Frequency Response

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Frequency Response.

$G(j\omega) = M_G \angle \varphi_G$, where.

M_G : the magnitude of $G(j\omega)$

φ_G : the angle of $G(j\omega)$

$M_G \angle \varphi_G$: The frequency response of the system &

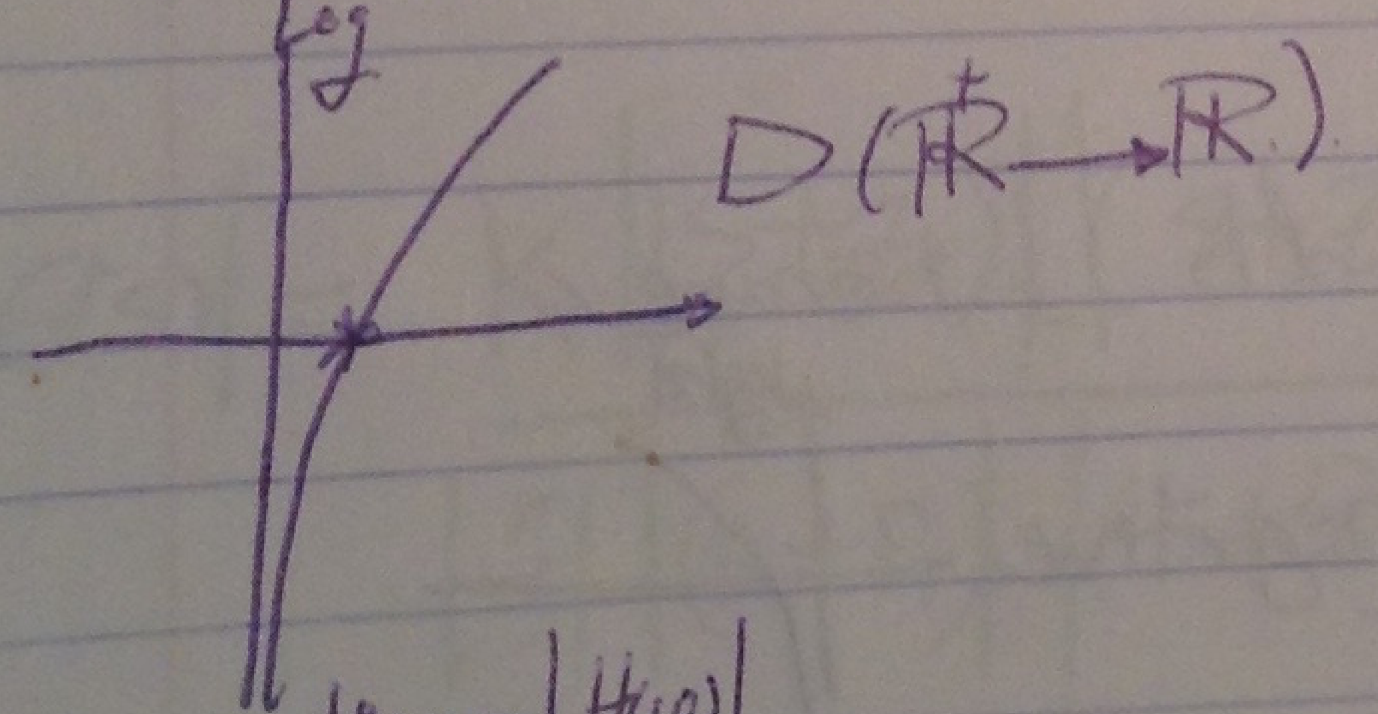
$$G(j\omega) = G(s)\Big|_{s \to j\omega}$$

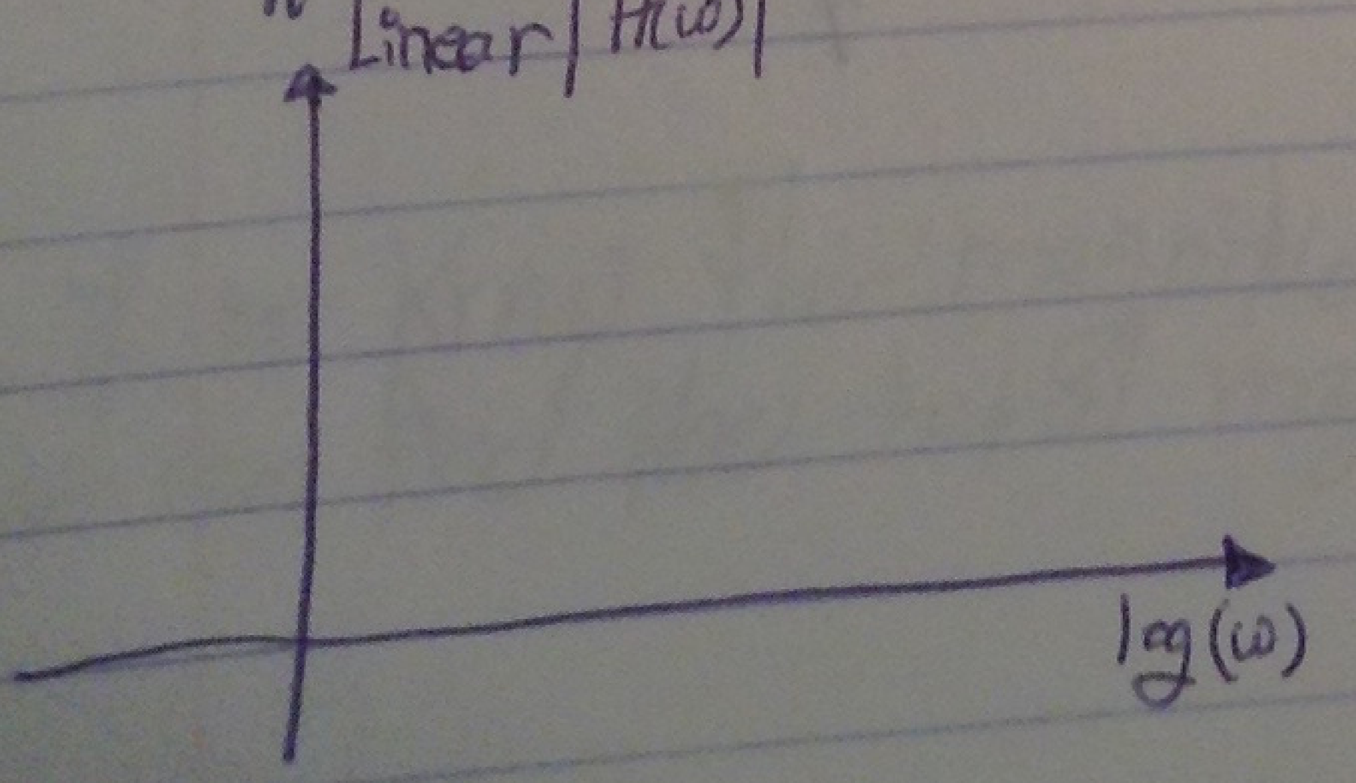
$$y(t) = \sum y_n(t)$$

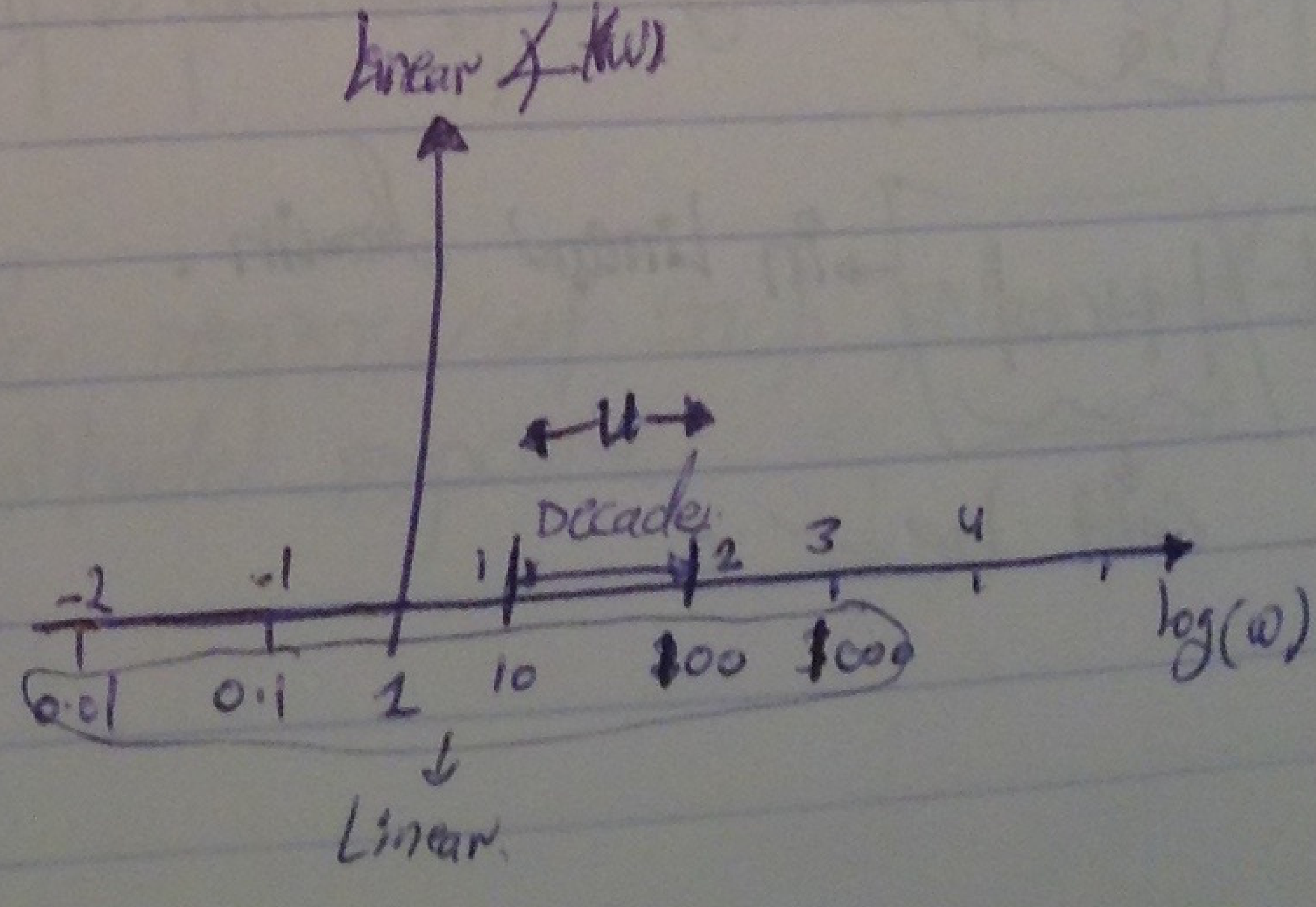
$$y_n = X_n |H(\omega)| \Big| \cos(\omega_n t + I + \measuredangle H(\omega)) \Big|_{n\omega_0}$$

- **Bode Plots.**

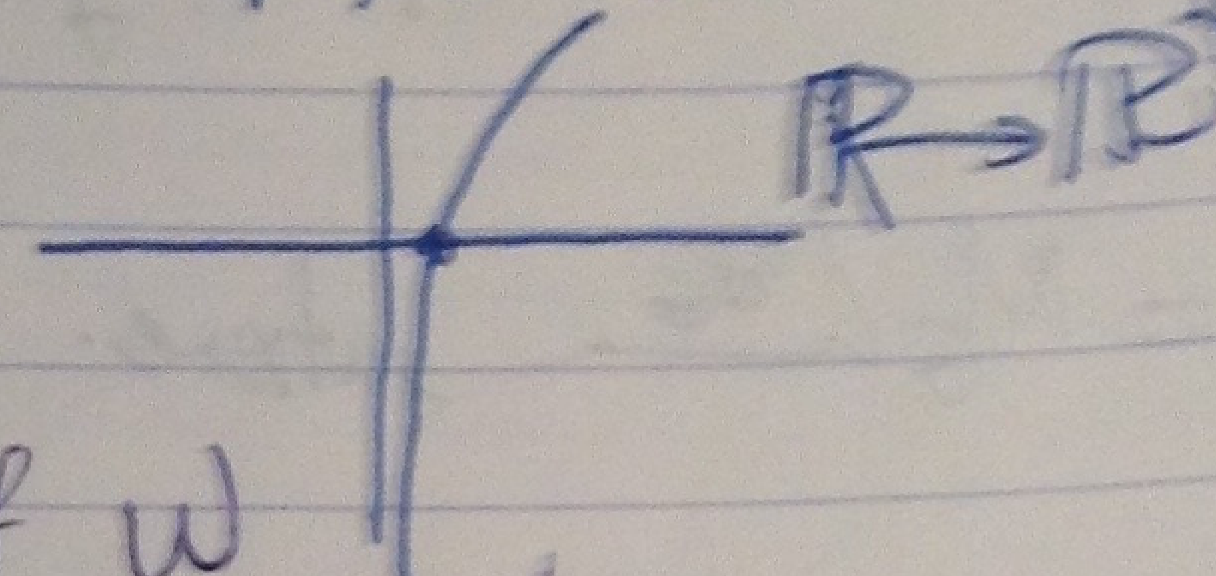
Bode plots: spectral (function of frequency (Depend of f).) representation using semi log refrence





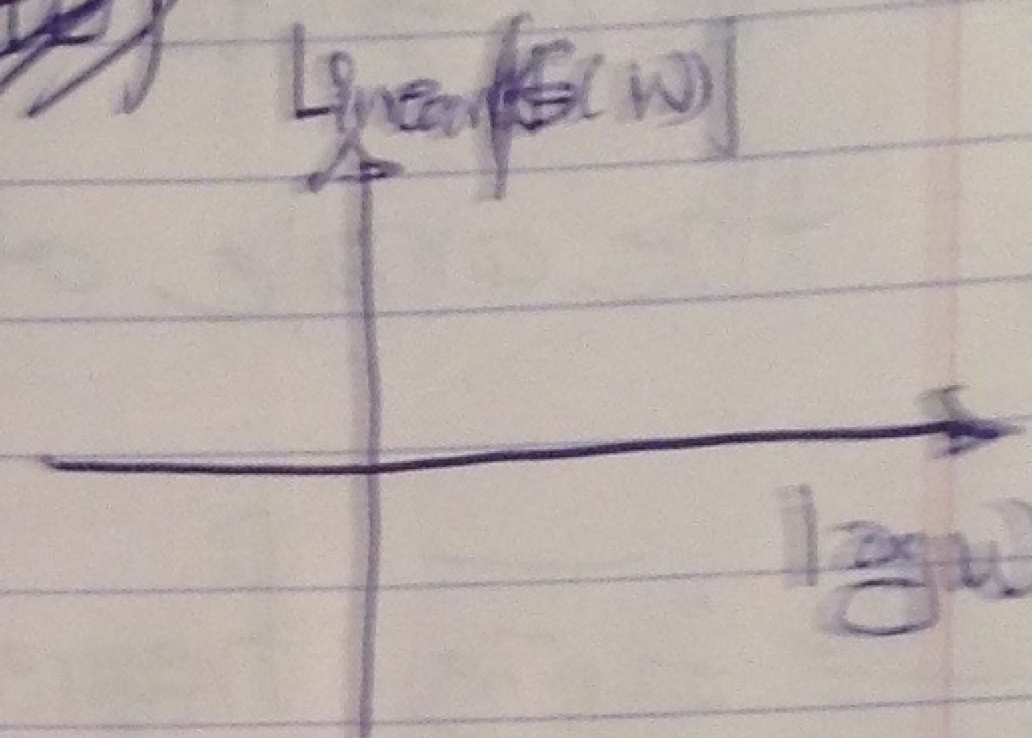


- Bode plots: is single side representation because we have positive domain. since



Semilog: one axis is $\underbrace{\text{Logarithm}}_{}$ — the axis of ω

and the other is $\underbrace{\text{Linear}}_{\text{the axis of magnitude}}$

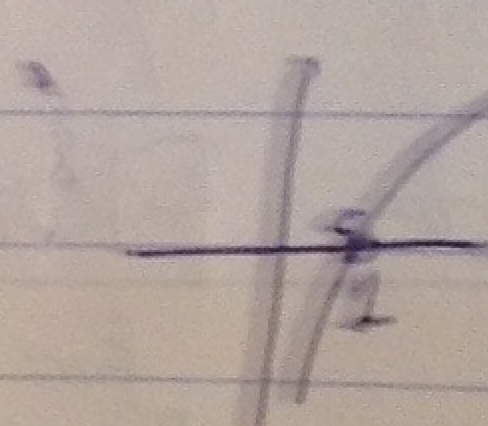


Linear: have a constant unit between each two points on its domain.

$$\begin{array}{ccccc} \vdash u_1 \dashv & \vdash u_2 \dashv & \vdash u_3 \dashv & & \\ 2 & 4 & 6 & 8 & 20 \end{array}$$

$$u_1 = u_2 = u_3$$

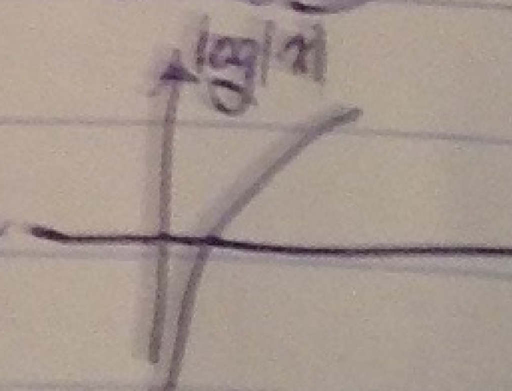
In logarith refrence we not have a constant unit.



- the origion for linear scale at $-\infty$
- " " " logarithmics " 1

- We can say 1db, 2db, -1db, -2db ; because

$1db = 20 \log|x|$ → may be positive or negative



$\underbrace{|H(\omega)|_{db}}_{\text{can be} < 0} \neq \boxed{|H(\omega)|}$ → in Linear domain. $|H(\omega)|$ can't be < 0

L.T.I

1. pole-zero form
2. time-constant form.

$$\text{pole-zero form} = \frac{\bar{K}\,\Pi(s+z_i)\,\Pi\, s^2+2\xi_{zi}\,\omega_{nzi}\,s+\omega^2_{nzi}}{\Pi(s+P_i)\,\Pi\,(s^2+2\xi_{Pi}\,\omega_{nPi}\,s+\omega^2_{nPi})}$$

$$\text{time-constant form} = \underbrace{K}_{\text{static gain}}\frac{\Pi_i(1+\tau_{zi}s)\,\Pi_i\left(\frac{s^2}{\omega^2_{nzi}}+\frac{2\xi_{zi}}{\omega_{nzi}}s+1\right)}{s^n\,\Pi_i(1+\tau_{Pi}s)\,\Pi\left(\frac{s^2}{\omega^2_{nPi}}+\frac{2\xi_{Pi}}{\omega_{nPi}}+1\right)}$$

- Consider the following transfer function

$$G(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)\text{------}(s+z_n)}{s^m(s+P_1)(s+P_2)(s+P_3)\text{------}(s+P_n)}$$

$$|G(s)| = \frac{K\,|(s+z_1)|\,|(s+z_2)|\,|(s+z_3)|\text{------}|(s+z_n)|}{|s^m|\,|(s+P_1)|\,|(s+P_2)|\,|(s+P_3)|\text{------}|(s+P_n)|}$$

- If we know the magnitude response of each pole and zero term we can find the total magnitude response.

- the magnitude can be calculate in dB. (decibles)
Vs $\log\omega$.

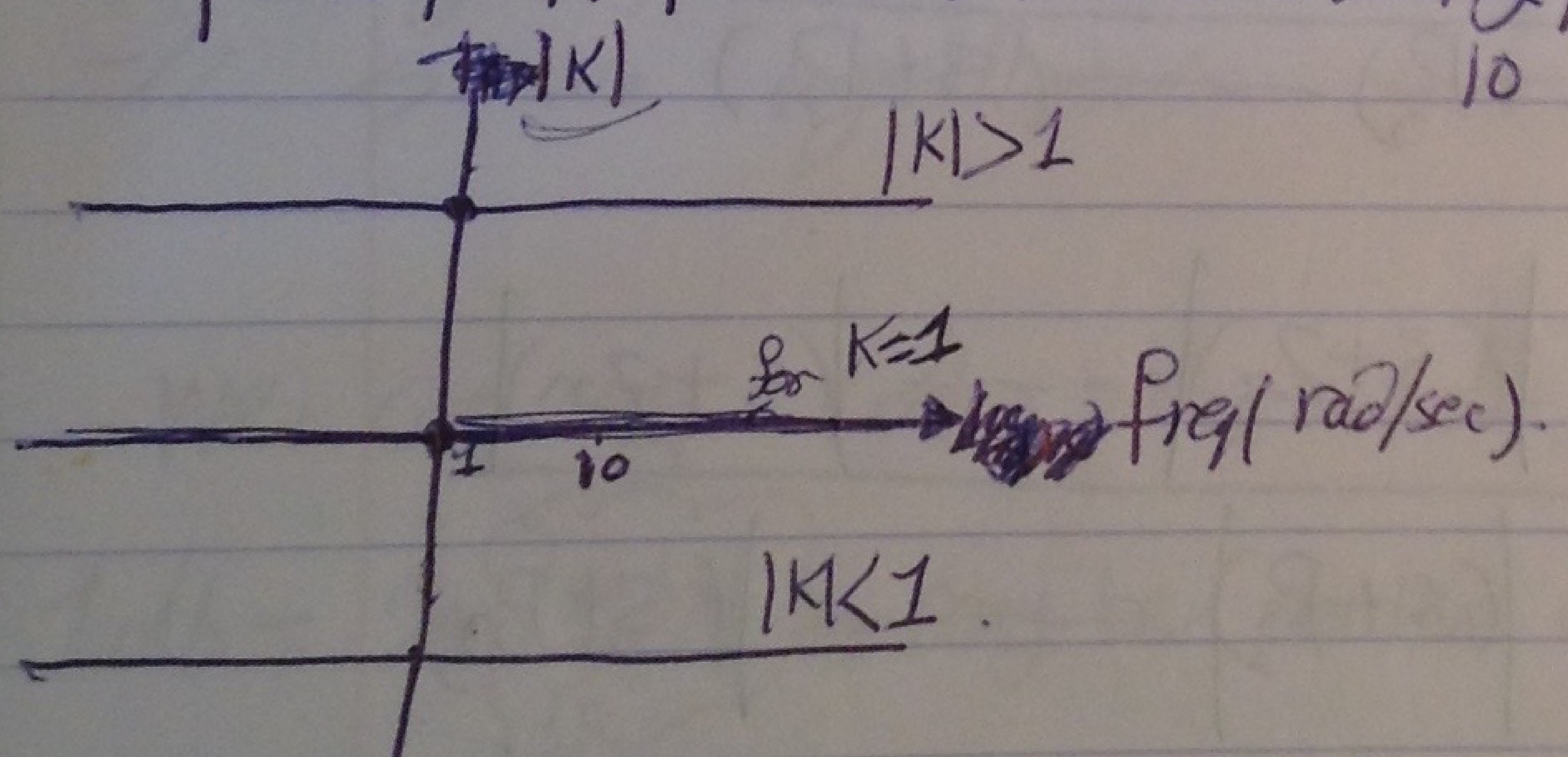
since $1\ dB = 20 \log \hat{M}^{\text{magnitude}}$

$$20\log|G(j\omega)| = 20\log K + 20\log|(s+z_1)| + 20\log|(s+z_2)| + \cdots - 20\log|(s+z_n)|$$
$$- 20\log|(s+P_1)| - 20\log|(s+P_2)| - 20\log|(s+P_3)| + \cdots - 20\log|(s+P_n)|$$

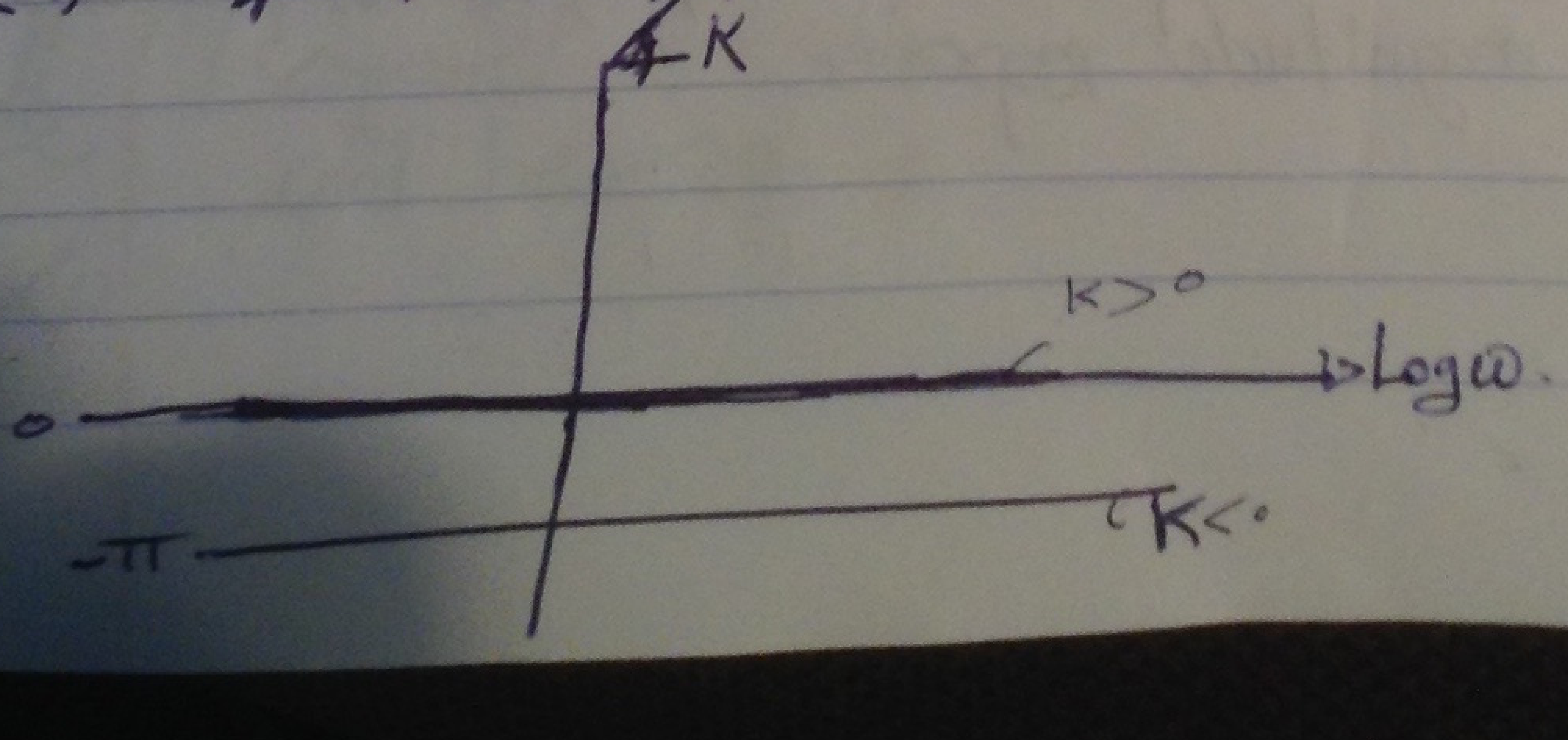
so if we can Knew the response of each term, the algebric sum would yield the total response in dB.

III Bode plots for $G(s) = K$.

$|G(s)| = |K| \longrightarrow$ in $db = 20\log_{10}|K|$ → for $|K|>0$, → for $|K|<0$



$\measuredangle G(s) = \measuredangle K$ → 0 when $K>0$, → π when $K<0$ $(-K = K\angle 180)$

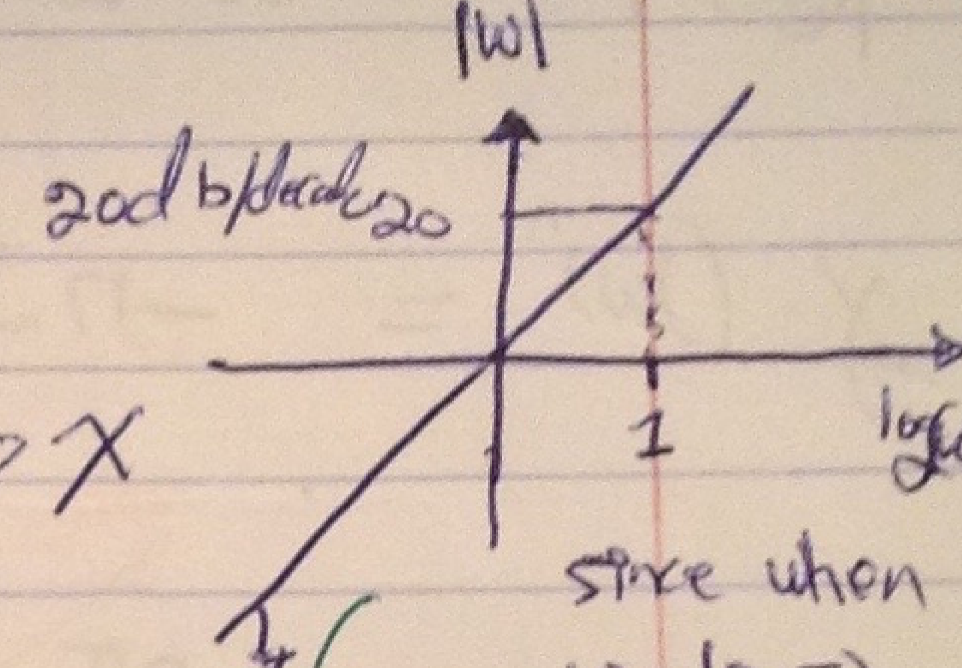


For $G(s) = s^n$.

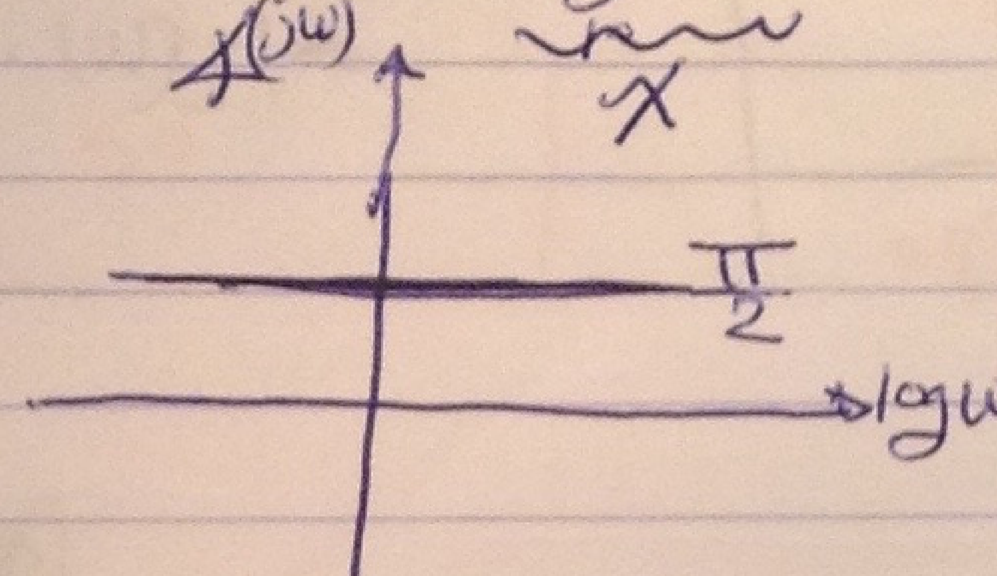
$G(j\omega) = (j\omega)^n$.

when $\boxed{n=1}$ (zero at origion).

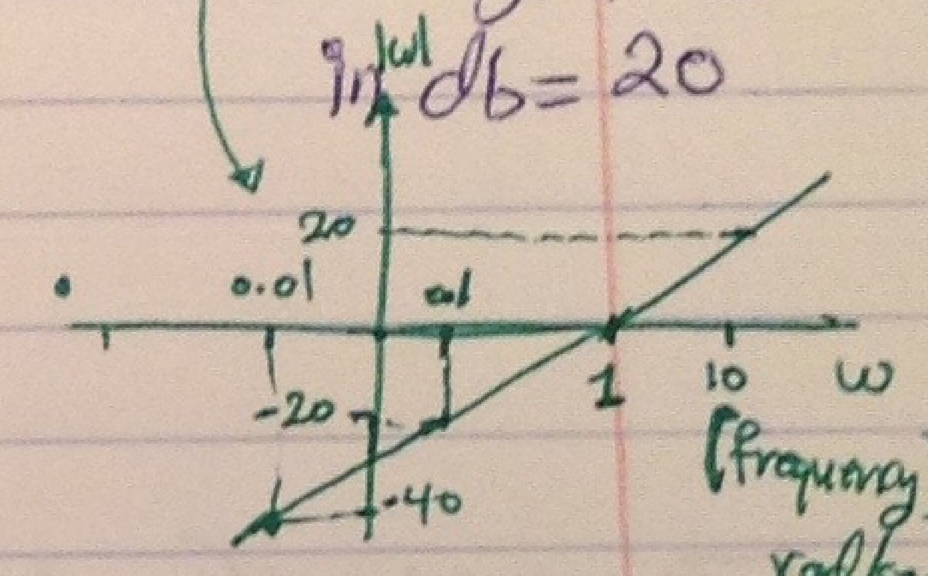
$|G(j\omega)| = |j\omega| = \omega \xrightarrow{\text{in db}} 20\underbrace{\log|\omega|}_{x} = 20x$

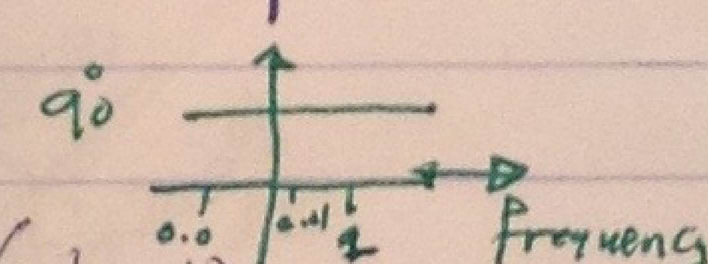


$\measuredangle(j\omega) = j = +\frac{\pi}{2}$



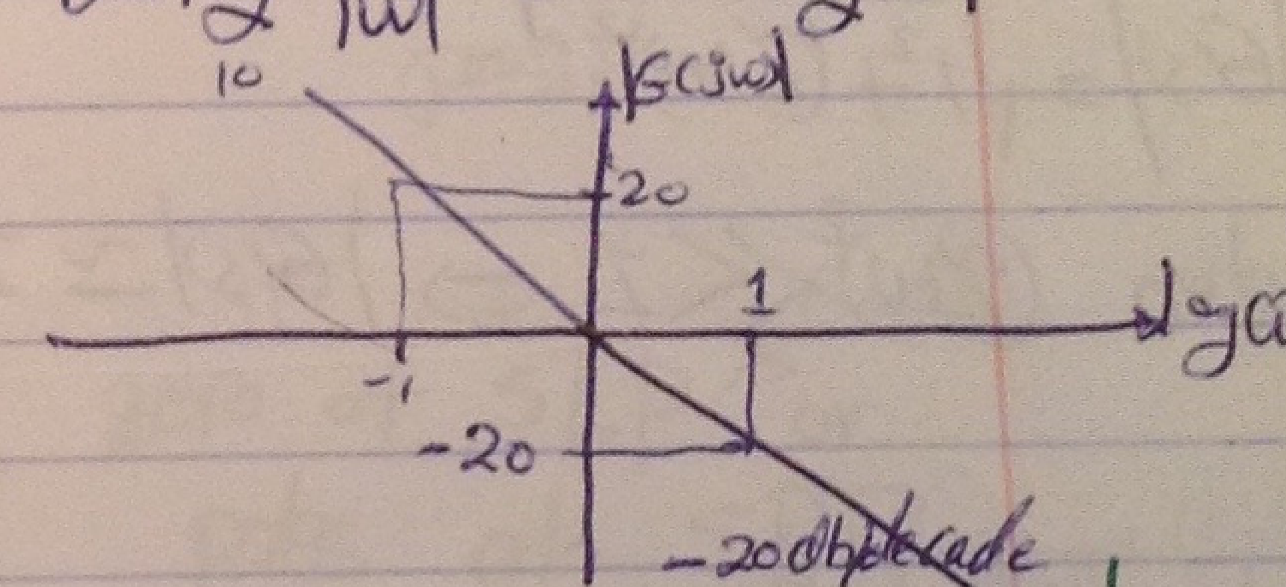
since when $\omega = 10 \Rightarrow \log|\omega| = 1$
in db $= 20$



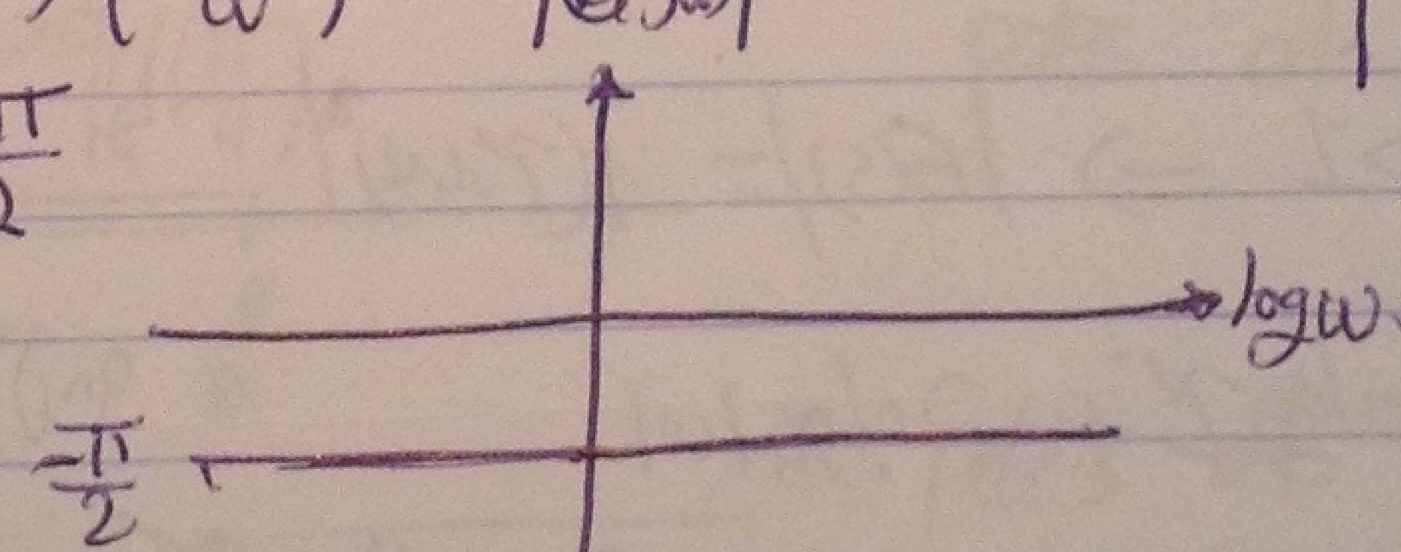


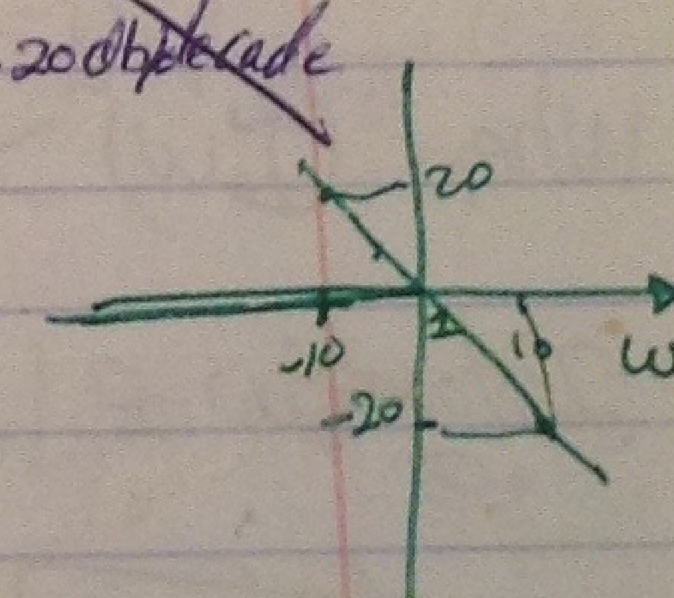
~~when~~ when $\boxed{n=-1}$ (pole at origion).

$|G(j\omega)| = |(j\omega)^{-1}| = \left|\frac{1}{j\omega}\right| = \frac{1}{|\omega|} \xrightarrow{\text{in db}} 20\log\frac{1}{|\omega|} = -20\overbrace{\log|\omega|}^{x}$



$\measuredangle(j\omega)^{-1} = \measuredangle\left(\frac{1}{j\omega}\right) = \measuredangle\left(\frac{-j}{\omega}\right)$
$= -\frac{\pi}{2}$

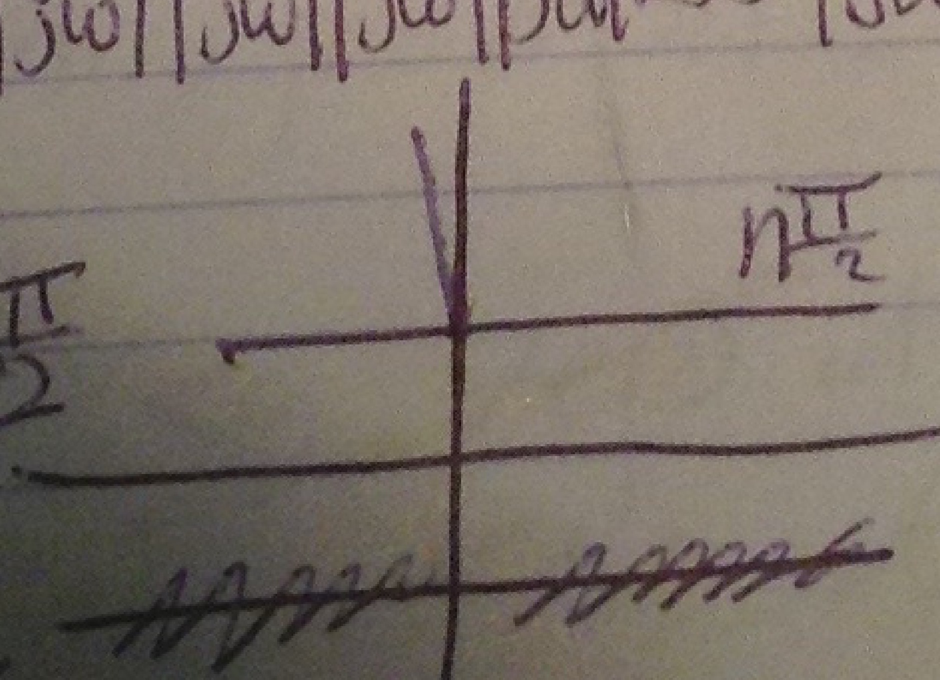


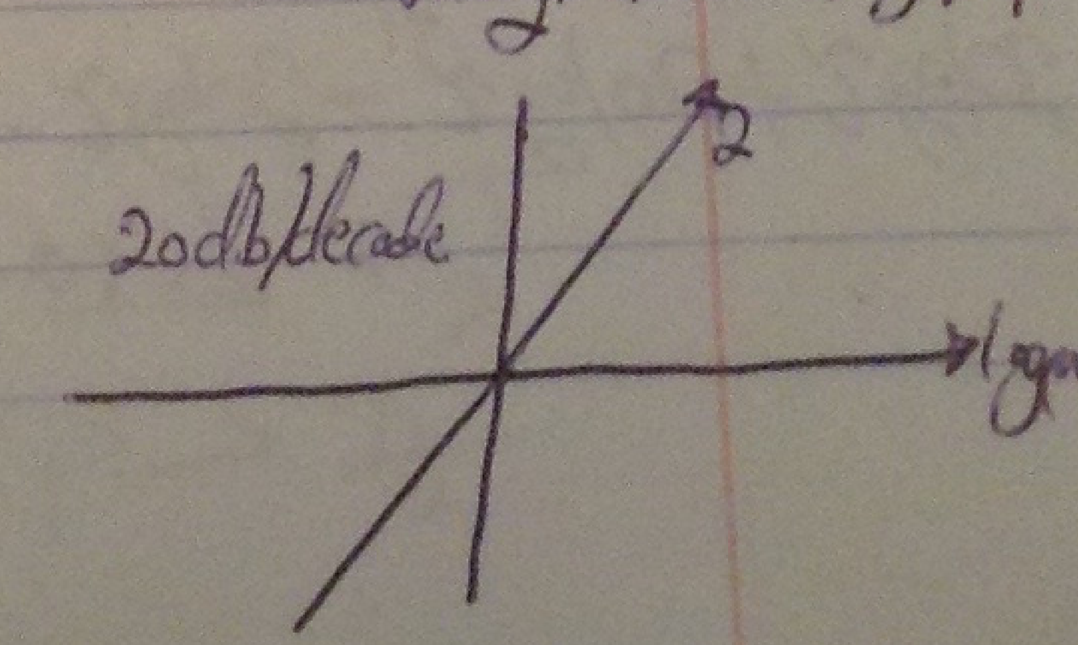


when $\boxed{n>1}$

$|G(j\omega)^n| = |(j\omega)^n| = |j\omega||j\omega||j\omega||j\omega|\cdots|j\omega| = \omega^n \xrightarrow{\text{in db}} 20\log\omega^n = 20n\log|\omega|$

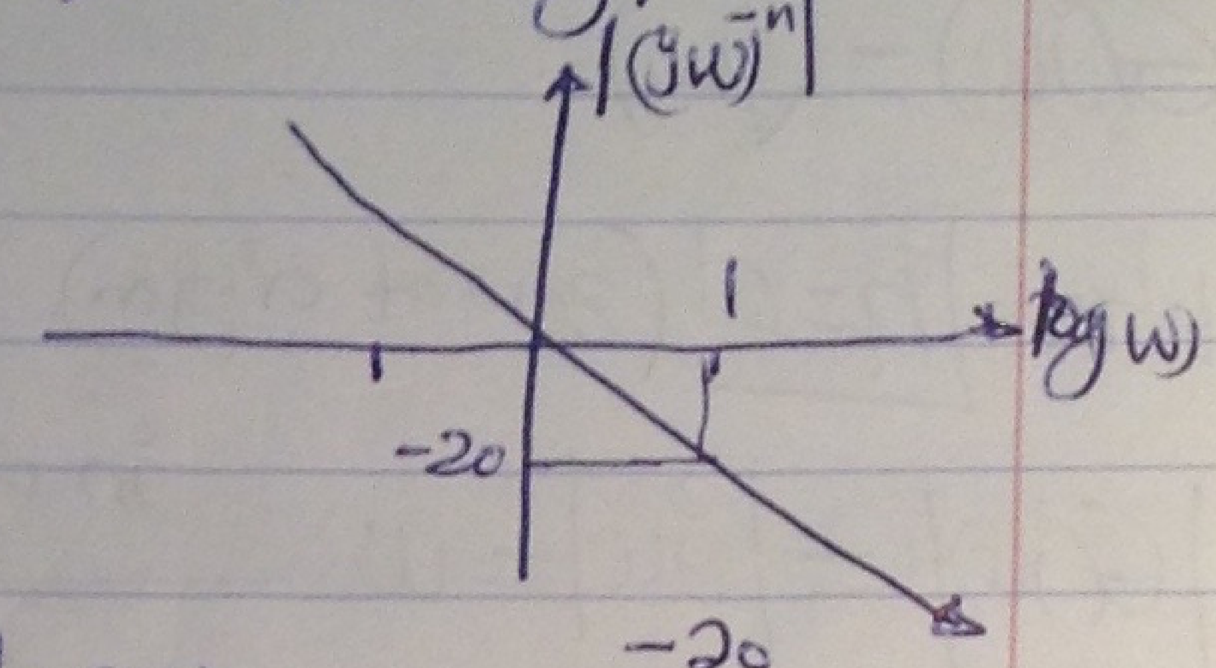
$\measuredangle((j\omega)^n) = n\frac{\pi}{2}$



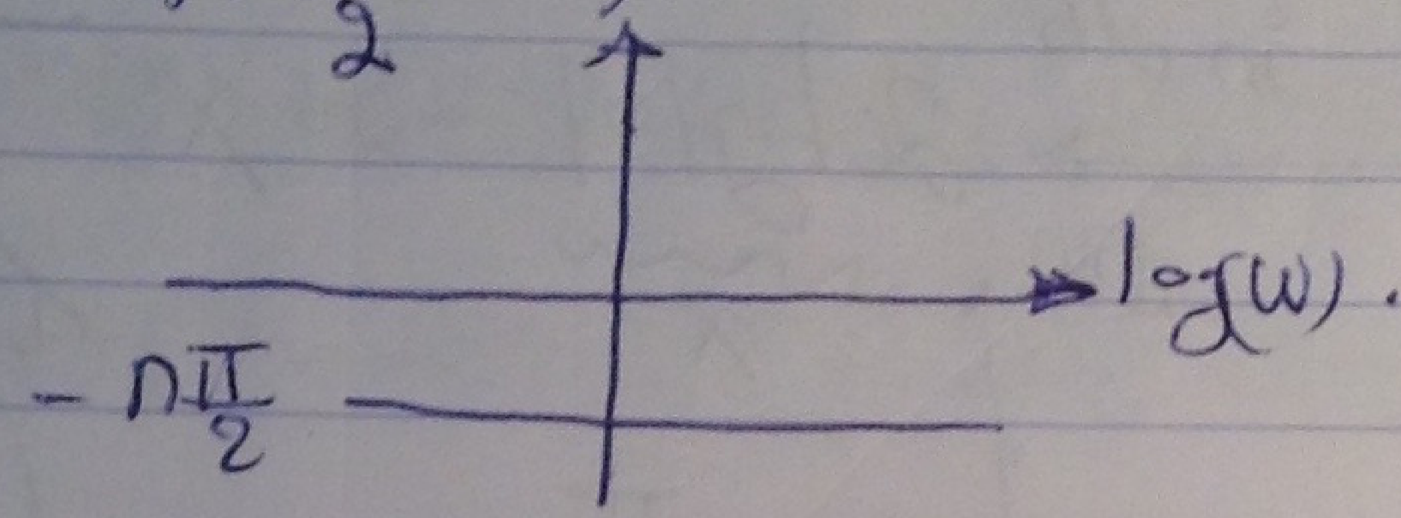


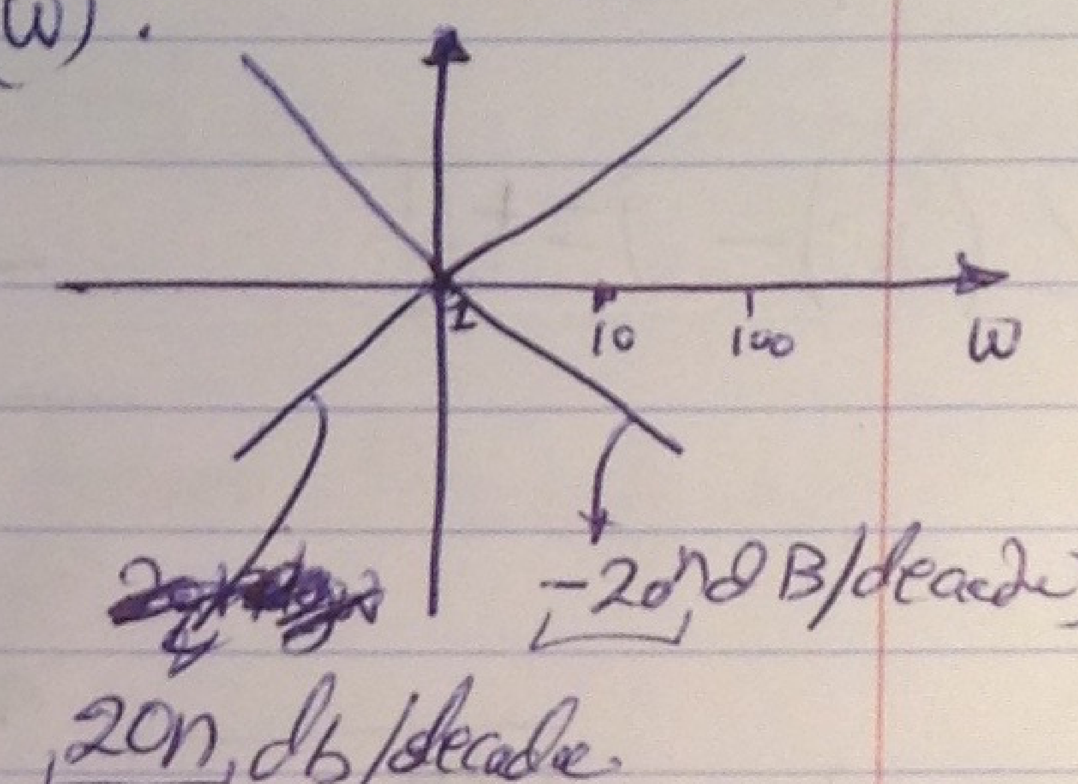
when $n<1$

$|(j\omega)^{-n}| = \frac{1}{\omega}\cdot\frac{1}{\omega}\cdot\frac{1}{\omega} = \omega^{-n} \xrightarrow{\text{in db}} -20n\log|\omega|$



$\angle (j\omega)^{-n} = -n\frac{\pi}{2}$



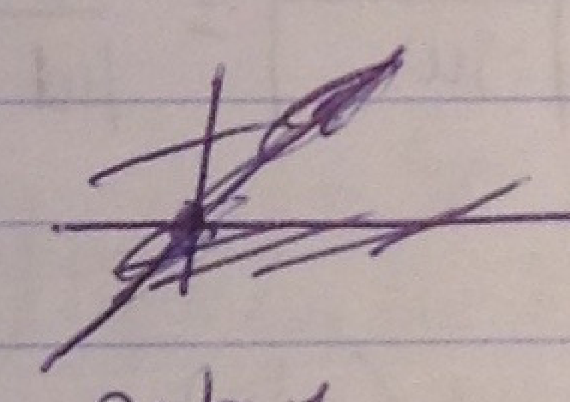


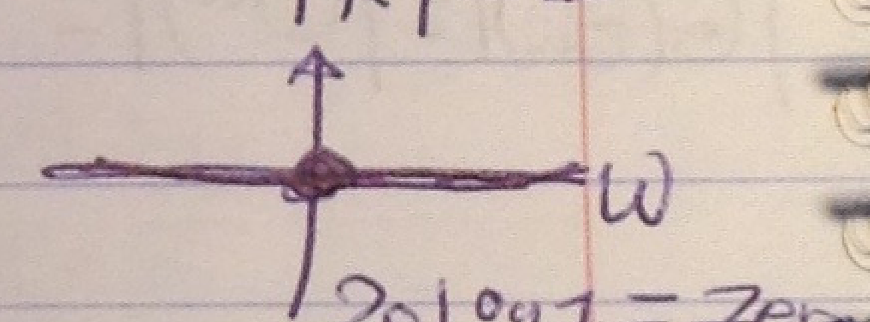
- For $G(s)=(1+j\tau\omega)^n$

$|G(s)| = \sqrt{1^2+(\tau\omega)^2}$

when $(\tau\omega)^2 \ll 1 \Rightarrow |G(s)| \simeq 1$

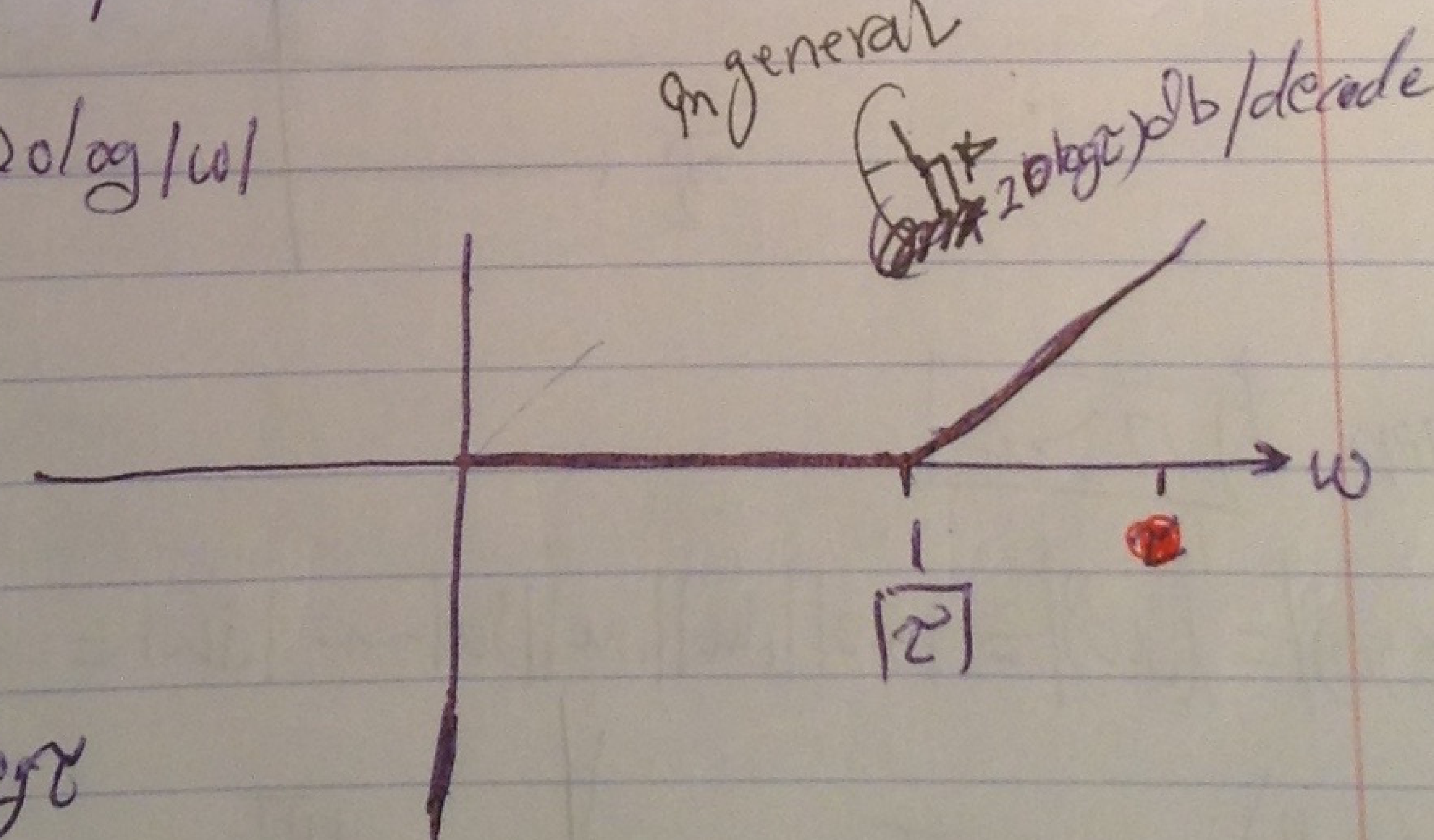
in db = 20log 1





When $(\tau\omega)^2 \gg 1 \Rightarrow |G(s)| = |(\tau\omega)| \xrightarrow{\text{in dB}}$

$20\log\tau\omega = 20\log\tau + 20\log|\omega|$

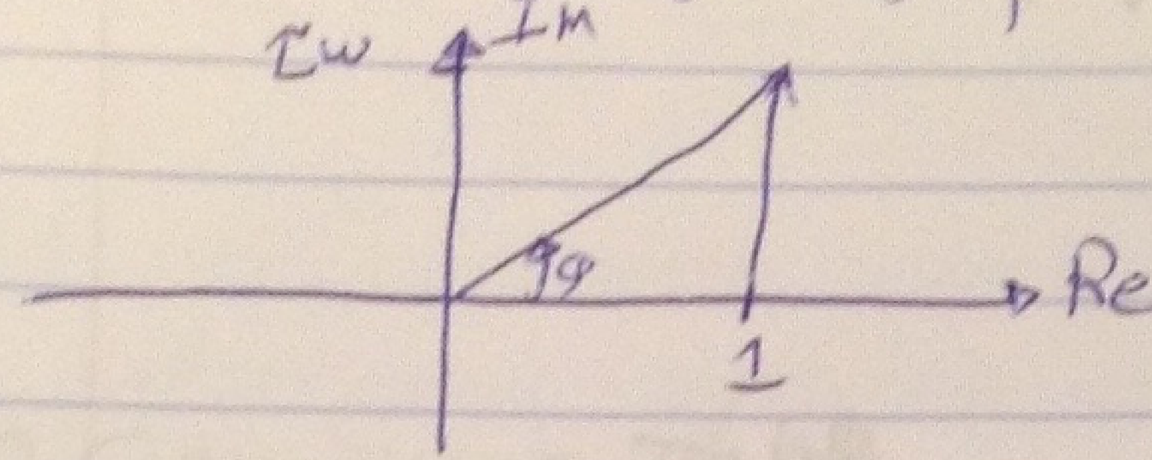


when $\boxed{\omega = \frac{1}{|\tau|}}$

$20\log\tau + 20\log|\frac{1}{\tau}|$

$20\log\tau + 20\log\tau^{-1} = 20\log\tau - 20\log\tau$
$= \text{Zero}$

To find the angle graph.



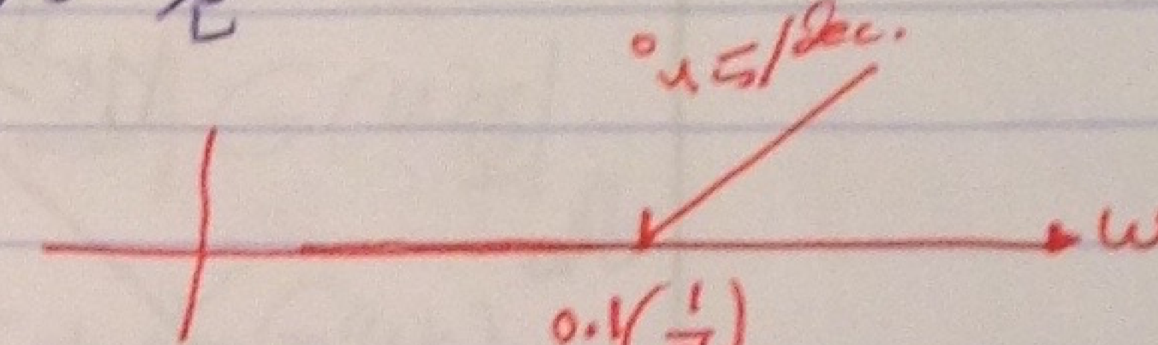
$\tan(\varphi) = (\tau w)$, at $w = \frac{1}{\tau}$

$\tan(\varphi) = 1$

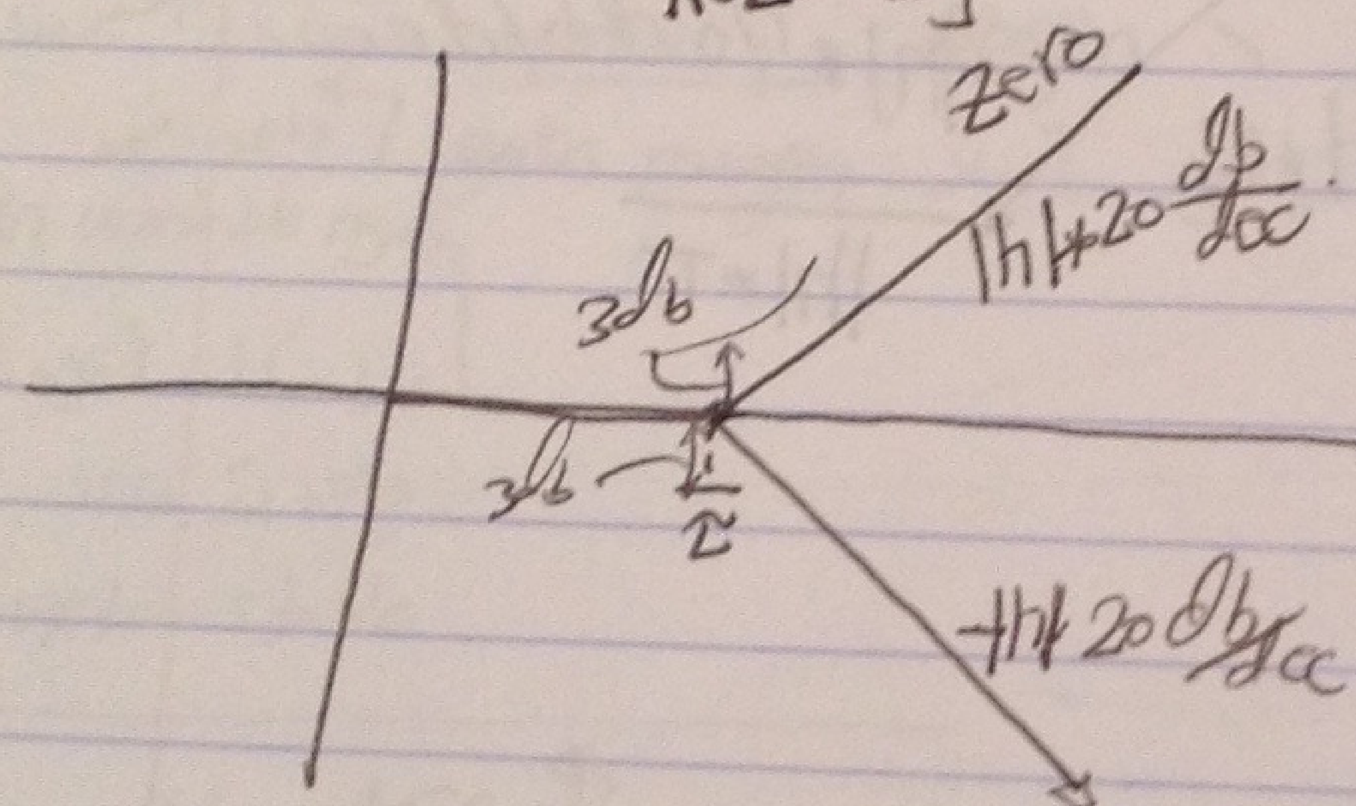
so

$\varphi = \pi/4 = \tan^{-1}(1)$

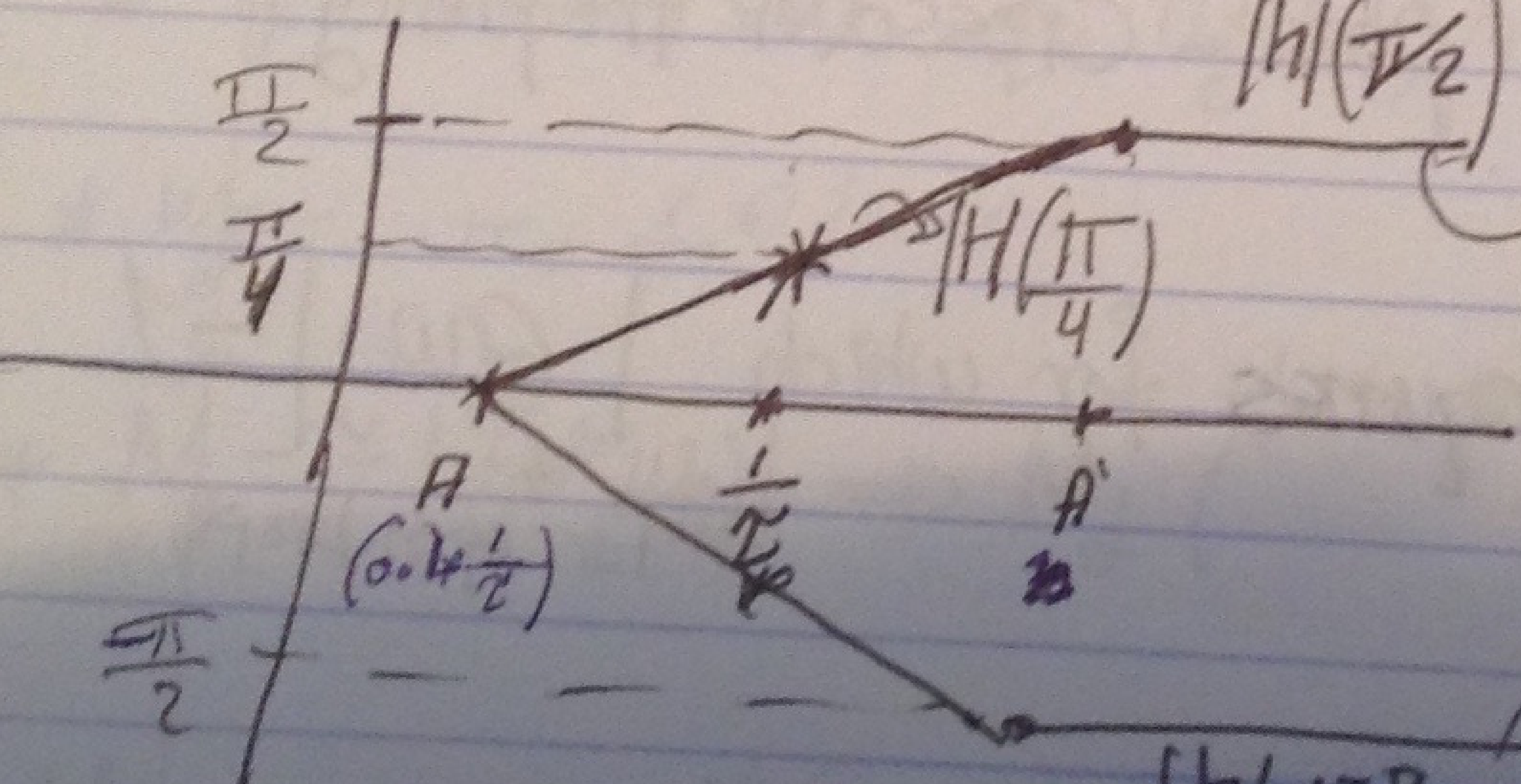
so



$\tau = \frac{-1}{Re[\text{poles}]}$



* single side representation ⇒ phase positive.



→ zero at SLP $(s+1)^1$ or poles at SRP $\frac{1}{(s-1)^1} = (s-1)^{-1}$

→ Poles at SLP or $\frac{1}{(s+1)^1} = (s+1)^{-1}$

Zero at SRP $(s-1)^1$

zero [illegible] poles [illegible]

[illegible]

on the semi left plane.

When $h = 1$, $\tau > 0$, zero at SLP $\rightarrow \frac{\pi}{2}$

$h = -1$, $\tau > 0$, poles at SLP. $\rightarrow -\frac{\pi}{2}$

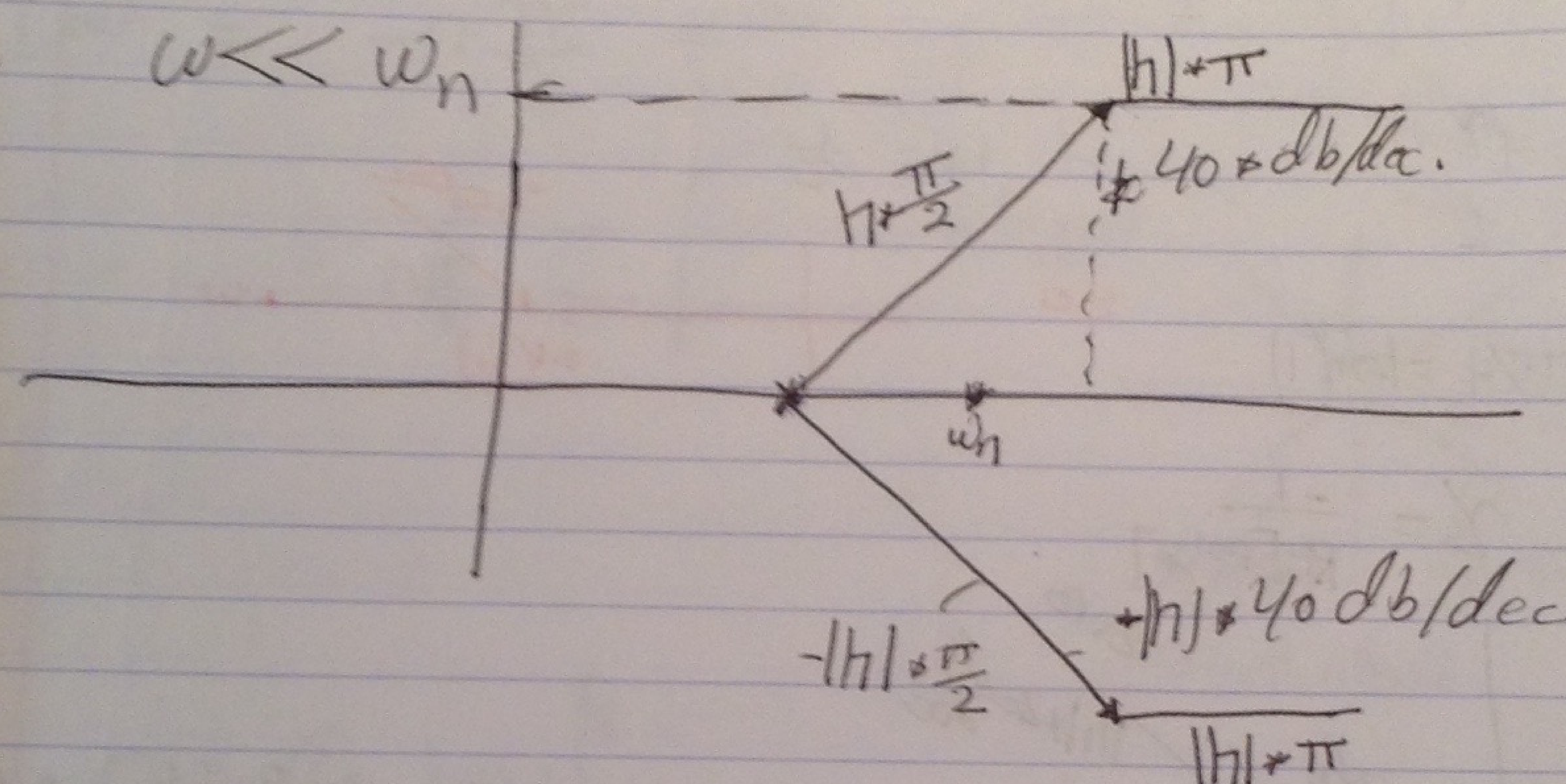
When $h = 1$, $\tau < 0$, zero at SRP $\rightarrow +\frac{\pi}{2}$

$h = -1$, $\tau < 0$, poles at SRP $\rightarrow \frac{\pi}{2}$

$$G(s) = \left(\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1\right)$$

double real poles

when $\omega << \omega_n$



Control

[I] Gain crossover frequency phase crossover frequency.

ω_c : gain " " " (frequencies for which $|G(\omega_c)| = 1$

for open loop.

$|G(\omega_c)|_{db} = 0$

gain crossover frequency.

ω_ϕ : phase cross over frequency : frequency for which

$\angle G(\omega_\phi) = -\pi$

* Draw the Bode plot (refer to).

[2] Gain margin $\left(M_g = \dfrac{1}{|G(\omega_\Phi)|}\right.$

$$M_{g_{db}} \overset{-ve}{=} -|G(\omega_\Phi)|_{db}$$

phase margin $= -\pi + \measuredangle G(\omega_c)$.

إذا كانت فوق ال $-\pi$ يكون في منطقة ال stability أما إذا كانت تحت ال $-\pi$ يكون unstable وكبيرة إذا — phase margin

- نستخدم ال gain crossover frequency ونقوم بحساب ال phase margin

واحد تحت in unstable region — إذا كان أكبر من واحد يكون ال system stable. — gain margin

و نستخدم ال phase crossover frequency ونقوم بحساب ال gain margin

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* $\omega_c = |G(\omega_c)| = 0 \Rightarrow \omega_c$: gain cross over frequency.

* $\omega_\Phi = -\pi + \measuredangle \Phi \,/\, |G(\omega)| = 0$

* $M_g = -|\cancel{G}(\omega_\Phi)| \Rightarrow M_g$ (نستعمل ال phase cross over freq لإيجاد ال gain margin)

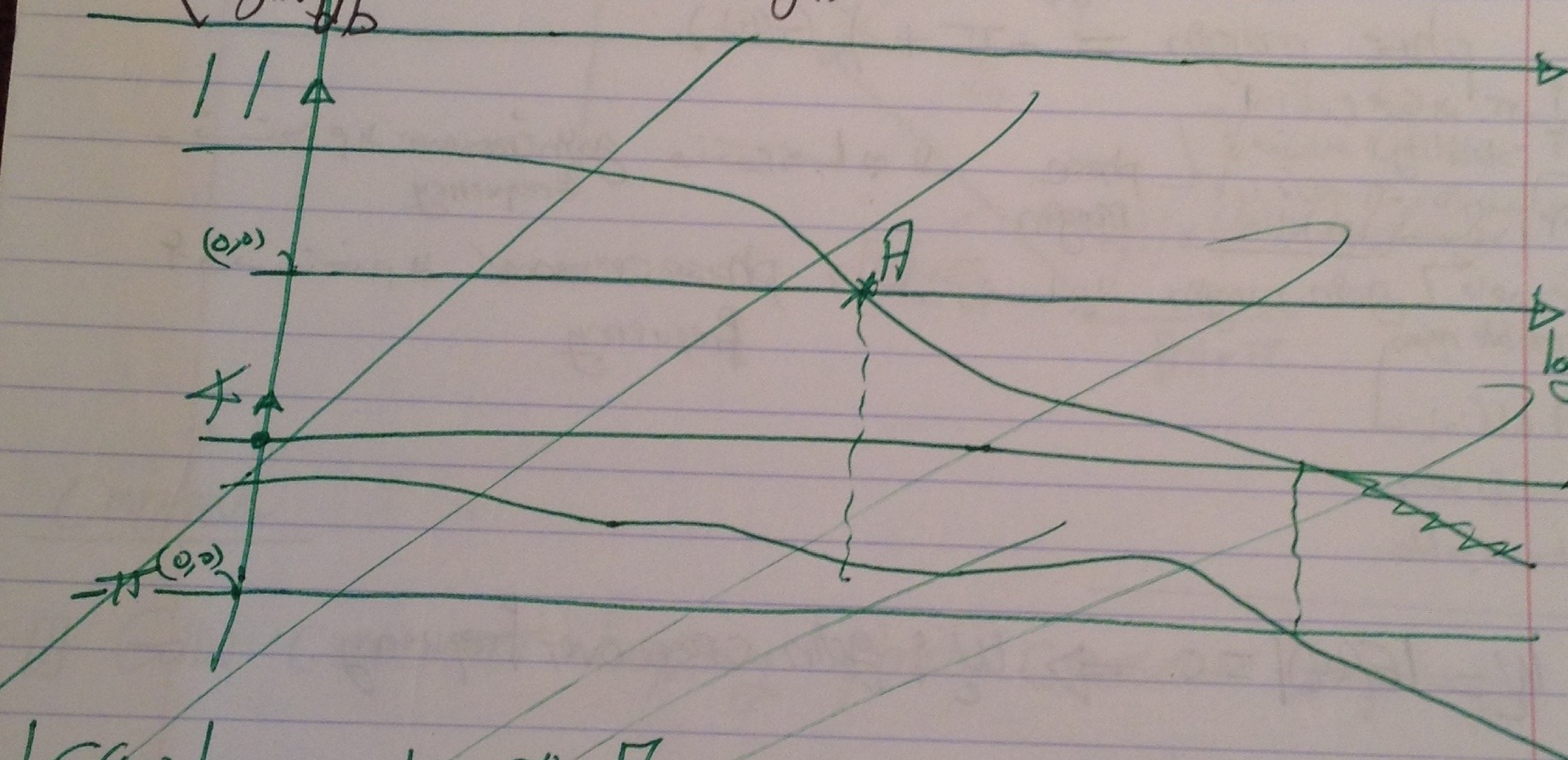
* $M_\Phi = -\pi + \measuredangle (G(\omega_c))$ (نستعمل ال gain cross over frequency لإيجاد ال phase margin)

Stability Theorem

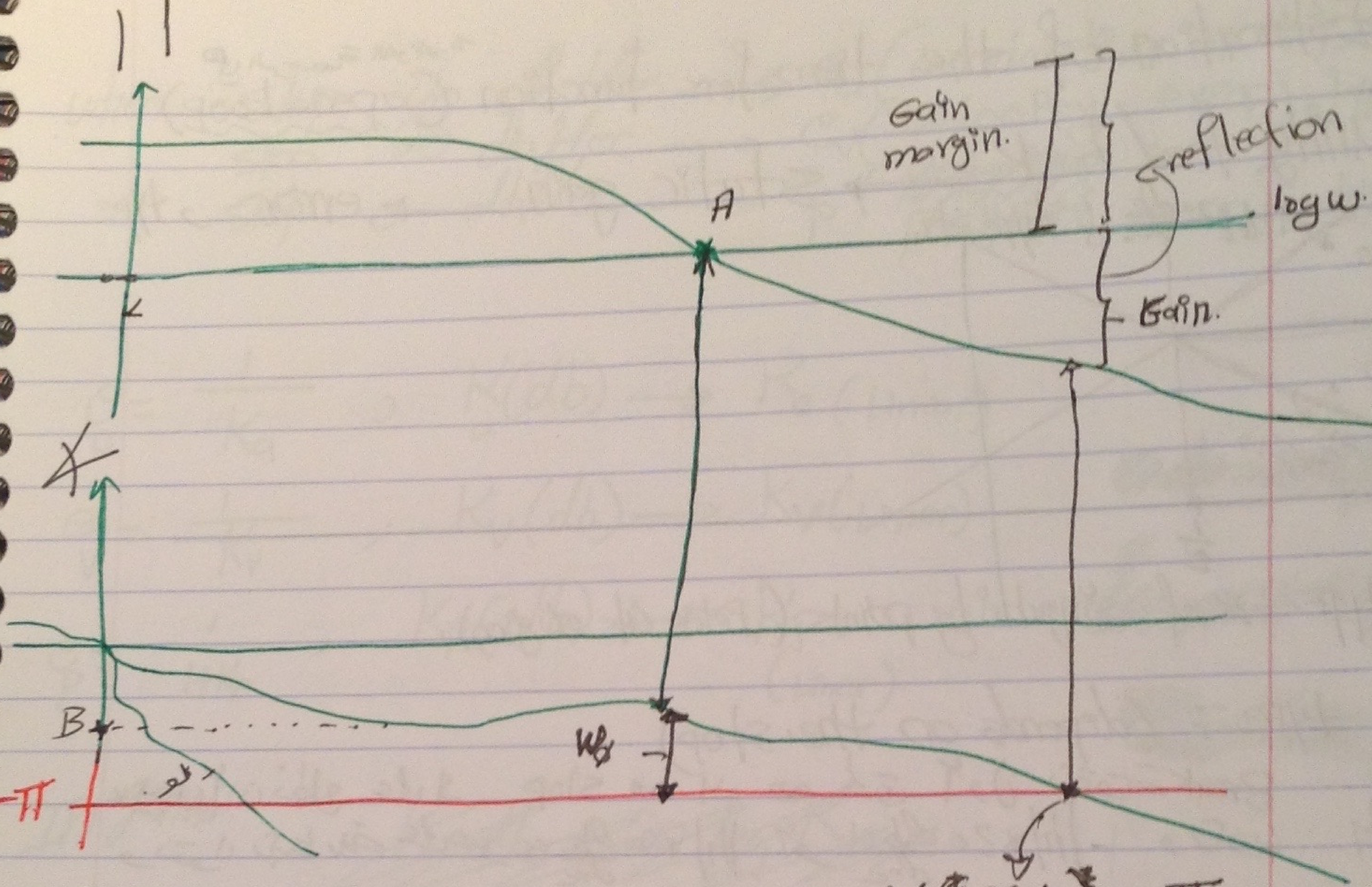
Given a L.T.I system with frequency response (open-Loop) the system is marginally stable $\Longleftrightarrow$ the phase margin is positive or the gain margin in db positive $\rightarrow$ $(M_g > 1)$.

* الرسمة تكون ال open loop) ~~...~~ والقول ال

ال system تكون marginally stable إذا كان ال phase margin > 0 أو ال (gain margin)db > 0



$|G(\omega_c)|_{db} = 0$ at poin A



$|G(w_c)| = 0$ @ point A &

phas margin @ point B

Phase margin positive or above $-\pi$

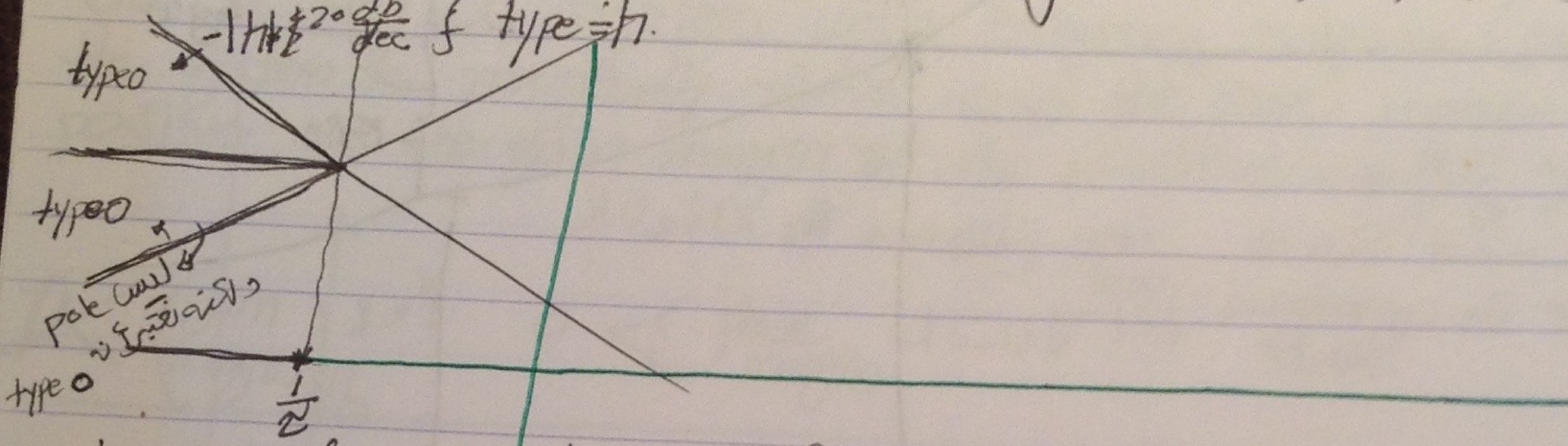
or gain margin in db > 0

so the system is Marginally stable

هي الرسمة الموجودة

- Estimation of the transfer function (open-Loop)

unity feed back ⟶ static gain ⟶ error , type



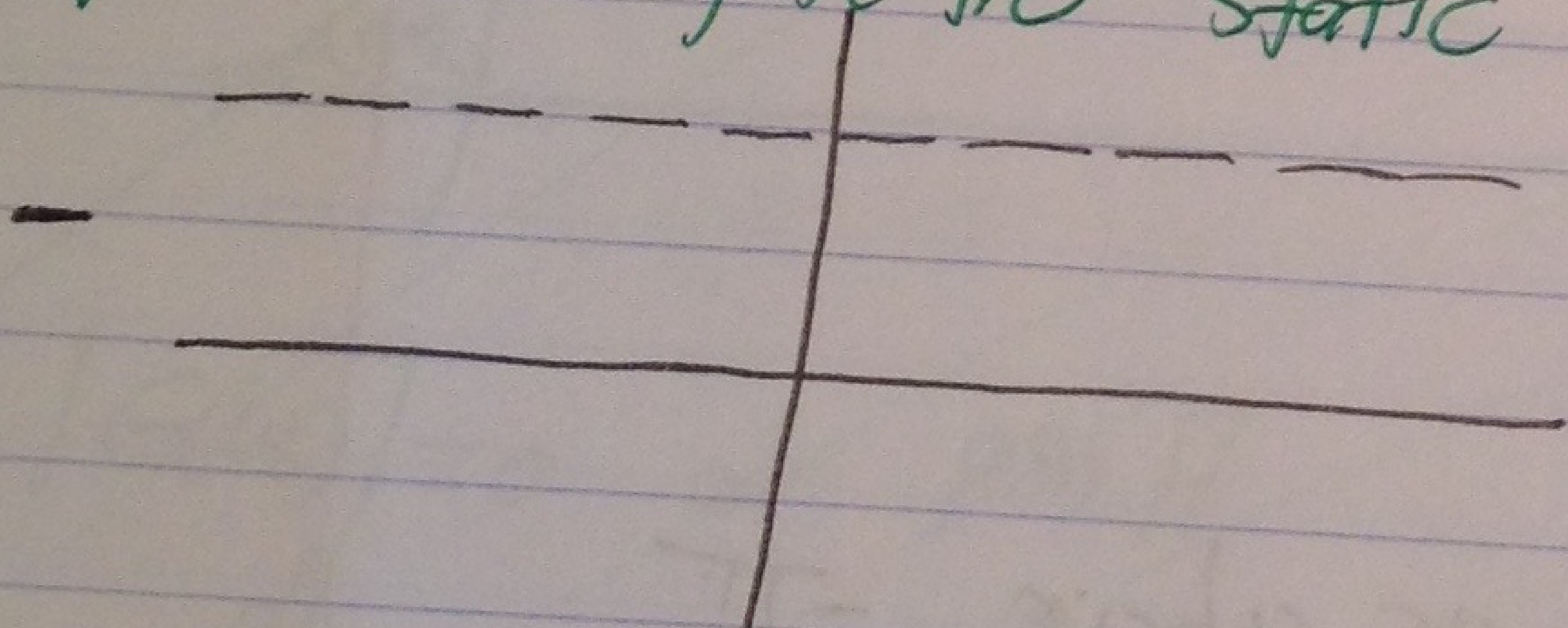
type : # of singularity points (Poles at origion).

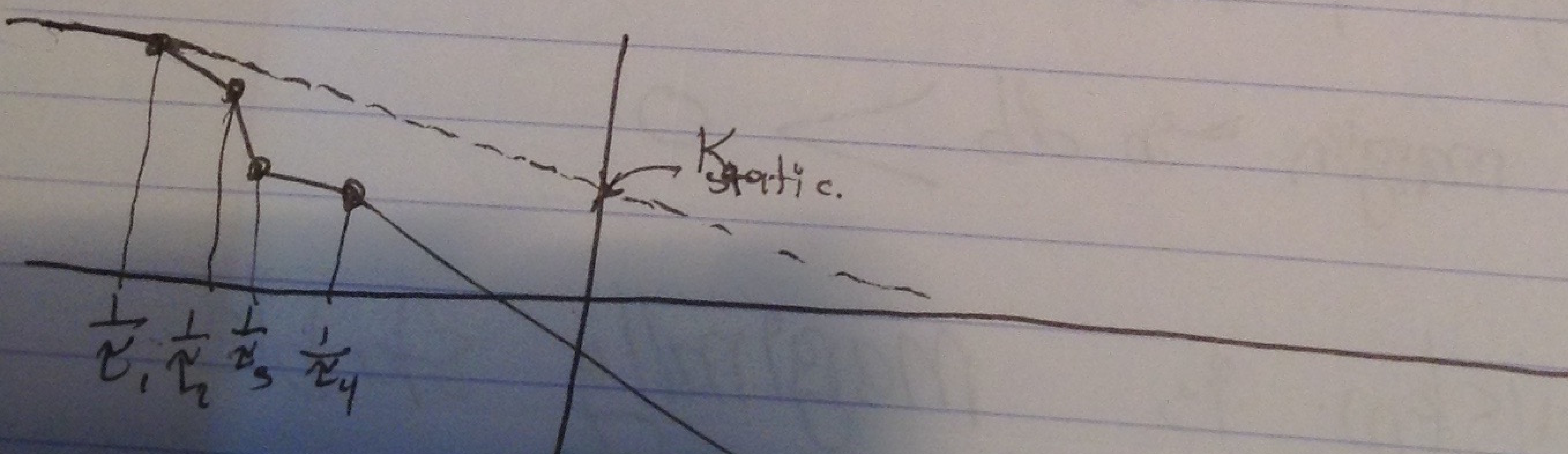
* type : (depends on the stop)

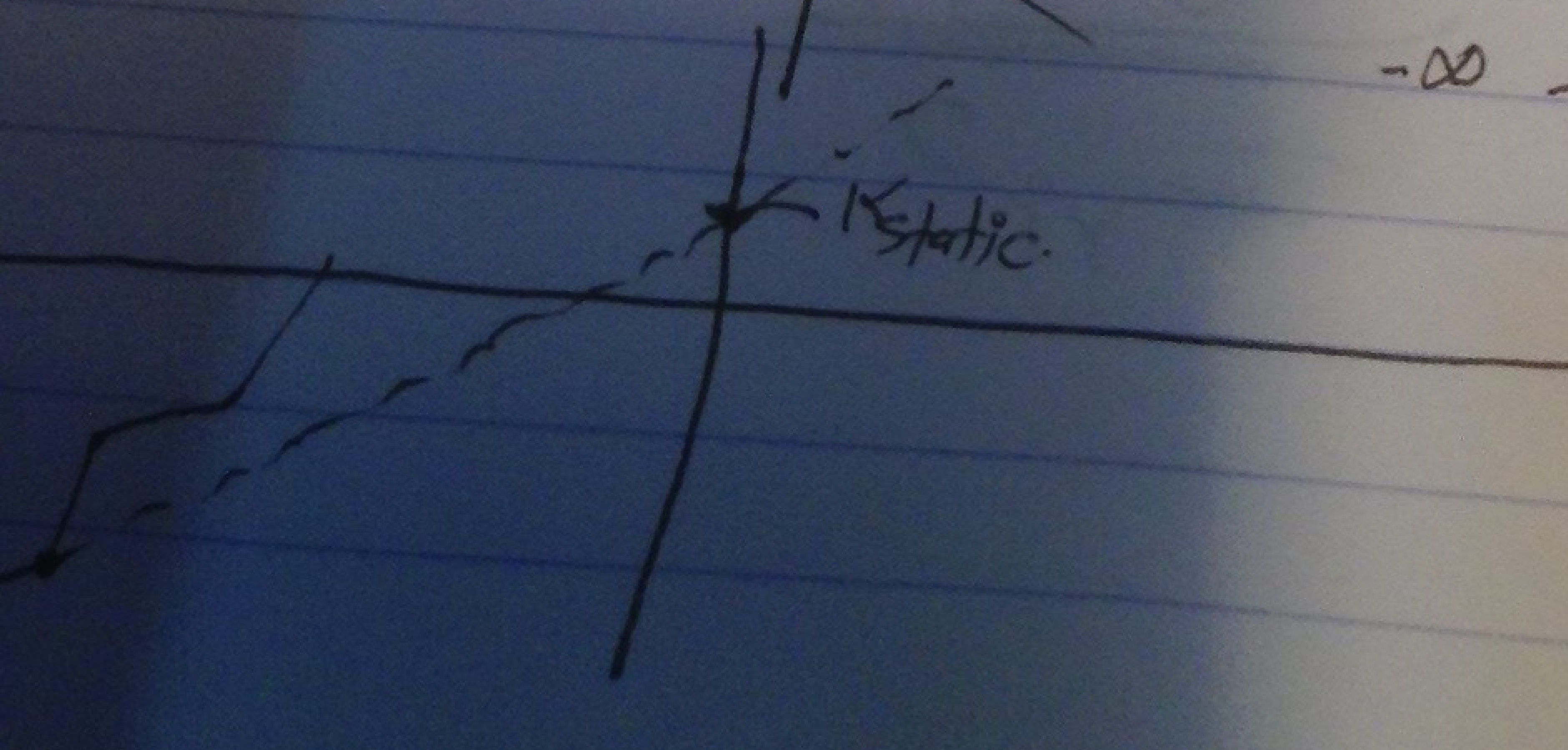
* يجب أن نطلع على الـ slop من الـ ∞ حتى أول نقطة Break
ونقول بداية على محور $|h|*20\frac{db}{dec}$ أو $-|h|*20\frac{db}{dec}$ وتكون h هي

type. الـ

* How to compute the static gain.







* نقوم بمد الخط الذي جاي من الـ -∞
ونرى أين يقطع ||