



دفتر محاضرات

تحليل أنظمة قوى I

ابدل غاية الجهد ببسّر لك غاية الهدى
(والذين جاهدوا فينا لنهدينهم سبلنا)

اعداد الطالب:

أحمد عيسى الحجج



WWW.SPARKTEAMFET.WEEBLY.COM

« Website

FACEBOOK.COM/SPARK.FET

« Page

FACEBOOK.COM/GROUPS/POWER.
COMMUNICATIONS.GROUP

« Group

Spark Team

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

تحية طيبة وبعد ...

اخواننا الطلبة الكرام ...

فأنا في فريق #سبارك نضع بين ايديكم مجموعة من الاوراق محاضرات التي نأمل من الله عز وجل أن تكون في ميزان حسنات من قام بها وتعب عليها ...

حيث أننا نفيديكم انه من قام بعمل هذه محاضرات هم ثلة رائعة من طلاب تخصص هندسة الطاقة الكهربائية في كلية الهندسة التكنولوجية / السليتكك ...

نود اعلامكم أن هذه الاوراق المختصرة لا تغني بأي شكل من الاشكال لأي مادة تخصصية ومرجعك الاول والأخير ألا وهو #الكتاب ...

اخوانكم في فريق #سبارك

لا تنسوننا من صالح الدعاء

تحليل 1

د. محمود عواد

CHAPTER 1:

The Power system Analysis...

#SPARK_TEAM

Introduction

1.3 MODERN POWER SYSTEM

The power system of today is a complex interconnected network as shown in Figure 1.1 (page 7). A power system can be subdivided into four major parts:

- Generation
- Transmission and Subtransmission
- Distribution
- Loads

1.3.1 GENERATION

Generators — One of the essential components of power systems is the three-phase ac generator known as synchronous generator or alternator. Synchronous generators have two synchronously rotating fields: One field is produced by the rotor driven at synchronous speed and excited by dc current. The other field is produced in the stator windings by the three-phase armature currents. The dc current for the rotor windings is provided by excitation systems. In the older units, the exciters are dc generators mounted on the same shaft, providing excitation through slip rings. Today's systems use ac generators with rotating rectifiers, known as brushless excitation systems. The generator excitation system maintains generator voltage and controls the reactive power flow. Because they lack the commutator, ac generators can generate high power at high voltage, typically 30 kV. In a power plant, the size of generators can vary from 50 MW to 1500 MW.

The source of the mechanical power, commonly known as the *prime mover*, may be hydraulic turbines at waterfalls, steam turbines whose energy comes from the burning of coal, gas and nuclear fuel, gas turbines, or occasionally internal combustion engines burning oil. The estimated installed generation capacity in 1998 for the United States is presented in Table 1.1.

Steam turbines operate at relatively high speeds of 3600 or 1800 rpm. The generators to which they are coupled are cylindrical rotor, two-pole for 3600 rpm or four-pole for 1800 rpm operation. Hydraulic turbines, particularly those operating with a low pressure, operate at low speed. Their generators are usually a salient type rotor with many poles. In a power station several generators are operated in parallel in the power grid to provide the total power needed. They are connected at a common point called a *bus*.

#SPARK_TEAM

Introduction

Transformers — Another major component of a power system is the transformer. It transfers power with very high efficiency from one level of voltage to another level. The power transferred to the secondary is almost the same as the primary, except for losses in the transformer, and the product VI on the secondary side is approximately the same as the primary side. Therefore, using a step-up transformer of turns ratio a will reduce the secondary current by a ratio of $1/a$. This will reduce losses in the line, which makes the transmission of power over long distances possible.

The insulation requirements and other practical design problems limit the generated voltage to low values, usually 30 kV. Thus, step-up transformers are used for transmission of power. At the receiving end of the transmission lines step-down transformers are used to reduce the voltage to suitable values for distribution or utilization. In a modern utility system, the power may undergo four or five transformations between generator and ultimate user.

1.3.2 TRANSMISSION AND SUBTRANSMISSION

The purpose of an overhead transmission network is to transfer electric energy from generating units at various locations to the distribution system which ultimately supplies the load. Transmission lines also interconnect neighboring utilities which permits not only economic dispatch of power within regions during normal conditions, but also the transfer of power between regions during emergencies.

Standard transmission voltages are established in the United States by the American National Standards Institute (ANSI). Transmission voltage lines operating at more than 60 kV are standardized at 69 kV, 115 kV, 138 kV, 161 kV, 230 kV, 345 kV, 500 kV, and 765 kV line-to-line. Transmission voltages above 230 kV are usually referred to as extra-high voltage (EHV).

Figure 1.1 shows an elementary diagram of a transmission and distribution system. High voltage transmission lines are terminated in substations, which are called *high-voltage substations*, *receiving substations*, or *primary substations*. The function of some substations is switching circuits in and out of service; they are referred to as *switching stations*. At the primary substations, the voltage is stepped down to a value more suitable for the next part of the journey toward the load. Very large industrial customers may be served from the transmission system.

The portion of the transmission system that connects the high-voltage substations through step-down transformers to the distribution substations are called the *subtransmission network*. There is no clear delineation between transmission and subtransmission voltage levels. Typically, the subtransmission voltage level ranges from 69 to 138 kV. Some large industrial customers may be served from the subtransmission system. Capacitor banks and reactor banks are usually installed in the substations for maintaining the transmission line voltage.

#SPARK_TEAM

#SPARK_TEAM

Introduction

1.3.3 DISTRIBUTION

The distribution system is that part which connects the distribution substations to the consumers' service-entrance equipment. The primary distribution lines are usually in the range of 4 to 34.5 kV and supply the load in a well-defined geographical area. Some small industrial customers are served directly by the primary feeders.

The secondary distribution network reduces the voltage for utilization by commercial and residential consumers. Lines and cables not exceeding a few hundred

feet in length then deliver power to the individual consumers. The secondary distribution serves most of the customers at levels of 240/120 V, single-phase, three-wire; 208Y/120 V, three-phase, four-wire; or 480Y/277 V, three-phase, four-wire. The power for a typical home is derived from a transformer that reduces the primary feeder voltage to 240/120 V using a three-wire line.

Distribution systems are both *overhead* and *underground*. The growth of underground distribution has been extremely rapid and as much as 70 percent of new residential construction is served underground.

1.3.4 LOADS

Loads of power systems are divided into industrial, commercial, and residential. Very large industrial loads may be served from the transmission system. Large industrial loads are served directly from the subtransmission network, and small industrial loads are served from the primary distribution network. The industrial loads are composite loads, and induction motors form a high proportion of these load. These composite loads are functions of voltage and frequency and form a major part of the system load. Commercial and residential loads consist largely of lighting, heating, and cooling. These loads are independent of frequency and consume negligibly small reactive power.

The real power of loads are expressed in terms of kilowatts or megawatts. The magnitude of load varies throughout the day, and power must be available to consumers on demand.

The daily-load curve of a utility is a composite of demands made by various classes of users. The greatest value of load during a 24-hr period is called the *peak* or *maximum demand*. Smaller peaking generators may be commissioned to meet the peak load that occurs for only a few hours. In order to assess the usefulness of the generating plant the *load factor* is defined. The load factor is the ratio of average load over a designated period of time to the peak load occurring in that period. Load factors may be given for a day, a month, or a year. The yearly, or annual load factor is the most useful since a year represents a full cycle of time. The daily load factor is

$$\text{Daily L.F.} = \frac{\text{average load}}{\text{peak load}} \quad (1.1)$$

Multiplying the numerator and denominator of (1.1) by a time period of 24 hr, we have

$$\text{Daily L.F.} = \frac{\text{average load} \times 24 \text{ hr}}{\text{peak load} \times 24 \text{ hr}} = \frac{\text{energy consumed during 24 hr}}{\text{peak load} \times 24 \text{ hr}} \quad (1.2)$$

The annual load factor is

$$\text{Annual L.F.} = \frac{\text{total annual energy}}{\text{peak load} \times 8760 \text{ hr}} \quad (1.3)$$

There are a few other factors used by utilities. *Utilization factor* is the ratio of maximum demand to the installed capacity, and *plant factor* is the ratio of annual energy generation to the plant capacity $\times 8760$ hr. These factors indicate how well the system capacity is utilized and operated.

1.4 SYSTEM PROTECTION

In addition to generators, transformers, and transmission lines, other devices are required for the satisfactory operation and protection of a power system. Some of the protective devices directly connected to the circuits are called *switchgear*. They include instrument transformers, circuit breakers, disconnect switches, fuses and lightning arresters. These devices are necessary to deenergize either for normal operation or on the occurrence of faults. The associated control equipment and protective relays are placed on *switchboard* in *control houses*.

1.5 ENERGY CONTROL CENTER

For reliable and economical operation of the power system it is necessary to monitor the entire system in a control center. The modern control center of today is called the *energy control center (ECC)*. Energy control centers are equipped with on-line computers performing all signal processing through the remote acquisition system. Computers work in a hierarchical structure to properly coordinate different functional requirements in normal as well as emergency conditions. Every energy control center contains a control console which consists of a visual display unit (VDU), keyboard, and light pen. Computers may give alarms as advance warnings to the operators (dispatchers) when deviation from the normal state occurs. The dispatcher makes judgments and decisions and executes them with the aid of a computer. Simulation tools and software packages written in high-level language are implemented for efficient operation and reliable control of the system. This is referred to as SCADA, an acronym for "supervisory control and data acquisition."

1.6 COMPUTER ANALYSIS

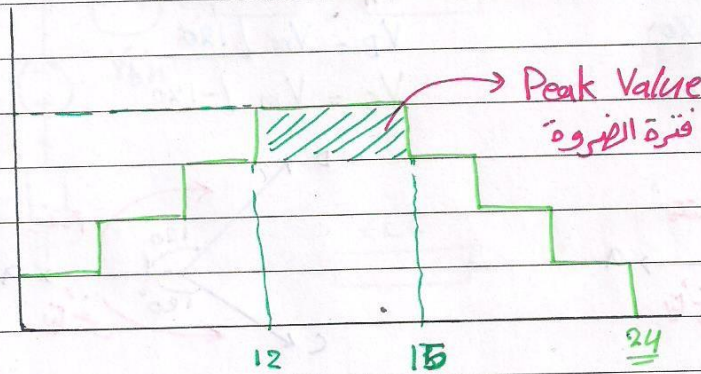
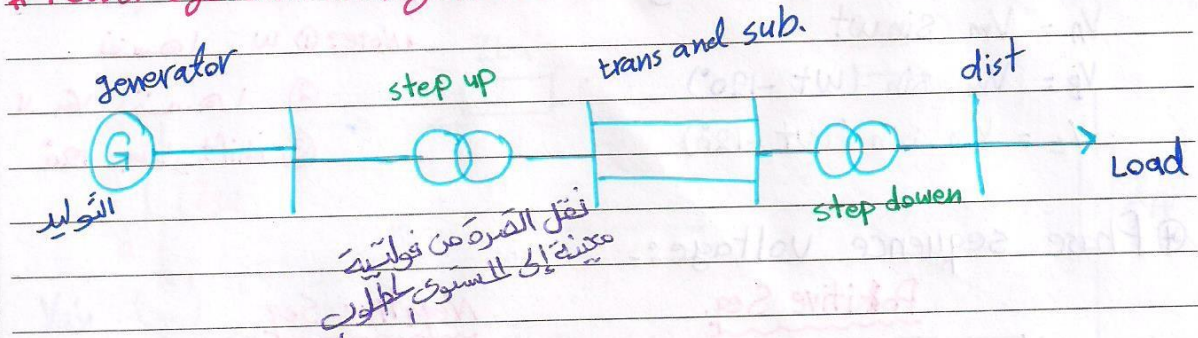
For a power system to be practical it must be safe, reliable, and economical. Thus many analyses must be performed to design and operate an electrical system. However, before going into system analysis we have to model all components of electrical power systems. Therefore, in this text, after reviewing the concepts of power and three-phase circuits, we will calculate the parameters of a multi-circuit transmission line. Then, we will model the transmission line and look at the performance of the transmission line. Since transformers and generators are a part of the system, we will model these devices. Design of a power system, its operation and expansion requires much analysis. This text presents methods of power system analysis with the aid of a personal computer and the use of *MATLAB*. The *MATLAB* environment permits a nearly direct transition from mathematical expression

#SPARK_TEAM

CH: 1

Power system analysis (PSA) الأحد 7.12.2016

* Power system analysis :-



$$P.F = \cos \varphi = \frac{P}{S} \Rightarrow P \approx S \approx (1)$$

أفضل كلما كانت أقرب إلى الواحد.

to simulation. Some of the basic analysis covered in this text are:

- Evaluation of transmission line parameters
- Transmission line performance and compensation
- Power flow analysis
- Economic scheduling of generation
- Synchronous machine transient analysis
- Balanced fault
- Symmetrical components and unbalanced fault
- Stability studies
- Power system control

#SPARK_TEAM

Three phase system

9/12/2016

الثلاثي

* Balanced 3-Phase system:

$$V_A = V_m \sin \omega t$$

$$V_B = V_m \sin (\omega t - 120^\circ)$$

$$V_C = V_m \sin (\omega t + 120^\circ)$$

+Note: ① ω نفسها

② V_m نفسها

③ shift بمقدار 120°

* Phase sequence voltage:-

Positive Seq.

$$V_A = V_m / 0$$

$$V_B = V_m / -120$$

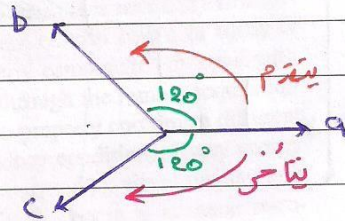
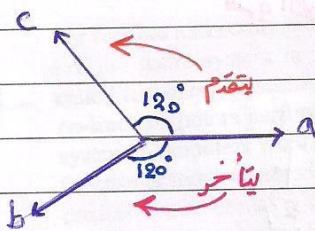
$$V_C = V_m / 120$$

Negative Seq.

$$V_A = V_m / 0$$

$$V_B = V_m / 120$$

$$V_C = V_m / -120$$



* Three phase connection:-

Source

Load

Δ

Δ

Δ

Δ

Δ

Δ

Δ

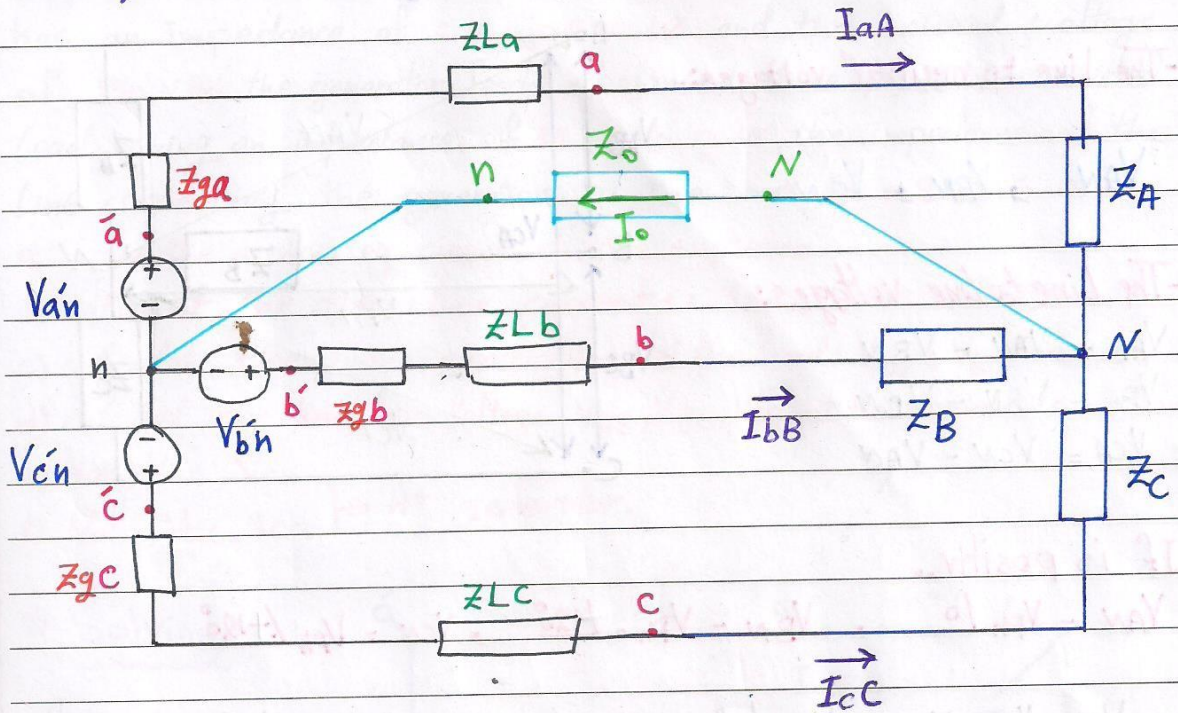
Δ

Notes: في أي سؤال إذا كان المطلوب generator أو Trans. line الطريقة تكون Y-Y

إذا كان Source وال Load يكون معيّنات خطية يصل على أي توصيلة

#SPARK_TEAM

Analysis Y-Y circuit:



Note:

السلك الواجب الـ N يظهر في حالة أن النظام غير متوازن وإذا كان متوازن لا يظهر

* يمكن تحويلها إلى one phase... ليسهل الحل *
نستنتج

① $Z_{ph} = Z_{ga} + Z_{La} + Z_A$

② $I_{aA} = \frac{V_{an}}{Z_{ph}}$, $I_{cC} = \frac{V_{cn}}{Z_{ph}}$

$I_{bB} = \frac{V_{bn}}{Z_{ph}}$

* ويمكن نعمل shift بمقدار 120° *

* مجموع التيارات في 3-Phase يساوي صفر *

Line to line and line to neutral volt. 9/2/2016

Line to line and line to neutral voltages :-

-The line to neutral voltages:-

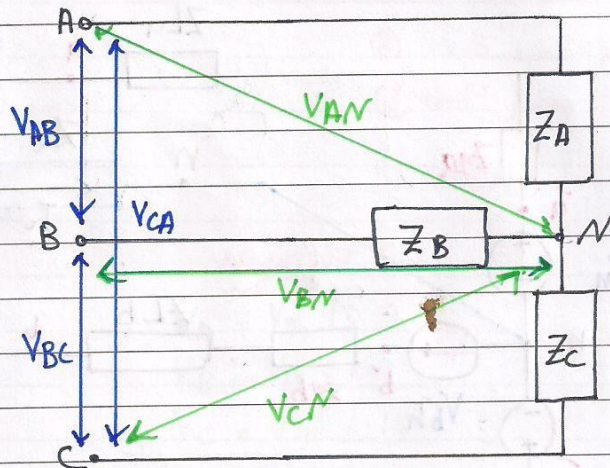
V_{AN} , V_{BN} , V_{CN}

-The line to line voltages:-

$$V_{AB} = V_{AN} - V_{BN}$$

$$V_{BC} = V_{BN} - V_{CN}$$

$$V_{CA} = V_{CN} - V_{AN}$$



If is positiv..

$$V_{AN} = V_{ph} \angle 0^\circ, \quad V_{BN} = V_{ph} \angle +120^\circ, \quad V_{CN} = V_{ph} \angle +120^\circ$$

$$\therefore V_{AB} = V_{AN} - V_{BN} = V_{ph} \angle 0^\circ - V_{ph} \angle -120^\circ = \sqrt{3} V_{ph} \angle 30^\circ$$

$$V_{BC} = \sqrt{3} V_{ph} \angle -90^\circ, \quad V_{CA} = \sqrt{3} V_{ph} \angle 150^\circ$$

*Note: V_{L-L} ^{تسبق} Lead V_p by 30°

*Note: If is negative seq

$$V_{AB} = \sqrt{3} V_{ph} \angle 30^\circ$$

$$V_{BC} = \sqrt{3} V_{ph} \angle 150^\circ$$

$$V_{CA} = \sqrt{3} V_{ph} \angle -90^\circ$$

*Note: ABC \rightarrow Positive
ACB \rightarrow Negative

*التحويل في نظام الفيز من 2 Phase الى 3 Phase والانتقال الفيز الثالث.

#SPARK_TEAM

Example.....

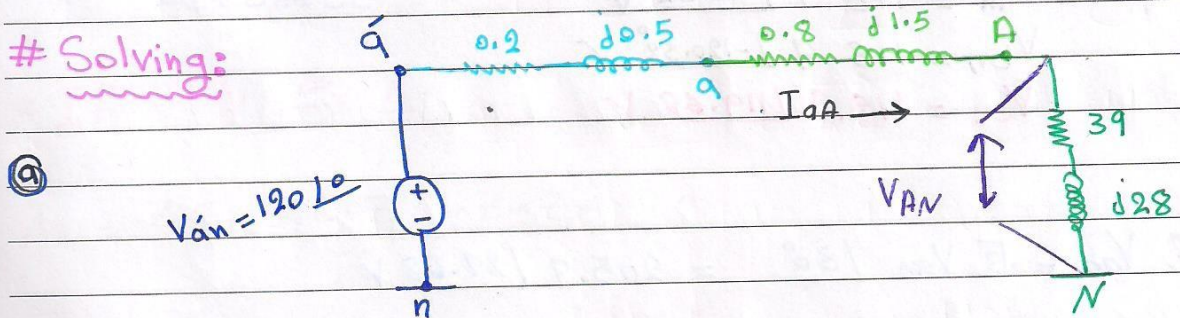
11/2/2016

Ex.1:

A balanced three phase Δ -connected generator with a positive sequence has an Impedance of $0.2 + j0.5 \Omega/\phi$ and the internal voltage of $120 \text{ V}/\phi$ the generator feeds a balanced three phase Δ -connected load having an impedance of $39 + j28 \Omega/\phi$ the impedance of the line connecting the generator to the load is $0.8 + j1.5 \Omega/\phi$.

- Draw the a phase eq. circuit of the system.
- calculate the three line currents: I_{aA} , I_{bB} , I_{cC} .
- calculate the three phase Voltage at the Load: V_{AN} , V_{BN} , V_{CN} .
- calculate the ~~three~~ line Voltage V_{AB} , V_{BC} , V_{CA} at the terminals of Load
- V_{an} , V_{bn} , V_{cn}
- V_{ab} , V_{bc} , V_{ca} } \rightarrow At generator.

Solving:



$$\textcircled{b} I_{aA} = \frac{120}{(0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28)} = 2.4 \angle -36.83^\circ \text{ A}$$

$$I_{bB} = 2.4 \angle -156.87^\circ \text{ A}$$

$$I_{cC} = 2.4 \angle 83.13^\circ \text{ A}$$

#SPARK_TEAM

Example

11/2/2016

$$\textcircled{a} V_{AN} = 2.4 \angle -36.87^\circ * (39 + j28) = 115.22 \angle -1.19^\circ \text{ V}$$

$$V_{BN} = 115.22 \angle -121.19^\circ \text{ V}$$

$$V_{CN} = 115.22 \angle 118.81^\circ \text{ V}$$

$$\textcircled{a} V_{AB} = \sqrt{3} \angle 30^\circ * V_{AN} = 199.58 \angle 28.81^\circ \text{ V}$$

$$V_{BC} = 199.58 \angle -91.19^\circ \text{ V}$$

$$V_{CA} = 199.58 \angle 148.81^\circ \text{ V}$$

$$\textcircled{a} V_{an} = 120 \angle 0^\circ - (0.2 + j0.5) * 2.4 \angle -36.87^\circ$$

$$V_{an} = 118.9 \angle -0.32^\circ \text{ V}$$

$$V_{bn} = 118.9 \angle -120.32^\circ \text{ V}$$

$$V_{cn} = 118.9 \angle 119.68^\circ \text{ V}$$

$$\textcircled{a} V_{ab} = \sqrt{3} V_{an} \angle 30^\circ = 205.9 \angle 29.68^\circ \text{ V}$$

$$V_{bc} = 205.9 \angle -90.32^\circ \text{ V}$$

$$V_{ca} = 205.9 \angle 149.68^\circ \text{ V}$$

* H.W: solve the same example but at
Negative seq.

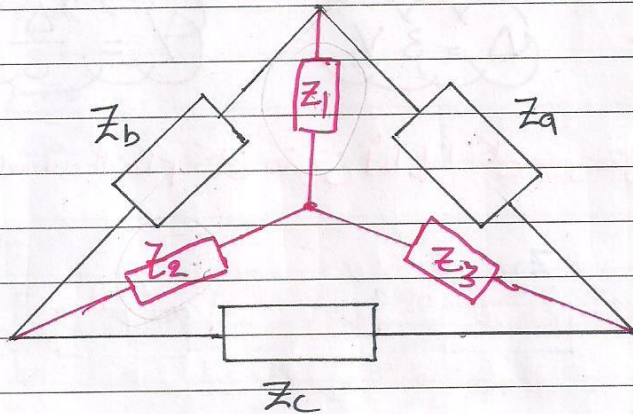
#SPARK_TEAM

connections

11/12/2016

الهندسة الكهربائية

*Transformation from Δ to Ystar :- $\Delta \rightarrow Y$



$$Z_1 = \frac{Z_a * Z_b}{Z_a + Z_b + Z_c}, \quad Z_2 = \frac{Z_b * Z_c}{Z_a + Z_b + Z_c}, \quad Z_3 = \frac{Z_a * Z_c}{Z_a + Z_b + Z_c}$$

* IF $Y \xrightarrow{to} \Delta$:-

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

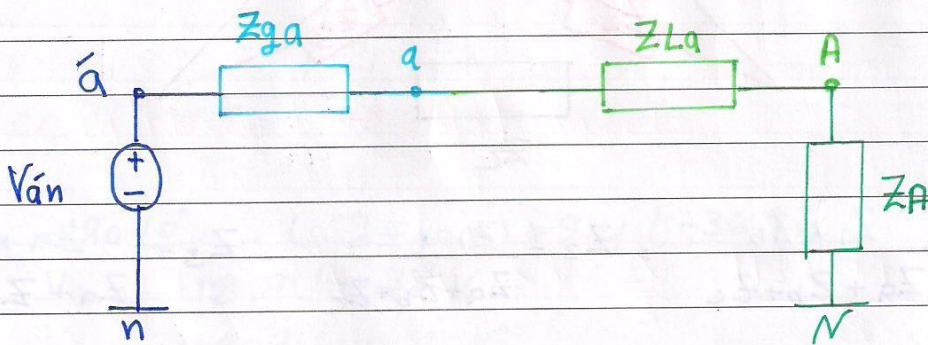
#SPARK_TEAM

Analysis of the Y-Δ circuits الخسيس 11/21/2016

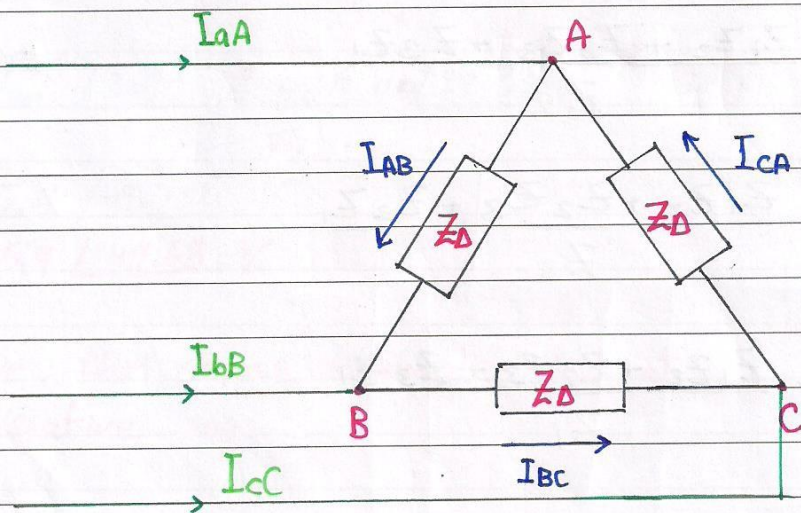
* Analysis of the Y-Δ circuit:

- At balanced: $\Delta = 3Y$ $Y = \frac{\Delta}{3}$

لهذه القوانين في حالة أن النظام كان متزن أما إذا كان غير متزن نحل على القوانين السابقة ...



* The Phase and line current for Δ-circuit:



#SPARK_TEAM

Analysis of the Y-Δ circuit.

الخميس 11/2/2018

*If is positive seq:

$$I_{AB} = I_{Ph} / 0^\circ$$

$$I_{BC} = I_{Ph} / -120^\circ$$

$$I_{CA} = I_{Ph} / 120^\circ$$

*at lead:

$$I_{oA} = I_{AB} - I_{CA} = I_{Ph} / 0^\circ - I_{Ph} / 120^\circ = \sqrt{3} I_{Ph} / 30^\circ$$

$$I_{oB} = I_{BC} - I_{AB} = \sqrt{3} I_{Ph} / -150^\circ$$

$$I_{oC} = I_{CA} - I_{BC} = \sqrt{3} I_{Ph} / 90^\circ$$

*في حالة الحمل للترتيب ←

#SPARK_TEAM

Example: 16/12/2016 ... s. L. V. K. I. I. I.

Ex: 9:

A balance Three phase Y-connected generator with positive sequence has an impedance $0.2 + j0.5 \Omega/\phi$ and the internal voltage of $120 V/\phi$ the Y-connected source feeds a Δ -connected load Through a distribution line having an impedance of $0.3 + j0.9 \Omega/\phi$ the load impedance is $118.5 + j85.5 \Omega/\phi$

① construct a single-phase eq. circuit of three phase system.

② calculate the line current; I_{AA}, I_{BB}, I_{CC} .

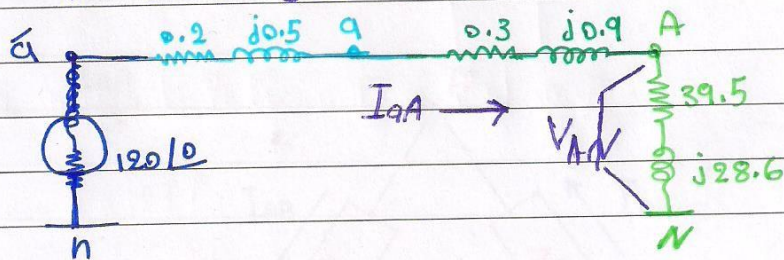
③ calculate the phase voltage at the load terminals.

④ // the phase currents at the load.

⑤ // the line Voltages at source terminals.

Solution:- ① $\Delta = 3Y \Rightarrow Y = \frac{\Delta}{3} = \frac{118.5 + j85.8}{3}$

$Y = 39.5 + j28.6 \Omega/\phi$



②

$$I_{AA} = \frac{120 \angle 0}{(0.2 + 0.3 + 39.5) + j(0.5 + 0.9 + 28.6)} = 2.4 \angle -36.87^\circ A$$

$$I_{BB} = 2.4 \angle -156.87^\circ A$$

$$I_{CC} = 2.4 \angle 83.13^\circ A$$

#SPARK_TEAM

16/21/2016

$$c) V_{AN} = 2.4 \angle -36.87^\circ * (39.5 + j28.6) = 117.04 \angle -0.96^\circ \text{ V}$$

$$V_{AB} = (\sqrt{3} \angle 30^\circ) (V_{AN}) = 202.72 \angle 29^\circ \text{ V}$$

$$V_{BC} = 202.72 \angle -91^\circ \text{ V}$$

$$V_{CA} = 202.72 \angle 149^\circ \text{ V}$$

$$d) I_{AB} = \left(\frac{1}{\sqrt{3}} \angle 30^\circ\right) I_{aA} = 2.4 \angle -36.87^\circ * \left(\frac{1}{\sqrt{3}} \angle 30^\circ\right) = 1.39 \angle -6.87^\circ \text{ A}$$

$$I_{BC} = 1.39 \angle -126.87^\circ \text{ A}$$

$$I_{CA} = 1.39 \angle 113.13^\circ \text{ A}$$

$$e) V_{an} = V_{An} - I_{aA} * Z_g = 120 \angle 0^\circ - (2.4 \angle -36.87^\circ * (0.2 + j0.5)) = 118.9 \angle -0.32^\circ \text{ V}$$

$$V_{ab} = (\sqrt{3} \angle 30^\circ) (V_{an}) = 205.94 \angle 29.68^\circ \text{ V}$$

$$V_{bc} = 205.94 \angle 90.32^\circ \text{ V}$$

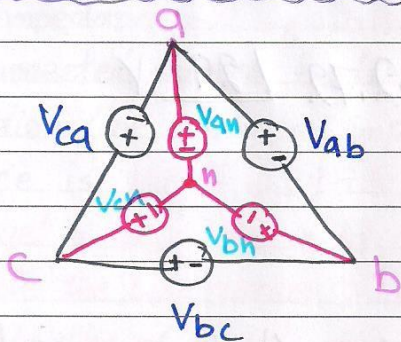
$$V_{ca} = 205.94 \angle 149.68^\circ \text{ V}$$

#SPARK_TEAM

#SPARK_TEAM

Power in Three phase system 18/2/2016 s.Litil

⊗ Δ-Y connection:



$$V_{an} = \frac{V_{ph}}{\sqrt{3}} \angle -30^\circ$$

$$V_{bn} = \frac{V_{ph}}{\sqrt{3}} \angle -150^\circ$$

$$V_{cn} = \frac{V_{ph}}{\sqrt{3}} \angle 90^\circ$$

⊗ Power in three phase system:-

Ⓘ Active power:- [KW]

$$P_T = 3P_{ph} = 3V_{ph}I_{ph}\cos\phi = \sqrt{3}V_L I_L \cos\phi = S_T \cos\phi$$

Ⓙ Reactive power:- [KVAR]

$$Q_T = 3Q_{ph} = 3V_{ph}I_{ph}\sin\phi = \sqrt{3}V_L I_L \sin\phi = S_T \sin\phi$$

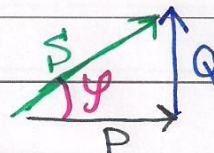
Ⓚ Apparent power:- [KVA]

$$S = 3|V_{ph}| |I_{ph}| = \sqrt{3} |V_L| |I_L|$$

Ⓛ Complex power:-

$$\dot{S} = \sqrt{3} V_L \dot{I}_L^* = 3V_{ph} \dot{I}_{ph}^* = P_T + jQ_T$$

Note:-
 $-Q \rightarrow$ Cap.
 $+Q \rightarrow$ Ind.



Examples...

9/12/2016

From Example 1: Calculate power in three phase Y-Y circuit.

- a) calculate the average Power Phase delivered to Y-conn. Load.
- b) calculate the total average power delivered to the load.
- c) calculate the total average power lost in the line.
- d) calculate the total average power lost in the generator.
- e) calculate the total number of magnetizing VARs absorbed by the load.
- f) calculate the total complex power delivered by the source.

Solution:

$$\textcircled{a} P_{Ph} = V_{Ph} I_{Ph} \cos \phi = 115.22 \times 2.4 \cos 53.68 = 224.64 \text{ watt}$$

OR $P_{Ph} = I_{Ph}^2 R_{Ph}$

$$\textcircled{b} P_T = 3P_{Ph} = 3 \times 224.64 = 673.92 \text{ watt} \quad \text{OR} \quad P_T = \sqrt{3} V_L I_L \cos \phi$$

$$\textcircled{c} P_g = 3 I_L^2 R_L = 3 \times (2.4)^2 \times (0.8) = 13.82 \text{ watt}$$

$$\textcircled{d} P_g = 3 I^2 R_g = (2.4)^2 \times 0.2 = 3.46 \text{ watt}$$

$$\textcircled{e} Q_T = Q_{Ph} = 3 V_{Ph} I_{Ph} \sin \phi = \sqrt{3} V_L I_L \sin \phi = \sqrt{3} \times (199.58) (2.4) \sin(53.68) = 483.84 \text{ VAR}$$

$$\textcircled{f} S_T = \sqrt{3} V_L I_L^* = 3 S_{Ph} = 3 (120)(2.4) / 36.87 = 691.20 + j 518.4 \text{ VA}$$

#SPARK_TEAM

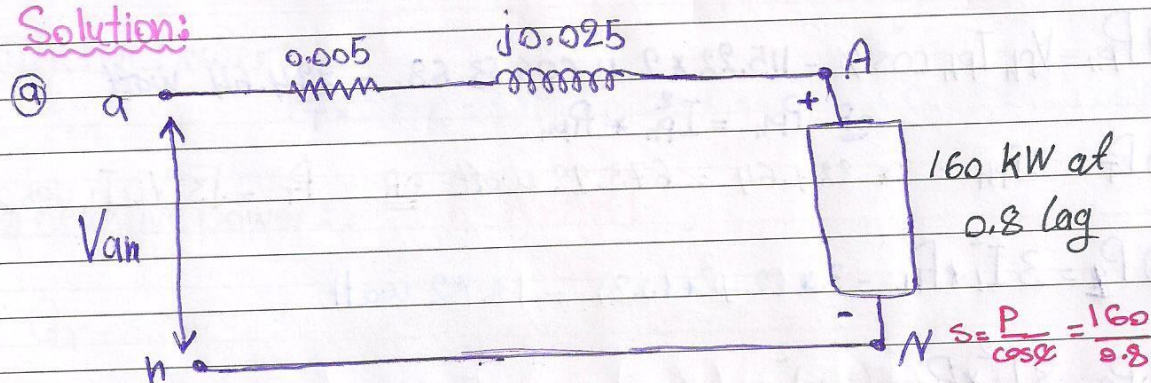
Example

9/9/2018

Ex 3: A balanced three phase requires 480 kW at lagging P.F of 0.8 the load is fed from a line voltage an impedance of $0.005 + j0.025 \Omega$ the line voltage at the terminal of load is 600V

- ⓐ calculate the magnitude of the voltage of the sending end of the line.
- ⓑ calculate a single eq. circuit of the system.
- ⓒ calculate the magnitude of the line current.
- ⓓ calculate the power factor at the sending end of the line

Solution:



ⓑ line current: $P_L = 3 I_L^2 R_L$

$$\Rightarrow \left(\frac{600}{\sqrt{3}}\right) I_{QA}^* = (160 + j120) \times 10^3$$

$$I_{QA}^* = 577.35 \angle 36.87^\circ$$

$$\Rightarrow I_{QA} = 577.35 \angle -36.87^\circ \text{ A}$$

#SPARK_TEAM

Examples...

21/2/2016

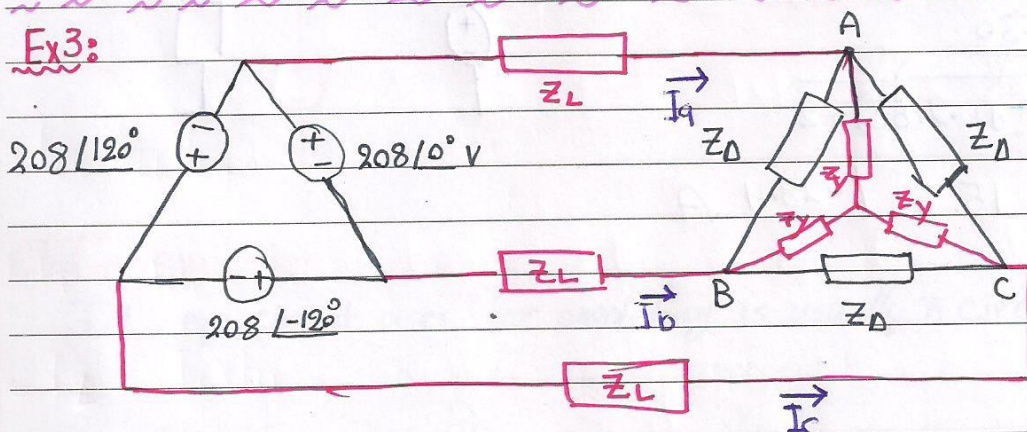
$$\textcircled{c} V_{an} = V_{AN} + I_{aA} * Z_L$$
$$= \left(\frac{600}{\sqrt{3}}\right) + ((0.005 + j0.025) * 577.35 / -36.87^\circ) = 357.54 / 1.57^\circ$$

$$V_L \equiv V_A = \sqrt{3} |V_{an}| = 619.23 \text{ V}$$

ⓐ V_{an} & I_{aA} :

$$\text{P.F.} = \cos(1.57 - (-36.87)) = 0.78 \text{ lagging}$$

Ex3:



* Find the line current I_a, I_b, I_c :

$$Z_{\Delta} = 12 - j15$$

$$Z_Y = 4 + j6$$

$$Z_L = 2 \Omega$$

#SPARK_TEAM

Example.....

21/2/2016

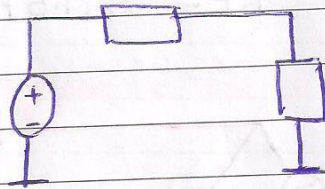
الجزء الثاني

Solution: convert the Δ -connected source to Y-connected.

$$V_{an} = \frac{V_{ph}}{\sqrt{3}} \angle -30^\circ = \frac{208}{\sqrt{3}} \angle -30^\circ = 120 \angle -30^\circ \text{ V}$$

$$Z = Z_Y \parallel \frac{Z_\Delta}{3} = (4 + j6) \parallel (4 - j5) = 5.723 - j0.2153$$

$$I_{aA} = \frac{120 \angle -30^\circ}{(5.723 - j0.2153) + 2}$$



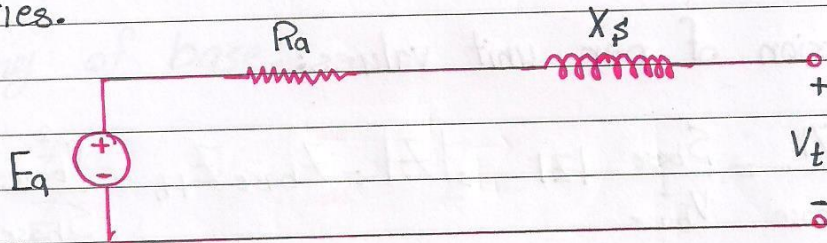
$$I_{aA} = 15.53 \angle -28.4^\circ \text{ A}$$

#SPARK_TEAM

* Power system components:

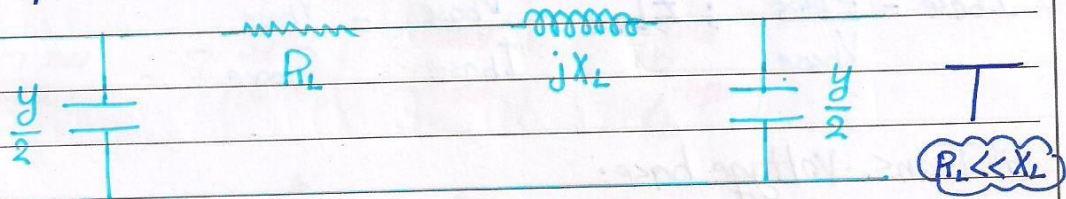
① synchronous generator:

For steady-state operation synch. generator can be represented by a circuit consists of per phase generated Voltage E_g and the per phase armature resistance R_a and synchronous reactance X_s in series.



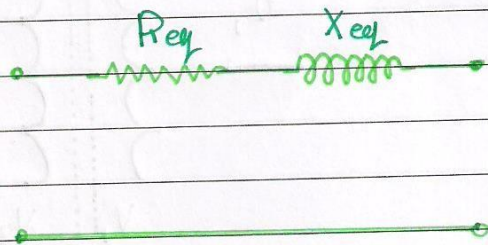
② Transmission line:

T.L. equ. circuit uses for modeling is usually π circuit.



③ Transformers:

Transformers are usually represented their equivalent approximate circuit.



Per unit system...

23/2/2018 5:15 AM

* Per Unit System:

① Per unit Value = $\frac{\text{Actual Value}}{\text{base Value}}$:

② $S_{ph} = \frac{|S|}{S_{base}}$ ③ $I_{ph} = \frac{|I|}{I_{base}}$ ④ $V_{ph} = \frac{|V|}{V_{base}}$ ⑤ $Z_{ph} = \frac{|Z|}{Z_{base}}$

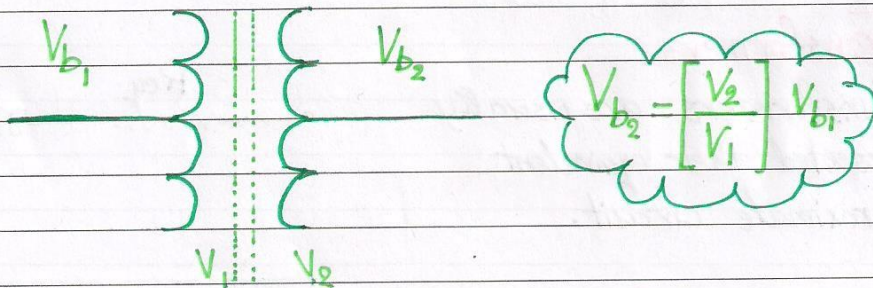
② Conversion of per unit values:

$$Z_{ph} = \frac{|Z|}{Z_{base}} = \frac{S_{base}}{V_{base}^2} |Z|, |Z| = Z_{base} Z_{ph} = \frac{V_{base}^2}{S_{base}} Z_{ph}$$

③ usually (S_{base}) and (V_{base}) take as base Values:

$$I_{base} = \frac{S_{base}}{V_{base}}; Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}$$

④ Trans. Voltage base:



#SPARK_TEAM

Per unit system...

23/2/2016

السيد

⑤ Per unit in three system:

$$S_b = \sqrt{3} V_b I_b ; \quad I_b = \frac{S_b}{\sqrt{3} V_b}$$

$$V_b = \sqrt{3} I_b Z_b \Rightarrow Z_b = \frac{V_b}{\sqrt{3} I_b} \quad \text{OR} \Rightarrow \frac{V_b^2}{S_b}$$

⑥ Change of base:

$$Z_{P.U.}^{\text{old}} = \frac{Z_{\Omega}}{Z_b^{\text{old}}} = \left[\frac{S_b^{\text{old}}}{(V_b^{\text{old}})^2} \right] Z_{\Omega}$$

$$Z_{P.U.}^{\text{new}} = \frac{Z_{\Omega}}{Z_b^{\text{new}}} = \left[\frac{S_b^{\text{new}}}{(V_b^{\text{new}})^2} \right] Z_{\Omega}$$

$$\Rightarrow Z_{P.U.}^{\text{new}} = Z_{P.U.}^{\text{old}} * \left(\frac{S_b^{\text{new}}}{S_b^{\text{old}}} \right) \left(\frac{V_b^{\text{old}}}{V_b^{\text{new}}} \right)^2$$

#SPARK_TEAM

Example

28/2/2018

Ex: The one line diagram of an unloaded power system is shown below. Reactance of the T.L. is shown on the diagram. The generator and transformers are rated follows:

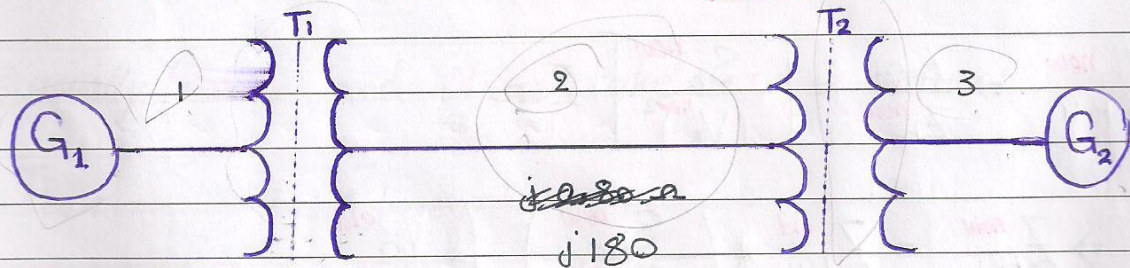
Generator I: 20 MVA, 13.8 KV, $X'' = 0.20$ per unit.

Generator II: 30 MVA, 18 KV, $X'' = 0.20$ per unit.

Trans I: 25 MVA, 220 Y / 13.8 Δ KV, $X = 10\%$

Trans II: 30 MVA, 220 Y / 18 Δ KV, $X = 10\%$.

Draw the impedance with all reactances chosen a base of 50 MVA; 13.8 KVA in the circuit of generator I



$$S_{base} = 50 \text{ MVA} = S_{b1}^{New} = S_{b2}^{New} = S_{b3}^{New}$$

$$V_{b1} = 13.8 \text{ KV} \rightarrow V_{b1}^{New}$$

$$V_{b2} = V_{b2}^{New} = \left(\frac{220}{13.8}\right) V_{b1} = \left(\frac{220}{13.8}\right) * 13.8 = 220 \text{ KV}$$

$$V_{b3} = V_{b3}^{New} = \left(\frac{18}{220}\right) * V_{b2} = \left(\frac{18}{220}\right) * 220 = 18 \text{ KV}$$

Example

29/2/2016

The corresponding base impedance in each region are:

$$Z_{b1}^{New} = \frac{(V_{b1}^{New})^2}{S_{b1}^{New}} = \frac{(13.8)^2}{50} = 3.8088 \Omega$$

$$Z_{b2}^{New} = \frac{(V_{b2}^{New})^2}{S_{b2}^{New}} = \frac{(220)^2}{50} = 968 \Omega$$

$$Z_{b3}^{New} = \frac{(V_{b3}^{New})^2}{S_{b3}^{New}} = \frac{(18)^2}{50} = 6.48 \Omega$$

$$* Z_{P.U}^{New} = Z_{P.U}^{old} \left[\frac{S_b^{New}}{S_b^{old}} \right] \left[\frac{V_b^{old}}{V_b^{New}} \right]^2$$

$$Z_{G1, P.U}^{New} = (j0.2) \left[\frac{50}{20} \right] \left[\frac{13.8}{13.8} \right]^2 = j0.5 \text{ p.u.}$$

$$Z_{T1, P.U}^{New} = (j0.1) \left[\frac{50}{25} \right] \left[\frac{13.8}{13.8} \right]^2 = j0.2 \text{ p.u.}$$

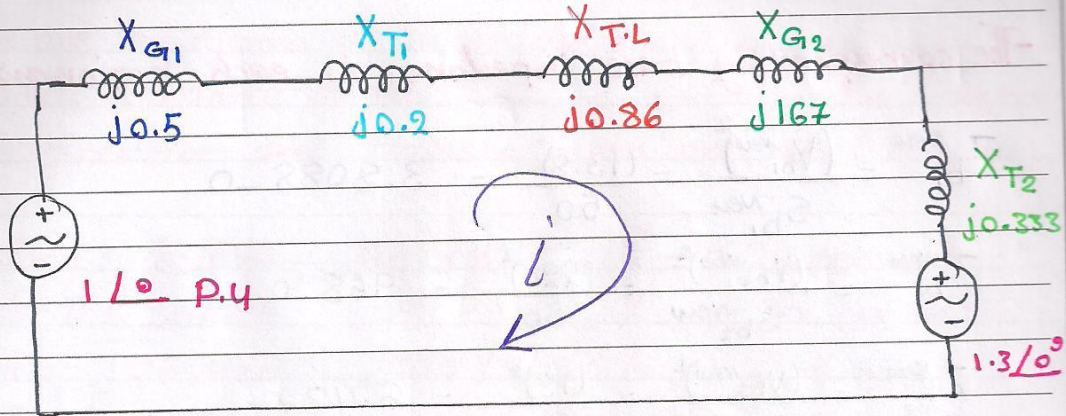
$$Z_{Line, P.U}^{New} = \frac{Z_{line}}{Z_{base}} = \frac{Z_{line}}{Z_{base}^{New}} = \frac{j0.180}{968} = j0.186 \text{ p.u.}$$

$$Z_{T2, P.U}^{New} = (j0.1) \left[\frac{59}{30} \right] \left[\frac{220}{220} \right]^2 = j0.167 \text{ p.u.}$$

$$Z_{G2, P.U}^{New} = (j0.2) \left[\frac{50}{30} \right] \left[\frac{18}{18} \right]^2 = j0.333 \text{ p.u.}$$

Example...

الأحرار 28/2/2016

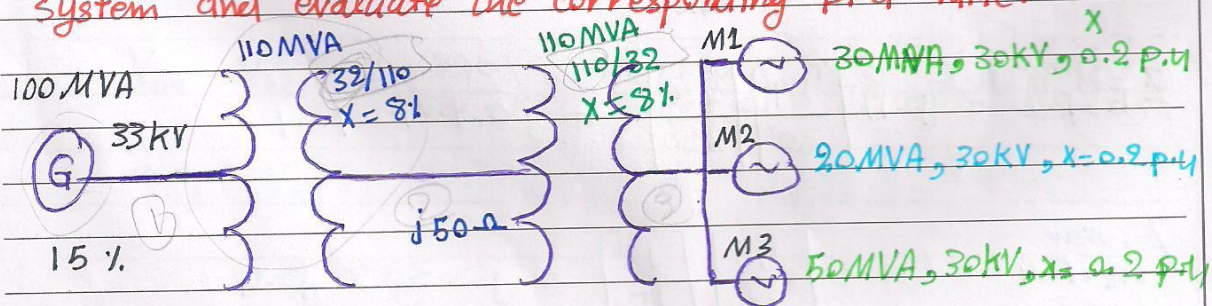


$$\frac{18}{13.8} = 1.3 \angle 0$$

$$I = \frac{1 \angle 0 - 1.3 \angle 0}{Z_{T.PH}}$$

Ex: A 100 MVA, 33 kV, 3-Phase generator has subtransient reactance of 15%. the generator is connected to the motors through a T.L and transformers:

Selecting the generator rating as the base quantities in the generator circuit determine the base in other parts of the system and evaluate the corresponding P.u value.



Example

1/3/2016 cl. Will

$$S_b = 100 \text{ MVA}, V_b = 33 \text{ kV}$$

Solution:

$$S_b = 100 \text{ MVA} = S_{b1}^{\text{New}} = S_{b2}^{\text{New}} = S_{b3}^{\text{New}}$$

$$V_{b1}^{\text{New}} = 33 \text{ kV}$$

$$V_{b2}^{\text{New}} = \left[\frac{110}{32} \right] * V_{b1}^{\text{New}} = \left[\frac{110}{32} \right] * 33 = 113.4375 \text{ kV}$$

$$V_{b3}^{\text{New}} = \left[\frac{33}{110} \right] V_{b2}^{\text{New}} = \left[\frac{33}{110} \right] * 113.4375 = 34.0313 \text{ kV}$$

$$Z_{b1}^{\text{New}} = \frac{(V_{b1}^{\text{New}})^2}{S_{b1}^{\text{New}}} = \frac{(33)^2}{100} = 10.89 \Omega$$

$$Z_{b2}^{\text{New}} = \frac{(V_{b2}^{\text{New}})^2}{S_{b2}^{\text{New}}} = \frac{(113.4375)^2}{100} = 128.68 \Omega$$

$$Z_{b3}^{\text{New}} = \frac{(V_{b3}^{\text{New}})^2}{(S_{b3}^{\text{New}})} = \frac{(34.0313)^2}{100} = 10.89 \Omega$$

$$Z_{g1}^{\text{New}} = Z_{P.U}^{\text{old}} \left[\frac{S_{b1}^{\text{New}}}{S_{b1}^{\text{old}}} \right] \left[\frac{V_{b1}^{\text{old}}}{V_{b1}^{\text{New}}} \right]^2 = j0.15 \left[\frac{100}{100} \right] \left[\frac{33}{33} \right]^2 = j0.15 \text{ p.u.}$$

$$Z_{T1}^{\text{New}} = j0.09 \left[\frac{100}{110} \right] * \left[\frac{32}{33} \right]^2 = j0.6838 \text{ p.u.}$$

$$Z_L^{\text{New}} = \frac{Z_{\text{line}}}{Z_{b2}} = \frac{j50}{128.68} = j0.3886 \text{ p.u.}$$

$$Z_{T2}^{\text{New}} = j0.09 \left[\frac{100}{110} \right] \left[\frac{110}{113.4375} \right]^2 = j0.6838 \text{ p.u.}$$

Example:

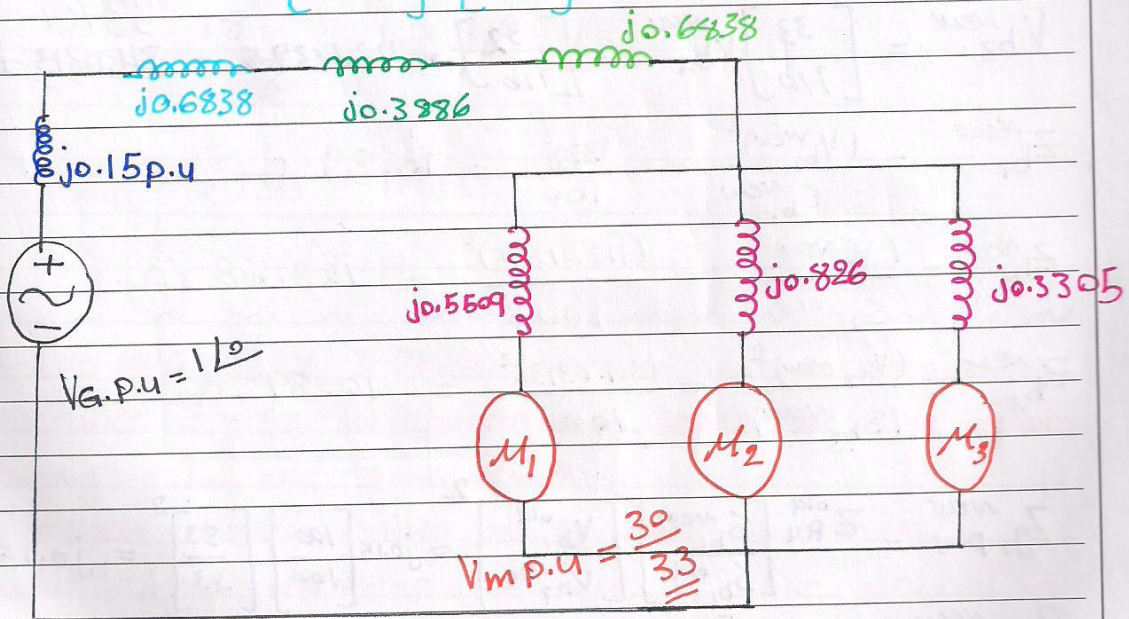
1/3/2016

s. Lilit

$$Z_{M1}^{New} \text{ p.u} = (j0.2) \begin{bmatrix} 100 \\ 30 \end{bmatrix} \begin{bmatrix} 30 \\ 33 \end{bmatrix}^2 = j0.5509 \text{ p.u}$$

$$Z_{M2}^{New} \text{ p.u} = (j0.2) \begin{bmatrix} 100 \\ 20 \end{bmatrix} \begin{bmatrix} 30 \\ 33 \end{bmatrix}^2 = j0.826 \text{ p.u}$$

$$Z_{M3}^{New} \text{ p.u} = (j0.2) \begin{bmatrix} 100 \\ 50 \end{bmatrix} \begin{bmatrix} 30 \\ 33 \end{bmatrix}^2 = j0.3305 \text{ p.u}$$



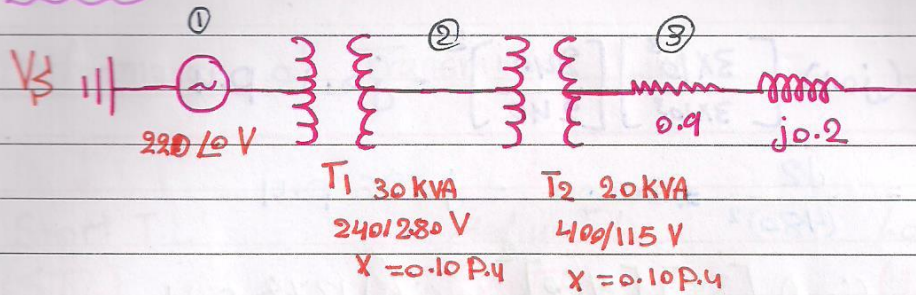
Note:

You can find the current...

Example:

3/3/2016

الأمير



Using base values of 30 kVA and 240 V in Zone 1; draw the per unit circuit and determine the per unit impedance and the per unit source, then calculate the Load current both in per unit and amperes.

Solution:

$$S_b = 30 \text{ kVA} = S_{b1}^{\text{new}} = S_{b2}^{\text{new}} = S_{b3}^{\text{new}}$$

$$V_{b1} = V_{b1}^{\text{new}} = 240 \text{ V}$$

$$V_{b2}^{\text{new}} = \left[\frac{480}{240} \right] * V_{b1}^{\text{new}} = 480 \text{ Volt}$$

$$V_{b3}^{\text{new}} = \left[\frac{115}{460} \right] * V_{b2}^{\text{new}} = 115 \text{ Volt}$$

$$Z_{b1}^{\text{new}} = \frac{(V_{b1}^{\text{new}})^2}{S_{b1}} = \frac{(240)^2}{30000} = 1.92 \Omega$$

$$Z_{b2}^{\text{new}} = \frac{(V_{b2}^{\text{new}})^2}{S_{b2}} = \frac{(480)^2}{30000} = 7.68 \Omega$$

$$Z_{b3}^{\text{new}} = \frac{(V_{b3}^{\text{new}})^2}{S_{b3}} = \frac{(115)^2}{30000} = 0.44 \Omega$$

Example -

3/31/2016

الجزء الثاني

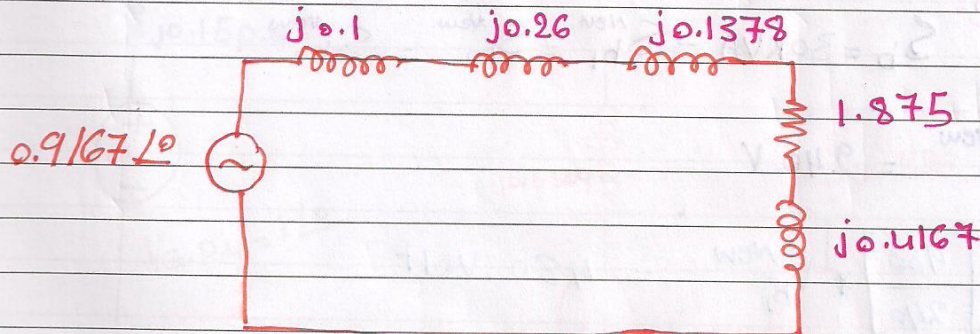
$$Z_{T1}^{new} \text{ p.u.} = (j0.1) \left[\frac{3 \times 10^3}{3 \times 10^3} \right] \left[\frac{240}{240} \right]^2 = j0.1 \text{ p.u.}$$

$$Z_{line}^{new} \text{ p.u.} = \frac{j2}{(480)^2} * 30000 = j0.26 \text{ p.u.}$$

$$Z_{T2}^{new} \text{ p.u.} = (j0.1) \left[\frac{30}{20} \right] \left[\frac{460}{480} \right]^2 = j0.11378 \text{ p.u.}$$

$$Z_{load} \text{ p.u.} = \frac{0.9 + j0.2}{(120)^2} * 30000 = 1.875 + j0.4167 \text{ p.u.}$$

$$V_s \text{ p.u.} = \frac{220}{240} = 0.9167 \angle 0^\circ \text{ p.u.}$$



$$I_{load} \text{ p.u.} = \frac{0.9167}{2086 \angle 26.01} = 0.4395 \angle -26.01 \text{ p.u.}$$

$$I_{actual} = (I_{load} \text{ p.u.}) (I_{base3}) \Rightarrow I_{base3} = \frac{S_{b3}}{V_{b3}} = 250$$

$$I_{actual} = (0.4395 \angle -26.01) (250) = 109.9 \angle -26.01 \text{ A}$$

Performance of T.L 20/3/2016 الخميس

* Performance of Transmission Line:

Short T.L

(STL)

80 km <

Medium T.L

(MTL)

80 km < M.T.L < 250 km

Long T.L

(LTL)

L.T.L > 250 km

* Important terms:

* Voltage Regulation:

The difference in voltage at the receiving end of a transmission line (T.L) between conditions of no load and full load is called "Voltage Regulation"

$$\% \text{age Voltage regulation} = \frac{V_s - V_R}{V_R} \times 100 \%$$

$$V_s = A V_r + B I_r$$

$$I_s = C V_r + D I_r$$

$$\rightarrow AD - BC = 1$$

* Transmission efficiency:

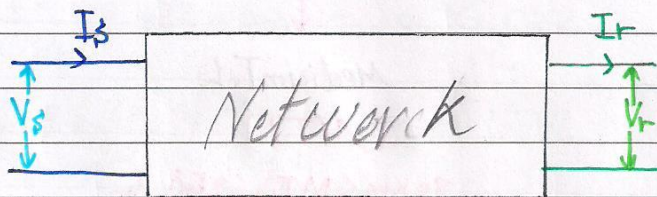
The ratio of receiving end power to the sending end power of T.L.

$$\% \text{age T.L eff. ; } \eta = \frac{\text{Res. end Power}}{\text{Sen. end Power}} \times 100$$

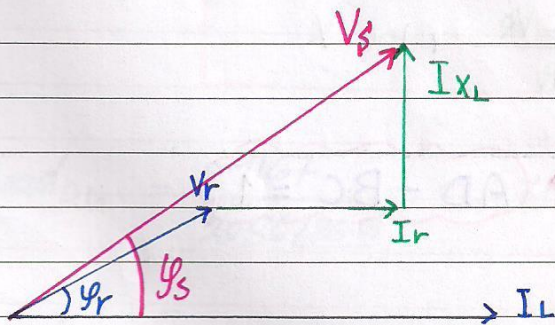
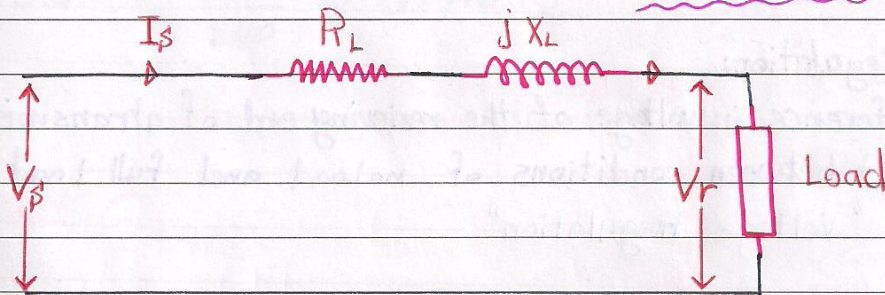
Short T.L

20/3/2016 الخميس

$$\eta = \frac{V_r I_r \cos \phi_r}{V_s I_s \cos \phi_s} \times 100\%$$



* Short Transmission Line :- (Single Phase)



$$\vec{V}_s = \vec{V}_r + \vec{I}Z$$

$$\vec{I}_s = \vec{I}_r$$

$$\begin{bmatrix} V_r \\ I_r \end{bmatrix} \begin{bmatrix} 1 \rightarrow A & Z \rightarrow B \\ 0 \leftarrow C & 1 \rightarrow D \end{bmatrix} = \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

$$Z \angle \phi = Z \cos \phi + j Z \sin \phi$$

Short T.L

الخميس 20/3/2016

$$* \% \text{age regulation} = \frac{V_s - V_r}{V_r} * 100\%$$

$$* \text{line losses} = I^2 * R$$

$$* \text{Power sent out} = V_r I_r \cos \phi_r + I^2 R$$

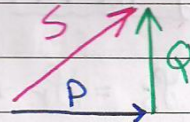
$$* \% \text{age T.L eff.} = \frac{V_r I_r \cos \phi_r}{V_r I_r \cos \phi_r + I^2 R} * 100\%$$

* effect of load P.f or regulation & efficiency

P.f \rightarrow Lag $\rightarrow V_r < V_s \rightarrow$ Voltage regulation positive. (+ve).

P.f \rightarrow Lead $\rightarrow V_r > V_s \rightarrow$ Voltage regulation negative. (-ve).

$$\text{P.f} = \cos \phi = \frac{P}{S} = \frac{R}{Z}$$



Ex: Single phase over head Transmission [OHT] delivers 1100kw at 33 KV out of 0.8 P.F lagging the total resistance and reactance of the line are 10Ω , 15Ω respectively. Determine:

- ① Sending end Voltage.
- ② Sending P.F.
- ③ Transmission efficiency.

Example

الخميس 2013/2016

*Solution:

$$Z_L = 10 + j15$$

$$P.F = \cos \psi = 0.8$$

$$I_s = I_r = I_L$$

$$V_s = V_r + I \cdot Z_L$$

$$P = VI \cos \psi$$

*line current:

$$I = \frac{1100 \cdot 10^3}{33 \cdot 10^3 \cdot 0.8} = 41.67 \text{ A}$$

$$\psi = \cos^{-1}(0.8) = 36.78^\circ$$

$$\Rightarrow V_s = V_r + I \cdot Z_L$$

$$= 33 \times 10^3 / 0 + (10 + j15)(41.67 \angle -36.78^\circ)$$

$$= 33.708 + j250 = 33.709 \angle 0.42^\circ \text{ V}$$

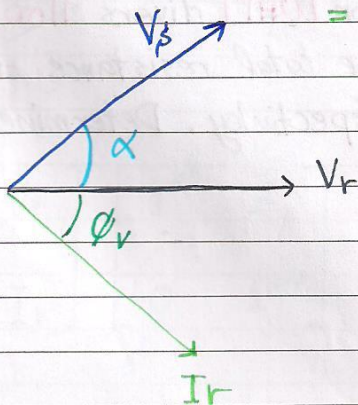
*Sending end P.f:

$$\psi_s = \psi_r + \alpha = 36.78^\circ + 0.42^\circ$$

$$\Rightarrow P.F = \cos(37.29^\circ)$$

$$= 0.7956 \text{ lagging}$$

لأنه inductive



Example...

20/13/2016

محمد!

$$* \text{Line Losses:} = I^2 R = (41.67)^2 \times 10 = 17.364 \text{ kW}$$

$$* \text{Power sent} = 1100 + 17.364 = 1117.364 \text{ kW}$$

$$\eta = \frac{1100}{1117.364} \times 100\% = \cancel{98.44\%} 98.44\%$$

Ex: A 3-Phase 50 Hz, 16 km long OHTL supplies 1000 kW at 11 kV P.F lagging the line resistance is 0.03 Ω per phase per km and line inductance is 0.7 mH per phase per km. calculate the sending end Voltage, Voltage regulation and efficiency of T.L

Solution:

$$R_L = 0.03 \times 16 = 0.48 \Omega$$

$$X_L = j 2\pi f L = j 2\pi \times 50 \times 0.7 \times 10^{-3} = j 0.22 \Omega$$

$$X_L = X_L \times 16 = j 0.22 \times 16 = j 3.52 \Omega$$

$$Z_L = 0.48 + j 3.52$$

$$V_r = \frac{11 \times 10^3}{\sqrt{3}} = 6351 \text{ V}$$

$$I = \frac{1000 \times 10^3}{3 \times V_r \times \cos \phi} = \frac{1000 \times 10^3}{3 \times 6351 \times 0.8} = 65.6 \text{ A}$$

$$* P = 3 V_{ph} I_{ph} \cos \phi$$

$$* P = \sqrt{3} V_L I_L \cos \phi$$

Medium T. h

الأحمر 23/3/2016

$$\begin{aligned}V_s &= V_r + I Z \\&= 6351 + (65.6 \angle -36.87^\circ) (0.48 + j 3.52) \\&= 6515 \angle 1.46^\circ \text{ V}\end{aligned}$$

$$\% \text{ age voltage reg.} = \frac{6515 - 6351}{6351} * 100\% = 2.58\%$$

$$\begin{aligned}\ast \text{ line losses:} &= 3 I^2 R \\&= 3 (65.6)^2 (0.48) \\&= 6.2 \text{ kW}\end{aligned}$$

$$\eta = \frac{1000}{1006.2} * 100\% = 99.38\%$$

* Medium Transmission Lines:

غير مألوف

- * The effect of capacitance can't be neglected.
- * The line capacitance must be taken into consideration.
- * The capacitance is uniformly distributed over the entire length of the line.
- * The most commonly used methods:

① End condenser method.

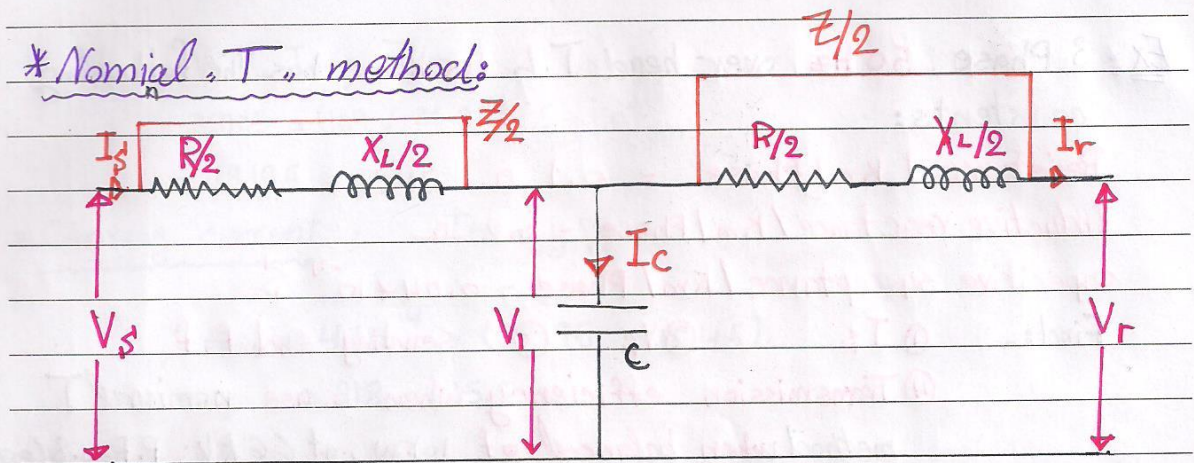
② Nominal T method.

③ Nominal π method.

Nominal T method

23/3/2016

* Nominal T method:



$$V_1 = V_r + I_r * \frac{Z}{2}$$

$$I_c = V_1 y$$

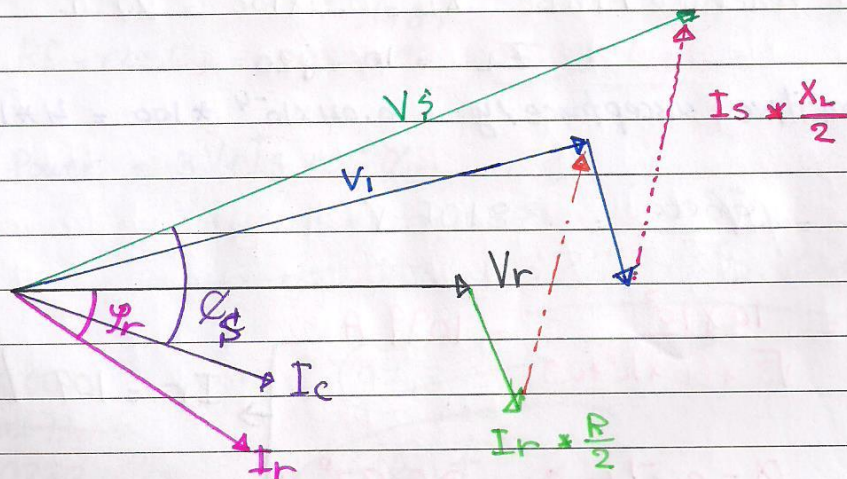
$$= V_1 j\omega C$$

$$= j2\pi f C V_1$$

$$I_s = I_r + I_c$$

$$V_s = I_s * \frac{Z}{2} + V_1$$

$$X_c = \frac{-j}{\omega C} \quad ; \quad y = \frac{1}{-jX_c}$$



Example...

23/3/2016

الأمثلة

Ex: 3 Phase, 50 Hz over head T.L 100 km has the following constants:

Resistance / km / Phase = 0.1 Ω

Inductive reactance / km / Phase = 0.2 Ω

capacitive susceptance / km / Phase = $0.04 \times 10^{-4} \text{ S}$

Find: ① I_s ② V_s ③ sending end P.F

④ Transmission efficiency when a use nominal T method when balanced of 10 kW, at 66 kV; P.F = 0.8 lag

Solution:

$$Z = R + jX \Rightarrow Y = G + jB$$

$$Y = \frac{1}{R + jX} * \frac{R - jX}{R - jX} = \frac{R}{R^2 + X^2} - j \frac{X}{R^2 + X^2}$$

conductance susceptance

* total resistance / Phase $R = 0.1 * 100 = 10 \Omega$

* total reactance / Phase $X_L = 0.2 * 100 = 20 \Omega$

$$Z_L = 10 + j20$$

* Capacitive susceptance / y = $0.04 * 10^{-4} * 100 = 4 * 10^{-4} \text{ S}$

* $V_r = \frac{66000}{\sqrt{3}} = 38105 \text{ V}$

* $I_r = \frac{10 * 10^3}{\sqrt{3} * 66 * 10^3 * 0.8} = 109 \text{ A}$

$I_r = 109 \angle -36.87^\circ \text{ A}$

$\theta = \cos^{-1}(0.8) = 36.87^\circ$

Example

23/3/2016

الأ

$$\begin{aligned} * V_1 &= V_r + I_r * Z/2 \\ &= 38105 + (109 \angle -36.87^\circ) (5 + j10) \\ &= 39195 + j545 \end{aligned}$$

* Charging current ; $I_c = jy V_1$

$$\begin{aligned} &\Rightarrow j * 4 \times 10^{-4} * (39195 + j545) \\ &\Rightarrow -0.218 + j15.6 \end{aligned}$$

$$* \bar{I}_s = \bar{I}_c + \bar{I}_r \rightarrow I_s = 100 \angle -29.47^\circ \text{ A}$$

$$\begin{aligned} * V_s &= V_r + I_s Z/2 \\ &= (39195 + j545) + (87 - j49.8) (5 + j10) \\ &= 40145 \angle 1.4^\circ \text{ V} \end{aligned}$$

$$* \text{line value} = 40145 * \sqrt{3} = 69.533 \text{ KV}$$

$$* \text{Sending end P.f} = \cos \phi_s = \cos 31.87^\circ = 0.858 \text{ lags.}$$

$$\begin{aligned} * \text{ending end Power} &= 3 V_s I_s \cos \phi_s \\ &= 3 * 40.145 * 100 * 0.858 \\ &= 10273.105 \text{ kW. } \underline{10373.329 \text{ kW}} \end{aligned}$$

$$\eta = \frac{10000}{10273.105} * 100\% = 97.34\%$$

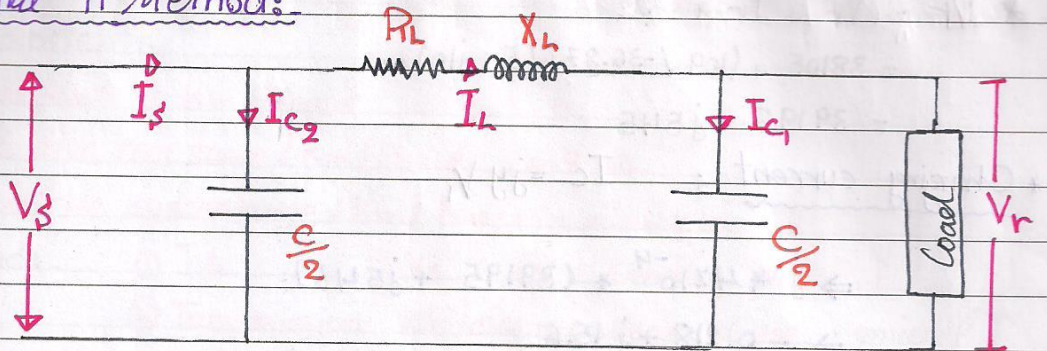
$$10333.329$$

$$\underline{96.77\%}$$

Nominal π Method

27/3.1.2016

* Nominal π Method:



$$* I_L = I_r + I_{c1}$$

$$* I_s = I_L + I_{c2}$$

$$* I_{c1} = j\pi f C V_r \rightarrow \frac{1}{y}$$

$$* I_{c2} = j\pi f C V_s \rightarrow \frac{1}{y}$$

$$I = \frac{V}{Z} = Y V$$

$$Y = \frac{1}{X_c} = \frac{1}{1/\omega C} = \omega C = 2\pi f C$$

$$\frac{Y}{2} = \pi f C$$

$$V_s = V_r + I_L Z$$

Ex: A 3 Phase, 50 Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0.1Ω , $j 0.5 \Omega$ and $3 \times 10^{-6} S$ per km per phase. If the line delivers 50 MW at 110 kV and 0.8 P.f lagging. Determine the sending end voltage and current. Assume a nominal π circuit for the line.

Nominal π method

الأحد 27/3/2016

Solution:

$$* V_r = \frac{110 \times 10^3}{\sqrt{3}} = 63.508 \text{ V}$$

$$* I_r = \frac{50 \times 10^6}{\sqrt{3} + 110 \times 10^3 + 0.8} = 328 \text{ A}$$

$$\rightarrow I_r = 328 \angle -36.87^\circ \text{ A}$$

$$* R_L = 0.1 \times 150 = 15 \Omega$$

$$* X_L = j0.5 \times 150 = j75 \Omega$$

$$* Y = j3 \times 10^{-6} \times 150 = j45 \times 10^{-5} \text{ S}$$

$$* I_L = I_r + I_{C1}$$

$$= 328 \angle -36.87^\circ + j14.3$$

$$= 262.4 - j182.5 \text{ A}$$

$$* V_s = V_r + I_L Z \rightarrow (15 + j75)$$

$$V_s = 63.508 + (262.4 - j182.5)(15 + j75)$$

$$= 82.881 \angle 11.47^\circ \text{ KV}$$

* line Voltage (V_s):

$$V_s = \frac{V_{s \text{ Ph}}}{\sqrt{3}} = 143.55 \text{ KV}$$

$$V_s = V_r + I_L Z$$

$$I_L = I_r + I_{C1}$$

$$I_{C1} = j\pi f V_r C$$

$$= j\pi \times 50 \times 63.508 \times C$$

$$* I_{C1} = V_r * \frac{Y}{2}$$

$$I_{C1} = \frac{63.508 * j45 \times 10^{-5}}{2}$$

$$I_{C1} = j14.3$$

Example ..

29/8/2018

الثلاثاء

Ex: A 100 km Long 3 Phase, 50 Hz T.L has following line constants:

$$\text{Resistance / Phase / km} = 0.1 \Omega$$

$$\text{Reactance / Phase / km} = 0.5 \Omega$$

$$\text{Susceptance / Phase / km} = 10 \times 10^{-6} \text{ S}$$

If the line supplies load of 20 MW of 0.9 P.f lagging at 66 kV at the receiving end, calculate by nominal π method:

① sending P.F. ③ Transmission eff.

② regulation.

Solution:

$$Z_L = 10 + j50$$

$$R = 0.1 \times 100 = 10 \Omega$$

$$X_L = 0.5 \times 100 = 50 \Omega$$

$$y = 10 \times 10^{-6} \times 100 = 10 \times 10^{-4} \text{ S}$$

$$V_r = \frac{66 \times 10^3}{\sqrt{3}} = 38.105 \text{ kV}$$

$$I_r = \frac{20 \times 10^6}{\sqrt{3} \times 66 \times 10^3 \times 0.9} = 195 \text{ A}$$

$$I_{c1} = V_r \times j \frac{y}{2} = 38.105 \times j \frac{10 \times 10^{-4}}{2} = j19 \text{ A}$$

$$\begin{aligned} I_L = I_r + I_{c1} &= 195 \angle -25.8^\circ + 19 \angle 90^\circ \\ &= 176 - j66 \text{ A} \end{aligned}$$

$$V_s = V_r + I_L Z = 38.105 + (176 - j66)(10 + j50) = 43.925 \angle 10.65^\circ \text{ V}$$

Example

27/3/2016 الثلاثاء

$$I_s = I_{c2} + I_L \quad ; \quad I_c = V_s j \frac{Y}{2}$$
$$= (43.925 / 10.65) (j \frac{10 \times 10^{-4}}{2})$$

$$I_s = (-4 + j21.6) + (176 - j66)$$
$$= -4 + j21.6$$

$$= 177.6 \angle -14.5^\circ \text{ A}$$

* sending end P.f = $\cos \phi$

$$= \cos(25.15) = 0.905 \text{ Lagging}$$

* Voltage regulation:

$$= \frac{V_s - V_r}{V_r} \times 100$$

$$= \frac{43929 - 38105}{38105} \times 100 = 15.27\%$$

* Sending end power:

$$= 3 V_s I_s \cos \phi$$

$$= 3(43925)(177.6)(0.905)$$

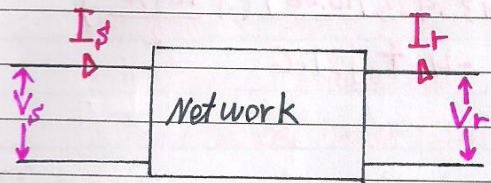
$$= 21.18 \text{ MW}$$

$$\eta = \frac{20}{21.18} \times 100 = 94\%$$

Generalised circuit

31/8/2016 الخسيس

* Generalised circuit constants of T.L:



$$V_s = \bar{A} \bar{V}_r + \bar{B} \bar{I}_r$$

$$I_s = \bar{C} \bar{V}_r + \bar{D} \bar{I}_r$$

① A, B, C, D → cons. (complex number)

② B → thm.

C → siemen.

③ for a given T.L $\bar{A} = \bar{D}$

④ for a given T.L $\bar{A}\bar{D} - \bar{B}\bar{C} = 1$

* Short T.L:

$$V_s = V_r + I_r Z$$

$$I_s = I_r$$

$$A = 1 ; B = Z$$

$$C = 0 ; D = 1$$

$$\left. \begin{array}{l} V_s = V_r + I_r Z \\ I_s = I_r \end{array} \right\} AD - BC = 1$$

* Medium T.L [T method]:

$$V_s = \left[1 + \frac{yZ}{z} \right] V_r + \left[\bar{Z} + \frac{y\bar{Z}^2}{z} \right] I_r$$

$$I_s = \underbrace{y}_{\rightarrow C} V_r + I_r \left[1 + \frac{yZ}{z} \right] \underbrace{\quad}_{\rightarrow D}$$

Example no.

314/2016

→ 11

* Medium T.L [π method]

$$\bar{V}_s = \bar{V}_r \left[1 + \frac{y\bar{Z}}{2} \right] + \bar{I}_r \bar{Z}$$

$$\bar{I}_s = \bar{V}_r y \left[1 + \frac{y\bar{Z}}{4} \right] + \bar{I}_r \left[1 + \frac{y\bar{Z}}{2} \right]$$

Ex: A balanced 3-Phase load of 30 MW is supplied at 132 kV, 50 Hz and 0.85 P.F lagging by means of a T.L. The series impedance of single conductor is $(20 + j52) \Omega$ and the total phase neutral admittance is $315 \times 10^{-6} \text{ S}$.

Using Nominal T Method. Determine:

- (A) The A, B, C and D constants of the line.
- (B) sending end Voltage.
- (C) regulation of the line.

⊕ Solution: $Z_L = 20 + j52$, $y = 315 \times 10^{-6}$

$$(A) \quad A = D = 1 + \frac{ZY}{2} = 1 + \frac{(20 + j52)(315 \times 10^{-6})}{2} = 0.992 \angle 0.18^\circ$$

$$B = Z \left[1 + \frac{ZY}{4} \right] = (20 + j52) \left[1 + \frac{(20 + j52)(315 \times 10^{-6})}{4} \right] = 55.5 \angle 69^\circ$$

$$C = y = 0.000315$$

Example 6.6 31212016 الأجر

$$V_r = \frac{132 \times 10^3}{\sqrt{3}} = 76210 \text{ V}$$

$$I_r = \frac{30 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0.85} = 154 \text{ A}$$

$$\theta = \cos^{-1}(0.85) = 31.7^\circ$$

$$\therefore I_r = 154 \angle -31.7^\circ \text{ A}$$

$$\begin{aligned} \textcircled{B} \quad V_s &= A V_r + B I_r \\ &= (0.992 \angle 10.18^\circ)(76210) + [(55.5 \angle 69^\circ)(154 \angle -31.7^\circ)] \\ &= 82.428 + j 5413 \end{aligned}$$

$$V_s = \sqrt{(82.428)^2 + (5413)^2} = 82.6 \text{ kV}$$

© At no load ($I_r = 0$):

$$V_s = A V_r + \overset{0}{\cancel{B I_r}} \Rightarrow V_r = \frac{V_s}{A}$$

$$\% \text{ Regulation} = \frac{(V_s/A) - V_r}{V_r} \times 100\%$$

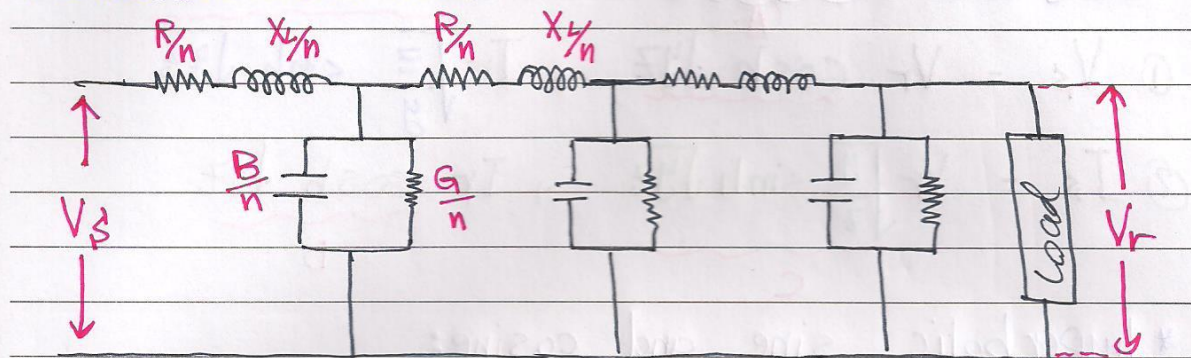
$$= 9.25\%$$

End...

Long Transmission Line...

7/14/2016

* Long Transmission Line:



- The line constant of the T.L are uniformly distributed over the entire length of the line.

- The leakage susceptance (B) and leakage conductance (G) are shunt elements.

- The leakage susceptance is due to the fact that capacitance exists between line and neutral.

- The leakage conductance takes into account the energy losses occurring through leakage over the insulators.

- The leakage current through shunt admittance is max at sending end and decreases at the receiving end at which point its value is zero.

Long T.L. (Rigorous method) ... 7/4/2016 ...

* Long T.L. [Rigorous method]:

$$\textcircled{1} V_s = V_r \underbrace{\cosh \sqrt{YZ}}_A + I_r \underbrace{\sqrt{\frac{Z}{Y}} \sinh \sqrt{YZ}}_B$$

$$\textcircled{2} I_s = V_r \underbrace{\sqrt{\frac{Y}{Z}} \sinh \sqrt{YZ}}_C + I_r \underbrace{\cosh \sqrt{YZ}}_D$$

* hyperbolic sine and cosine:

$$\textcircled{1} \cosh \sqrt{YZ} = \left[1 + \frac{ZY}{2} + \frac{Z^2 Y^2}{24} + \dots \right]$$

$$\textcircled{2} \sinh \sqrt{YZ} = \left[\sqrt{YZ} + \frac{(YZ)^{\frac{3}{2}}}{6} + \dots \right]$$

$\textcircled{3} \gamma = \sqrt{YZ}$ Propagation constant.

$\textcircled{4} Z_c = \sqrt{\frac{Z}{Y}}$ characteristic impedance.

$$A = D = \cosh \sqrt{YZ}$$

$$B = \sqrt{\frac{Z}{Y}} \sinh \sqrt{YZ}$$

$$C = \sqrt{\frac{Y}{Z}} \sinh \sqrt{YZ}$$

Symmetrical fault [Three Phase short circuit] الذئ

⊛ Symmetrical fault [Three phase short circuit]:

fault studies $\left\{ \begin{array}{l} \rightarrow \text{Bus Voltage.} \\ \rightarrow \text{Current during fault.} \end{array} \right.$

- Three phase fault information is used to select and set phase relays. Fault studies are used for proper choice of CB's.

- During fault, loads current can be neglected because voltage dip very low so that current drawn by loads can be neglected in comparison to fault currents.

- The magnitude of the fault current depends on the internal impedance of the synchronous generator and the impedance of the intervening circuit.

- The purpose of fault studies generator behaviour can be divided into three different periods:

- 1- The subtransient period, lasting only for the first few cycles.
- 2- The transient period, covering a relatively longer time.
- 3- Steady state period.

Symmetrical fault

~~The~~ A nother important point: That the circuit breakers MVA capacity is based of three phase fault MVA CCIB manufactured in standareed size, 250; 500; 750 MVA etcl.

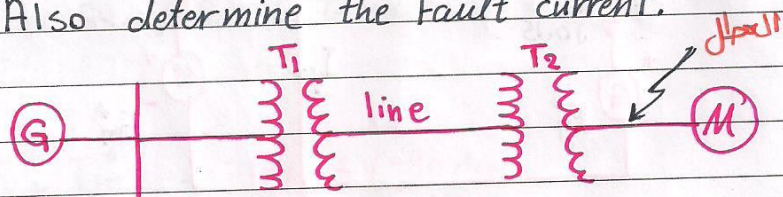
For three phase fault calculation following assumptions are made

- ① The emf's of all generator are 1.0 p.u.
- ② charging capacitances of the T.L are ignored.
- ③ shunt elements in the transformer model are neglected.

Example 6.6

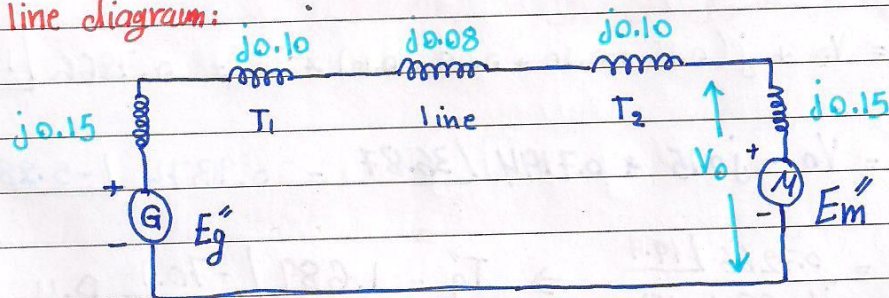
مثال 6.6

Example: A synchronous generator and a synch. motor each rated 20 MVA ; 12.66 kV having 15% subtransient reactance are connected through transformer and a line, the transformer are rated 20 MVA ; $12.66 / 66 \text{ kV}$ and $66 / 12.66 \text{ kV}$ with leakage reactance of 10% each. The line has a reactance of 8% on a base of 20 MVA ; 66 kV . The motor is drawing 10 MW at 0.8 leading P.F and a terminal voltage 11 kV . When a symmetrical fault occurs at the motor current. Also determine the fault current.



Solution:

Single line diagram:



→ Prefault eq. circuit

- Prefault Voltage:

$$V_0 = \frac{11}{12.66} \angle 0 = 0.8688 \angle 0 \text{ p.u.}$$

Example 6.6 ٤٤

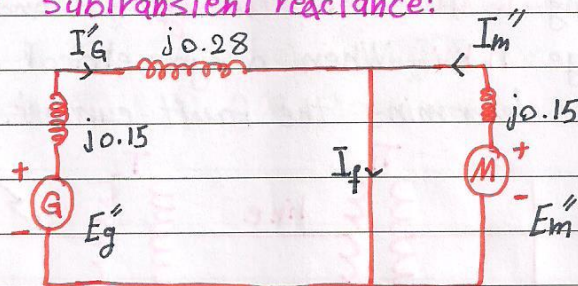
$$\text{Load} = \frac{10}{20} = 0.50 \text{ P.u}$$

- Prefault current:

* Note: $P = IV \cos \phi$

$$I_0 = \frac{0.50}{0.8688 * 0.8} \angle 36.87^\circ = 0.7194 \angle 36.87^\circ \text{ P.u.}$$

- Voltage behind subtransient reactance:



eq. circuit during fault

$$E_g'' = V_0 + j(0.15 + 0.10 + 0.08 + 0.10) * I_0 = 0.7266 \angle 19.9^\circ \text{ P.u}$$

$$E_m'' = V_0 - j0.15 * 0.7194 \angle 36.87^\circ = 0.9374 \angle -5.28^\circ \text{ P.u}$$

$$I_g'' = \frac{0.7266 \angle 19.9^\circ}{j(0.28 + 0.15)} \Rightarrow I_g'' = 1.689 \angle -70.1^\circ \text{ P.u.}$$

$$I_m'' = \frac{0.9374 \angle -5.28^\circ}{j0.15} = 6.25 \angle -95.28^\circ \text{ P.u}$$

$$I_f = I_g'' + I_m'' = -j7.811 \text{ P.u}$$

Example

المثال الثاني

base current [Generator and Motor]:

$$I_b = \frac{20 \times 10^3}{\sqrt{3} \times 12.66} = 912,085 \text{ A}$$

$$I_g'' = 912,085 \times 1.689 \angle -70.1 \\ = 1540.5 \angle -70.1 \text{ A}$$

$$I_m'' = 912,085 \times 6.25 \angle -95.28 \\ = 5700.15 \angle -95.28^\circ$$

$$I_f = I_g'' + I_m'' = 7124.3 \angle -90 \text{ A}$$

* حل نفس السؤال على نظرية (Thevenin).

Rated MVA interrupting capacity of a CB

*Rated MVA interrupting capacity of a CB:

-The circuit breakers rating requires the computation of rated momentary current and rated symmetrical interrupting current. Computation of symmetrical short circuit current requires subtransient reactances for symmetrical machine.

-The interrupting current of a CB is inversely proportional to the operating voltage over a certain range, i.e.

$$I_{or} = I_r \times \frac{V_r}{V_{or}}$$

Where:

I_{or} : current at operating voltage.

I_r : current at rated voltage.

V_r : rated voltage.

V_{or} : operating voltage.

-Note that operating voltage cannot exceed the max. design value. Also rated interrupting current can't exceed the rated max. interrupting current.

-Three phase rated interrupting MVA capacity a CB is given as:

Example 666..... \rightarrow الأ

$$(MVA)_{\text{rated}} = \sqrt{3} |V_{\text{line}}| \times |I_{\text{line}}|_{\text{ric}}$$

Where:

V_{line} : rated line voltage.

I_{line} : rated interrupting current.

- Three phase SC MVA to be interrupting, where

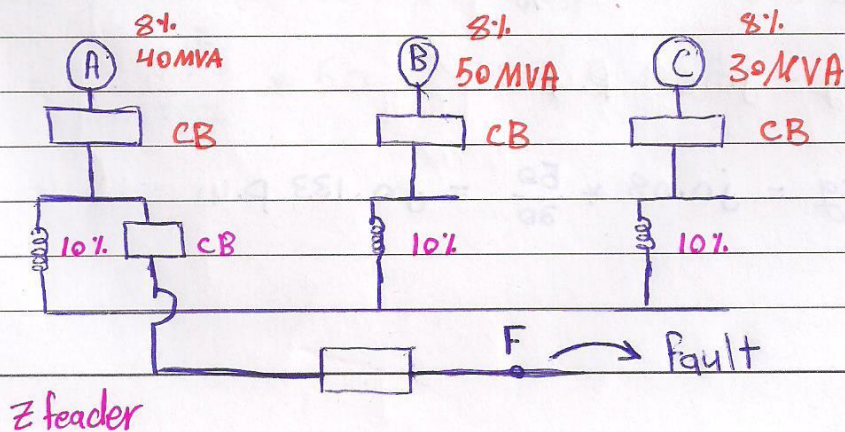
$$SC \text{ MAV} (3\phi) = \sqrt{3} |E_0| |I_{sc}| * (MAV)_{\text{base}}$$

*Where:

E_0 : prefault voltage (kV).

I_{sc} : SC current (KA).

Example: Three 11.2 KV generators are inter connected, by a tie ber through current limiting reactors. A three Phase feeder is supplied from the bus ber of generator A at a line Voltage 11.2 KV. Impedance of feeder is $[0.12 + j0.24]$ ohm per phase. compute the max MVA that can be feel into a symmetrical SC at the for end of the feeder.



Example...

11

* Generator reactance:

$$X_{Ag} = 8\% = 0.08 \text{ pu}; X_{Bg} = X_{Cg} = 0.08 \text{ p.u}$$

* Feeder reactance:

$$X_A = X_B = X_C = 10\% = 0.10 \text{ p.u}$$

* Feeder impedance:

$$Z_{\text{feeder}} = [0.12 + j0.24]$$

→ Choose a base value 50 MVA; 11.2 kV

$$\text{base Impedance} = \frac{(11.2)^2}{50} = 2.5088 \Omega$$

$$Z_{\text{feeder}} (\text{p.u}) = \frac{(0.12 + j0.24)}{2.5088} = 10.07478 + j0.0956 \Omega$$

$$X_{Ag} = j0.08 * \frac{50}{40} = j0.10 \text{ p.u}$$

$$X_{Bg} = j0.08 \text{ p.u}$$

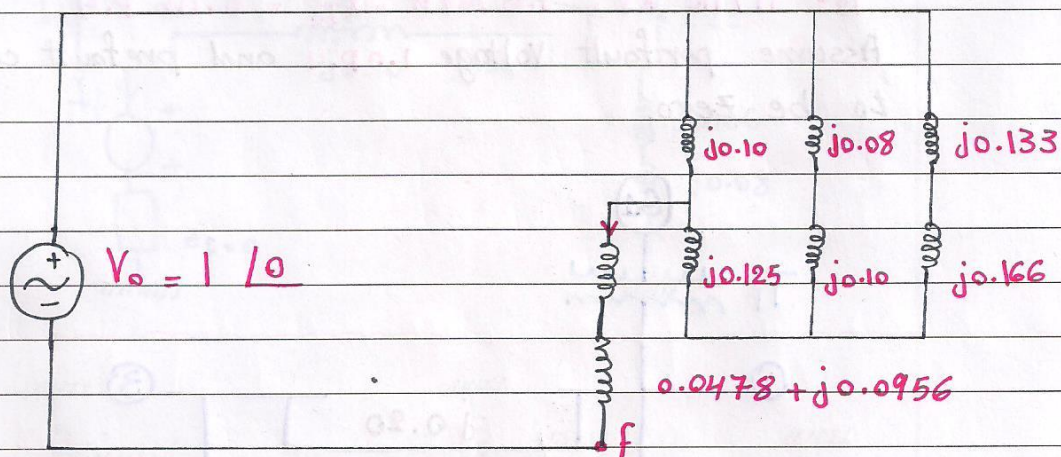
$$X_{Cg} = j0.08 * \frac{50}{30} = j0.133 \text{ p.u}$$

Example 6.66

$$X_A = j0.10 \times \frac{50}{40} = j0.125 \text{ p.u.}$$

$$X_B = j0.10 \text{ p.u.}$$

$$X_C = j0.10 \times \frac{50}{30} = j0.166 \text{ p.u.}$$



$$Z_{eq} = 0.1727 \angle 73.94^\circ$$

$$MVA = |V_o| |I_f| * (MVA)_{base} \Rightarrow |V_o| \frac{|V_o|}{|Z|} * (MVA)_{base}$$

$$MVA = \frac{(1)^2}{0.1727} * 50 = 289.5 \text{ MVA}$$

Example

26/4/2016

Example: A 4 bus sample power system. Perform the short circuit analysis for a three phase solid fault on bus 4.

Data are given below:

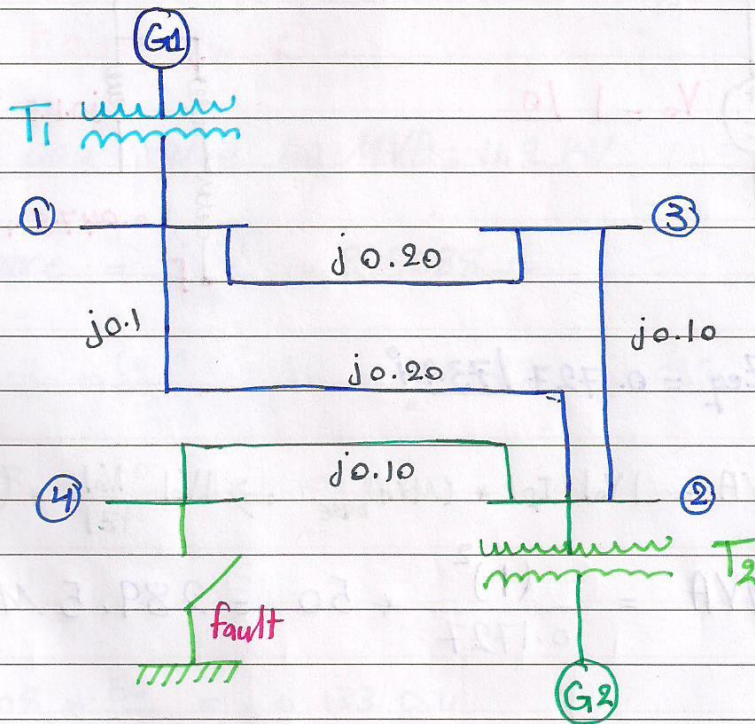
G_1 : 11.2 KV ; 100 MVA ; $X'_{g1} = 0.08$ P.u

G_2 : 11.2 KV ; 100 MVA ; $X'_{g2} = 0.08$ P.u

T_1 : 11/110 KV ; 100 MVA ; $X'_{T1} = 0.06$ P.u

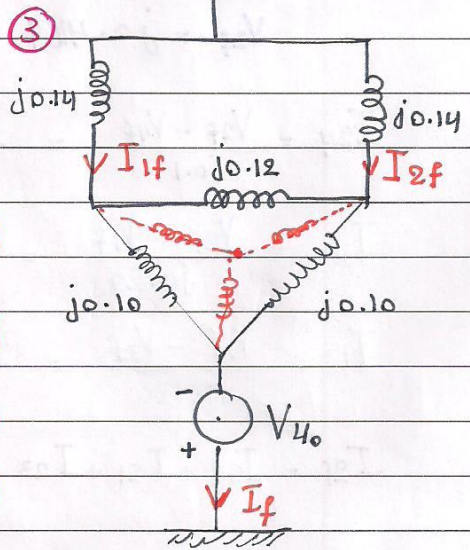
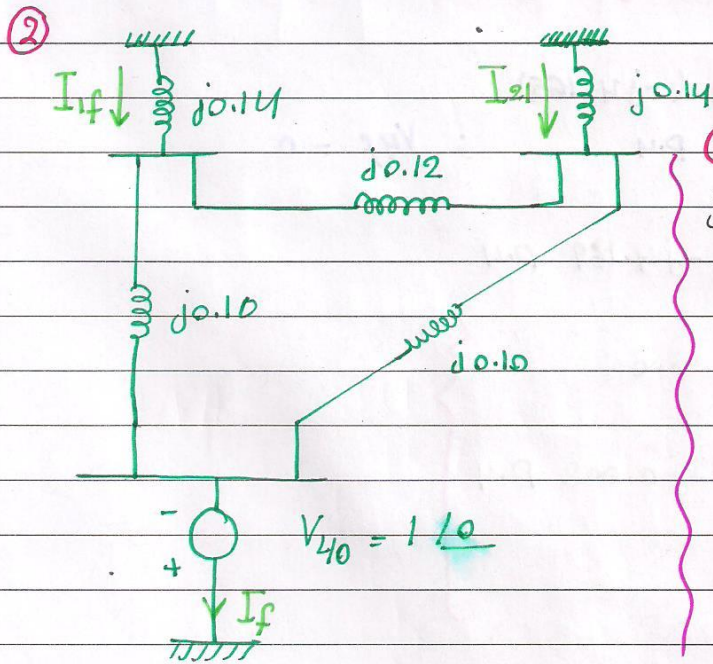
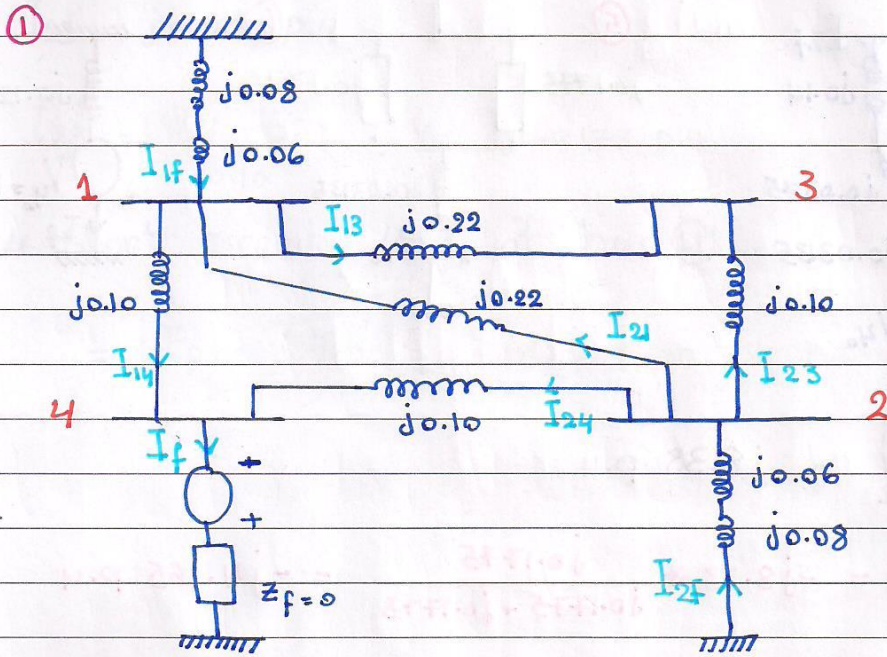
T_2 : 11/110 KV ; 100 MVA ; $X'_{T2} = 0.06$ P.u

Assume per fault Voltage 1.0 P.u and pre fault current to be zero.



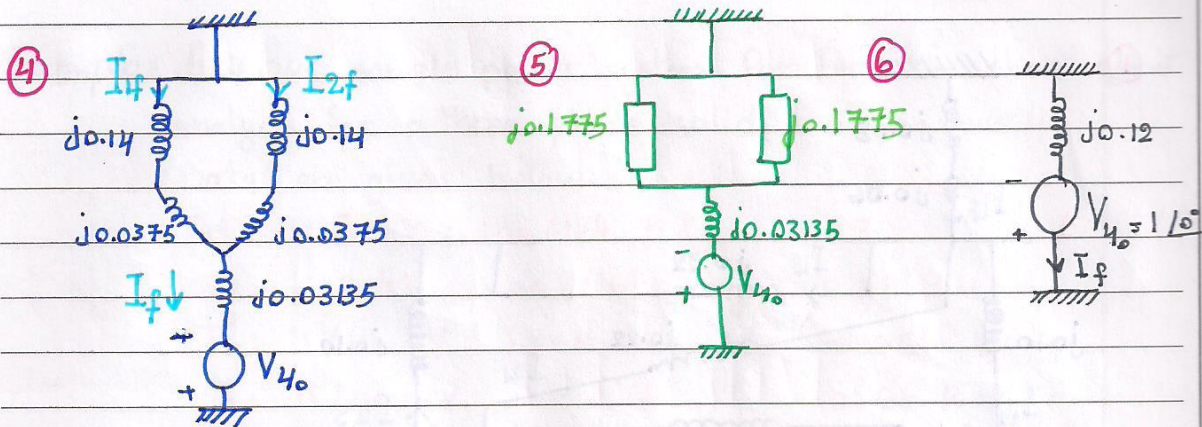
Example 6.6

التاريخ 26/4/2016



Example 6.6

26/4/2016



$$I_{3f} = \frac{1 \angle 0^\circ}{j0.12} = -j8.33 \text{ p.u.}$$

$$I_{1f} = I_{2f} = -j8.33 * \frac{j0.1775}{j0.1775 + j0.1775} = -j4.165 \text{ p.u.}$$

$$\text{or } \frac{E_{g1} - V_{1f}}{j0.14} = I_{1f} = -j4.165 \text{ p.u.}$$

$$1 - V_{2f} = j0.14 (-j4.165)$$

$$V_{2f} = j0.4169 \text{ p.u.} \quad ; \quad V_{4f} = 0$$

$$I_{24} = \frac{V_{2f} - V_{4f}}{j0.1} = -j4.169 \text{ p.u.}$$

$$I_{21} = \frac{V_{2f} - V_{1f}}{j0.20} = 0.0$$

$$I_{23} = \frac{V_{1f} - V_{3f}}{j0.20} = -j0.002 \text{ p.u.}$$

$$I_{2f} = I_{24} + I_{21} + I_{23}$$

Example 66

26/4/2016

$$I_{23} = j0.004 \text{ p.u.}$$

$$I_{23} = \frac{V_{2f} - V_{3f}}{j0.10} \Rightarrow V_{3f} = 0.4173 \text{ p.u.}$$

Short circuit MVA at bus (4)

$$= |I_f| * (MVA)_{base} = 8.33 * 100 = 833 \text{ MVA}$$

Power Flow Analysis

28/4/2016

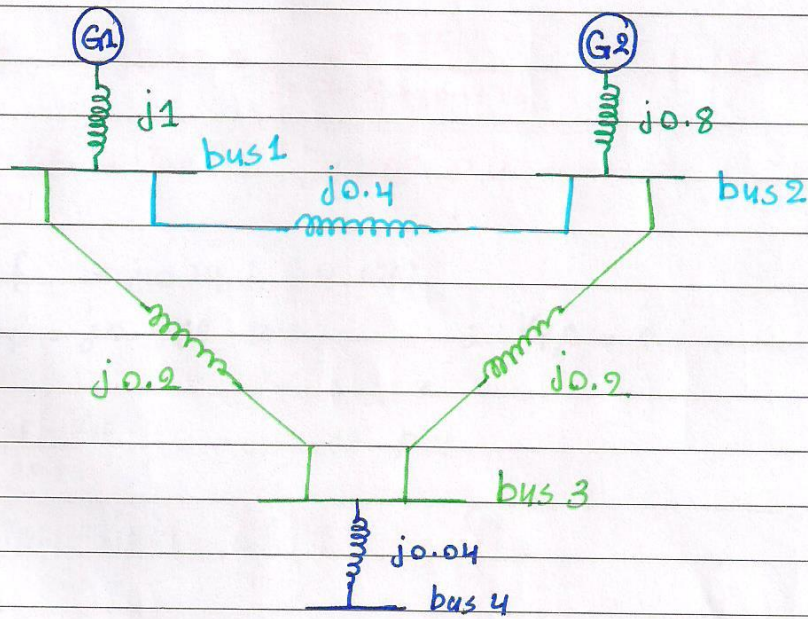
الحفيس

* Power Flow Analysis [P.F.L]:

The power flow is the backbone of the power system operation, analysis and design, it is necessary for planning, operation, economic scheduling and exchange power between utilities.

$$[I] = [Y][V]$$

* bus Admittance Matrix or Y_{bus} :

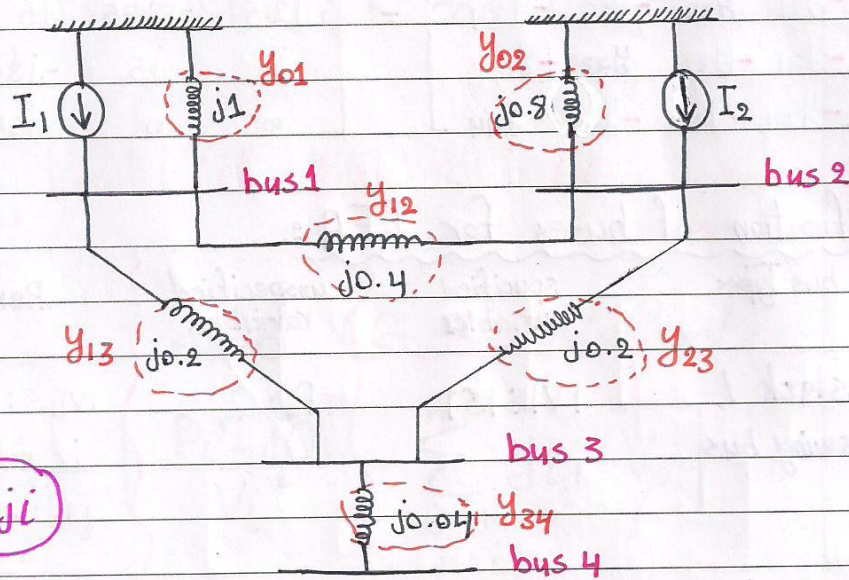


- Formulate the bus admittance for the network
shunt admittance are ignored.

Power flow analysis

28/4/2016 الخميس

* تحويل مصدر الفولتية إلى مصدر تيار



$y_{ij} = y_{ji}$

$y_{11} = y_{01} + y_{12} + y_{13}$

$y_{22} = y_{02} + y_{21} + y_{23}$

$y_{33} = y_{31} + y_{32} + y_{34}$

$y_{44} = y_{43}$

$y_{01} = \frac{1}{j1} = -j1$

$y_{23} = y_{32} = \frac{1}{j0.2} = -j5$

$y_{02} = \frac{1}{j0.8} = -j1.25$

$y_{34} = y_{43} = \frac{1}{j0.04} = -j25$

$y_{12} = y_{21} = \frac{1}{j0.4} = -j2.5$

$y_{13} = y_{31} = \frac{1}{j0.2} = -j5$

Classification of buses الخفيس 281.412016

y_{11}	$-y_{12}$	$-y_{13}$	$-y_{14}$	$-j8.5$	$j2.5$	$j5$	0
$-y_{21}$	y_{22}	$-y_{23}$	$-y_{24}$	$j2.5$	$-j8.75$	$j5$	0
$-y_{31}$	$-y_{32}$	y_{33}	$-y_{34}$	$j5$	$j5$	$-j85$	$j25$
$-y_{41}$	$-y_{42}$	$-y_{43}$	y_{44}	0	0	$j25$	$-j25$

* Classification of buses for LFA:

No.	bus Types	specified variables	unspecified variables	Remarks
1.	slack / swing bus	$ V ; S $	$P_g; Q_g$	$ V , S $ are assumed if not specified as 1.0 and 0°
2.	Generator / PV bus	$P_g; V $	$Q_g; \delta$	A generator is present at the machine bus
3.	load / PQ bus	$P_g; Q_g$	$ V ; \delta$	About 80% buses are of PQ type.
4.	Voltage controlled bus	$P_g; Q_g; V $	$\delta; a$	"a" is the % tap change in tap-changing transformer.

Power flow equation 3/5/2016 الثلاثاء

$$P_i = \sum_{j=1}^n |V_i| |V_j| |y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$i = 1, 2, \dots, n$

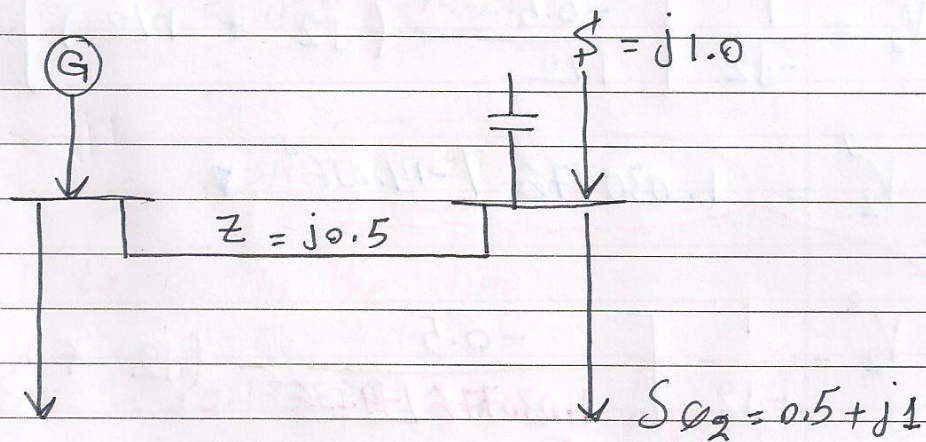
$$Q_i = \sum_{j=1}^n |V_i| |V_j| |y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

power flow eq

* Gauss - Seidel (G.S) Method:

$$V_i = \frac{1}{y_{ij}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^n y_{ij} V_j \right] ; i = 2, 3, 4, \dots$$

Example: - obtain the voltage at bus 2 For the sample system using G.S method if $V_1 = 1 \angle 0$ p.u



الثلاثاء 3/5/2015

Solution:

$$S_2 = j1 - (0.5 + j1) = -0.5 \text{ p.u}$$

$$y = \frac{1}{z} = \frac{1}{j0.5} = -j2$$

$$y_{\text{bus}} = \begin{bmatrix} y_{11} & -y_{12} \\ -y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_i = \frac{1}{y_{ij}} \left[\frac{P_i - jQ_i}{V_i} - \sum_{\substack{j=1 \\ j \neq i}}^n y_{ij} V_j \right]$$

$$V_2 = 1 \angle 0 \text{ p.u}$$

$$V_2^1 = \frac{1}{-j2} \left[\frac{-0.5}{1 \angle 0} - (j2 * 1 \angle 0) \right]$$

$$V_2^1 = 1.030776 \angle -14.036^\circ$$

$$V_2^2 = \frac{1}{-j2} \left[\frac{-0.5}{1.030776 \angle -14.036} - (j2 * 1 \angle 0) \right]$$

3.15/2016... الساتر

$$V_2^3 = 0.970261 \angle -14.931^\circ \text{ p.u.}$$

$$V_2^4 = 0.966237 \angle -14.931^\circ \text{ p.u.}$$

$$V_2^5 = 0.966237 \angle -14.931^\circ \text{ p.u.}$$

* To compute line flow:

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1 \angle 0^\circ - 0.966237 \angle -14.931^\circ}{j0.5}$$

$$= 0.517472 \angle -14.931^\circ$$

$$S_{12} = V_1 I_{12}^* = 1 \angle 0^\circ * 0.517472 \angle 14.931^\circ = 0.5 + j0.133329 \text{ p.u.}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = 0.517472 \angle -14.931^\circ$$

$$S_{21} = V_2 I_{21}^* = 0.517472 \angle 14.931^\circ$$

* The total losses in the line...

$$S_{12} + S_{21} = j0.133329 \text{ p.u.}$$

Encl₀₀₀

Final

گیا ہے

۱۰

8

Symmetrical Fault

8.1 INTRODUCTION

This chapter is devoted to the analysis of symmetrical three-phase fault or balanced fault. This type of fault can be defined as the simultaneous short circuit across all the three phases. This type of fault occurs infrequently, for example, when a mechanical excavator cuts quickly through a whole cable, or when a line, which has been made safe for maintenance by clamping all the three phases to earth is accidentally made alive or when due to slow fault clearance, an earth fault spreads across to the other two phases. This type of fault generally leads to most severe fault current flow against which the system must be protected. Fault studies form an important part of power system analysis and the problem consists of determining bus voltage and line current during faults. The three phase fault information is used to select and set phase relays. Fault studies are used for proper choice of circuit breakers and protective relaying. A power system network comprises synchronous generators, transformers, transmission lines and loads. During fault, loads current can be neglected because voltages dip very low so that current drawn by loads can be neglected in comparison to fault currents. The magnitude of the fault current depends on the internal impedance of the synchronous generator and the impedance of the intervening circuit. We have seen in Chapter-4 that for the purpose of fault studies, generator behaviour can be divided into three different periods: (i) the subtransient period, lasting only for the first few cycles; (ii) the transient period, covering a relatively longer time and (iii) steady state period.

Another important point is that the circuit breakers rated MVA breaking capacity is based on three phase fault MVA. In fact high precision is not necessary when calculating the three phase fault level because circuit breakers are manufactured in standard sizes, e.g., 250, 500, 750 MVA etc. Generally for three phase fault calculation, following assumptions are made:

1. The emfs of all generators are $1 \angle 0^\circ$ pu. This assumption simplify the problem and it means that the voltage is at its nominal value and the system is operating at no load at the time of fault. Since all emfs are equal and in phase, all the generators can be replaced by a single generator.
2. Charging capacitances of the transmission line are ignored.
3. Shunt elements in the transformer model are neglected.

Example 8.1: A synchronous generator and a synchronous motor each rated 20 MVA, 12.66 KV having 15% subtransient reactance are connected through transformers and a line as shown in Fig. 8.1. The transformers are rated 20 MVA, 12.66/66 KV and 66/12.66 KV with leakage reactance of 10% each. The line has a reactance of 8% on a base of 20 MVA, 66 KV. The motor

is drawing 10 MW at 0.80 leading power factor and a terminal voltage 11 KV when a symmetrical three-phase fault occurs at the motor terminals. Determine the generator and motor currents. Also determine the fault current.

Solution:

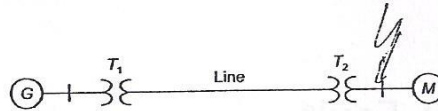


Fig. 8.1: Single line diagram.

All reactances are given on a base of 20 MVA and appropriate voltages.

Prefault voltage $V_0 = \frac{11}{12.66} \angle 0^\circ = 0.8688 \angle 0^\circ \text{ pu.}$

Load = 10 MW, 0.80 power factor (leading) = $\frac{10}{20} = 0.50 \text{ pu.}$

Prefault current $I_0 = \frac{0.50}{0.8688 \times 0.80} \angle 36.87^\circ$

$\therefore I_0 = 0.7194 \angle 36.87^\circ \text{ pu}$

Prefault equivalent circuit is shown in Fig. 8.2

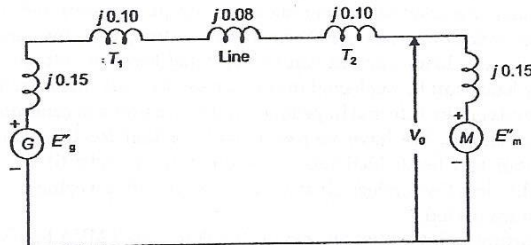


Fig. 8.2: Prefault equivalent circuit of Example 8.1.

Equivalent circuit during fault is shown in Fig. 8.3.

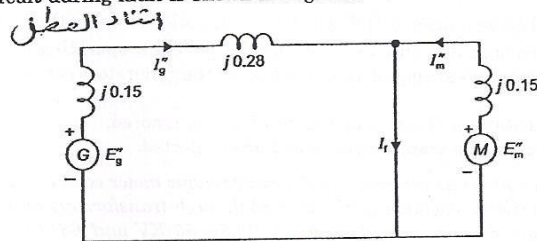


Fig. 8.3: Equivalent circuit during fault.

From Fig. 8.2, voltage behind subtransient reactance (generator)

$$E_g'' = V_0 + j(0.15 + 0.10 + 0.08 + 0.10) \times I_0$$

$$E_g'' = 0.8688 \angle 0^\circ + j0.43 \times 0.7194 \angle 36.87^\circ$$

$$\therefore E_g'' = 0.7266 \angle 19.9^\circ \text{ pu} \quad \checkmark$$

Similarly,

$$E_m'' = 0.8688 \angle 0^\circ - j0.15 \times 0.7194 \angle 36.87^\circ$$

$$\therefore E_m'' = 0.9374 \angle -5.28^\circ \text{ pu} \quad \checkmark$$

From Fig. 8.3,

$$I_g'' = \frac{E_g''}{j(0.15 + 0.28)} = \frac{0.7266 \angle 19.9^\circ}{0.43 \angle 90^\circ}$$

$$\therefore I_g'' = 1.689 \angle -70.1^\circ \text{ pu} \quad \checkmark$$

$$\therefore I_g'' = (0.575 - j1.588) \text{ pu}$$

$$I_m'' = \frac{E_m''}{j0.15} = \frac{0.9374 \angle -5.28^\circ}{0.15 \angle 90^\circ}$$

$$\therefore I_m'' = 6.25 \angle -95.28^\circ \text{ pu} \quad \checkmark$$

$$\therefore I_m'' = (-0.575 - j6.223) \text{ pu}$$

Fault current

$$I_f = I_g'' + I_m'' = 0.575 - j1.588 - 0.575 - j6.223$$

$$\therefore I_f = -j7.811 \text{ pu} \quad \checkmark$$

Base current (generator and motor)

$$I_B = \frac{20 \times 1000}{\sqrt{3} \times 12.66} = 912.085 \text{ Amp.} \quad \checkmark$$

$$\therefore I_g'' = 912.085 \times 1.689 \angle -70.1^\circ = 1540.5 \angle -70.1^\circ \text{ Amp.} \quad \checkmark$$

$$\therefore I_m'' = 912.085 \times 6.25 \angle -95.28^\circ = 5700.5 \angle -95.28^\circ \text{ Amp.} \quad \checkmark$$

$$\therefore I_f = 912.085 \times (-j7.811) = 7124.3 \angle -90^\circ \text{ Amp.}$$

Example 8.2: Solve Ex-8.1 using Thevenin's Theorem.

Solution: The detailed derivation for this is given in Chapter-4, Section-4.8.2.

Figure 8.4 shows the Thevenin's equivalent of the system feeding the fault impedance.

$$X'' = j(0.1 + 0.08 + 0.01) = j0.28$$

$$X_{dg}'' = j0.15, X_{dm}'' = j0.15$$

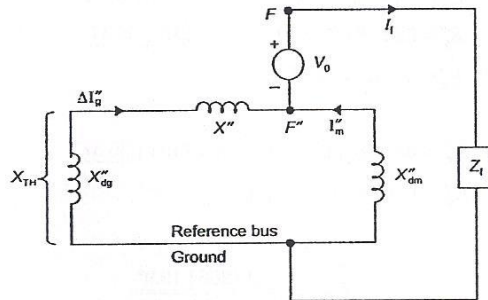


Fig. 8.4: Thevenin equivalent circuit of Example-8.1.

$$X_{dg}'' + X'' = j(0.15 + 0.28) = j0.43$$

$$\therefore X_{TH} = \frac{(X_{dg}'' + X'')(X_{dg}'')}{(X_{dg}'' + X'' + X_{dm}'')} = \frac{j0.43 \times j0.15}{j(0.43 + 0.15)}$$

$$X_{TH} = j0.1112 \text{ pu}$$

$$\therefore I_f = \frac{V_0}{(Z_f + X_{TH})} = \frac{0.8688 \angle 0^\circ}{j0.1112} \text{ [since } z_f = z_0]$$

$$\therefore I_j = -j7.811 \text{ pu.}$$

Change in generator current

$$\Delta I_g'' = I_f \times \frac{X_{dm}''}{(X_{dg}'' + X'' + X_{dm}'')}$$

$$\therefore \Delta I_g'' = -j7.811 \times \frac{j0.15}{j(0.15 + 0.28 + 0.15)}$$

$$\Delta I_g'' = -j2.02 \text{ pu}$$

Similarly,

$$\Delta I_m'' = -j7.811 \times \frac{j(0.15 + 0.28)}{j0.58}$$

$$\Delta I_m'' = -j5.79 \text{ pu}$$

Therefore,

$$I_g'' = \Delta I_g'' + I_0 = -j2.02 + 0.7194 \angle 36.87^\circ$$

$$\therefore I_g'' = (0.575 - j1.589) \text{ pu}$$

$$I_m'' = \Delta I_m'' - I_0 = -j5.79 - 0.7194 \angle 36.87^\circ$$

$$\therefore I_m'' = (-0.575 - j6.221) \text{ pu.}$$

8.2 RATED MVA INTERRUPTING CAPACITY OF A CIRCUIT BREAKER

The circuit breakers rating requires the computation of rated momentary current and rated symmetrical interrupting current computation of symmetrical short circuit current requires subtransient reactances for synchronous machines. RMS value of momentary current is then computed by multiplying the symmetrical momentary current by a factor of 1.60 to consider the presence of DC off-set current.

The interrupting current of a circuit breaker is inversely proportional to the operating voltage over a certain range, i.e.,

$$I_{ov} = I_r \times \frac{V_r}{V_{ov}} \quad \dots(8.1)$$

Where

- I_{ov} = current at operating voltage
- I_r = current at rated voltage
- V_r = rated voltage
- V_{ov} = operating voltage

Note that operating voltage cannot exceed the maximum design value. Also rated interrupting current cannot exceed the rated maximum interrupting current.

Therefore, three phase rated interrupting MVA capacity of a circuit breaker is given as

$$(MVA)_{rated-3\phi} = \sqrt{3} |V_{line}|_r \times |I_{line}|_{ric} \quad \dots(8.2)$$

where

- $|V_{line}|_r$ = rated line voltage (kV)
- $|I_{line}|_{ric}$ = rated interrupting current (KA)

Thus, three phase short circuit MVA to be interrupted, where

$$SC \text{ MVA } (3\phi) = \sqrt{3} |E_o| |I_{sc}| \times (MVA)_{Base} \quad \dots(8.3)$$

where

- $|E_o|$ = pre-fault voltage (kV)
- $|I_{sc}|$ = short circuit current (KA)

Note that $MVA_{rated-3\phi}$ is to be more than or equal to the SC MVA (3 ϕ) required to be interrupted. A three phase fault which is very rare gives the highest short circuit MVA and a circuit breaker must be capable of interrupting it.

Example 8.3: Three 11.2 KV generators are interconnected as shown in Fig. 8.5 by a tie-bar through current limiting reactors. A three phase feeder is supplied from the bus bar of generator A at a line voltage 11.2 KV. Impedance of the feeder is (0.12 + j0.24) ohm per phase. Compute the maximum MVA that can be fed into a symmetrical short circuit at the far end of the feeder.

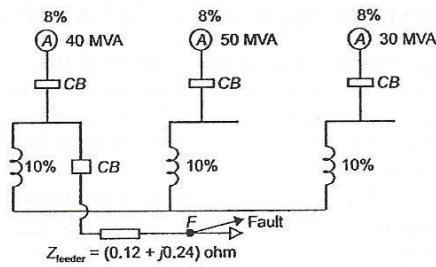


Fig. 8.5: Circuit diagram of Example 8.3.

Solution: Generator reactance

$$x_{Ag} = 8\% = 0.08 \text{ pu}, x_{Bg} = x_{Cg} = 0.08 \text{ pu}$$

Reactor reactance

$$x_A = x_B = x_C = 10\% = 0.10 \text{ pu}$$

Feeder impedance

$$Z_{feeder} = (0.12 + j0.24) \text{ ohm.}$$

choose a base 50 MVA, 11.2 KV

$$\text{Base impedance } Z_B = \frac{(11.2)^2}{50} \text{ ohm} = 2.5088 \text{ ohm}$$

$$\therefore Z_{feeder} (\text{pu}) = \frac{Z_{feeder} (\text{ohm})}{Z_B} = \frac{(0.12 + j0.24)}{2.5088}$$

$$\therefore Z_{feeder} (\text{pu}) = (0.0478 + j0.0956) \text{ pu.}$$

$$x_{Ag} = j0.08 \times \frac{50}{40} = j0.10 \text{ pu}$$

$$x_{Bg} = j0.08 \text{ pu}$$

$$x_{Cg} = j0.08 \times \frac{50}{30} = j0.133 \text{ pu}$$

$$x_A = j0.10 \times \frac{50}{40} = j0.125 \text{ pu}$$

$$x_B = j0.10 \text{ pu}$$

$$x_C = j0.10 \times \frac{50}{30} = j0.166 \text{ pu}$$

Assume a zero pre-fault current (i.e., no load pre-fault condition). Circuit model for the fault calculation is given in Fig. 8.5(a).

$$Z = 0.0478 + j0.0956 + j \frac{0.10 \times 0.2375}{0.3375}$$

$$\therefore Z = 0.1727 \angle 73.94^\circ \text{ pu.}$$

$$\text{Short circuit MVA} = |V_0| |I_f| \times (\text{MVA})_{\text{Base}}$$

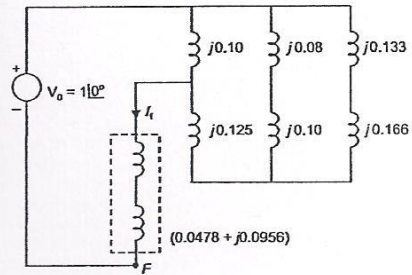


Fig. 8.5(a)

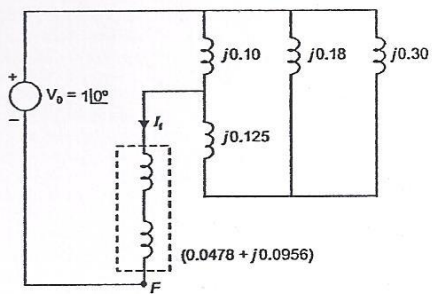


Fig. 8.5(b)

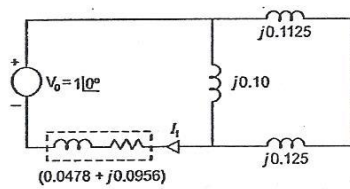


Fig. 8.5(c)

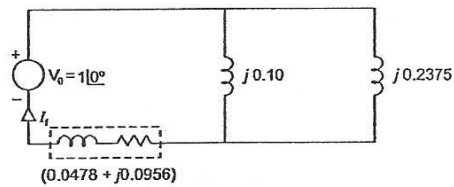


Fig. 8.5(d)

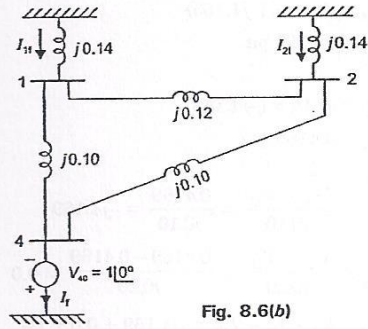


Fig. 8.6(b)

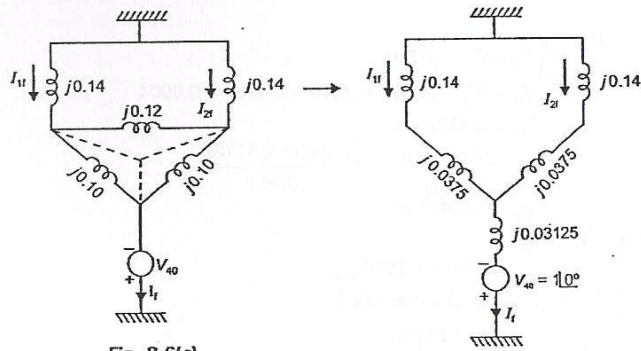


Fig. 8.6(c)

Fig. 8.6(d)

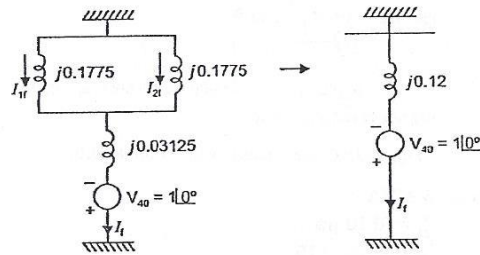


Fig. 8.6(e)

Fig. 8.6(f)

$$\begin{aligned} \therefore 1 - V_{1f} &= j0.14 \times (-j4.165) \\ \therefore V_{1f} &= 0.4169 \text{ pu.} \end{aligned}$$

Similarly

$$\begin{aligned} \therefore 1 - V_{2f} &= j0.14 \times (-j4.165) \\ \therefore V_{2f} &= 0.4169 \text{ pu.} \\ V_{4f} &= 0.0 \\ I_{24} &= \frac{V_{2f} - V_{4f}}{j0.10} = \frac{0.4169}{j0.10} = -j4.169 \\ I_{21} &= \frac{V_{2f} - V_{1f}}{j0.20} = \frac{0.4169 - 0.4169}{j0.20} = 0.0 \\ I_{2f} &= I_{24} + I_{21} + I_{23} = -j4.169 + 0.0 + I_{23} \\ \therefore -j4.165 &= -j4.169 + I_{23} \\ \therefore I_{23} &= j0.004 \text{ pu.} \end{aligned}$$

Now

$$\begin{aligned} \frac{V_{2f} - V_{3f}}{j0.10} &= I_{23} = j0.004 \\ \therefore V_{3f} &= V_{2f} - j0.004 \times j0.10 = 0.4169 + 0.0004 \\ \therefore V_{3f} &= 0.4173 \text{ pu.} \\ I_{13} &= \frac{V_{1f} - V_{3f}}{Z_{12}} = \frac{(0.4169 - 0.4173)}{j0.20} \\ \therefore I_{13} &= -j0.002 \text{ pu} \end{aligned}$$

SC MVA at bus 4

$$\begin{aligned} &= |I_f| \times (\text{MVA})_{\text{Base}} \\ &= 8.33 \times 100 \text{ MVA} \\ &= 833 \text{ MVA} \end{aligned}$$

Example 8.5: Two generators G1 and G2 are rated 15 MVA, 11 KV and 10 MVA, 11 KV respectively. The generators are connected to a transformer as shown in Fig. 8.7. Calculate the subtransient current in each generator when a three phase fault occurs on the high voltage side of the transformer.

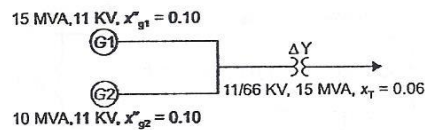


Fig. 8.7: Circuit diagram of Example 8.5.

Solution: Choose a base 15 MVA

$$\begin{aligned} x''_{g1} &= j0.10 \text{ pu} \\ x''_{g2} &= j0.10 \times \frac{15}{10} = j0.15 \text{ pu} \end{aligned}$$

$$x_T = j0.06 \text{ pu}$$

$$I_f = \frac{V_0}{j0.12} = \frac{1}{j0.12} = -j8.33 \text{ pu}$$

$$I_{E1} = \frac{j0.15}{j(0.1+0.15)} \times (-j8.33)$$

$$= -j5.0 \text{ pu}$$

$$I_{E2} = \frac{j0.10}{j(0.1+0.15)} \times (-j8.33) = -j3.33 \text{ pu}$$

Base current

$$I_B = \frac{15 \times 1000}{\sqrt{3} \times 11} = 787.3 \text{ Amp.}$$

$$\therefore I_{E1} = -j5 \times 787.3 = -j3.936 \text{ KA.}$$

$$I_{E2} = -j3.33 \times 787.3 = -j2.621 \text{ KA.}$$

$$I_f = -j8.33 \times 787.3 = -j6.557 \text{ KA.}$$

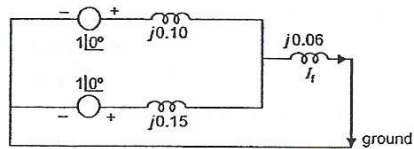


Fig. 8.7(a)

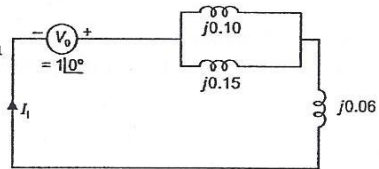


Fig. 8.7(b)

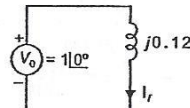


Fig. 8.7(c)

8.3 CURRENT LIMITING REACTORS

The short circuit current is large enough to do considerable damage mechanically and thermally. The interrupting capacities of circuit breakers to handle such current would be very large. To reduce this high fault current, artificial reactances are sometimes connected between bus sections. These current limiting reactors are usually consist of insulated copper strip embedded in concrete formers. This is necessary to withstand the high mechanical forces produced by the current in the neighbouring conductors.

Example 8.6: The estimated short circuit MVA at the bus bars of a generating station-1 is 900 MVA and at another generating station-2 of 600 MVA. Generator voltage at each station is 11.2 KV. The two stations are interconnected by a reactor of reactance 1 ohm per phase. Compute the fault MVA at each station.

Solution:

SC MVA of generating station-1 = 900 MVA

SC MVA of generating station-2 = 600 MVA

Assume base MVA = 100

$$\therefore x_1 = \frac{\text{Base MVA}}{\text{SC MVA}} = \frac{100}{900} = 0.111 \text{ pu}$$

$$\therefore x_2 = \frac{100}{600} = 0.166 \text{ pu}$$

Base current

$$I_B = \frac{100 \times 1000}{\sqrt{3} \times 11.2} = 5154.9 \text{ Amp.}$$

Per unit reactance of reactor

$$x_R = \frac{1 \times 100}{(11.2)^2} = 0.797 \text{ pu}$$

Figure 8.8 shows the pu impedance diagram.

When fault occurs at generating station-1, pu impedance diagram is shown in Fig. 8.8(a)

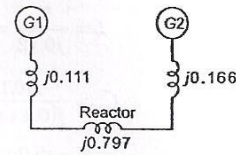


Fig. 8.8: circuit diagram.

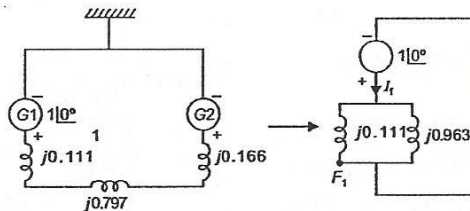


Fig. 8.8(a)

Fig. 8.8(b)

$$x_{eq1} = j \frac{0.111 \times 0.963}{1.074} = j0.0995 \text{ pu}$$

$$I_{f1} = \frac{1}{j0.0995} = -j10.047 \text{ pu}$$

$$\text{SC MVA} = 10.047 \times 100 = 1004.7 \text{ MVA}$$

When fault occurs at generating station-2

$$x_{eq2} = j \frac{0.166 \times 0.908}{1.074} = j0.1403 \text{ pu}$$

$$I_{f2} = \frac{1}{j0.1403} = -j7.125 \text{ pu}$$

$$\text{SC MVA} = 7.125 \times 100 = 712.5 \text{ MVA.}$$

Example 8.7: A 50 MVA generator with a reactance of 0.10 pu is connected to a bus-bar. A 25 MVA transformer with a reactance of 0.05 pu is also connected through a bus-bar reactor of 0.10 pu to the same bus-bar. Both these reactances are based on 25 MVA rating. If a feeder taken out from the bus-bar through a circuit breaker develops a line to ground fault, what should be the rating of circuit breaker?

Solution: Circuit connection is shown in Fig. 8.9.

Set base MVA = 50

$$x_g = j0.10 \text{ pu}$$

$$x_R = j0.10 \times \frac{50}{25} = j0.20 \text{ pu}$$

$$x_T = j0.05 \times \frac{50}{25} = j0.10 \text{ pu}$$

$$\therefore x_{eq} = \frac{x_g (x_T + x_R)}{(x_g + x_T + x_R)} = j \frac{0.10 \times (0.10 + 0.20)}{(0.10 + 0.20 + 0.10)}$$

$$\therefore x_{eq} = j0.075 \text{ pu}$$

$$\begin{aligned} \text{Therefore SC MVA} &= \frac{\text{Base MVA}}{x_{eq}} = \frac{50}{0.075} \\ &= 667 \text{ MVA. Ans.} \end{aligned}$$

Example 8.8: Determine the ohmic value of the current limiting reactor per phase external to a 30 MVA, 11 KV, 50 Hz, three phase synchronous generator which can limit the current on short circuit to 6 times the full load current. The reactance of the synchronous generator is 0.06 pu.

Solution: Given that

$$\frac{\text{Full load current}}{\text{Short circuit current}} = \frac{1}{6}$$

$$x_g = j0.06 \text{ pu}$$

External reactance required per phase

$$= j \left(\frac{1}{6} - 0.06 \right) = j0.1066 \text{ pu.}$$

Full load current,

$$I_n = \frac{30 \times 1000}{\sqrt{3} \times 11} = 1574.6 \text{ Amp.}$$

$$\text{Per unit reactance} = \frac{I x_R}{V}$$

$$\therefore 0.1066 = \frac{I_n \times X_R}{11 \times 1000 / \sqrt{3}}$$

$$\therefore x_R = 0.43 \text{ ohm. Ans.}$$

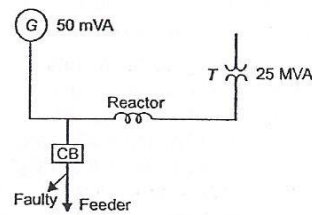


Fig. 8.9: Circuit diagram of Example-8.7.

Example 8.9: Two generating stations are connected together through transformers and a transmission line as shown in Fig. 8.10. If a three phase fault occurs as shown in Fig. 8.10, calculate the fault current.

- G1 : 11 KV, 40 MVA, 15%
- G2 : 11 KV, 20 MVA, 10%
- G3 : 11 KV, 20 MVA, 10%
- T₁ : 40 MVA, 11/66 KV, 15%
- T₂ : 40 MVA, 66/11 KV, 15%
- T₃ : 5 MVA, 11/6.6 KV, 8%
- Line reactance = 40 ohm.

Solution:

Set Base MVA = 40, Base Voltage = 11 KV

$$\therefore x_{g1} = j0.15 \text{ pu,}$$

$$x_{g2} = j \frac{40}{20} \times 0.10 = j0.20 \text{ pu}$$

$$x_{g3} = j0.10 \times \frac{40}{20} = j0.20 \text{ pu}$$

$$x_{T1} = j0.15 \text{ pu}$$

$$x_{T2} = j0.15 \text{ pu}$$

$$x_{T3} = j0.08 \times \frac{40}{5} = j0.64 \text{ pu}$$

$$x_{\text{line}} = j40 \times \frac{40}{(66)^2} = j0.367 \text{ pu.}$$

Circuit model for fault calculation is shown in Fig. 8.10 (a).

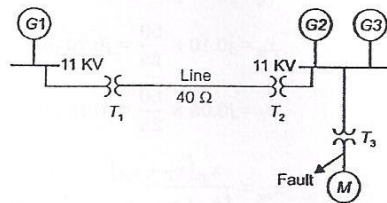


Fig. 8.10: Circuit diagram of Example 8.9.

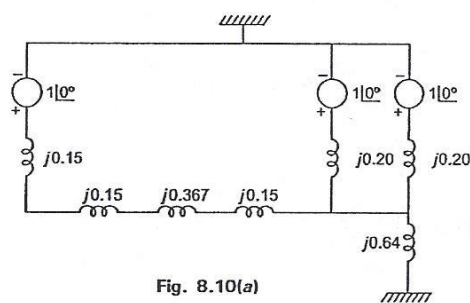


Fig. 8.10(a)

$$\therefore I_f = \frac{1 \angle 0^\circ}{j0.729} = -j1.37 \text{ pu}$$

$$\begin{aligned} \text{Base current } I_B &= \frac{40 \times 1000}{\sqrt{3} \times 11} \\ &= 2099.45 \text{ Amp} \end{aligned}$$

$$\begin{aligned} \therefore |I_f| &= 1.37 \times 2099.45 \\ &= 2.876 \text{ KA. Ans.} \end{aligned}$$

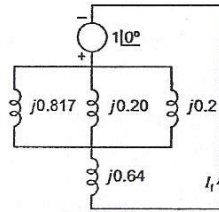


Fig. 8.10(b)

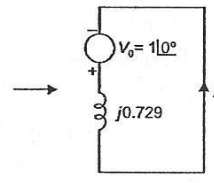


Fig. 8.10(c)

Example 8.10: A generating station consists of two 100 MVA generators with 6% reactance each and one 150 MVA generator with 8% reactance as shown in Fig. 8.11. These generators are connected to a common bus bar from which loads are taken through a number of 50 MVA, step up transformers each having 5% reactance. Compute the rating of circuit breaker on (i) low voltage side and (ii) on high voltage side.

Solution:

Set base power = 150 MVA.

$$x_{g1} = x_{g2} = j0.06 \times \frac{150}{100} = j0.09 \text{ pu}$$

$$x_{g3} = j0.08 \text{ pu, } x_T = j0.05 \times \frac{150}{100} = j0.15 \text{ pu.}$$

(i) If the fault occurs on low voltage side current will be restricted by the reactance of three generators in parallel.

$$\therefore \frac{1}{x_{eq}} = \frac{1}{j0.09} + \frac{1}{j0.09} + \frac{1}{j0.08}$$

$$\therefore x_{eq} = j0.0288 \text{ pu.}$$

$$\text{SC MVA on low voltage side} = \frac{150}{0.0288} = 5208 \text{ MVA.}$$

(ii) On the high voltage side,

$$x_{eq} = j(0.0288 + 0.15) = j0.1788 \text{ pu}$$

$$\text{SC MVA} = \frac{150}{0.1788} = 840 \text{ MVA.}$$

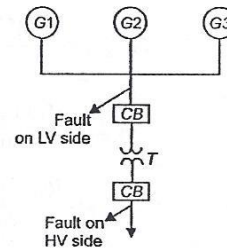


Fig. 8.11: Circuit diagram of Example 8.10.

Example 8.11: A radial power system network is shown in Fig. 8.12, a three phase balanced fault occurs at F. Determine the fault current and the line voltage at 11.8 KV bus under fault condition.

Solution:

Let Base MVA = 12

Base Voltage = 11.8 KV.

$$x_{g1} = j0.12 \text{ pu}, \quad x_{g2} = j0.15 \text{ pu}$$

$$x_{T1} = j0.12 \text{ pu},$$

$$x_{T2} = j0.08 \times \frac{12}{3} = j0.32 \text{ pu}$$

Base voltage for line-1 is 33 KV.

Base voltage for line-2 is 6.6 KV.

$$Z_{B, \text{line-1}} = \frac{(33)^2}{12} = 90.75 \text{ ohm.}$$

$$Z_{B, \text{line-2}} = \frac{(6.6)^2}{12} = 3.63 \text{ ohm.}$$

$$\therefore Z_{\text{line-1}} = \frac{(9.45 + j12.6)}{90.75} = (0.104 + j0.139) \text{ pu}$$

$$Z_{\text{line-2}} = \frac{(0.54 + j0.40)}{3.63} = (0.148 + j0.11) \text{ pu}$$

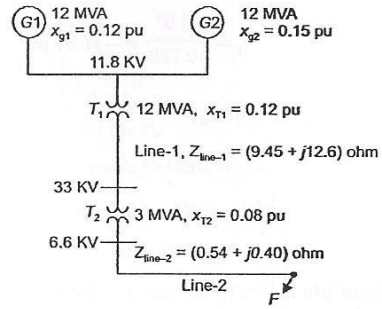


Fig. 8.12: Radial power system network.

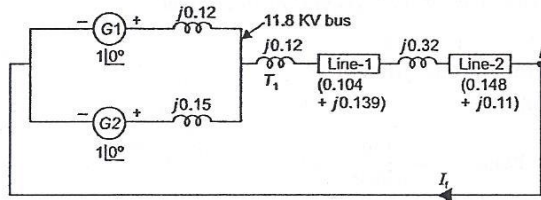


Fig. 8.12(a)

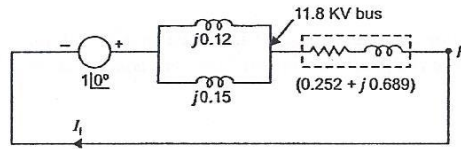


Fig. 8.12(b)

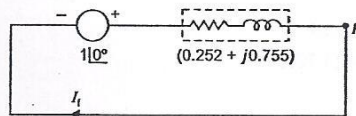


Fig. 8.12(c)

$$\text{Base current } I_B = \frac{12 \times 1000}{\sqrt{3} \times 6.6} = 1049.7 \text{ Amp.}$$

$$\text{Now } I_f = \frac{1 \angle 0^\circ}{(0.252 + j0.755)} = 1.256 \angle -71.5^\circ \text{ pu}$$

$$\therefore I_f = 1.256 \angle -71.5^\circ \times 1049.7$$

$$\therefore I_f = 1318.4 \angle -71.5^\circ \text{ Amp.}$$

$$\begin{aligned} \text{Total impedance between F and 11.8 KV bus} \\ = (0.252 + j0.689) \text{ pu} \end{aligned}$$

$$\begin{aligned} \text{Voltage at 11.8 KV bus} \\ = 1.256 \angle -71.5^\circ \times (0.252 + j0.689) \\ = 0.921 \angle -1.6^\circ \text{ pu} \\ = 0.921 \angle -1.6^\circ \times 11.8 \text{ KV} \\ = 10.86 \angle -1.6^\circ \text{ KV. Ans.} \end{aligned}$$

Example 8.12: A 100 MVA, 11 KV generator with $x_g'' = 0.20 \text{ pu}$ is connected through a transformer, and line to a bus bar that supplies three identical motor as shown in Fig. 8.13 and each motor has $x_m'' = 0.20 \text{ pu}$ and $x_m' = 0.25 \text{ pu}$ on a base of 20 MVA, 33 KV. The bus voltage at the motors is 33 KV when a three phase balanced fault occurs at the point F. Calculate

- Subtransient current in the fault.
- Subtransient current in the circuit breaker B.
- Momentary current in the circuit breaker B.
- The current to be interrupted by circuit breaker B in (i) 2 cycles (ii) 3 cycles (iii) 5 cycles (iv) 8 cycles

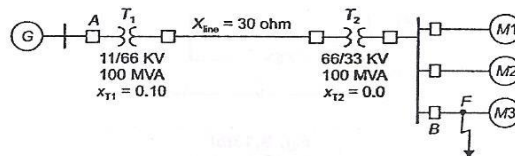


Fig. 8.13: Circuit diagram of Example 8.12.

Solution:

Let Base MVA = 100

Base Voltage = 11 KV.

$$x_g'' = j0.20 \text{ pu.}$$

$$x_m'' = x_{m1}'' = x_{m2}'' = x_{m3}'' = j0.2 \times \frac{100}{20} = j1.0 \text{ pu.}$$

$$x_m' = x_{m1}' = x_{m2}' = x_{m3}' = j0.25 \times \frac{100}{20} = j1.25 \text{ pu.}$$

$$x_{T1} = x_{T2} = j0.10 \text{ pu}$$

$$x_{line} = 30 \times \frac{100}{(66)^2} = j0.688 \text{ pu.}$$

(a) The circuit model of the system for fault calculation is given in Fig. 8.13(a).

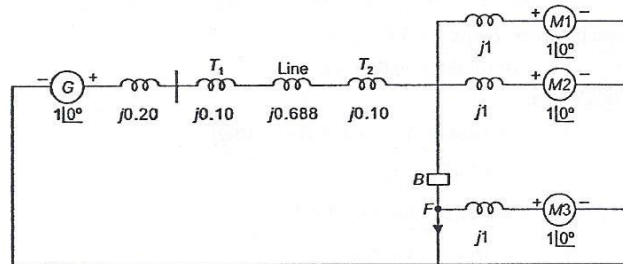


Fig. 8.13(a)

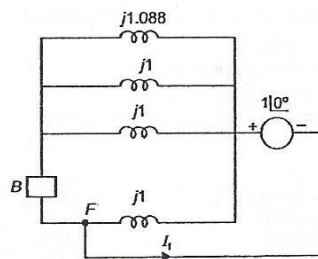


Fig. 8.13(b)

$$\therefore x_{eq} = \frac{j}{3.919} = j0.255$$

$$\therefore I_f = \frac{1 \angle 0^\circ}{j0.255} = -j3.919 \text{ pu.}$$

Base current for 33 KV circuit

$$I_B = \frac{100 \times 1000}{\sqrt{3} \times 33} = 1.75 \text{ KA.}$$

$$\therefore |I_f| = 3.919 \times 1.75 = 6.85 \text{ KA.}$$

(b) Current through circuit breaker B is,

$$I_{fB} = \frac{2}{j1} + \frac{1}{j1.088} = -j2.919 \text{ pu}$$

$$\therefore |I_{fB}| = 2.919 \times 1.75 = 5.108 \text{ KA.}$$

(c) Momentary current can be calculated by multiplying the symmetrical momentary current by a factor of 1.6 to account for the presence of DC off-set current.

$$\begin{aligned} \therefore \text{Momentary current through breaker B} \\ = 1.6 \times 5.108 \text{ KA} = 8.17 \text{ KA.} \end{aligned}$$

(d) For computing the current to be interrupted by the breaker, motor x_m'' ($x_m'' = j1.0$) is now replaced by x_m' ($x_m' = j1.25$ pu). The equivalent circuit is shown in Fig. 8.13(c).

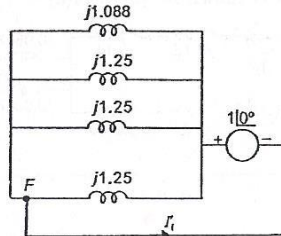


Fig. 8.13(c)

$$x_{eq} = j0.3012$$

Current to be interrupted by the breaker

$$I_f' = \frac{1}{j0.3012} = -j3.32 \text{ pu}$$

Allowance is made for the DC off-set value by multiplying with a factor of (i) 1.4 for 2 cycles (ii) 1.2 for 3 cycles (iii) 1.1 for 5 cycles (iv) 1.0 for 8 cycles.

Therefore, current to be interrupted as:

- (i) $1.4 \times 3.32 \times 1.75 = 8.134 \text{ KA}$
- (ii) $1.2 \times 3.32 \times 1.75 = 6.972 \text{ KA}$
- (iii) $1.1 \times 3.32 \times 1.75 = 6.391 \text{ KA}$
- (iv) $1.0 \times 3.32 \times 1.75 = 5.81 \text{ KA}$

Example 8.13: Fig. 8.14 shows a generating station feeding a 220 KV system. Determine the total fault current, fault level and fault current supplied by each generator for a three phase fault at the receiving end of the line.

- G1 : 11 KV, 100 MVA, $x'_{g1} = j0.15$
- G2 : 11 KV, 75 MVA, $x'_{g2} = j0.125$
- T1 : 100 MVA, $x_{T1} = j0.10$, 11/220 KV
- T2 : 75 MVA, $x_{T2} = j0.08$, 11/220 KV

Solution:

Let base MVA = 100, Base voltage = 11 KV.

$$x'_{g1} = j0.15, \quad x_{T1} = j0.10$$

$$x'_{g2} = j0.125 \times \frac{100}{75} = j0.166$$

$$x_{T2} = j0.08 \times \frac{100}{75} = j0.106$$

Per unit reactance of each line

$$= j42 \times \frac{100}{(220)^2} = j0.0867 \text{ pu.}$$

Single line reactance diagram is shown in Fig. 8.14(a)

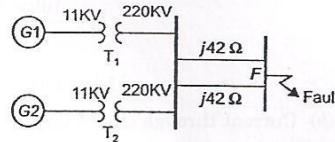


Fig. 8.14: Sample network of Example 8.13.

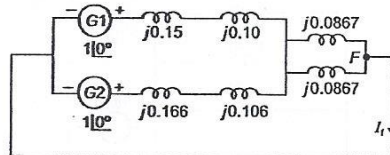


Fig. 8.14(a)

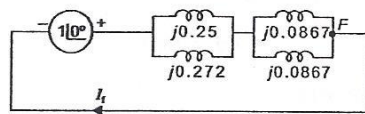


Fig. 8.14(b)

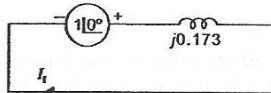


Fig. 8.14(c)

$$\therefore I_f = \frac{1}{j0.173} = -j5.78 \text{ pu}$$

Base current for 220 KV side

$$I_B = \frac{100 \times 1000}{\sqrt{3} \times 220} = 262.43 \text{ Amp.}$$

$$\therefore |I_f| = 5.78 \times 262.43 = 1.516 \text{ KA.}$$

$$\text{Fault level} = 5.78 \text{ pu} = 5.78 \times 100 = 578 \text{ MVA.}$$

Base current on 11 KV side

$$= I_B \times \left(\frac{220}{11}\right) = 262.43 \times \left(\frac{220}{11}\right) \\ = 5248.6 \text{ Amp.}$$

Fault current supplied by the two generators

$$= 5248.6 \times (-j5.78) = 30.34 \angle -90^\circ \text{ KA}$$

$$\therefore I_{fg1} = \frac{0.272}{0.522} \times 30.34 \angle -90^\circ \text{ KA}$$

$$\therefore I_{fg1} = 15.8 \angle -90^\circ \text{ KA}$$

$$I_{fg2} = \frac{0.25}{0.522} \times 30.34 \angle -90^\circ \text{ KA}$$

$$\therefore I_{fg2} = 14.53 \angle -90^\circ \text{ KA}$$

Example 8.14: Fig. 8.15 shows a system having four synchronous generators each rated 11.2 KV, 60 MVA and each having a subtransient reactance of 16%. Find (a) fault level for a fault on one of the feeders (near the bus with $x = 0$). (b) the reactance of the current limiting reactor x_R to limit the fault level to 860 MVA for a fault on one of the feeders near the bus.

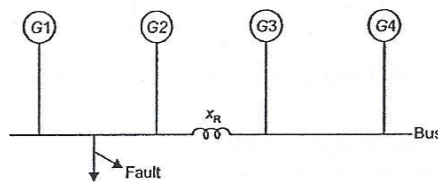


Fig. 8.15: Sample power system of Example 8.14.

Solution:

Set Base MVA = 60, Base voltage = 11.2 KV.

$$x_{g1}'' = x_{g2}'' = x_{g3}'' = x_{g4}'' = 16\% = 0.16 \text{ pu}$$

Circuit model under fault condition is shown in Fig. 8.15(a)

$$x_{eq} = j \frac{0.16}{4} = j0.04$$

$$(a) \text{ fault level} = \frac{1}{0.04} = 25.0 \text{ pu} = 25 \times 60 \text{ MVA} \\ = 1500 \text{ MVA. Ans.}$$

(b) The generators G1 and G2 will supply $\frac{1}{2} \times 1500 = 750$ MVA, directly to the fault. Therefore, the fault MVA from G3 and G4 must be limited to $(860 - 750) = 110$ MVA. The reactance of G3 and G4 together is $\frac{0.16}{2} = 0.08$ pu.

Thus,

$$\frac{1}{x_R + 0.08} = \frac{110}{60}$$

$$\therefore x_R = 0.465 \text{ pu}$$

$$\text{Base impedance} = \frac{(11.2)^2}{60} = 2.09 \text{ ohm}$$

$$\therefore x_R = 0.465 \times 2.09 = 0.97 \text{ ohm.}$$

Example 8.15: Fig. 8.16 shows a power system network. Each of the alternators G1 and G2 is rated at 125 MVA, 11 KV and has a subtransient reactance of 0.21 pu. Each of the transformers is rated at 125 MVA, 11/132 KV and has a leakage reactance of 0.06 pu. Find (a) fault MVA and (b) fault current for a fault at bus 5.

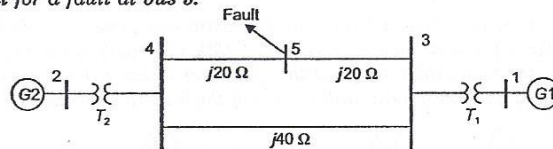


Fig. 8.16: Power system network of Example 8.15.

Solution:

Set Base MVA = 125, Base Voltage = 11 KV

Base voltage for transmission line = 132 KV

$$\text{Base impedance for the transmission line} = \frac{(132)^2}{125} \text{ ohm.} \\ = 139.392 \text{ ohm.}$$

$$\therefore x_{34} = j \frac{40}{139.392} = j0.286 \text{ pu,}$$

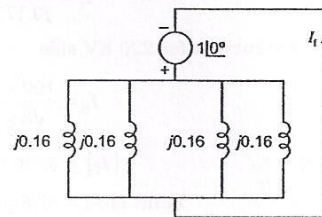


Fig. 8.15(a)

$$x_{45} = x_{35} = j0.143 \text{ pu.}$$

$$x_{g1} = x_{g2} = j0.21 \text{ pu., } x_{T1} = x_{T2} = j0.06 \text{ pu.}$$

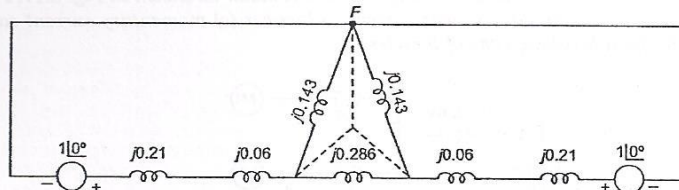


Fig. 8.16(a)

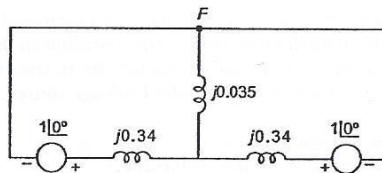


Fig. 8.16(b)

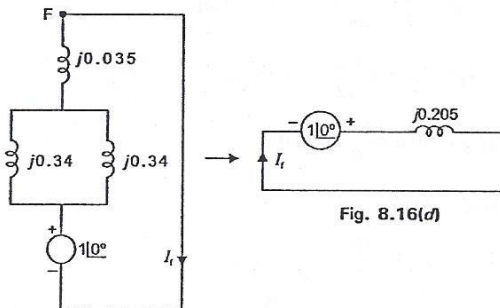


Fig. 8.16(c)

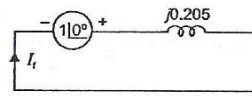


Fig. 8.16(d)

(a) Fault level = $\frac{1}{0.205} \times 125 = 610 \text{ MVA. Ans.}$

(b) $I_f = \frac{1\angle 0^\circ}{j0.205} = \frac{-j}{0.205} \text{ pu.}$

$$\therefore I_f = \frac{-j}{0.205} \times \frac{125 \times 1000}{\sqrt{3} \times 132}$$

$\therefore I_f = -j2.66 \text{ KA. Ans.}$

Example 8.16: A 12 MVA, 132/6.6 KV, transformer having a reactance of 0.15 pu is fed from an infinite bus. The transformer feeds two motor each 6 MVA, 6.6 KV. Each motors has a transient reactance of 0.14 pu and a subtransient reactance of 0.30 pu based on its own rating. A three phase balanced fault occurs at the terminals of one motor as shown in Fig. 8.17. Find (a) subtransient fault current (b) subtransient current in breaker (c) momentary current rating of breaker D which has a breaking time of 5 cycles.

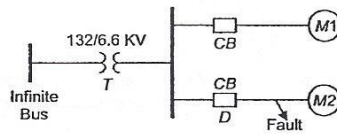


Fig. 8.17: Power system network of Example 8.16.

Solution: If the external power system is very large as compared to the system under consideration of any installation, disturbances within the installation do not affect the voltage and frequency of the external power system. Under this situation, the external power source is known as infinite bus and can be represented by an ideal voltage source, i.e., a constant voltage with zero impedance.

Let base MVA = 100, base voltage = 132 KV.

Therefore, on the motor bus bar, base voltage is 6.6 KV.

∴ Base current,

$$I_B = \frac{100 \times 1000}{\sqrt{3} \times 6.6} = 8747.7 \text{ Amp.}$$

$$x_T = j0.15 \times \frac{100}{12} = j1.25 \text{ pu.}$$

$$x'_{m1} = j0.3 \times \frac{100}{6} = j5.0 \text{ pu}$$

$$x'_{m2} = j5 \text{ pu}$$

$$x''_{m1} = j0.4 \times \frac{100}{6} = j6.67 \text{ pu}$$

$$x''_{m2} = j6.67 \text{ pu.}$$

(a) Circuit model under fault condition (Subtransient condition) is shown in Fig. 8.17(a).

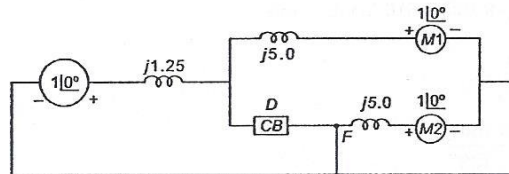


Fig. 8.17(a): Circuit model under subtransient condition.

Subtransient fault current,

$$I_f = \left(\frac{j|0^\circ}{j0.833} \right) \times 8747.7$$

$$= 10.5 \angle -90^\circ \text{ KA.}$$

- (b) Subtransient current through breaker *D* is the current from infinite bus and motor *M*₁.

Fault current from infinite bus

$$= \frac{1 \angle 0^\circ}{j1.25} = -j0.8 \text{ pu}$$

Fault current from motor *M*₁

$$= \frac{1 \angle 0^\circ}{j5.0} = -j0.20 \text{ pu}$$

Fault current through circuit breaker *D*

$$= -j0.8 - j0.2 = -j1.0 \text{ pu}$$

$$= -j1.0 \times 8747.7 = 8.74 \angle -90^\circ \text{ KA.}$$

- (c) To find the momentary current through the breaker, it is necessary to calculate the dc-off set current. However, empirical method for momentary current = 1.6 times symmetrical fault current.

$$\therefore \text{momentary current} = 1.6 \times 10.5 \angle -90^\circ \text{ KA.}$$

$$= 16.8 \angle -90^\circ \text{ KA. Amp.}$$

- (d) Fig. 8.17(d) shows the circuit model under transient condition.

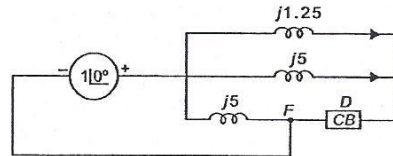


Fig. 8.17(b)

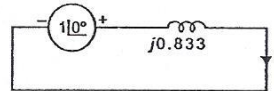


Fig. 8.17(c)

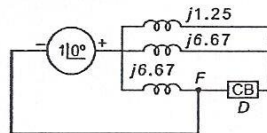


Fig. 8.17(d): Circuit model under transient condition.

Current interrupted by breaker *D*

$$= \frac{1}{j1.25} + \frac{1}{j6.67} = -j0.95 \text{ pu}$$

$$= -j0.95 \times 8747.7 = 8.31 \angle -90^\circ \text{ KA.}$$

However, effect of dc off-set can be included by using a multiplying factor of 1.1. Therefore current to be interrupted by breaker

$$= 1.1 \times 8.31 \angle -90^\circ = 9.14 \angle -90^\circ \text{ KA.}$$

(d:6)

Handwritten signature: Ahmad

Handwritten text: د. شورو واد

Handwritten text: = MS

Handwritten text in a circle: 2/4

Final

CHAPTER 3

LOAD FLOW ANALYSIS

[CONTENTS: Review of solution of equations, direct and iterative methods, classification of buses, importance of slack bus and Y_{BUS} based analysis, constraints involved, load flow equations, GS method: algorithms for finding the unknowns, concept of acceleration of convergence, NR method- algorithms for finding the unknowns, tap changing transformers, Fast decoupled load flow, illustrative examples]

REVIEW OF NUMERICAL SOLUTION OF EQUATIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

1. Solution Linear equations:

*** Direct methods:**

- Cramer's (Determinant) Method,
- Gauss Elimination Method (only for smaller systems),
- LU Factorization (more preferred method), etc.

*** Iterative methods:**

- Gauss Method
- Gauss-Siedel Method (for diagonally dominant systems)

2. Solution of Nonlinear equations:

Iterative methods only:

- Gauss-Siedel Method (for smaller systems)
- Newton-Raphson Method (if corrections for variables are small)

3. Solution of differential equations:

Iterative methods only:

- Euler and Modified Euler method,
- RK IV-order method,
- Milne's predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:

- Selection of initial solution/ estimates
- Determination of fresh/ new estimates during each iteration
- Selection of number of iterations as per tolerance limit
- Time per iteration and total time of solution as per the solution method selected
- Convergence and divergence criteria of the iterative solution
- Choice of the Acceleration factor of convergence, etc.

A comparison of the above solution methods is as under:

- In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate.
- The iterative methods are more useful when the diagonal elements of the coefficient matrix are large in comparison with the off diagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.
- The Newton-Raphson method is very much useful for solution of non-linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

LOAD FLOW STUDIES

Introduction: Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system.

Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow

studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load-flow studies play a vital role in power system studies.

Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:

- The Kirchhoff's relations holding good,
- Capability limits of reactive power sources,
- Tap-setting range of tap-changing transformers,
- Specified power interchange between interconnected systems,
- Selection of initial values, acceleration factor, convergence limit, etc.

Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1. Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$ V , \delta$	P_G, Q_G	$ V , \delta$: are assumed if not specified as 1.0 and 0°
2	Generator/ Machine/ PV Bus	$P_G, V $	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$ V , \delta$	About 80% buses are of PQ type
4	Voltage Controlled Bus	$P_G, Q_G, V $	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 0^0 , as per flat-start procedure of iterative solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of Y_{BUS} based LFA: The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load flow analysis. It is a complex, square and symmetric matrix and hence only $n(n+1)/2$ elements of Y_{BUS} need to be stored for a n-bus system. Further, in the Y_{BUS} matrix, $Y_{ij} = 0$, if an incident element is not present in the system connecting the buses 'i' and 'j'. since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, Y_{BUS} of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

$$\text{Percentage sparsity of a given matrix of } n^{\text{th}} \text{ order} = \frac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$$

$$S = (Z / n^2) \times 100 \% \quad (1)$$

The percentage sparsity of Y_{BUS} , in practice, could be as high as 80-90%, especially for very large, practical power systems. This sparsity feature of Y_{BUS} is extensively

used in reducing the load flow calculations and in minimizing the memory required to store the coefficient matrices. This is due to the fact that only the non-zero elements Y_{BUS} can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While Y_{BUS} is thus highly sparse, its inverse, Z_{BUS} , the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus i , the complex power S_i (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \quad (2)$$

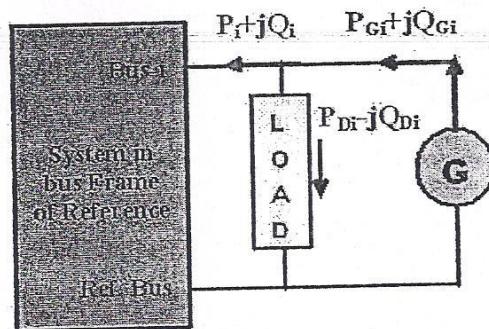


Fig.1 power flows at a bus-i

where S_i = net complex power injected into bus i , S_{Gi} = complex power injected by the generator at bus i , and S_{Di} = complex power drawn by the load at bus i . According to conservation of complex power, at any bus i , the complex power injected into the

bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

$$S_i = \sum S_{ij} \quad \forall i = 1, 2, \dots, n \quad (3)$$

where S_{ij} is the sum over all lines connected to the bus and n is the number of buses in the system (excluding the ground). The bus current injected at the bus- i is defined as

$$I_i = I_{Gi} - I_{Di} \quad \forall i = 1, 2, \dots, n \quad (4)$$

where I_{Gi} is the current injected by the generator at the bus and I_{Di} is the current drawn by the load (demand) at that bus. In the bus frame of reference

$$I_{BUS} = Y_{BUS} V_{BUS} \quad (5)$$

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \text{ is the vector of currents injected at the buses,}$$

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \text{ is the vector of complex bus voltages.}$$

Equation (5) can be considered as

$$I_i = \sum_{j=1}^n Y_{ij} V_j \quad \forall i = 1, 2, \dots, n \quad (6)$$

The complex power S_i is given by

$$\begin{aligned} S_i &= V_i I_i^* \\ &= V_i \left(\sum_{j=1}^n Y_{ij} V_j \right)^* \\ &= V_i \left(\sum_{j=1}^n Y_{ij}^* V_j^* \right) \end{aligned} \quad (7)$$

Let $V_i \triangleq |V_i| \angle \delta_i = |V_i| (\cos \delta_i + j \sin \delta_i)$
 $\delta_{ij} = \delta_i - \delta_j$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^n |V_i| |V_j| (\cos \delta_{ij} + j \sin \delta_{ij}) (G_{ij} - j B_{ij}) \quad (8)$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^n |V_i| |V_j| (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \quad (9) \checkmark$$

$$Q_i = \sum_{j=1}^n |V_i| |V_j| (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \quad (10) \checkmark$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

$$\text{as } Y_{ij} = |Y_{ij}| \angle \theta_{ij} \quad (11)$$

Again, we get from (7),

$$S_i = |V_i| \angle \delta_i \sum_{j=1}^n |Y_{ij}| \angle -\theta_{ij} |V_j| \angle -\delta_j \quad (12)$$

The real part of (12) gives P_i .

$$\begin{aligned} P_i &= |V_i| \sum_{j=1}^n |Y_{ij}| |V_j| \cos(-\theta_{ij} + \delta_i - \delta_j) \\ &= |V_i| \sum_{j=1}^n |Y_{ij}| |V_j| \cos-(\theta_{ij} - \delta_i + \delta_j) \quad \text{or} \end{aligned}$$

$$P_i = \sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad \forall i = 1, 2, \dots, n, \quad (13) \checkmark$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^n |Y_{ij}| |V_j| \sin-(\theta_{ij} - \delta_i + \delta_j) \quad \text{or}$$

$$Q_i = -\sum_{j=1}^n |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad \forall i = 1, 2, \dots, n \quad (14) \checkmark$$

Equations (9)-(10) and (13)-(14) are the 'power flow equations' or the 'load flow equations' in two alternative forms, corresponding to the n-bus system, where each bus- i is characterized by four variables, P_i , Q_i , $|V_i|$, and δ_i . Thus a total of $4n$ variables are involved in these equations. The load flow equations can be solved for

any $2n$ unknowns, if the other $2n$ variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

DATA FOR LOAD FLOW

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

System data: It includes: number of buses- n , number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as 0°), tolerance limit, base MVA, and maximum permissible number of iterations.

Generator bus data: For every PV bus i , the data required includes the bus number, active power generation P_{Gi} , the specified voltage magnitude $|V_{i,sp}|$, minimum reactive power limit $Q_{i,min}$, and maximum reactive power limit $Q_{i,max}$.

Load data: For all loads the data required includes the the bus number, active power demand P_{Di} , and the reactive power demand Q_{Di} .

Transmission line data: For every transmission line connected between buses i and k the data includes the starting bus number i , ending bus number k , resistance of the line, reactance of the line and the half line charging admittance.

Transformer data:

For every transformer connected between buses i and k the data to be given includes: the starting bus number i , ending bus number k , resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio a .

Shunt element data: The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance ($G_{sh} + j B_{sh}$).

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated

till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only: $(P_G, Q_G) \rightarrow V, S$

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n-1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^n Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^n Y_{ij} V_j \right) \quad (15)$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^n Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij} V_j \quad (16)$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij} V_j \right] \quad \forall i = 2, 3, \dots, n \quad (17) \checkmark$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss-Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up

convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix Y_{BUS} . This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,...n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0 \angle 0^\circ$. This is normally referred as the flat start solution.
4. Update the voltages. In any $(k+1)^{th}$ iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^n Y_{ij} V_j^{(k)} \right] \quad \forall i=2,3,\dots,n \quad (18)$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3,...(i-1) in the current $(k+1)^{th}$ iteration. Hence these values are used. For buses $(i+1) \dots n$, values from previous, k^{th} iteration are used.

5. Continue iterations till

$$|\Delta V_i^{(k+1)}| = |V_i^{(k+1)} - V_i^{(k)}| < \epsilon \quad \forall i=2,3,\dots,n \quad (19)$$

Where, ϵ is the tolerance value. Generally it is customary to use a value of 0.0001 pu.

6. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^* \left(\sum_{j=1}^n Y_{1j} V_j \right) \quad (20)$$

7. Compute all line flows.
8. The complex power loss in the line is given by $S_{ik} + S_{ki}$. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Q_i to be used in (18). From (15) we have

$$Q_i = -\text{Im} \left\{ V_i^* \sum_{j=1}^n Y_{ij} V_j \right\}$$

Where Im stands for the imaginary part. At any $(k+1)^{\text{st}}$ iteration, at the PV bus- i ,

$$Q_i^{(k+1)} = -\text{Im} \left\{ (V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^n Y_{ij} V_j^{(k)} \right\} \quad (21)$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)
2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$
3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \quad (22)$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e. $Q_i^{(k+1)}$ computed using (21) is either less than $Q_{i,\min}$ or greater than $Q_{i,\max}$, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the $(k+1)^{\text{st}}$ iteration and the voltage is calculated with the value of Q_i set as follows:

$$\begin{array}{ll} \text{If } Q_i < Q_{i,\min} & \text{If } Q_i > Q_{i,\max} \\ \text{Then } Q_i = Q_{i,\min}. & \text{Then } Q_i = Q_{i,\max}. \end{array} \quad (23)$$

If in the subsequent iteration, if Q_i falls within the limits, then the bus can be switched back to PV status.

Acceleration of convergence

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if

the correction in voltage at each bus is accelerated, by multiplying with a constant α , called the acceleration factor. In the $(k+1)^{\text{st}}$ iteration we can let

$$V_i^{(k+1)}(\text{accelerate } d) = V_i^{(k)} + \alpha (V_i^{(k+1)} - V_i^{(k)}) \quad (24)$$

where α is a real number. When $\alpha = 1$, the value of $V_i^{(k+1)}$ is the computed value. If $1 < \alpha < 2$, then the value computed is extrapolated. Generally α is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

Examples on GS load flow analysis:

Example-1: Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss-Seidel method, if $V_1 = 1 \angle 0^\circ$ pu.

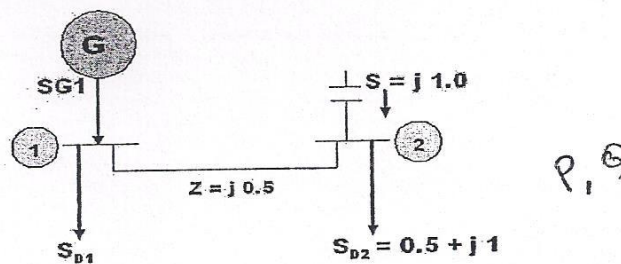


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j 1.0) = -0.5 \text{ pu.}$$

$$V_1 = 1 \angle 0^\circ$$

$$Y_{\text{BUS}} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^{(k)})^*} - Y_{21} V_1 \right]$$

$$y = \frac{1}{j0.5} = -j2$$

$$V_i = \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^n y_{ij} V_j \right]$$

$i = 2, 3, \dots, n$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j 0.0 = 1 \angle 0^\circ$ pu.

$$\begin{aligned}
V_2^1 &= \frac{1}{-j2} \left[\frac{-0.5}{1\angle 0^\circ} - (j2 \times 1\angle 0^\circ) \right] \\
&= 1.0 - j0.25 = 1.030776 \angle -14.036^\circ \\
V_2^2 &= \frac{1}{-j2} \left[\frac{-0.5}{1.030776\angle 14.036^\circ} - (j2 \times 1\angle 0^\circ) \right] \\
&= 0.94118 - j0.23529 = 0.970145 \angle -14.036^\circ \\
V_2^3 &= \frac{1}{-j2} \left[\frac{-0.5}{0.970145\angle 14.036^\circ} - (j2 \times 1\angle 0^\circ) \right] \\
&= 0.9375 - j0.249999 = 0.970261 \angle -14.931^\circ \\
V_2^4 &= \frac{1}{-j2} \left[\frac{-0.5}{0.970261\angle 14.931^\circ} - (j2 \times 1\angle 0^\circ) \right] \\
&= 0.933612 - j0.248963 = 0.966237 \angle -14.931^\circ \\
V_2^5 &= \frac{1}{-j2} \left[\frac{-0.5}{0.966237\angle 14.931^\circ} - (j2 \times 1\angle 0^\circ) \right] \\
&= 0.933335 - j0.25 = 0.966237 \angle -14.995^\circ
\end{aligned}$$

Since the difference in the voltage magnitudes is less than 10^{-6} pu, the iterations can be stopped. To compute line flow

$$\begin{aligned}
I_{12} &= \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^\circ - 0.966237\angle -14.995^\circ}{j0.5} \\
&= 0.517472 \angle -14.931^\circ \\
S_{12} &= V_1 I_{12}^* = 1\angle 0^\circ \times 0.517472 \angle 14.931^\circ \\
&= 0.5 + j0.133329 \text{ pu} \\
I_{21} &= \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^\circ - 1\angle 0^\circ}{j0.5} \\
&= 0.517472 \angle -194.93^\circ \\
S_{21} &= V_2 I_{21}^* = -0.5 + j0.0 \text{ pu}
\end{aligned}$$

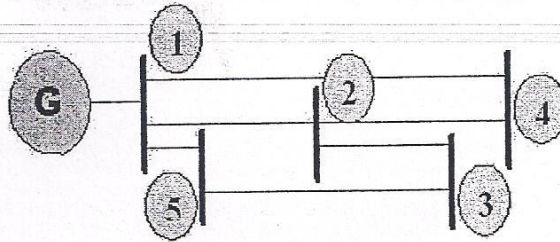
The total loss in the line is given by

$$S_{12} + S_{21} = j0.133329 \text{ pu}$$

Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_c}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$ V_{sp} $ (pu)	δ
1	-	-	-	-	1.02	0°
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

→ slack bus
→ PQ
→ PV
→ PQ
→ PQ

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

$$P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$$

$$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$$

$$\text{Similarly } P_4 + jQ_4 = -0.4 - j0.1, \quad P_5 + jQ_5 = -0.6 - j0.2$$

The Y_{bus} formed by the rule of inspection is given by:

$$Y_{bus} = \begin{bmatrix} 2.15685 & -0.58823 & 0.0+j0.0 & -0.39215 & -1.17647 \\ -j8.62744 & +j2.35294 & & +j1.56862 & +j4.70588 \\ -0.58823 & 2.35293 & -1.17647 & -0.58823 & 0.0+j0.0 \\ +j2.35294 & -j9.41176 & +j4.70588 & +j2.35294 & \\ 0.0+j0.0 & -1.17647 & 2.35294 & 0.0+j0.0 & -1.17647 \\ & +j4.70588 & -j9.41176 & & +j4.70588 \\ -0.39215 & -0.58823 & 0.0+j0.0 & 0.98038 & 0.0+j0.0 \\ +j1.56862 & +j2.35294 & & -j3.92156 & \\ -1.17647 & 0.0+j0.0 & -1.17647 & 0.0+j0.0 & 2.35294 \\ +j4.70588 & & +j4.70588 & & -j9.41176 \end{bmatrix}$$

The voltages at all PQ buses are assumed to be equal to $1+j0.0$ pu. The slack bus voltage is taken to be $V_1^0 = 1.02+j0.0$ in all iterations.

$$\begin{aligned} V_2^1 &= \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{0*}} - Y_{21} V_1^0 - Y_{23} V_3^0 - Y_{24} V_4^0 - Y_{25} V_5^0 \right] \\ &= \frac{1}{Y_{22}} \left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294) \times 1.02 \angle 0^\circ\} \right. \\ &\quad \left. - \{(-1.17647 + j4.70588) \times 1.04 \angle 0^\circ\} - \{(-0.58823 + j2.35294) \times 1.0 \angle 0^\circ\} \right] \\ &= 0.98140 \angle -3.0665^\circ = 0.97999 - j0.0525 \end{aligned}$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3 . This can be done as under:

$$\begin{aligned} Q_3 &= |V_3| |V_1| (G_{31} \sin \delta_{31} - B_{31} \cos \delta_{31}) + |V_3| |V_2| (G_{32} \sin \delta_{32} - B_{32} \cos \delta_{32}) \\ &\quad + |V_3|^2 (G_{33} \sin \delta_{33} - B_{33} \cos \delta_{33}) + |V_3| |V_4| (G_{34} \sin \delta_{34} - B_{34} \cos \delta_{34}) \\ &\quad + |V_3| |V_5| (G_{35} \sin \delta_{35} - B_{35} \cos \delta_{35}) \end{aligned}$$

We note that $\delta_1 = 0^\circ$; $\delta_2 = -3.0665^\circ$; $\delta_3 = 0^\circ$; $\delta_4 = 0^\circ$ and $\delta_5 = 0^\circ$

$$\therefore \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^\circ (\delta_{ik} = \delta_i - \delta_k); \quad \delta_{32} = 3.0665^\circ$$

$$\begin{aligned} Q_3 &= 1.04 [1.02 (0.0+j0.0) + 0.9814 \{-1.17647 \times \sin(3.0665^\circ) - 4.70588 \\ &\quad \times \cos(3.0665^\circ)\} + 1.04 \{-9.41176 \times \cos(0^\circ)\} + 1.0 \{0.0 + j0.0\} + 1.0 \{-4.70588 \times \cos(0^\circ)\}] \\ &= 1.04 [-4.6735 + 9.78823 - 4.70588] = 0.425204 \text{ pu.} \end{aligned}$$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31} V_1^0 - Y_{32} V_2^1 - Y_{34} V_4^0 - Y_{35} V_5^0 \right]$$

$$\begin{aligned}
&= \frac{1}{Y_{33}} \left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588) \times (0.98140 \angle -3.0665^\circ)\} \right. \\
&\quad \left. - \{(-1.17647 + j4.70588) \times (1 \angle 0^\circ)\} \right] \\
&= 1.05569 \angle 3.077^\circ = 1.0541 + j0.05666 \text{ pu.}
\end{aligned}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is

$$\text{computed as: } V_3^1 = 1.04 \angle 3.077^\circ \text{ pu}$$

$$\begin{aligned}
V_4^1 &= \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{V_4^{0*}} - Y_{41} V_1^0 - Y_{42} V_2^1 - Y_{43} V_3^1 - Y_{45} V_5^0 \right] \\
&= \frac{1}{Y_{44}} \left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862) \times 1.02 \angle 0^\circ\} \right. \\
&\quad \left. - \{(-0.58823 + j2.35294) \times (0.98140 \angle -3.0665^\circ)\} \right] \\
&= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715 \angle -7.303^\circ \text{ pu} = 0.94796 - j0.12149
\end{aligned}$$

$$\begin{aligned}
V_5^1 &= \frac{1}{Y_{55}} \left[\frac{P_5 - jQ_5}{V_5^{0*}} - Y_{51} V_1^0 - Y_{52} V_2^1 - Y_{53} V_3^1 - Y_{54} V_4^1 \right] \\
&= \frac{1}{Y_{55}} \left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588) \times 1.02 \angle 0^\circ\} \right. \\
&\quad \left. - \{(-1.17647 + j4.70588) \times 1.04 \angle 3.077^\circ\} \right] \\
&= 0.994618 \angle -1.56^\circ = 0.994249 - j0.027
\end{aligned}$$

Thus at end of 1st iteration, we have,

$$\begin{aligned}
V_1 &= 1.02 \angle 0^\circ \text{ pu} & V_2 &= 0.98140 \angle -3.066^\circ \text{ pu} \\
V_3 &= 1.04 \angle 3.077^\circ \text{ pu} & V_4 &= 0.955715 \angle -7.303^\circ \text{ pu} \\
\text{and } V_5 &= 0.994618 \angle -1.56^\circ \text{ pu}
\end{aligned}$$

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

- (i) All buses except bus 1 are PQ Buses
- (ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu
- (iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and $0.25 \leq Q_2 \leq 1.0$ pu.

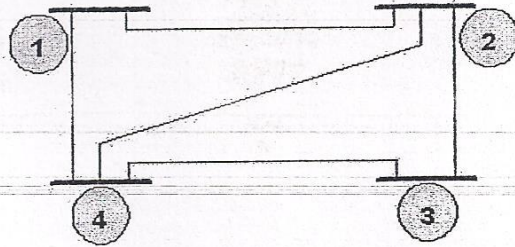


Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	-	-	$1.04 \angle 0^\circ$
2	0.5	-0.2	-
3	-1.0	0.5	-
4	-0.3	-0.1	-

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$$Y_{BUS} = \begin{bmatrix} 3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 & 0 \\ -2.0 + j6.0 & 3.666 - j11.0 & -0.666 + j2.0 & -1.0 + j3.0 \\ -1.0 + j3.0 & -0.666 + j2.0 & 3.666 - j11.0 & -2.0 + j6.0 \\ 0 & -1.0 + j3.0 & -2.0 + j6.0 & 3.0 - j9.0 \end{bmatrix}$$

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0 \angle 0^\circ$ pu.

$$V_2^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{0*}} - Y_{21} V_1^0 - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$\begin{aligned}
&= \frac{1}{Y_{22}} \left[\frac{0.5 + j0.2}{1.0 - j0.0} - \{(-2.0 + j6.0) \times (1.04 \angle 0^\circ)\} \right. \\
&\quad \left. - \{(-0.666 + j2.0) \times (1.0 \angle 0^\circ)\} - \{(-1.0 + j3.0) \times (1.0 \angle 0^\circ)\} \right] \\
&= 1.02014 \angle 2.605^\circ
\end{aligned}$$

$$\begin{aligned}
V_3^1 &= \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31} V_1^0 - Y_{32} V_2^1 - Y_{34} V_4^0 \right] \\
&= \frac{1}{Y_{33}} \left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0) \times (1.04 \angle 0^\circ)\} \right. \\
&\quad \left. - \{(-0.666 + j2.0) \times (1.02014 \angle 2.605^\circ)\} - \{(-2.0 + j6.0) \times (1.0 \angle 0^\circ)\} \right] \\
&= 1.03108 \angle -4.831^\circ
\end{aligned}$$

$$\begin{aligned}
V_4^1 &= \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{V_4^{0*}} - Y_{41} V_1^0 - Y_{42} V_2^1 - Y_{43} V_3^1 \right] \\
&= \frac{1}{Y_{44}} \left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0) \times (1.02014 \angle 2.605^\circ)\} \right. \\
&\quad \left. - \{(-2.0 + j6.0) \times (1.03108 \angle -4.831^\circ)\} \right] \\
&= 1.02467 \angle -0.51^\circ
\end{aligned}$$

Hence

$$\begin{aligned}
V_1^1 &= 1.04 \angle 0^\circ \text{ pu} & V_2^1 &= 1.02014 \angle 2.605^\circ \text{ pu} \\
V_3^1 &= 1.03108 \angle -4.831^\circ \text{ pu} & V_4^1 &= 1.02467 \angle -0.51^\circ \text{ pu}
\end{aligned}$$

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2 .

$$\begin{aligned}
Q_2 &= |V_2| \left[|V_1| (G_{21} \sin \delta_{21} - B_{21} \cos \delta_{21}) + |V_2| (G_{22} \sin \delta_{22} - B_{22} \cos \delta_{22}) \right. \\
&\quad \left. + |V_3| (G_{23} \sin \delta_{23} - B_{23} \cos \delta_{23}) + |V_4| (G_{24} \sin \delta_{24} - B_{24} \cos \delta_{24}) \right] \\
&= 1.04 [1.04 \{-6.0\} + 1.04 \{11.0\} + 1.0 \{-2.0\} + 1.0 \{-3.0\}] = 0.208 \text{ pu.}
\end{aligned}$$

$$\begin{aligned}
V_2^1 &= \frac{1}{Y_{22}} \left[\frac{0.5 - j0.208}{1.04 \angle 0^\circ} - \{(-2.0 + j6.0) \times (1.04 \angle 0^\circ)\} \right. \\
&\quad \left. - \{(-0.666 + j2.0) \times (1.0 \angle 0^\circ)\} - \{(-1.0 + j3.0) \times (1.0 \angle 0^\circ)\} \right] \\
&= 1.051288 + j0.033883
\end{aligned}$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04 \angle 1.846^\circ$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{-1.0 - j0.5}{1.0 \angle 0^\circ} - \{(-1.0 + j3.0) \times (1.04 \angle 0.0^\circ)\} \right. \\ \left. - \{(-0.666 + j2.0) \times (1.04 \angle 1.846^\circ)\} - \{(-2.0 + j6.0) \times (1.0 \angle 0^\circ)\} \right] \\ = 1.035587 \angle -4.951^\circ \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}} \left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0) \times (1.04 \angle 1.846^\circ)\} \right. \\ \left. - \{(-2.0 + j6.0) \times (1.035587 \angle -4.951^\circ)\} \right] \\ = 0.9985 \angle -0.178^\circ$$

Hence at end of 1st iteration we have:

$$V_1^1 = 1.04 \angle 0^\circ \text{ pu} \quad V_2^1 = 1.04 \angle 1.846^\circ \text{ pu} \\ V_3^1 = 1.035587 \angle -4.951^\circ \text{ pu} \quad V_4^1 = 0.9985 \angle -0.178^\circ \text{ pu}$$

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \leq Q_2 \leq 1$ pu. If $0.25 \leq Q_2 \leq 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2 . The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^\circ \text{ pu} \quad V_2^1 = 1.05645 \angle 1.849^\circ \text{ pu} \\ V_3^1 = 1.038546 \angle -4.933^\circ \text{ pu} \quad V_4^1 = 1.081446 \angle 4.896^\circ \text{ pu}$$

Limitations of GS load flow analysis:

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

نحن .. لا نكتب لكي نسمع التصفيق
ولسنا مجرد فريق زائدة في الجامعة ، نحن ..
هواءٌ يُنتَفَسُ ، وأحلامٌ ترافقك دائما ، لنرسم التفاضل ما
استطعنا ، نحن اسرة مترابطة معا لتحقيق حلم كل مسلم