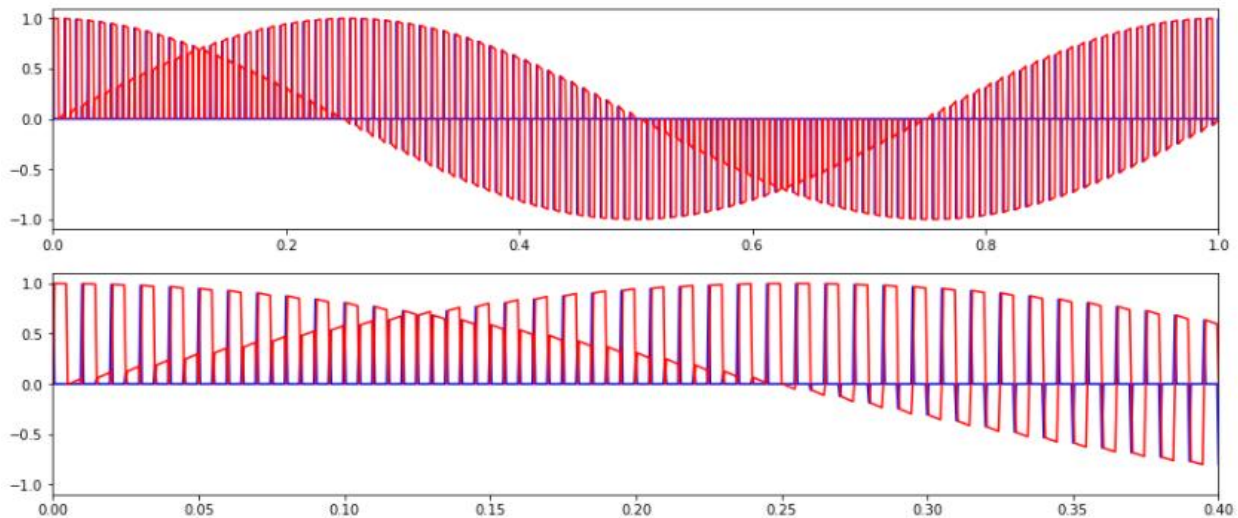


Figure 29: multiplexed the two signals and zoomed-in

- **Note:** We can see from figure29 that each independent signal appears in fraction of time while the other is off, where the signals are sent respectively once Cos signal and once Sin signal. In order to recovered the original message of each signal, we must separate the pulses for two signal from each other and then insert them on the Low Pass Filter.

Exercise:

3- Let change $f_m=3$:

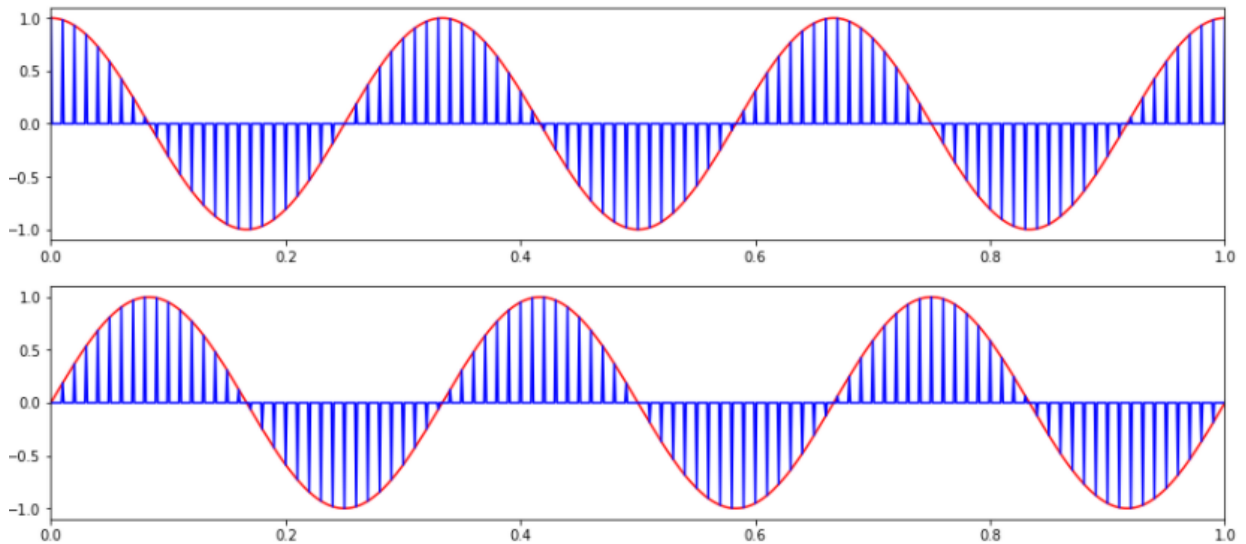


Figure 30: $m_1(t)=\cos(t)$ and $m_2(t)=\sin(t)$ when $f_m=3$

- **Note:** we can show when increase f_m this effects on the value of the period of the cos and sin signals and affects the value of the BW, where when the value of f_m change the period of the signal decreased by $1/f_m$ and the pluses are close to each other. While change f_m it causes a difference in the period of the multiplexed signal and this different due to the difference of the cos and sin signals periodic. But multiplexing process it will stay each indication appears independent at a fraction of the time while the other is off and that appears in the figure below.

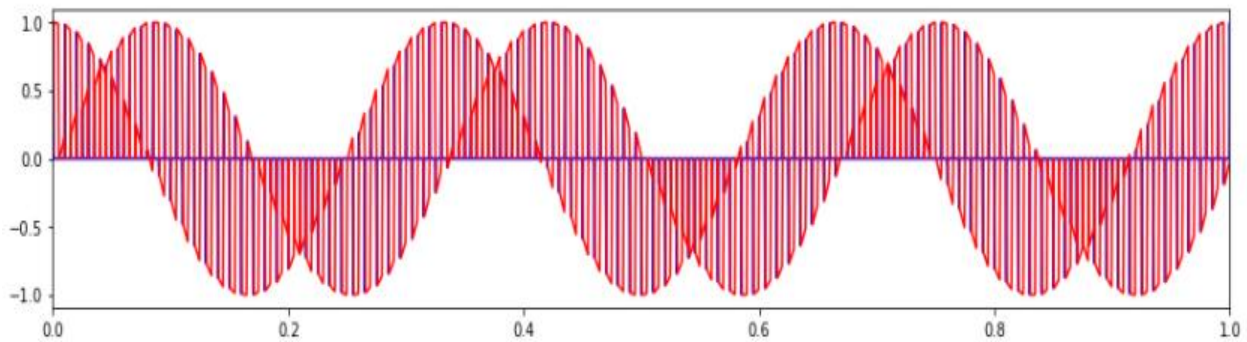


Figure 31: multiplexed the two signals when $f_m=3$

3. Conclusion:

In conclusion, we were able to understand the Working mechanism of Pulse Amplitude Modulation and understand the difference between another type of sampling theorem where the sampling theorem specifies the minimum-sampling rate at which a continuous-time signal needs to be uniformly sampled so that the original signal can be completely reconstructed by these samples alone. We conclude from all the results shown above that when $F_s \geq 2W$ case1 and case3 it's satisfy Nyquist rate so we could reconstruct the message signal by applying a LPF with cut-off frequency = $F_s/2$, but when $F_s < 2W$ in case3 it doesn't satisfy the Nyquist rate so we could not recover or reconstruct the message signal. Also, we were able to understand the division multiplexing process and effect of changing the parameters on the multiplexing signal. Finally, the experiment ran smoothly using the Colab and our results were logical and convincing.