Design Using Root Locus

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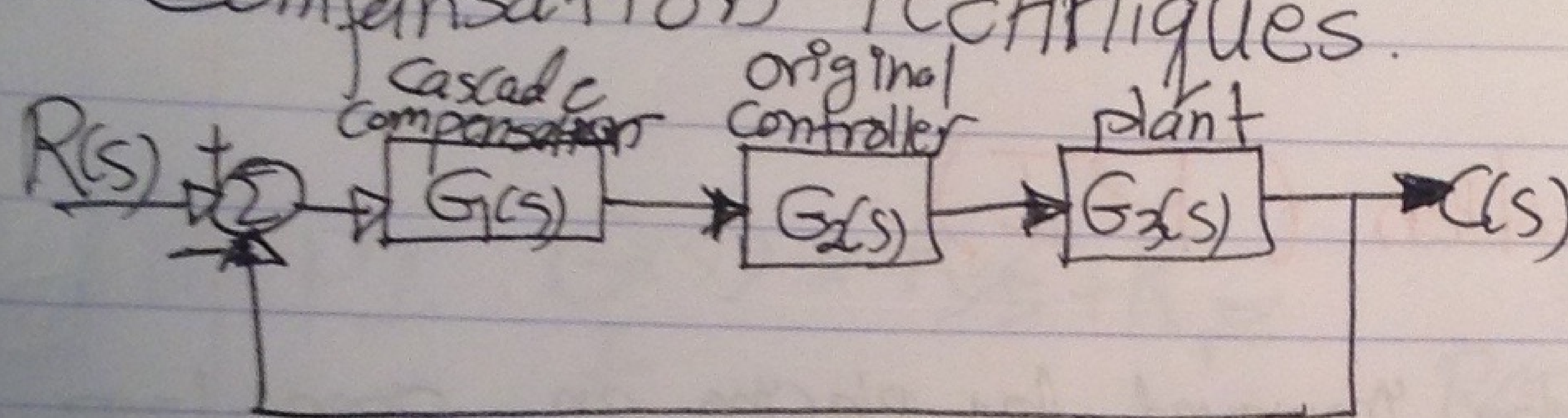
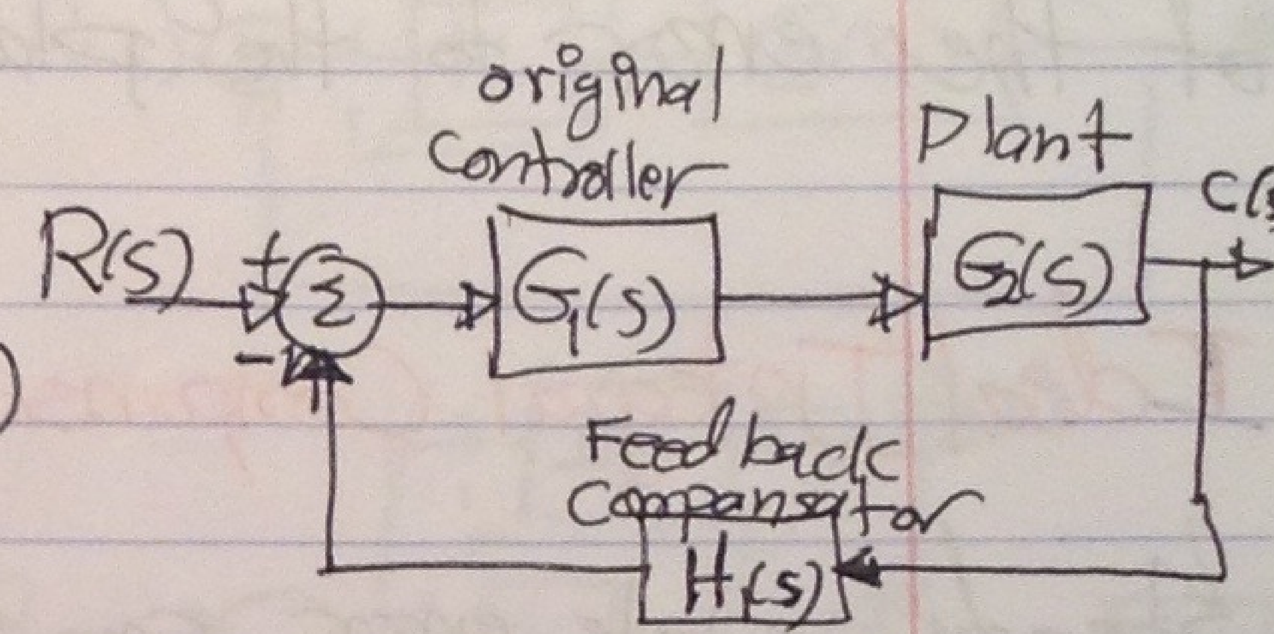
Design via Root Locus.

- The transient response is improved with the addition of differentiation, and steady state error is improved with the addition of integreation in the forward path.

- Compansation techniques.



Cascade



Feedback.

- Ideal compansators : Compansators that use pure integration for improving steady state error or pure differentiation for improving transien response.

R, L, C.

- By Active compansator is that steady state error is reduced to zero (such as amplifier).
- By passive compansator is not driven the steady state error to zero

■ Improving Steady State Error via Cascade Compensation.

- [1] Ideal integral compensation. => increasing the system type and reducing the error to zero. (place the pole at the origin).

[2] Not used pure integration => place the pole near the origin so not derive the steady state to zero.

- Proportional control systems & systems that feed the error forward to the plant.

- Integral control system: " " " " integral of the error to the plant.

- Derivative control system: " " " " " derivative of the error to the plant.

Ideal Integral Compensation (PI):

Steady-state error can be improved by placing an open-loop pole at the origin, because this increase the system type by one.

- Lag compensation:

~~altho~~ Although the ideal compensator drives the steady state error to zero a lag compensator with a pole that is not at the origin will improve the static error constant by a factor equal to Z_c/P_c.

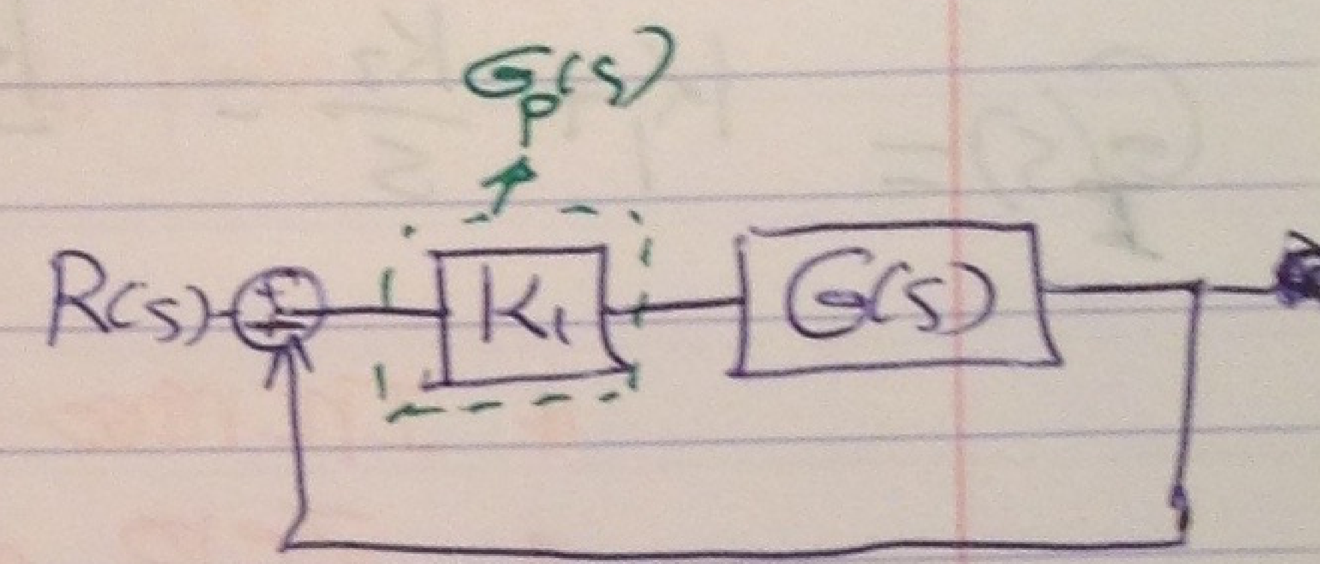
so $\frac{K_{PN}}{K_{PO}} = \frac{Z_c}{P_c}$ $|Z_c| > |P|$

Dynamic Controller — Controllers ال كل -

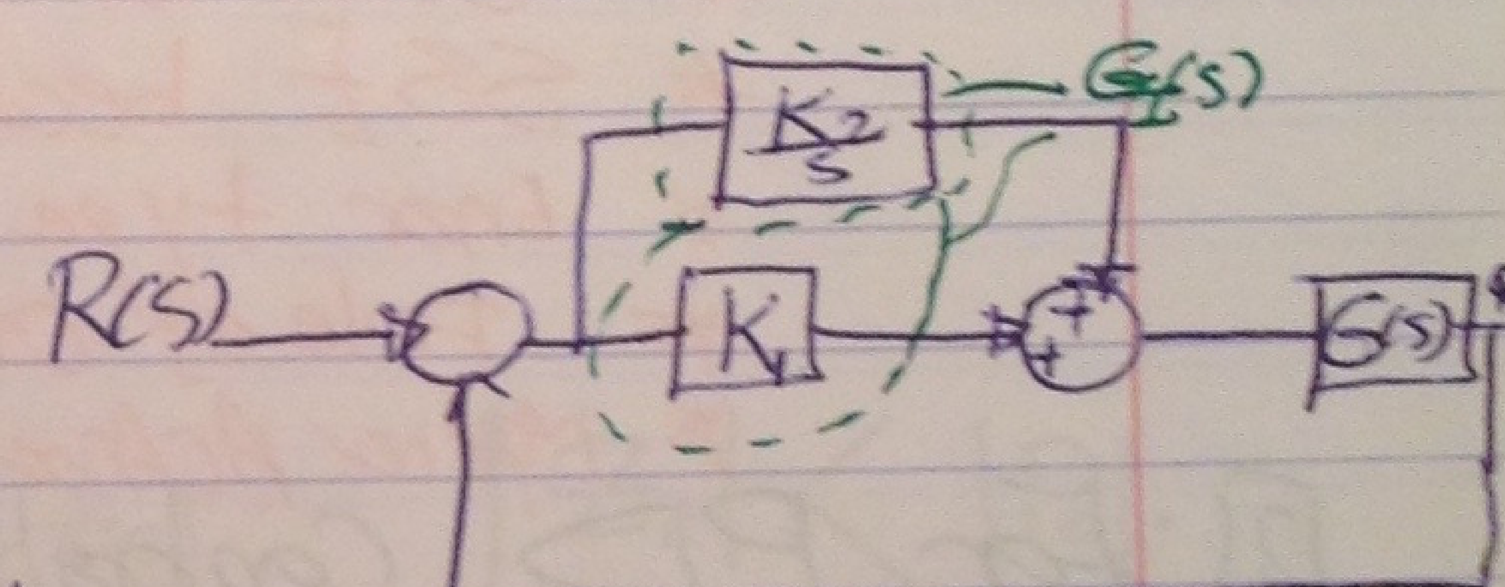
Transfer function & Block diagrams for the Controller

1 P

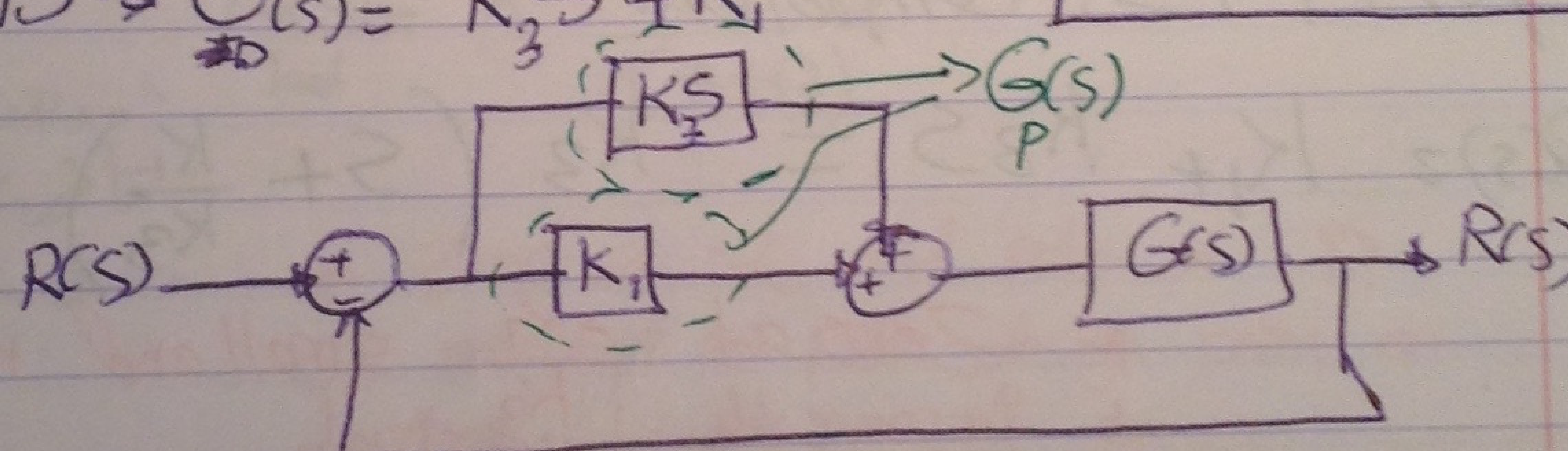
$$G_P(s) = K_1 \equiv$$



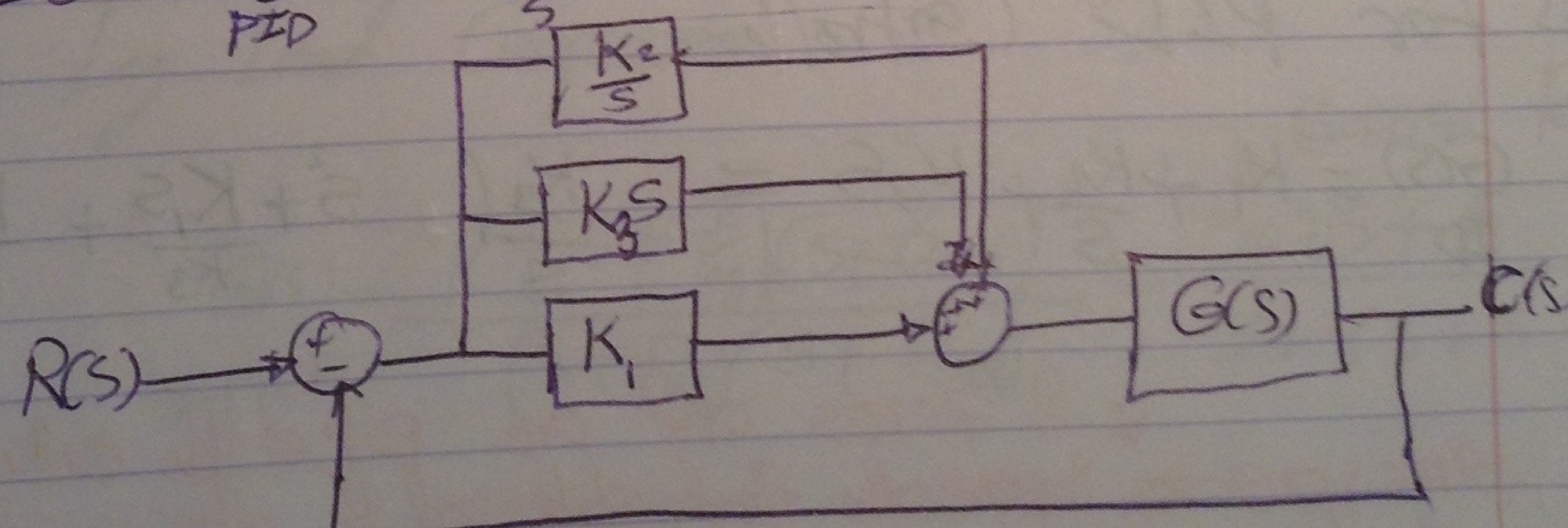
2 PI $G_I(s) = K_2/s + K_1 \equiv$



3 PD $G_{PD}(s) = K_3 s + K_1$



4 PID $G_{PID}(s) = \frac{K_2}{s} + K_3 s + K_1$



- Note on the transfer function for each controller

[1] For PI Controller

$$G_{PI}(s) = K_1 + \frac{K_2}{s} = \frac{K_1 s + K_2}{s} = \frac{K_1\left(s + \frac{K_2}{K_1}\right)}{(s+0)}$$

* Increase the system type
* Zero at $\boxed{-\frac{K_2}{K_1}}$ small and negative
* SSE becomes zero
* Error type increase
* pole at origin.
* require Active Components to implement.

[2] For PD Controller.

$$G_{P}(s) = K_1 + K_3 s = K_3\left(s + \frac{K_1}{K_3}\right)$$

* Zero at $-\frac{K_1}{K_3}$ small and negative
* improve the transient
* require Active Components to implement.

[3] For PID Controller

$$G_{PID}(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_3}{s}\left(s^2 + \frac{K_1 s}{K_3} + \frac{K_2}{K_3}\right)$$

* 2 zeros and one pole at the origin.

since one to improve the steady state and the other to improve the transient.

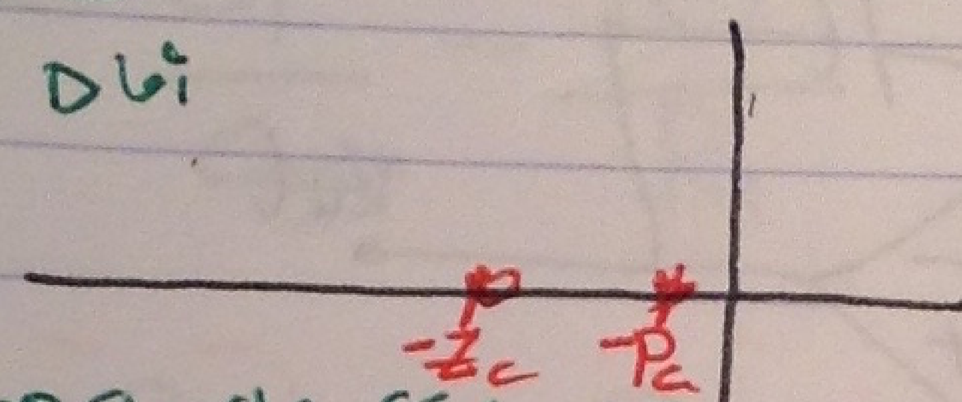
For

$\boxed{4}$ Lag Compensator

$$G_L(s) = K\,\frac{(s+Z_c)}{(s+P_c)} \qquad |Z_c| > |P_c|$$

وذلك حتى نحقق ideal
بديل I ، ويتحكم في
ال I ... delay (lag)
أما D

* pole is close to origine at $-P_c$
* zero left to the pole



حتى نكون حصلنا على أكبر Lag.

$\boxed{5}$ For Lead Compensator

$$G_{Lead}(s) = K\,\frac{(s+Z_c)}{(s+P_c)} \qquad |Z_c| < |P_c|.$$

* ولكن يكون ال Z_c
تأثيره أقوى من ال P_c
وذلك لنكون محصلة هي
Lead.

* Zero at $-Z_c$ and pole at $-P_c$ are selected to put design point on root Locus.

$\boxed{6}$ For Lag-Lead Compensator:

General form.

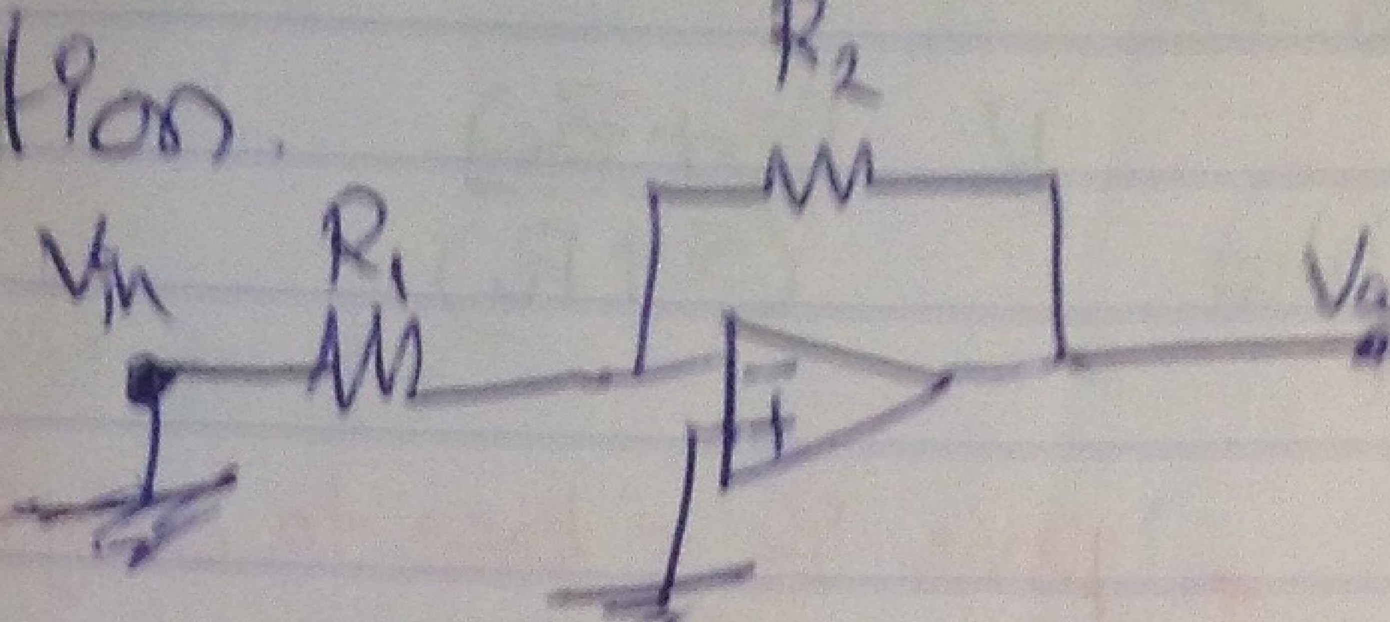
$$G_{lag\text{-}lead}(s) = \frac{K\,(s+Z_{lag})(s+Z_{lead})}{(s+P_{lag})(s+P_{lead})} = \frac{s^2+\gamma s+\delta}{s^2+\gamma s+\delta}$$

* Lag pole at $-P_{lag}$ & lag zero at $-Z_{lag}$ to improve the ~~transient~~ SSE

* Lead pole at $-P_{lead}$ & Lead zero at $-Z_{lead}$ to improve the ~~SSE~~ transient.

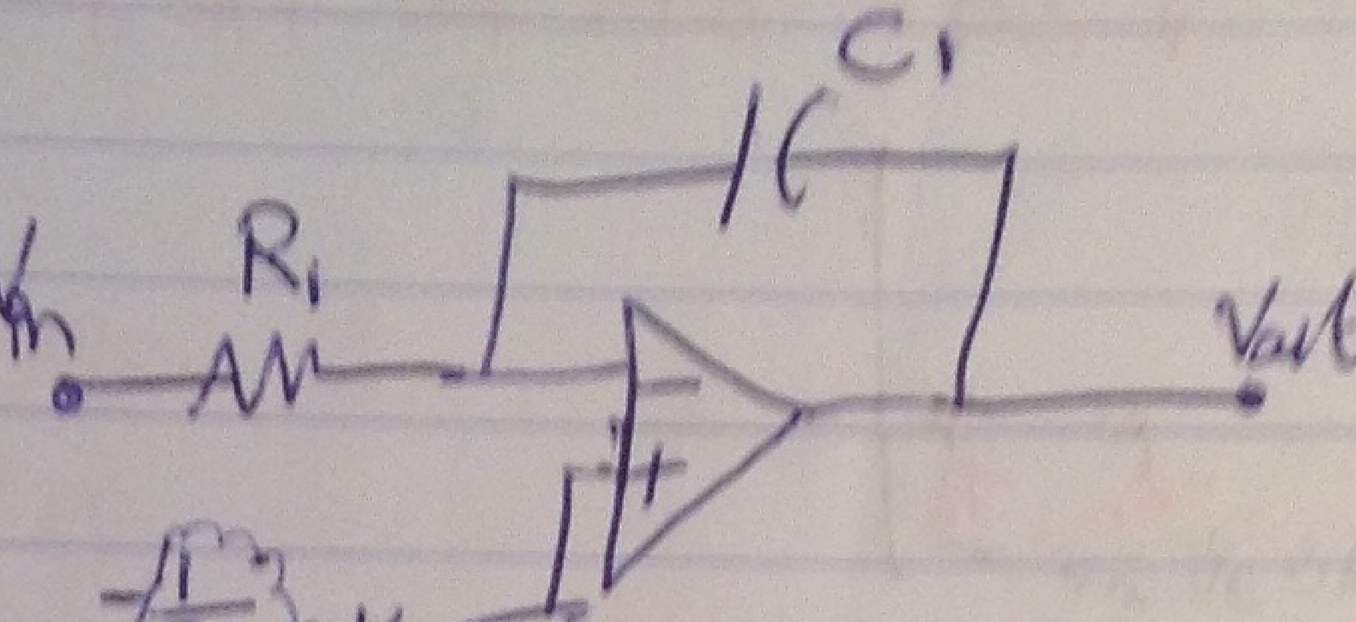
- physical realaization

P- Controller



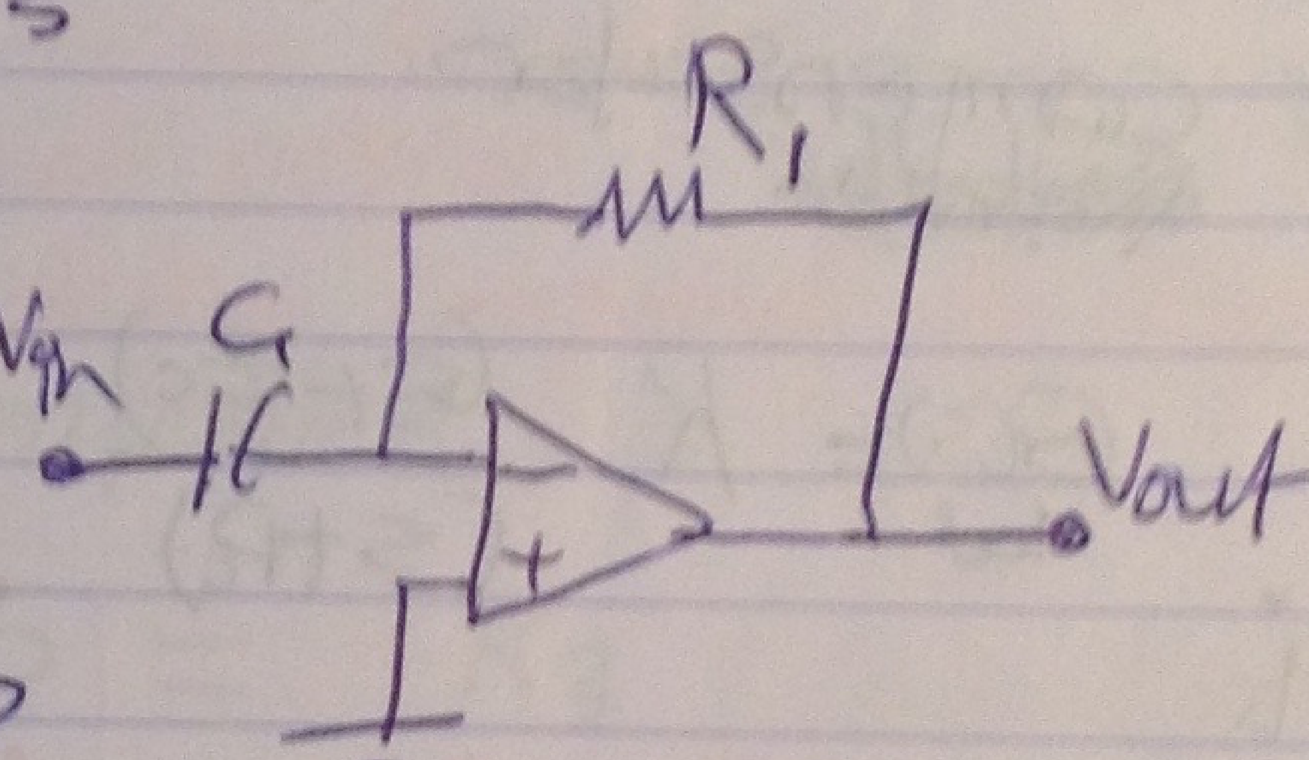
$G_P(s) = \frac{-R_2}{R_1}$

I-Controller



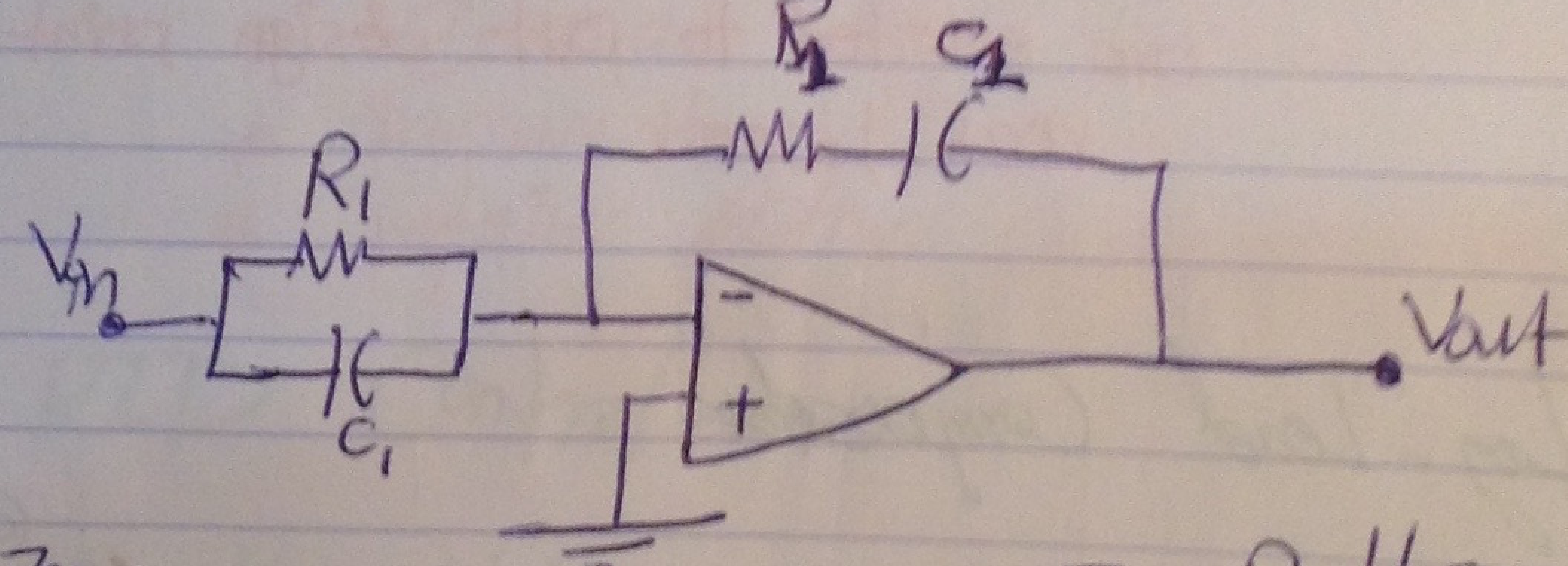
$G_I(s) = \frac{-\frac{1}{sC}}{R_1} = \frac{-\frac{1}{\boxed{R_1C_1}}K}{s}$

D- controller



$G_D(s) = \frac{-R_1}{\frac{1}{sC_1}} = -\underbrace{\boxed{R_1C_1}}_{K} s$

- PID



$G_{ID}(s) = \frac{-Z_2}{Z_1}$

$= \frac{-\left(R_2 + \frac{1}{sC_2}\right)}{\frac{R_1}{1+sC_1R_1}}$

$= \frac{-\left(\frac{1+sC_2R_2}{sC_2}\right)}{\frac{R_1}{1+sR_1C_1}}$

$Z_1 = R_1 // C_1$

$= \frac{R_1 \cdot \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}}$

$= \frac{R_1}{sC_1R_1+1}$

$Z_2 = R_2 + \frac{1}{sC_2}$

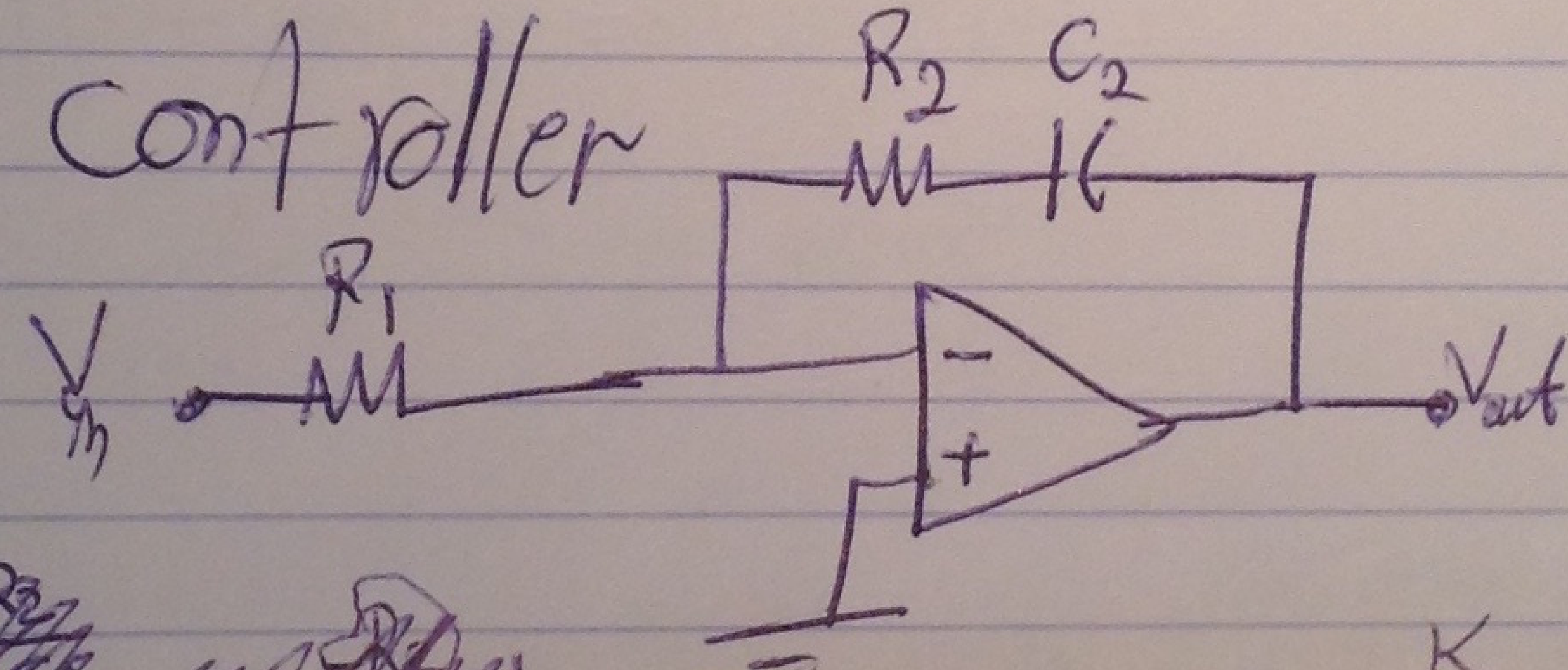
$$= -\left(\frac{1+R_2C_2S}{SC_2}\right) * \frac{1+R_1C_1S}{R_1}$$

$$= \left(\frac{-1}{SC_2} - \frac{R_2\cancel{C_2S}}{\cancel{C_2S}}\right) * \frac{1}{R_1} + \frac{\cancel{R_1}C_1S}{\cancel{R_1}}$$

$$= \frac{-1}{R_1C_2S} - \frac{C_1}{C_2} - \frac{R_2}{R_1} - R_2C_1S$$

$$= -\left[\frac{C_1}{C_2} + \frac{R_2}{R_1} + R_2C_1S + \frac{1}{R_1C_2S}\right]$$

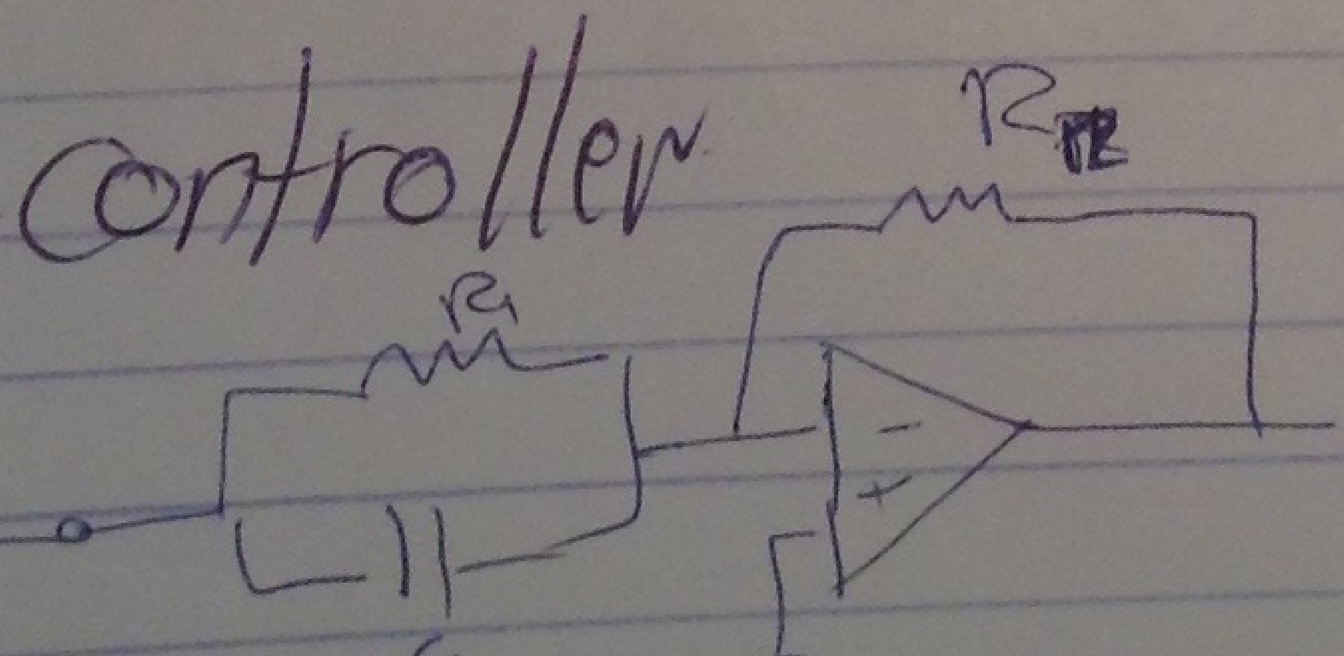
-PI- controller



$$G(S) = \frac{-(R_2 + \frac{1}{SC_2})}{R_1} = -\left(\frac{R_2C_2S+1}{SC_2R_1}\right) = \underbrace{\frac{-R_2}{R_1}}_{K}\,\frac{\left(S + \underbrace{\frac{1}{R_2C_2}}_{Z_B}\right)}{(S)}$$

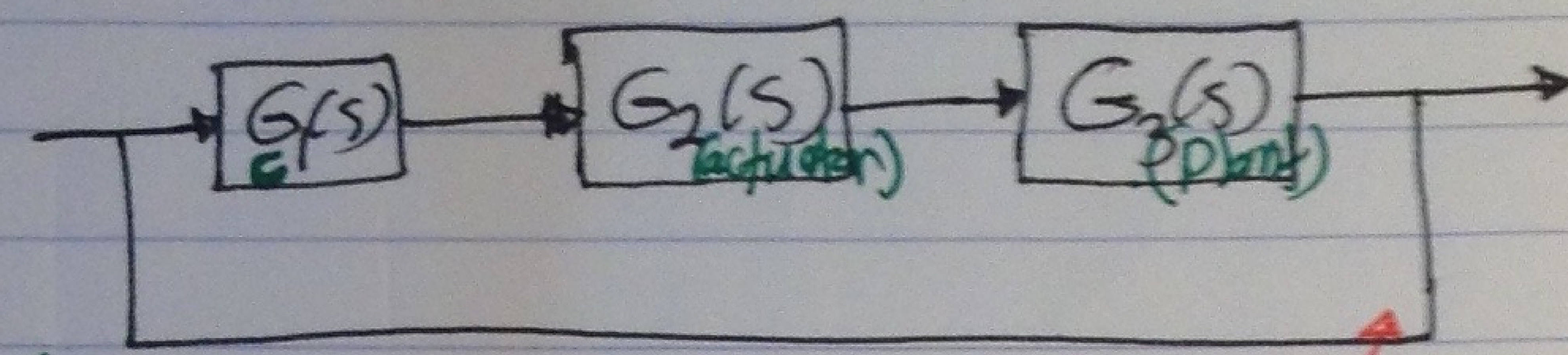
(S) pole at origin.

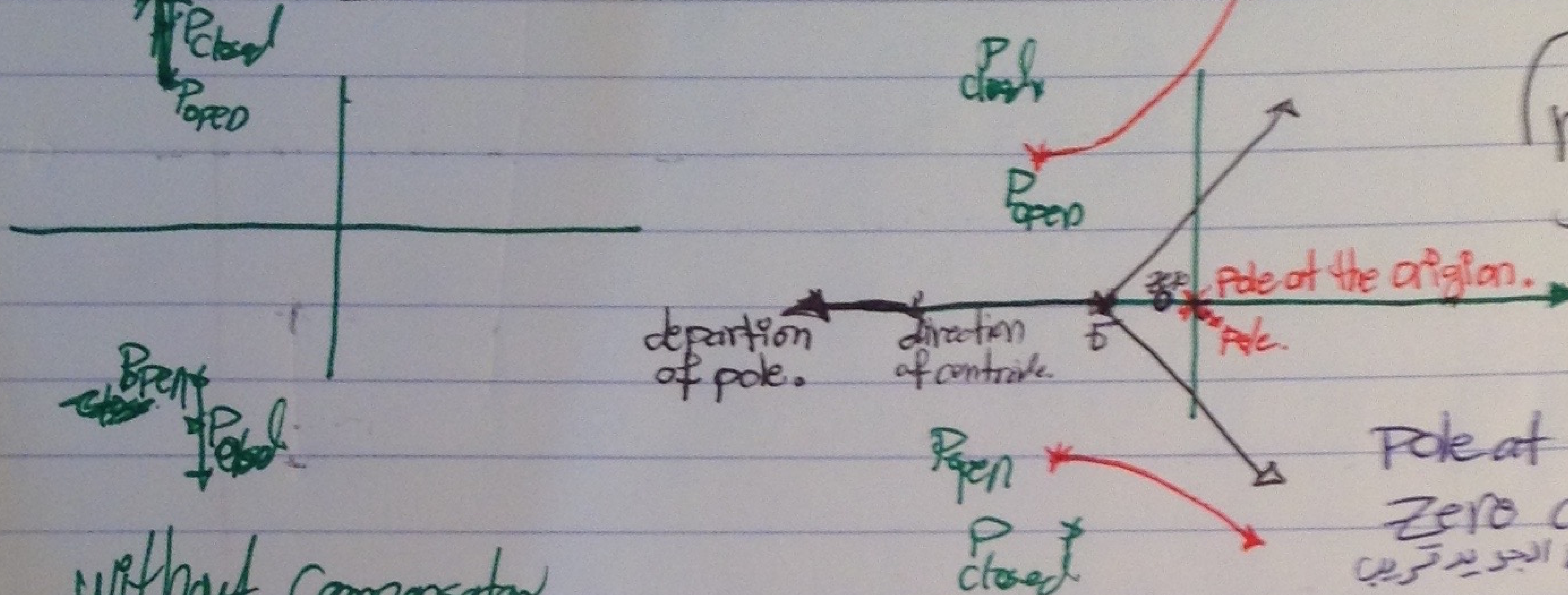
-PD-controller



??

- PI ~~compensator~~ controller : In direct path.





without compensator

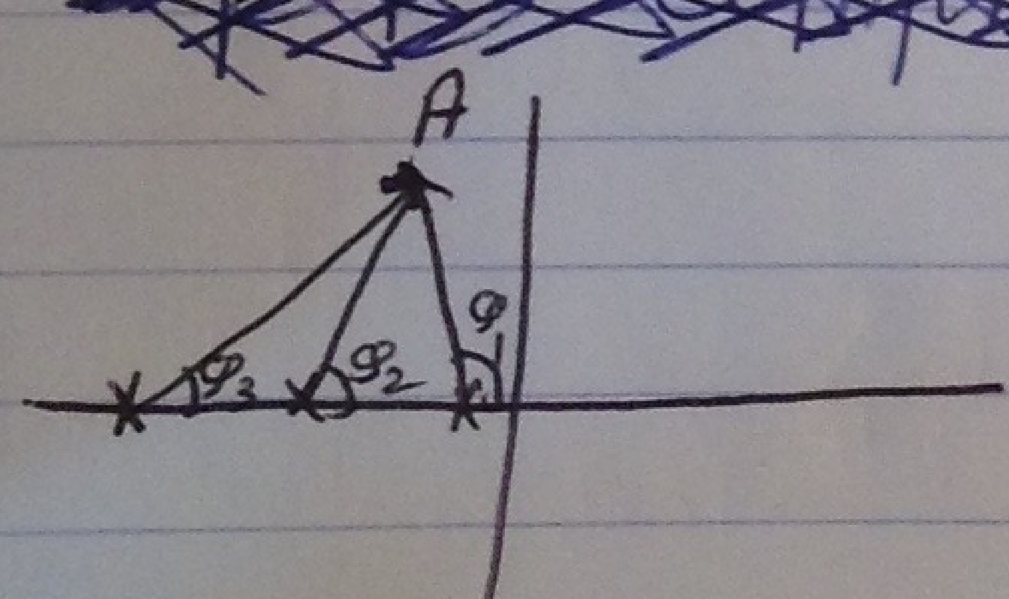
(تغيير الـ root locus وبالتالي تغيير الـ SSE لأنه الأصغر.

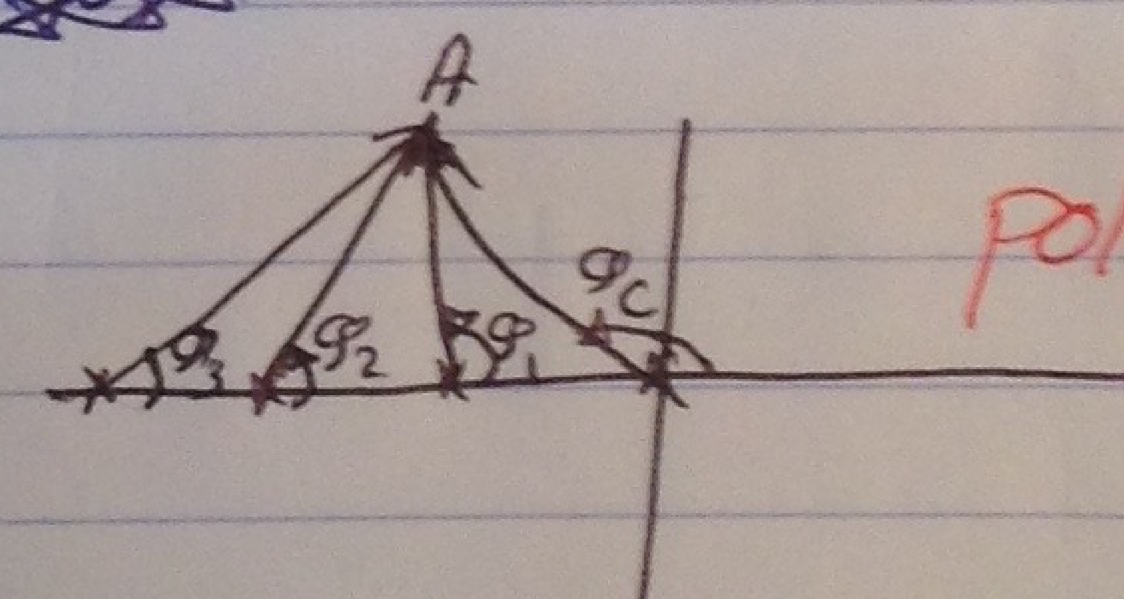
Pole at origion & Zero close to it
لحتى يكون الـ root locus الجديد قريب من القديم.

Increase the system type

SSE = 0

~~PD compensator~~

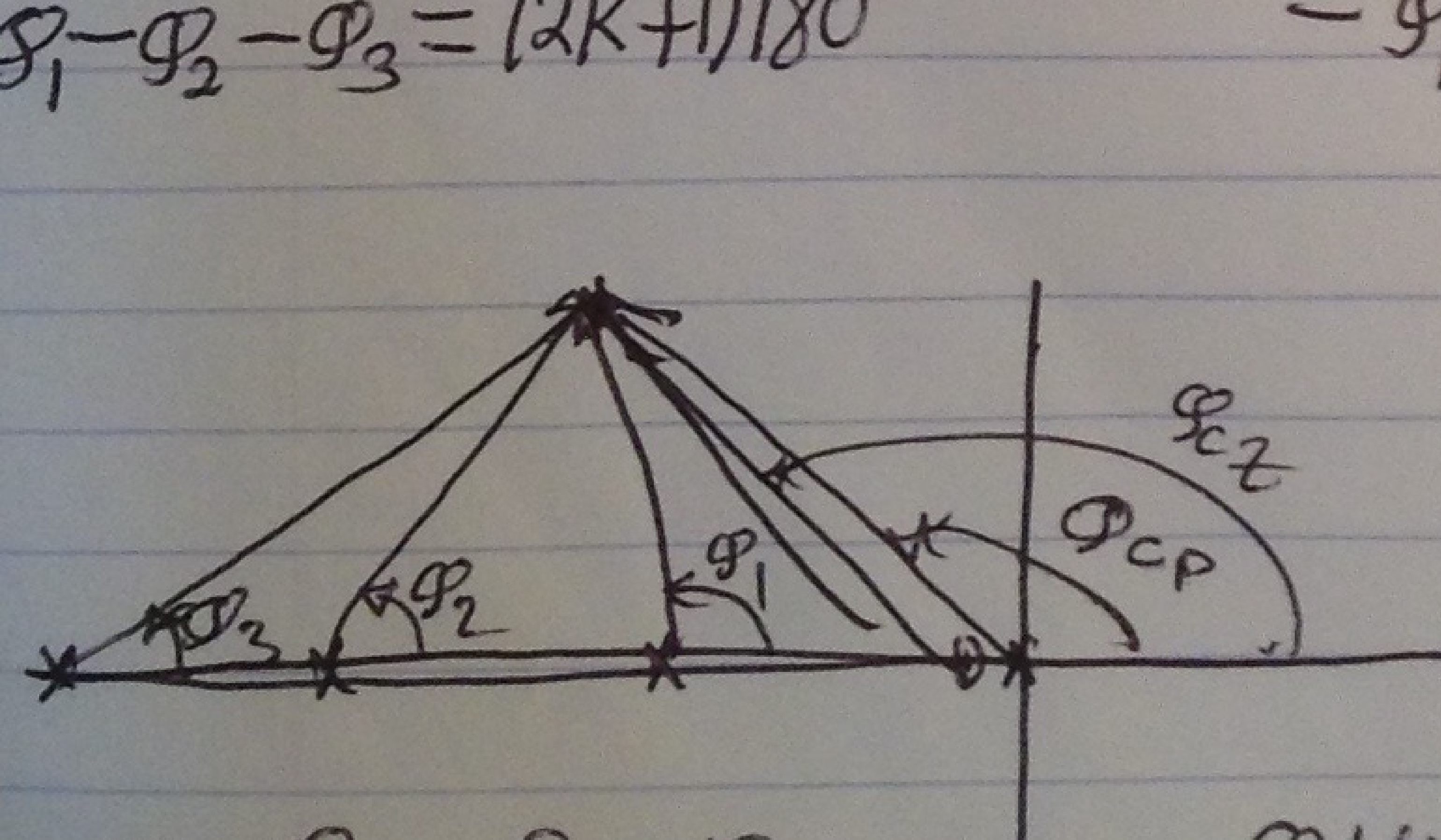




pole at the origion without close Zero

$$-\varphi_1-\varphi_2-\varphi_3=(2K+1)180$$

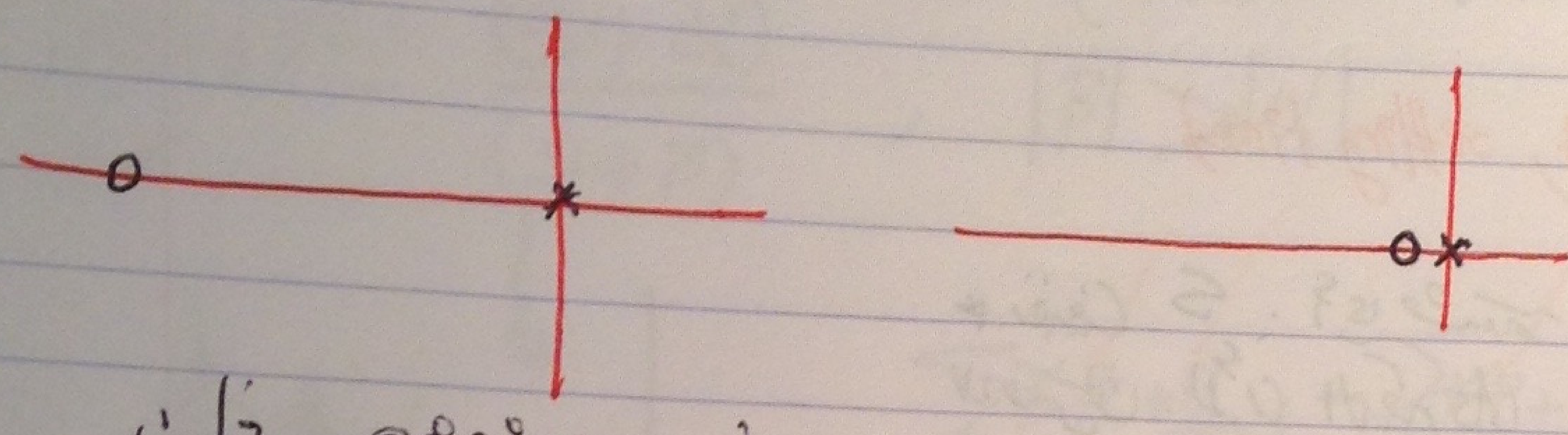
$$-\varphi_1-\varphi_2-\varphi_3\neq(2K+1)180$$



pole at the origion with close Zero

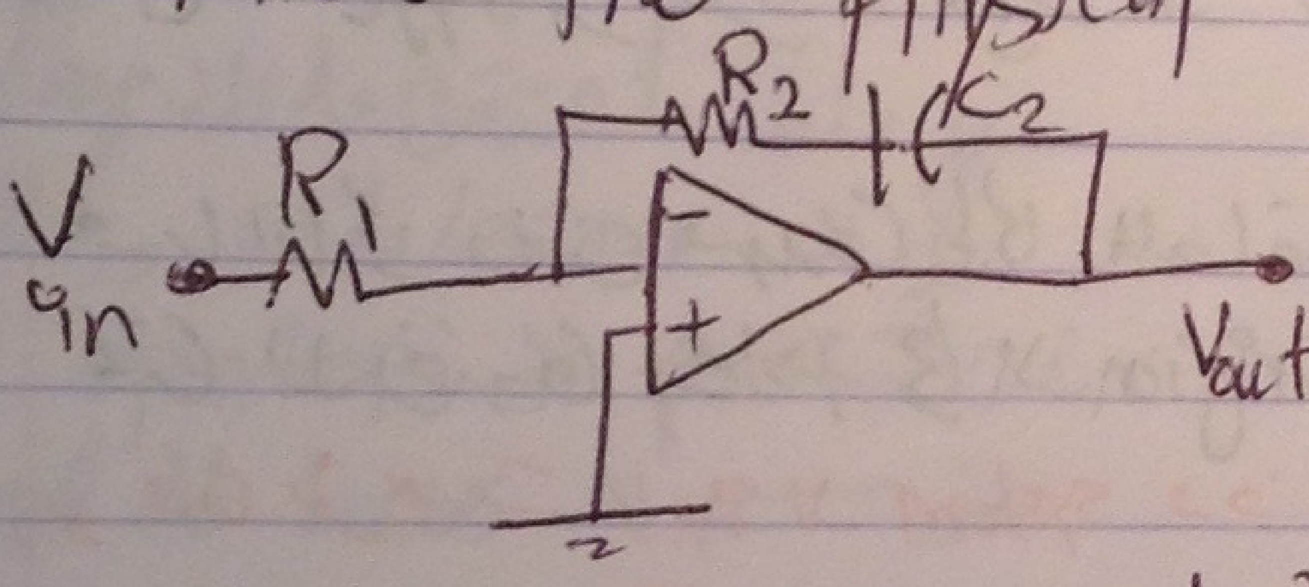
$$-\varphi_1-\varphi_2-\varphi_3-\varphi_{pc}+\varphi_{zc}\simeq(2K+1)180$$

If we have two case.



* كل ما قربنا ال zero على ال pole الذي موجود في ال origin فإن قيمة ال C في ال senthesis تزيد . أي أنه كل ما قلت المسافة تزيد ال capacitor.

We know the physical realization for PI



$$G_{PI}(s) = \frac{-R_2}{R_1} \frac{\left(s + \frac{1}{R_2 C_2}\right)}{s}$$

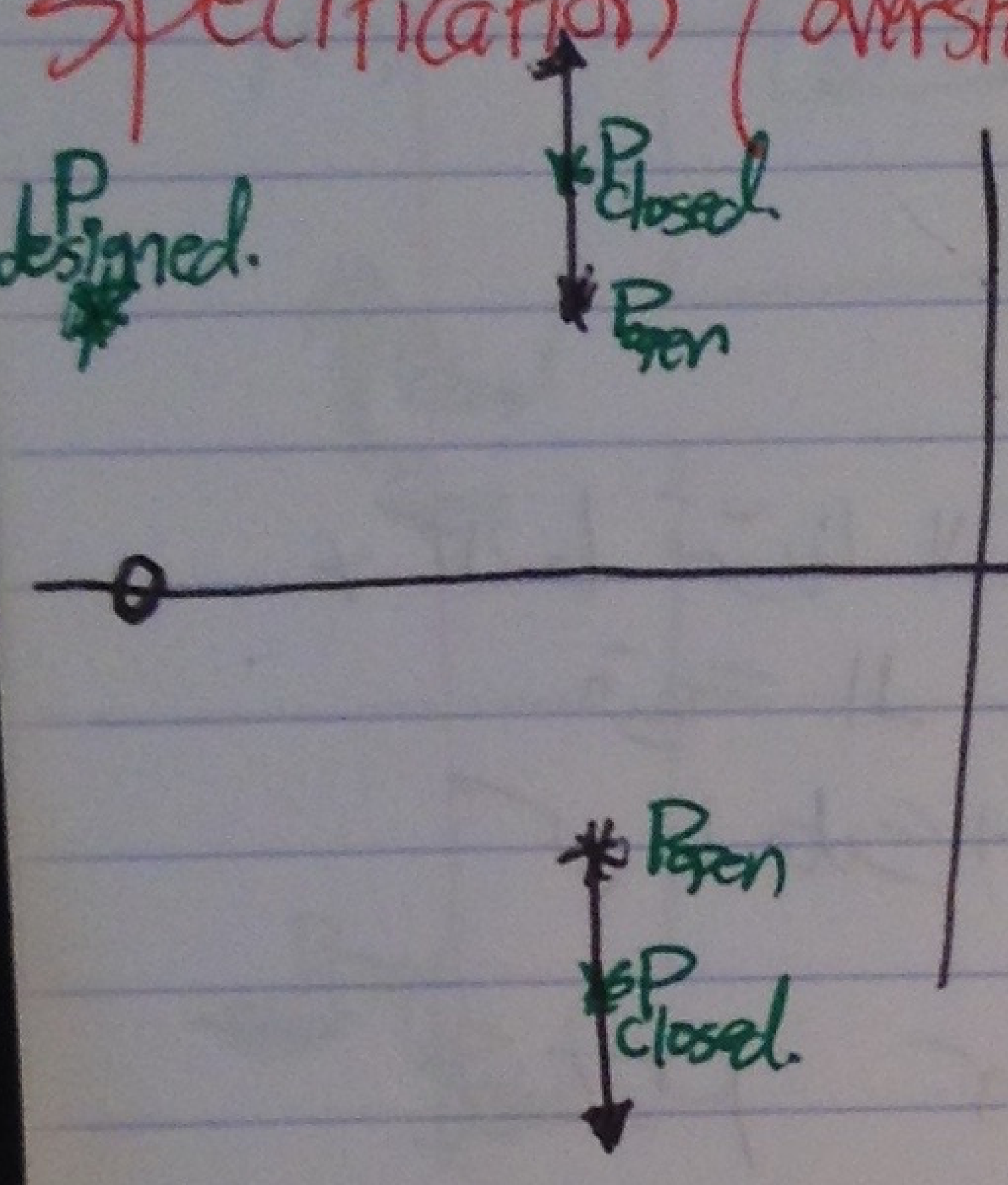
- If we have $Z_c = 0.001 \Rightarrow \frac{1}{R_2 C_2} = 0.001$, let $R = 1k\Omega$
compensator

$$C_2 = \frac{1}{0.001 \times 1 \times 10^3} = \frac{1}{10^{-3} \times 10^3} = 1F$$

If we have $Z_c = 0.01 \Rightarrow \frac{1}{R_2 C_2} = 0.01 \Rightarrow C_2 = \frac{1}{10^3 \times 10^{-2}} = 0.1F$

so as the value of zero decrease the capacitor value increase.

PD-Controller : We want to chang the transient specification (overshoot, settling time).

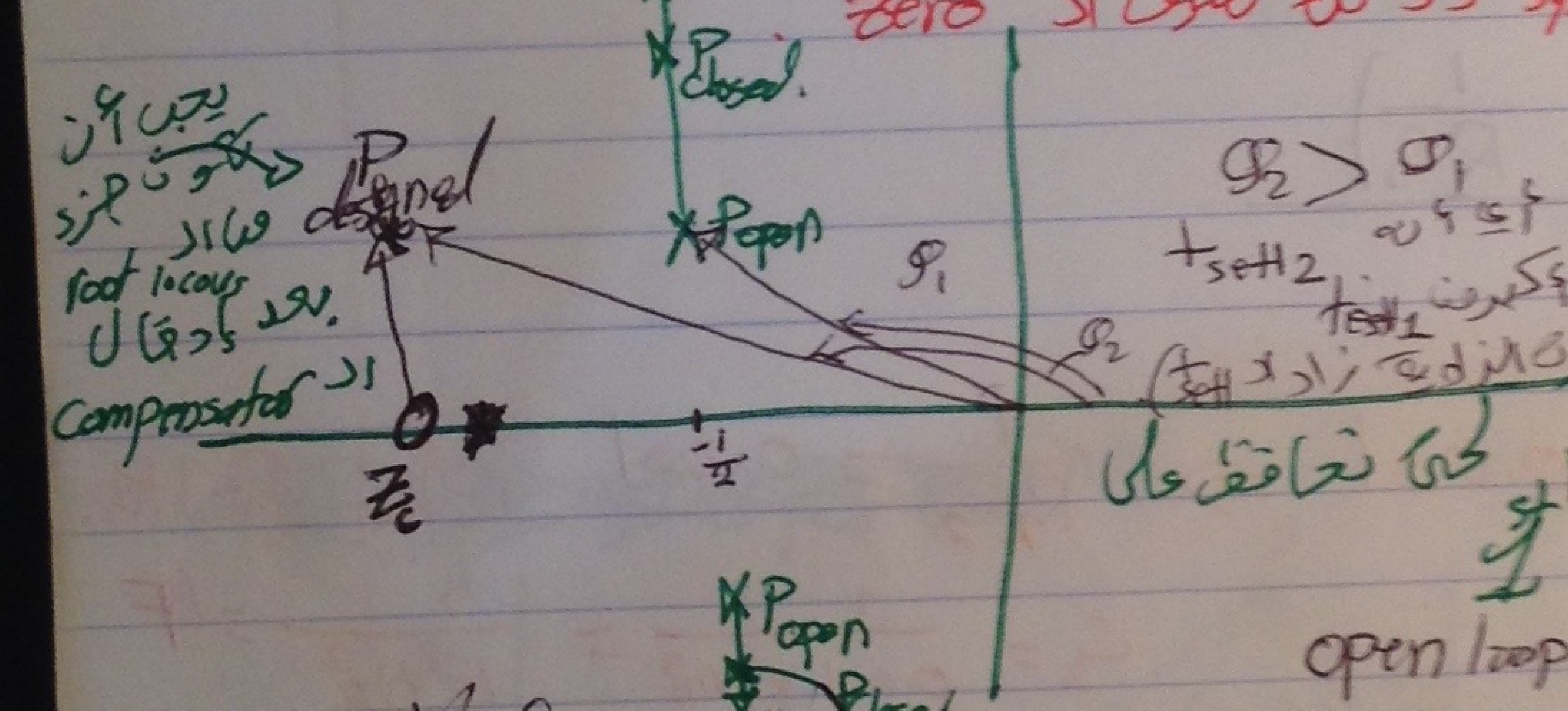


* نضيف S. أي مشتقة للاقتران. لأن التغير على المشتقة أسرع من التغير على الاقتران الأصلي

- إذا كان الـ system هو typ1 واشتققناه فإن يصبح type 0 و تزداد قيمة الـ error.

و بالتالي نحن نريد أن نحافظ على الـ type و أيضاً تحسين الـ transient.
نحن لا نضيف pole في الـ origin لأنه سوف يزداد الـ error.

* نحن نريد تسريع الـ system و لذلك نضيف الـ zero



$\theta_2 > \theta_1$
إذن t_{sett2} أكبر من t_{sett1}
(كلما زادت الزاوية زاد الـ t_{sett})

نحن نحافظ على 1st order implementation

additional الـ zero تضعه على الـ real axis

* Z_c نضيفه على الـ open loop system.

$$G_p(s) = K(S + Z_c)$$

أدخل zero.

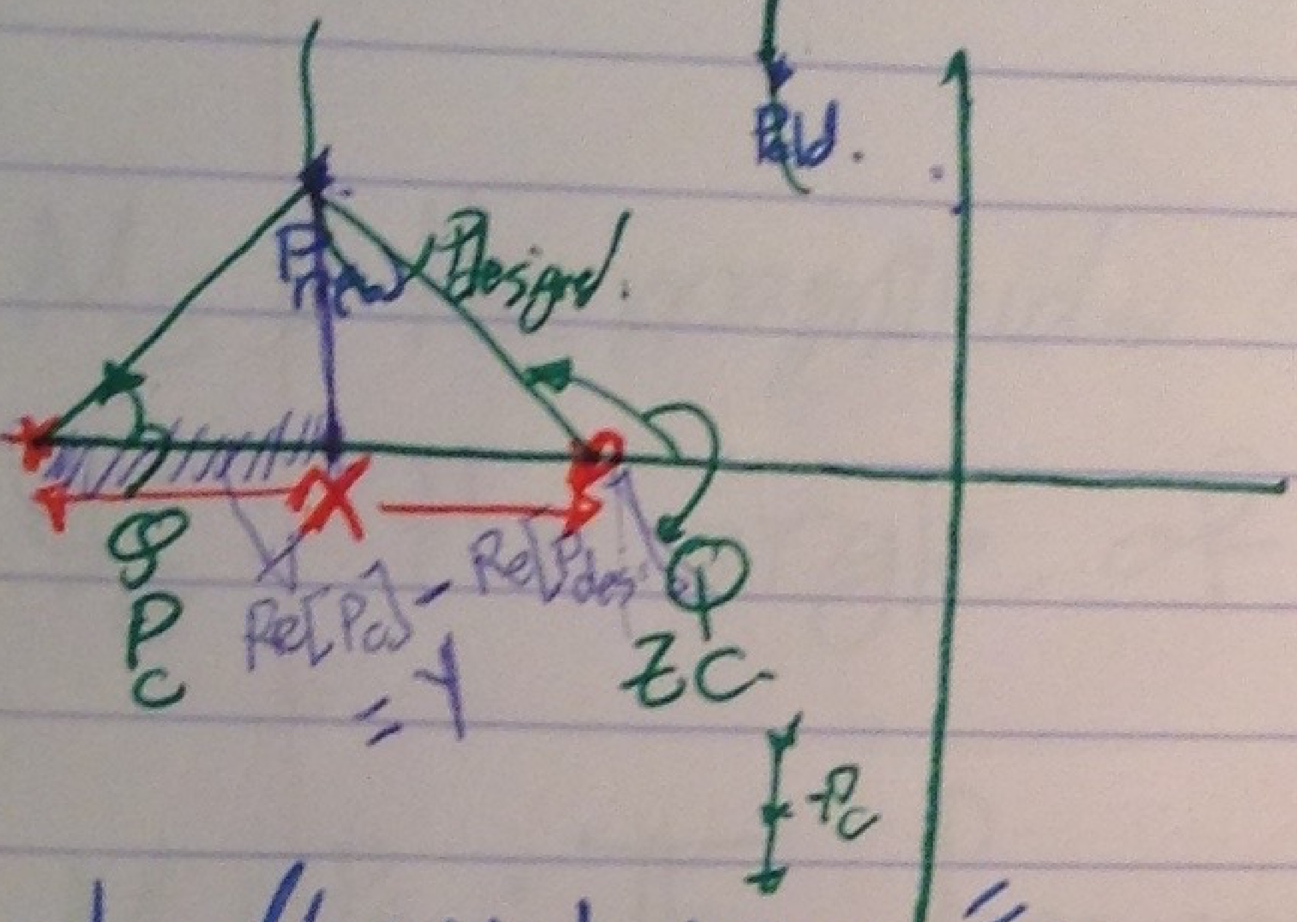
* نفترض أن كان الـ $t_{sett} = 8$ و نريد أن يكون $t_{sett,designed} = 4$

- نحن نريد أن الـ P designed جزء من الـ new root locus

يجب أن مجموع زوايا الـ poles و الـ (زاوية) P designed = (2k+1)π.

Lead compensator (لا نستخدم Amp ، لا نستخدم Power supply).

$$G(s)_{Lead} = \frac{(s+\alpha)}{(s+\beta)} , \quad |\beta| > |\alpha|$$



$$\sum \angle Z_K + P_{desired} - \sum \angle P_K + P_{designed} - \varphi_{P_C} + \varphi_{ZC} = (2\ell+1)\pi$$

for open loop, for closed loop, for open loop, for closed loop, one unknown

by Assumption.

$$= \tan^{-1}\left(\frac{Re(P_{des})}{Y}\right)$$

* دائماً نشتغل على الـ closed loop في الـ root Locous وذلك نحن نستخدم الـ open loop لحتى نرسم الـ Locus للـ closed.

$$\text{موقع الـ } Z_C = Re[\;] - X$$

* يجب أن نفحص الـ SSE بعدما حسّنا في الـ transient.
- كلما كانت Z_c و P_c أقرب على بعض يكون SSE أقرب للقديم.

- Lag-lead compensator.

lag : نحافظ على الـ transient
lead : نحصل على $P_{desired}$.

الـ pole أقرب ما يمكن على الـ origin وبالتالي نحافظ على $P_{desired}$.