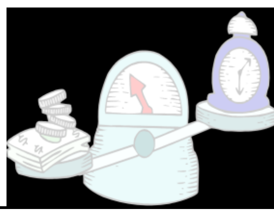
Instrumentation and Measurements ENEE4304

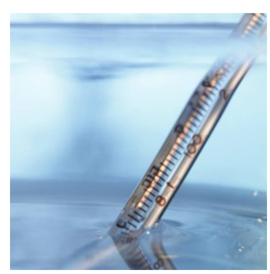
L4

Dynamic-Performance-Characteristics



Dynamic Performance

- The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and the time when the instrument output attains a steady value in response.
- Because dynamic signals vary with time, the measurement system must be able to respond fast enough to keep up with the input signal.
- Further, we need to understand how the input signal is applied to the sensor because that plays a role in system response.



- Zero Order Systems
- Non-Zero Order Systems
  - 1st order
  - 2nd order
  - ….
  - Nth order

- In any linear, time-invariant measuring system, the following general relation can be written between input and output for time t > 0:

$$a_n \frac{d^n q_0}{dt^n} + a_{n-1} \frac{d^{n-1} q_0}{dt^{n-1}} + \cdots + a_1 \frac{dq_0}{dt} + a_0 q_0$$
$$= b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \cdots + b_1 \frac{dq_i}{dt} + b_0 q_i$$

- where qi is the measured quantity, q_0 is the output reading and $a_0 \ldots a_n$, $b_0 \ldots b_m$ are constants.
- only certain special, simplified cases of it are applicable in normal measurement situations.
- The major point of importance is to have a practical appreciation of the manner in which various types of instruments respond when the measurand applied to them varies.

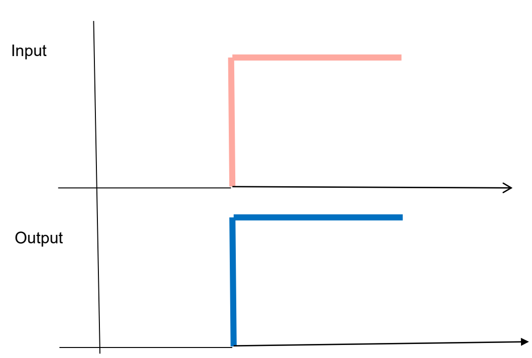
- If we limit consideration to that of step changes in the measured quantity only, then equation reduces to:

$$a_n \frac{d^n q_0}{dt^n} + a_{n-1} \frac{d^{n-1} q_0}{dt^{n-1}} + \cdots + a_1 \frac{dq_0}{dt} + a_0 q_0 = b_0 q_i$$

- Further simplification can be made by taking certain special cases of the equation, which collectively apply to nearly all measurement systems.

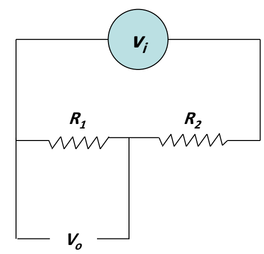
Zero Order Systems

❑ In general, systems without a storage or dissipative capability may be modeled as zero order system



Electrical Zero-Order Systems

- **In a zero-order system, the output responds to the input signal change instantaneously.**

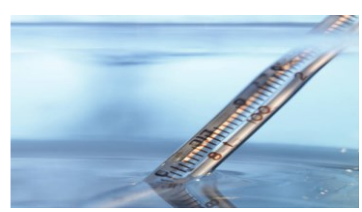


$$v_o = iR_1 = \frac{v_i}{R_1 + R_2} R_1$$

$$v_o = \frac{R_1}{R_1 + R_2} v_i$$

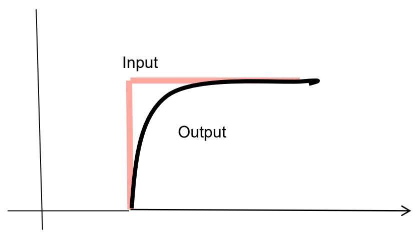
Non-zero Order Systems

- Measurement systems that contain storage or dissipative elements do not respond instantaneously to changes in input.
- In the bulb thermometer, when the ambient temperature changes, the liquid inside the bulb will need to store a certain amount of energy in order for it to reach the temperature of the environment.
- The temperature of the bulb sensor changes with time until this equilibrium is reached, which accounts physically for its lag in response.



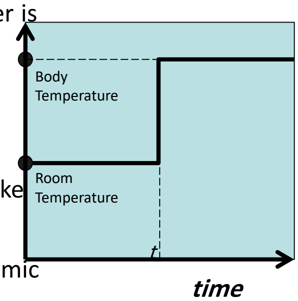
Non-zero Order Systems

- In general, systems with a storage or dissipative capability but negligible inertial forces may be modeled using a first-order differential equation.



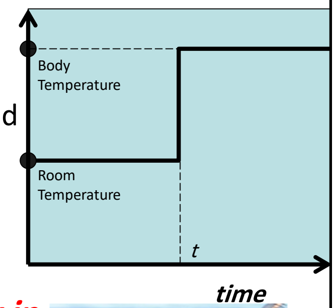
First Order Systems

- Consider the time response of a bulb thermometer for measuring body temperature.
- The thermometer, initially at room temperature, is placed under the tongue.
- Body temperature itself is constant (static) during the measurement, but the input signal to the thermometer is suddenly changed from room temperature to body temperature. This, is a step change in the measured signal.
- The thermometer must gain energy from its new environment to reach thermal equilibrium, and this take a finite amount of time.
- The ability of any measurement system to follow dynamic signals is a characteristic of the measuring system components.



First Order Systems

- Suppose a bulb thermometer originally at temperature T_o is suddenly exposed to a fluid temperature T_∞.
- Develop a model that simulates the thermometer output response.



Rate of energy stored = Rate of energy in

$$\dot{E}_{stored} = Q_{in}$$

$$mC_p \frac{dT}{dt} = hA(T_\infty - T)$$

$$mC_p \frac{dT}{dt} + hAT = hAT_\infty$$

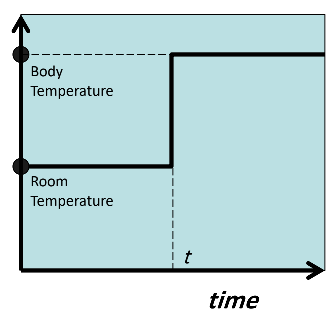
$$\frac{mC_p}{hA}\frac{dT}{dt} + T = T_\infty$$

- *m- Mass*
- *Cp- specific heat*
- *A- contact area ; h – heat transfer coeff.*

First Order Systems: ***Bulb Thermometer***

$$\frac{mC_p}{hA}\frac{dT}{dt} + T = T_\infty$$

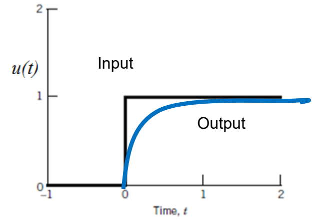
$$\tau \frac{dT}{dt} + T = T_\infty$$



- The ratio ***mCp/hA*** has a units of seconds and is called the time constant , *τ.*
- *m- sensor mass*
- *Cp- specific heat of sensor material*
- *A- contact area ; h – heat transfer coeff.*

1st Order Systems

- Examples:
  - Bulb Thermometer
  - RC Circuits
  - Terminal velocity



- Mathematical Model:

$$\tau\frac{dx}{dt}+x=f(t)$$

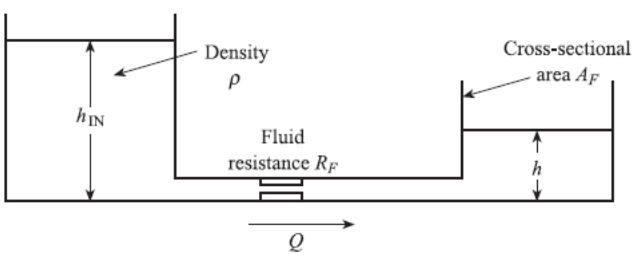
τ: Time constant

$f(t)$: Input (quantity to be measured)

x: Output (instrument response)

First Order System –Fluidic *(FIO-for info only)*

Fluidic



Volume flow rate $Q=\frac{1}{R_F}(P_{IN}-P)$

Pressures $P_{IN}=h_{IN}\rho g,\ P=h\rho g$

$$A_F\frac{dh}{dt}=Q=\frac{\rho g}{R_F}(h_{IN}-h)$$

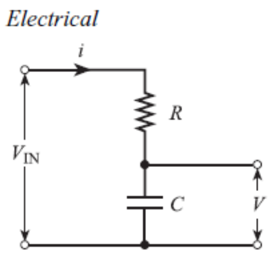
$$\frac{A_F R_F}{\rho g}\frac{dh}{dt}+h=h_{IN}$$

i.e.

$$\tau_F\frac{dh}{dt}+h=h_{IN},\ \tau_F=\frac{A_F R_F}{\rho g}$$

$$\tau_F=\frac{A_F R_F}{\rho g}=R_F C_F;\ R_F=R_F,\quad C_F=\frac{A_F}{\rho g}$$

First Order System -Electrical



$$\tau_E = RC = R_E C_E; \quad R_E = R, \quad C_E = C$$

$$V_{IN} - V = iR$$

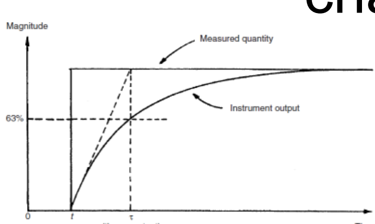
Charge $q = CV$, current $i = \frac{dq}{dt} = \frac{C dV}{dt}$

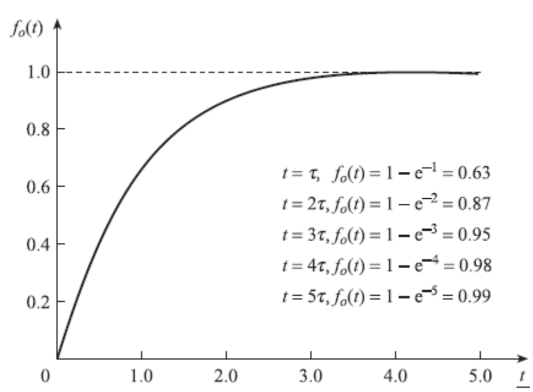
$$RC\frac{dV}{dt} + V = V_{IN}$$

i.e.

$$\tau_E \frac{dV}{dt} + V = V_{IN}, \ \tau_E = RC$$

First order instrument characteristic





2nd Order Systems

- ❑ Example:
  - ❑ Spring – mass damper
  - ❑ RLC Circuits
  - ❑ Accelerometers

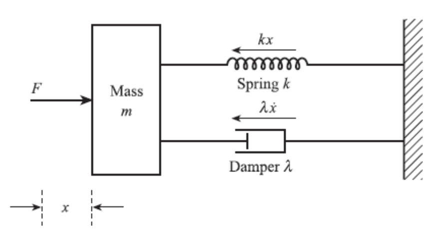
- ❑ Mathematical Model:

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n{}^2x = f(t)$$

ζ	Damping ratio (dimensionless)
ω_n	Natural frequency (1/s)
$f(t)$:	Input (quantity to be measured)
x:	Output (instrument response)

Mass–spring–damper model of elastic force sensor (2nd order)

- The elastic sensor which converts a force input F into a displacement output x, is a good example of a second-order element.
- The diagram is a conceptual model of the element, which incorporates***: a mass m [kg], a spring of stiffness k [N/m], and a damper of constant λ [N.s/m]***
- The system is initially at rest at time t = 0− so that the initial velocity x(0−) = 0 and the initial acceleration y(0−) = 0.

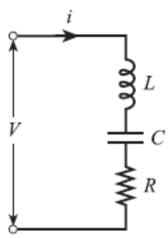


Second Order System -Electrical

- Transfer function for a second-order element

$$G(s) = \frac{1}{\frac{1}{\omega_n^2}s^2 + \frac{2\xi}{\omega_n}s + 1}$$

- Similar to series RLC circuit



$$V = iR + \frac{q}{C} + L\frac{di}{dt}$$

where $i = \frac{dq}{dt}$ (q = charge on the capacitance)

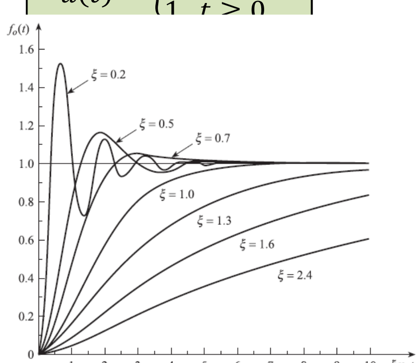
thus $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = V$

$\omega_n = 1/\sqrt{LC}$ and $\xi = (R/2)\sqrt{C/L}$.

2nd Order Systems with step input

$$f(t) = Ku(t)$$

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



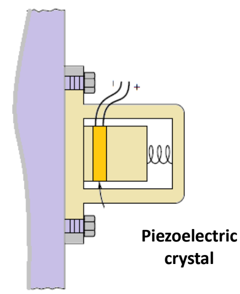
$\xi < 1$, underdamping; $\xi = 1$, critical damping; $\xi > 1$, overdamping

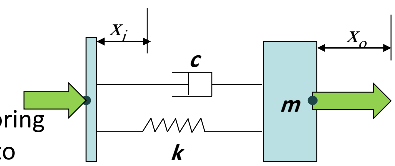
$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + {\omega_n}^2x = Af(t)$$

ζ	Damping ratio (dimensionless)
ω_n	Natural frequency (1/s)
$f(t)$:	Input (quantity to be measured)
x:	Output (instrument response)
A:	Arbitrary constant

Example – Automobile Accelerometer

- Consider the accelerometer used in seismic and vibration engineering to determine the motion of large bodies to which the accelerometer is attached.
- The acceleration of the large body places the piezoelectric crystal into compression or tension, causing a surface charge to develop on the crystal.
- The charge is proportional to the motion. As the large body moves, the mass of the accelerometer will move with an inertial response.
- The stiffness of the spring, k, provides a restoring force to move the accelerometer mass back to equilibrium while internal frictional damping, c, opposes any displacement away from equilibrium.





Can be used in airbag deployment system