Transmission Line Modeling

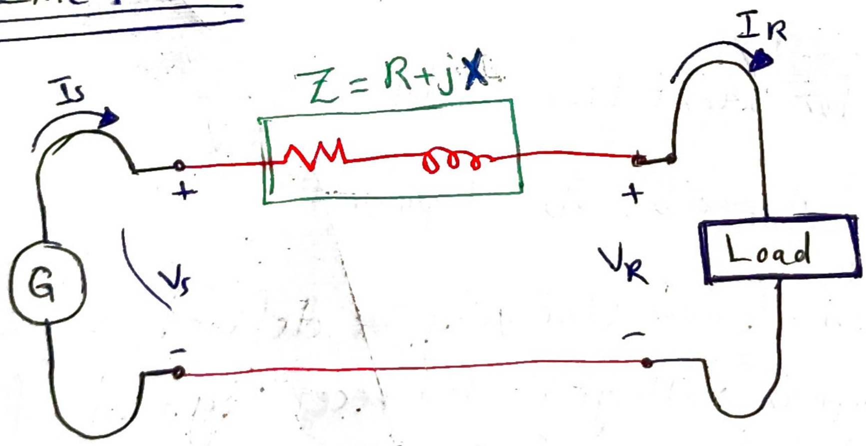
- Short Line Model (Less than 80 km)
- Medium Line Model (80km < L < 250 km)
- Long Line Model (L ≥ 250 km)

» Lumped parameter system.

» Distributed parameter system.

- we use Lumped parameters which give good accuracy for short lines and for lines of medium length.
- If an overhead line is classified as short, shunt capacitance is so small that it can be omitted entirely with little loss of accuracy, and we need to consider only the series resistance R and the series inductance L for the total length of the Line.

[1] Short Line Model :-



» line length < 80 km

» Generally MV / LV Lin

» Capacitance cab be neglected

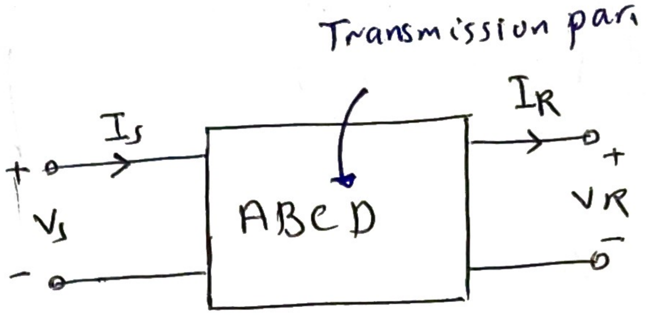
$$Z = (r + j\omega L)\,l$$
$$= R + jX$$

where r and L are the per-phase resistance and inductance per unit length, respectively, and l is the line length.

The phase voltage at the sending end is

$$V_s = V_R + Z I_R \quad \text{—— } (1)$$

$$I_s = I_R$$



Two-port representation of a T.L

$$V_s = A V_R + B I_R$$
$$I_s = C V_R + D I_R$$
$$\Rightarrow \begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

ثنائي

Since we are dealing with a linear passive, bilateral two-port network, the ~~det~~ of the transmission matrix is unity :-

$$AD - BC = 1$$

$$\Rightarrow \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

- According to (1) for short line model

$A = 1$ per unit, $B = Z\,\Omega$, $C = 0\,S$, $D = 1$ per unit

- Voltage regulation of the line may be defined as the percentage change in voltage at the receiving end of the line (expressed as percent of full load voltage) in going from no-load to full load.

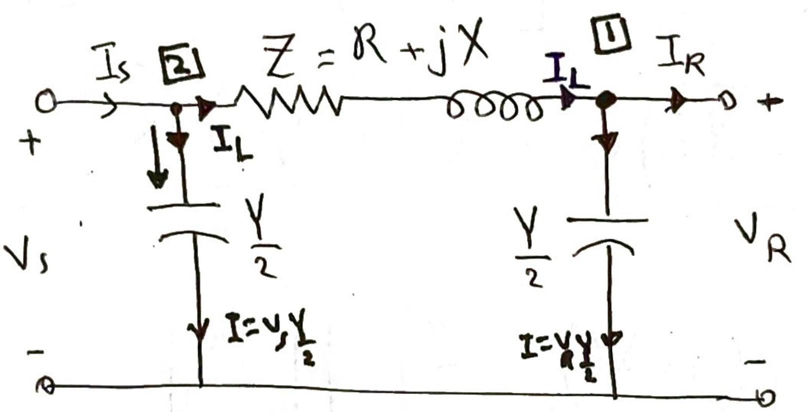
$$\boxed{\text{Percent } VR = \frac{|V_{R(NL)}| - |V_{R(FL)}|}{|V_{R(FL)}|} * 100}$$

Voltage regulation is a measure of line voltage drop.

At no Load $I_R = 0 \Rightarrow V_{R(NL)} = \frac{V_s}{A}$ A=1 for Short Line.

Medium Line Model

- $80km < \text{Length} < 250km$.
- As the length of line increases, the line charging current becomes appreciable and the shunt capacitance must be considered.
- For medium length lines, half of the shunt capacitance may be considered to be lumped at each end of the line. This is referred to as the nominal π model and is shown in Figure below:-



$$\boxed{I_L = I_R + V_R \frac{Y}{2}}$$

in terms of

$V_s(V_R, I_R)$

$I_s(V_R, I_R)$

$Z \equiv$ total series impedance of the Line.

$Y \equiv$ total shunt admittance of the Line.

$$Y = (g + j\omega c)l$$

Under normal conditions, the shunt conductance per unit length, which represents the leakage current over the insulators and due to corona, is negligible and g is assumed to be zero. C is the line to neutral capacitance per km, and l is the line length.

1. $V_s = V_R + Z I_L$

$$= V_R + Z\Big(\overbrace{I_R + V_R \cdot \frac{Y}{2}}^{I_L}\Big)$$

$$\boxed{V_s = \left(1+\frac{YZ}{2}\right)V_R + Z I_R}$$

$$\boxed{\begin{aligned} V_s &= A V_R + B I_R \\ I_s &= C V_R + D I_R \end{aligned}}$$

2. $I_s = I_L + V_s \cdot \frac{Y}{2}$

$$= \Big(\overbrace{I_R + V_R \cdot \frac{Y}{2}}^{I_L}\Big) + \frac{V_s Y}{2}$$

$$= I_R + \boxed{\frac{V_R Y}{2}} + \left[\left(1+\frac{YZ}{2}\right)V_R + Z I_R\right]\frac{Y}{2}$$

$$\boxed{I_s = Y\left(1+\frac{YZ}{4}\right)V_R + \left(1+\frac{YZ}{2}\right)I_R}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \left(1+\frac{YZ}{2}\right) & Z \\ Y\left(1+\frac{YZ}{4}\right) & \left(1+\frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

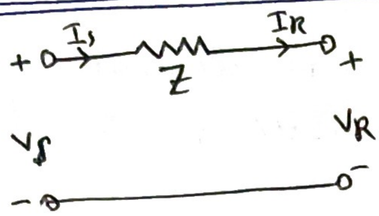
$A = D = 1 + \frac{YZ}{2}$ per unit

since the π model is a symmetrical two-port network $(A = D)$

$B = Z$ Ω

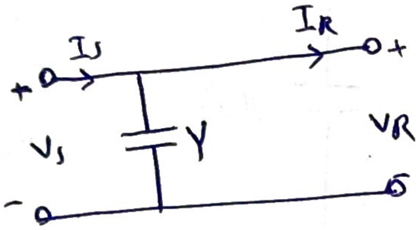
$C = Y\left(1+\frac{YZ}{4}\right)$ S

ABCD Matrix



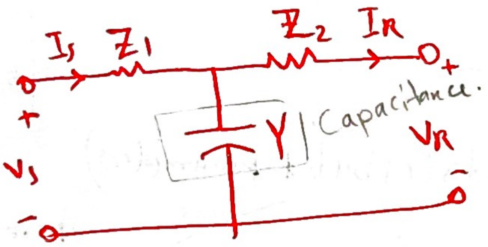
$$\left[\begin{array}{c|c} 1 & Z \\ \hline 0 & 1 \end{array}\right]$$

Short line



$$\left[\begin{array}{c|c} 1 & 0 \\ \hline Y & 1 \end{array}\right]$$

Compensat for reactive power

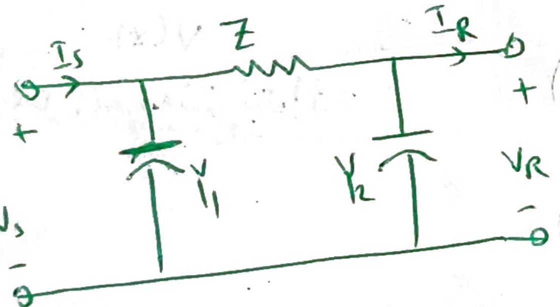


T- circuit

$$\left[\begin{array}{c|c} (1+YZ_1) & (Z_1+Z_2+YZ_1Z_2) \\ \hline Y & (1+YZ_2) \end{array}\right]$$

$\boxed{AD-BC=1}$

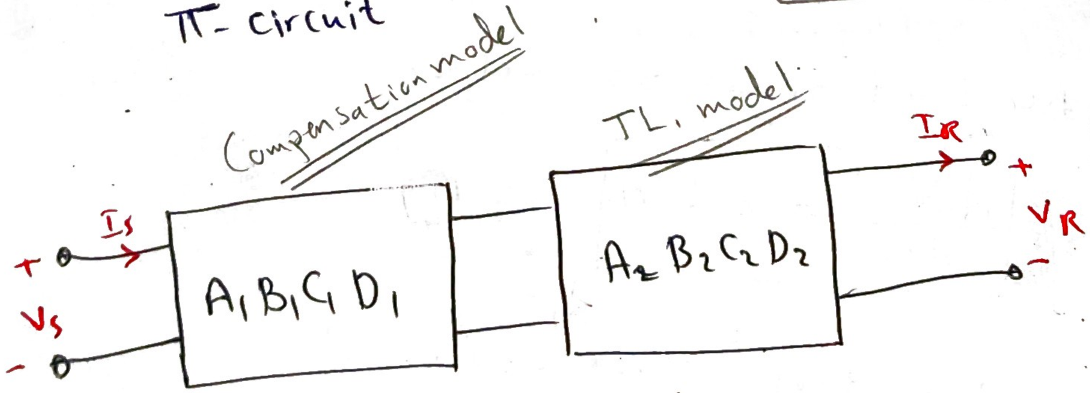
we can ... it inst... of using ...



π- circuit

$$\left[\begin{array}{c|c} (1+Y_2Z) & Z \\ \hline (Y_1+Y_2+Y_1Y_2Z) & (1+Y_1Z) \end{array}\right]$$

$\boxed{AD-BC=1}$

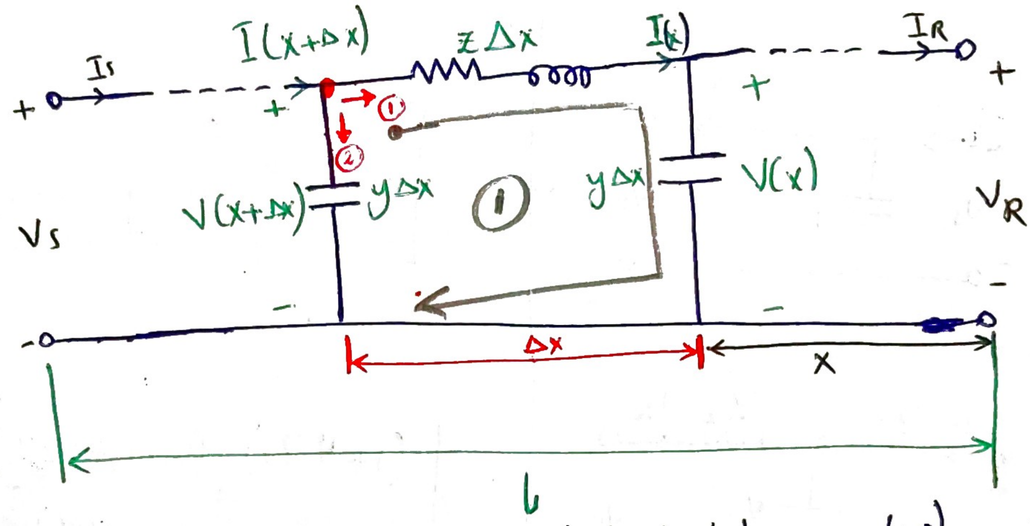


Cascaded networks

$$\left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right]\left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right] = \left[\begin{array}{c|c} (A_1A_2+B_1C_2) & (A_1B_2+B_1D_2) \\ \hline (C_1A_2+D_1C_2) & (C_1B_2+D_1D_2) \end{array}\right]$$

[3] Long Line Model :-

* For the short and medium length Lines ~~reasonably~~ app. accurate models were obtained by assuming the Line parameters to be Lumped. For lines 250 km and longer and for a more accurate solution the exact effect of the distributed parameters must be considered.



$\left(V\big|_{x=l}\right) = V_s$

$V_R = \left(V\big|_{x=0}\right)$

(Long-line with distributed parameters)

target → $V(x)$, $I(x)$

بدلالة par المعروضة بالرأس

- $z = r + j\omega L$
- $y = g + j\omega C$

KVL

[1] $V(x+\Delta x) = z\Delta x\, I(x) + V(x)$

$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = z\, I(x)$$

Taking the limit as $\Delta x \to 0$, we have

$$\boxed{\frac{dV(x)}{dx} = z\, I(x)} \quad \text{—— } (1)$$

KCL

[2] $I(x+\Delta x) = I(x) + y\Delta x\, V(x+\Delta x)$

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = y\, V(x+\Delta x)$$

lim $\Delta x \to 0$

$$\frac{dI(x)}{dx} = y\, V(x) \rightarrow (2)$$

and from (1) ⟹

نرجع للمعادلة رقم (1)

$$\frac{dV(x)}{dx} = z\,I(x) \quad \text{— ①} \qquad \text{from ①} \qquad \text{return to:}$$

نشتق

$$\boxed{\frac{d^2V(x)}{dx^2} = z\,\frac{dI(x)}{dx}}$$

substituting

$$\Rightarrow \frac{dI(x)}{dx} = y\,V(x) \quad \text{من معادله ②} \quad ② \qquad \text{from ②} \qquad \text{return to:}$$

$$\frac{d^2V(x)}{dx^2} = z\,\underbrace{\boxed{\frac{dI(x)}{dx}}}_{yV_x}$$

$$\frac{d^2V(x)}{dx^2} = \boxed{zy}\,V(x)$$

$$zy = \gamma^2 \,,$$

$$\boxed{\frac{d^2V(x)}{dx^2} - \gamma^2 V(x) = 0}$$

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

where $\gamma \equiv$ propagation constant $= \sqrt{zy} = \alpha + j\beta$

attenuation constant (α), phase constant (β).

$$= \sqrt{\underbrace{(r + j\omega L)}_{z}\,\underbrace{(g + j\omega C)}_{y}}$$

$\therefore$

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

we want to find I_x

$$I(x) = \frac{1}{z}\frac{dV(x)}{dx} \equiv \text{from — ①}$$

$$= \frac{\gamma}{z}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right)$$

$$= \sqrt{\frac{y}{z}}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right)$$

$$= \frac{1}{Z_c}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right), \quad Z_c \equiv \text{characteristic impedance}$$

$$Z_c = \sqrt{z/y}$$

$\boxed{\begin{matrix} V_x \\ I_x \end{matrix}}$ → $V(x)$ ✓, $I(x)$?

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$$I(x) = \frac{1}{Z_c}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right)$$

$A_1 = ?!$, $A_2 = ?!$

Two boundary conditions:-

$\Rightarrow$ at $x = 0$

① $V(x) = V_R$

② $I(x) = I_R$

$\Longrightarrow$ $x=0$

$$V_R = A_1 + A_2$$

$$I_R = \frac{A_1 - A_2}{Z_c}$$

$$\Rightarrow \quad A_1 = \frac{V_R + Z_c I_R}{2}$$

$$A_2 = \frac{V_R - Z_c I_R}{2}$$

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$I(x) = \frac{\frac{V_R}{Z_c} + I_R}{2} e^{\gamma x} \mp \frac{\frac{V_R}{Z_c} - I_R}{2} e^{-\gamma x}$$

Rearranged

$V(x) = \square\, V_R + \square\, I_R$

$I(x) = \square\, V_R + \square\, I_R$

$$V(x) = \square\, V_R + \square\, I_R$$
$$I(x) = \square\, V_R + \square\, I_R$$

$\cosh \gamma x$ $\sinh \gamma x$

$$V(x) = \frac{e^{\gamma x} + e^{-\gamma x}}{2} V_R + Z_c \frac{e^{\gamma x} - e^{-\gamma x}}{2} I_R$$

$$I(x) = \frac{1}{Z_c} \frac{e^{\gamma x} - e^{-\gamma x}}{2} V_R + \frac{e^{\gamma x} + e^{-\gamma x}}{2} I_R$$

$\sinh \gamma x$ $\cosh \gamma x$

$$V(x) = \cosh \gamma x \, V_R + Z_c \sinh \gamma x \, I_R$$

$$\Rightarrow \quad I(x) = \frac{1}{Z_c} \sinh \gamma x \, V_R + \cosh \gamma x \, I_R$$

We are particularly interested in the relation between the sending end and the receiving end of the line.

Setting $x = l$

$V(l) = V_s$

$I(l) = I_s$

$$\Rightarrow \boxed{\begin{aligned} V_s &= \cosh \gamma l \, V_R + Z_c \sinh \gamma l \, I_R \\ I_s &= \frac{1}{Z_c} \sinh \gamma l \, V_R + \cosh \gamma l \, I_R \end{aligned}} \quad \text{—— } 1$$

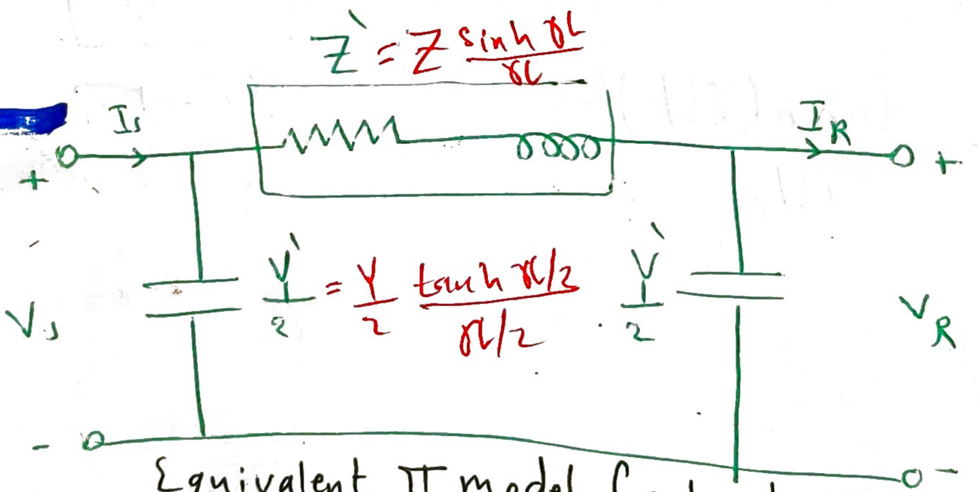
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_c \sinh \gamma l \\ \frac{1}{Z_c} \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

(ABCD matrix)

before

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} (1 + \frac{YZ}{2}) & Z \\ Y(1 + \frac{YZ}{4}) & (1 + \frac{YZ}{2}) \end{bmatrix}$$

note that, as before, $A = D$ and $AD - BC = 1$.



Equivalent π model for long length Line.

$$\boxed{\begin{aligned} V_s &= \left(1 + \frac{Z'Y'}{2}\right) V_R + Z' I_R \\ I_s &= Y'\left(1 + \frac{Z'Y'}{4}\right) V_R + \left(1 + \frac{Z'Y'}{2}\right) I_R \end{aligned}} \quad \text{—— } 2$$

$Z_c \sinh \gamma l$

$\cosh \gamma l$

Comparing 1 with 2

① $Z' = Z_c \sinh \gamma l$

$= \sqrt{\frac{z}{y}} \sinh \gamma l$

$= zl \frac{\sinh \gamma l}{\sqrt{zy}\, l} = Z \frac{\sinh \gamma l}{\gamma l}$

② $\cosh \gamma l = 1 + \frac{Z' Y'}{2}$

$\cosh \gamma l = 1 + \frac{(Z_c \sinh \gamma l\, Y')}{2} = 1 + Z_c \sinh \gamma l \cdot \frac{Y'}{2} = \cosh \gamma l$

$\frac{Y'}{2} = \frac{1}{Z_c} \cdot \boxed{\frac{\cosh \gamma l - 1}{\sinh \gamma l}}$ ← $\tanh \frac{\gamma l}{2}$

$= \frac{1}{Z_c} \tanh \frac{\gamma l}{2}$

$\boxed{Y = yl}$

$Z_c = \sqrt{z/y}$

$= \frac{Y}{2} \frac{\tanh(\gamma l/2)}{\gamma l/2}$

$= \frac{yl}{2} \frac{\tanh(\gamma l/2)}{\frac{\sqrt{zy}\, l}{2}}$

Note:-

$\cosh(\gamma l) = \cosh(\alpha l) \cdot \cos(\beta l) + j \sinh(\alpha l) \cdot \sin(\beta l)$

$\sinh(\gamma l) = \sinh(\alpha l) \cdot \cos(\beta l) + j \cosh(\alpha l) \cdot \sin(\beta l)$

Loss Less Line :-

→ A, B, C, D par.
→ Z', $\frac{Y'}{2}$ (model)

loss-less
- صعب cosh
- بدون approx. idea
- good idea in power flow analysis.

$z = j\omega L \quad \Omega/m \qquad (r = 0)$

$y = j\omega C \quad S/m \qquad (g = 0)$

$$Z_s = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \equiv \text{Surge Impedance}$$

purely resistive.

lossless Line
- A B C D par
- Z' $\frac{Y'}{2}$ model
- wave length λ
- SIL

$$\gamma = \sqrt{zy} = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = j\beta \ m^{-1}$$

purely imag.

γ: real α attenuation constant; Imag β phase constant

$\beta = \omega\sqrt{LC}$ = phase constant ; $\alpha = 0$ since there is no loss in the line.

ABCD Parameters (Lossless Line) :-

$$* \ A(x) = D(x) = \cosh(\gamma x) = \cosh(j\beta x) = \frac{e^{j\beta x} + e^{-j\beta x}}{2} = \cos(\beta x) \text{ per unit}$$

not hyp. Function

note
$$\sinh(\gamma x) = \sinh(j\beta x) = j\frac{e^{j\beta x} - e^{-j\beta x}}{2j} = j\sin(\beta x) \text{ per unit}$$

(not) hyp function

$$* \ B(x) = Z_c \sinh(\gamma x) = j Z_c \sin(\beta x)$$
$$= j\sqrt{\frac{L}{C}} \cdot \sin(\beta x) \ \Omega$$

$$* \ C(x) = \frac{\sinh(\gamma x)}{Z_c} = \frac{j\sin(\beta x)}{\sqrt{\frac{L}{C}}} \ S$$

π-model for Loss-less line →

model :-

$$\odot\quad Z' = Z_c \sinh \gamma l \quad \leftarrow j\sin(\beta l)$$

$$= j\,\underbrace{Z_c \sin(\beta l)}$$

$$= j\,X'$$

$$\odot\quad \frac{Y'}{2} = \frac{Y}{2}\,\frac{\tanh \frac{\gamma l}{2}}{\gamma l/2} \quad (\gamma = j\beta) \quad = \frac{Y}{2}\,\frac{\tanh(j\,\beta l/2)}{j\,\beta l/2}$$

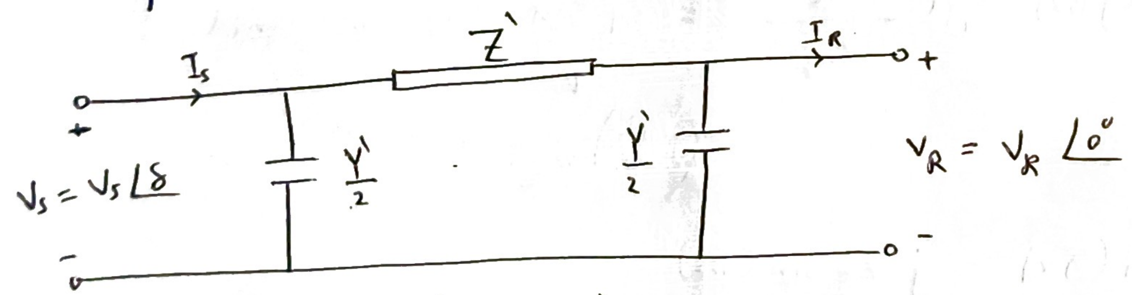
$$= \frac{Y}{2}\,\frac{\sinh(j\,\beta l/2)}{(j\frac{\beta l}{2})\cosh(\frac{j\beta l}{2})}$$

$$= \left(\frac{j\omega C l}{2}\right)\frac{\cancel{j}\,\sin(\beta l/2)}{(\cancel{j}\frac{\beta l}{2})\,\cos(\beta l/2)}$$

$$= \frac{j\omega C l}{2}\,\frac{\tan(\beta l/2)}{\beta l/2}$$

$$= \frac{j\omega C' l}{2}$$

π- Equivalent Circuit ((Loss Less Line)) :-



$$Z' = (j\omega L l)\left(\frac{\sin \beta l}{\beta l}\right) = j X' \ \Omega$$

$$\frac{Y'}{2} = \left(\frac{j\omega C l}{2}\right)\frac{\tan(\beta l/2)}{(\beta l/2)} = \frac{j\omega C' l}{2} \ S$$

For a lossless line :-

$$V(x) = A(x)\,V_R + B(x)\,I_R$$
$$= \cos(\beta x)\,V_R + j\,Z_c \sin(\beta x)\,I_R$$

$$I(x) = C(x)\,V_R + D(x)\,I_R$$
$$= \frac{j\sin(\beta x)}{Z_c}\,V_R + \cos(\beta x)\,I_R$$

Wave Length ((Loss Less Line)) :- A wavelength is the distance required to change the phase of the voltage or current by 2π radians or $360°$.

$$v = \frac{w}{\beta} = \frac{2\pi f}{\beta}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{w\sqrt{LC}} = \frac{1}{f\sqrt{LC}} \quad m$$

* The expression for the inductance per unit length L and capacitance per unit length C of a transmission line were derived in previous chapter. When the internal flux linkage of a conductor is neglected $GMR_L = GMR_C$

$$r' = r$$

$$\lambda \cong \frac{1}{f\sqrt{\mu_o \varepsilon_o}}$$

$\mu_o = 4\pi * 10^{-7}$

$\varepsilon_o = 8.85 * 10^{-12}$

$\Rightarrow \lambda = 6000\,km$, for $50\,Hz$

$$\Rightarrow f\lambda = v = \frac{1}{\sqrt{LC}} \cong \frac{1}{\sqrt{\mu_o \varepsilon_o}} \cong 3 * 10^{8}\ m/sec.$$

$\equiv$ Velocity of propagation of voltage and current waves on loss-less Line

Surge Impedance Loading :- (SIL) is the power delivered by a lossless line to a load resistance equal to the surge impedance Z_c.

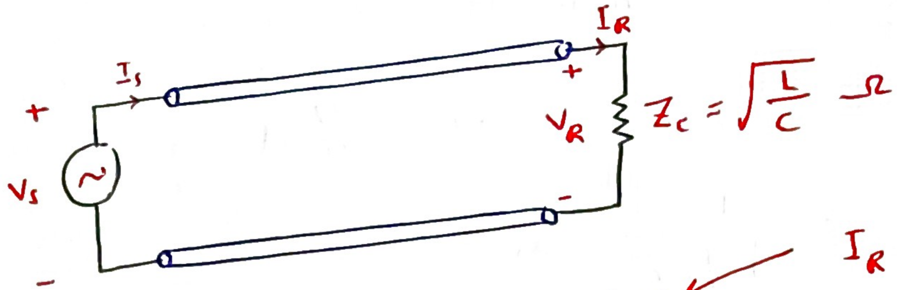
$$\boxed{Z_c = \sqrt{L/C}}$$

* $V(x) = A(x) V_R + B(x) I_R$

$$= \cos(\beta x) V_R + j Z_c \sin(\beta x) I_R$$

* $I(x) = C(x) V_R + D(x) I_R$

$$= j\frac{\sin(\beta x)}{Z_c} V_R + \cos(\beta x) I_R$$



$I_R = \frac{V_R}{Z_c}$

① $V(x) = \cos(\beta x)\, V_R + j\, Z_c \sin(\beta x)\, I_R$

$= \cos(\beta x)\, V_R + j\, Z_c \sin(\beta x) \left(\frac{V_R}{Z_c}\right)$

$= \underbrace{\left[\cos(\beta x) + j \sin(\beta x)\right]}_{e^{j\beta x}} V_R$

$= e^{j\beta x}\, V_R$ volts.

$|V(x)| = |V_R|$ volts ; Voltage is constant along the line.

② $I(x) = j \frac{\sin(\beta x)}{Z_c} V_R + \cos(\beta x) \frac{V_R}{Z_c}$

$= \left[\cos \beta x + j \sin \beta x\right] \frac{V_R}{Z_c}$

$= \left[e^{j\beta x}\right] \frac{V_R}{Z_c}$ A.

$S(x) = P(x) + j\, Q(x) = V(x)\, I^*(x)$

$= \left[e^{j\beta x}\, V_R\right]\left[\frac{e^{j\beta x}\, V_R}{Z_c}\right]^*$

$= \frac{|V_R|^2}{Z_c}$; Real power along the line is constant and reactive power flow is zero.

• at rated line voltage

$SIL = \frac{V_{rated}^2}{Z_c}$

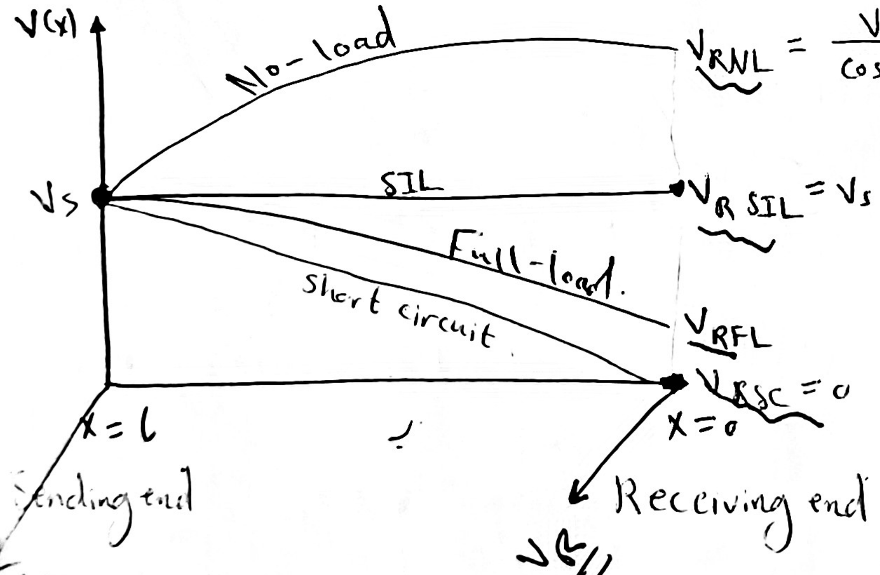
$\boxed{L-L}$

Vrated (kv)	$Z_c = \sqrt{L/C}$	$SIL = V_{rated}^2 / Z_c$ (MW)
230	380	140
345	285	420
500	250	1000
765	257	2280

Voltage Profiles:-

$V_{NL}(x) = [\cos(\beta x)]\, V_{RNL}$

$V_{SC}(x) = \bar{Z}_c \sin\beta x \; I_{RSC}$



, $I_{R\,NL} = 0$

$V_{NL}(x) = (\cos\beta x)\, V_{RNL}$

Voltage profiles of an uncompensated lossless line with fixed sending end voltage.

example 5.5
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$$\boxed{V(x) = \cos(\beta x)\, V_R + j Z_c \sin(\beta x)\, I_R}$$

① no-load, $I_{RNL} = 0$

$V_{NL}(x) = \cos(\beta x)\, V_{RNL}$

$V_{RNL} = \frac{V_S}{\cos(\beta l)}$

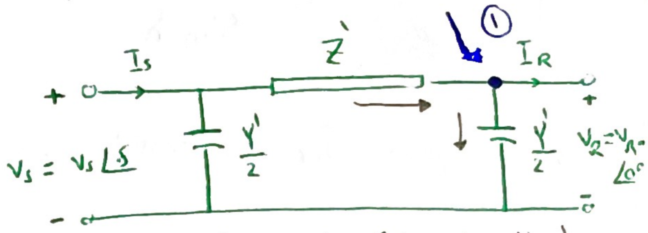
② for short circuit at the load $V_{RSC} = 0$

$V_{SC}(x) = (Z_c \sin\beta x)\, I_{RSC}$

Steady-State Stability Limit :

KCL at node (1)

$$I_R = \frac{V_S - V_R}{Z'} - \frac{Y'}{2} V_R$$



$Z' = jX'$ (loss-less line)

$\frac{Y'}{2} = j\frac{\omega c l}{2}$

$$= \frac{V_s e^{j\delta} - V_R}{jX'} - j\frac{\omega c l}{2} V_R$$

complex power at the receiving end

$$S_R = V_R I_R^* = V_R \left(\frac{V_s e^{j\delta} - V_R}{jX} \right)^* + j\frac{\omega c l}{2} V_R^2$$

$$= V_R \frac{j}{j} \left(\frac{V_s e^{-j\delta} - V_R}{-jX'} \right) + j\frac{\omega c l}{2} V_R^2$$

$$\boxed{A e^{j\theta} = A\cos\theta + jA\sin\theta}$$

$$= \frac{j V_R V_s \cos\delta + V_R V_s \sin\delta - j V_R^2}{X'} + j\frac{\omega c l}{2} V_R^2$$

real power

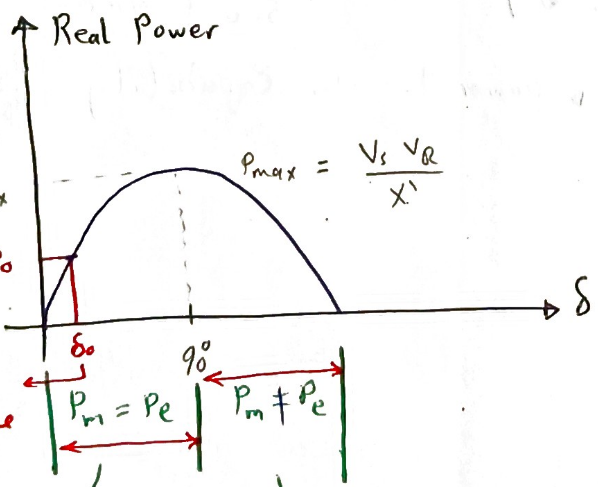
$$P = \underbrace{P_s = P_R}_{\text{loss-less line}} = Re(S_R) = \frac{V_R V_s}{X'} \sin\delta \quad W$$

real part

$\sin\delta$ → max when $\delta = 90°$

$P_{max} = \frac{V_R V_s}{X'}$ W , max power that can be transmitted over this T.L.

sync. machine connected to the system supply P_0



The power to be Transmitted

voltage angle for the machine

The machine will operate in stable region

→ The machine will be unstable.

Steady-State Stability limit
if an attempt were made to exceed this limit, then the machine would loss synchronism

notes :-

(1) $P_{max} = \frac{V_s V_R}{X'}$ ← ثابت ← نقلل

$V_s \cong V_R \cong 1$ per unit

bundel

GMR↑ , L↓ , X↓ , P_{max}↑

allow you to Transmitt more power on the T.L.

(2) mechanical input → [ ] → electrical output

input ↑ , δ ↑ , P_0 ↑

In tearms of SIL

$$P = \frac{V_R \, V_s}{X'} \sin\delta$$

real power for loss-less line.

$\odot \; Z' = Z_c \sinh \gamma l$

$= j \underbrace{Z_c \sin(\beta l)}$

$= j X'$

$\odot \; \lambda = \frac{2\pi}{\beta} \Rightarrow \beta = \frac{2\pi}{\lambda}$

$$= \frac{V_s \, V_R \, \sin\delta}{Z_c \sin \beta l}$$

$$= \left(\frac{V_s . V_R}{Z_c}\right) \cdot \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

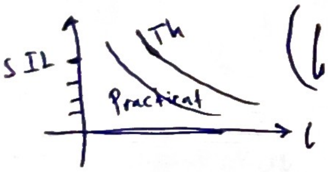
$$= \left(\frac{V_s}{V_{rated}}\right)\left(\frac{V_R}{V_{rated}}\right) \cdot \left(\frac{V^2_{rated}}{Z_c}\right) \cdot \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

$$= (V_{s.p.u})(V_{R.p.u})(SIL) \cdot \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)} \quad W$$

$$P_{max} = \frac{V_{s.p.u} \, V_{R.p.u} \, SIL}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

of t Line.

Voltage kV	SIL (MW)	Typical Thermal Rating (MW)
230	150	400
345	400	1200
500	900	2600



$(l \uparrow \; , \; P_{max} \downarrow)$

power transfer capability

Maximum Power Flow (Lossy Line):-

real / img

$$A = \cosh(\gamma l) = A \angle \theta_A$$

real / img

$$B = Z' = Z' \angle \theta_Z$$

$$I_R = \frac{V_s - A V_R}{B} = \frac{V_s e^{j\delta} - A V_R e^{j\theta_A}}{Z' e^{j\theta_Z}}$$

$$S_R = P_R + jQ_R = V_R^* I_R^* = V_R \left[\frac{V_s e^{j(\delta - \theta_Z)} - A V_R e^{j(\theta_A - \theta_Z)}}{Z'} \right]^*$$

$$= \frac{V_R V_s}{Z'} e^{j(\theta_Z - \delta)} - \frac{A V_R^2}{Z'} e^{j(\theta_Z - \theta_A)}$$

$$P_R = Re(S_R) = \underbrace{\frac{V_R V_s}{Z'} \cos(\theta_Z - \delta)}_{\text{same as previous}} - \underbrace{\frac{A V_R^2}{Z'} \cos(\theta_Z - \theta_A)}_{\text{راح نقللها هذا الجزء}}$$

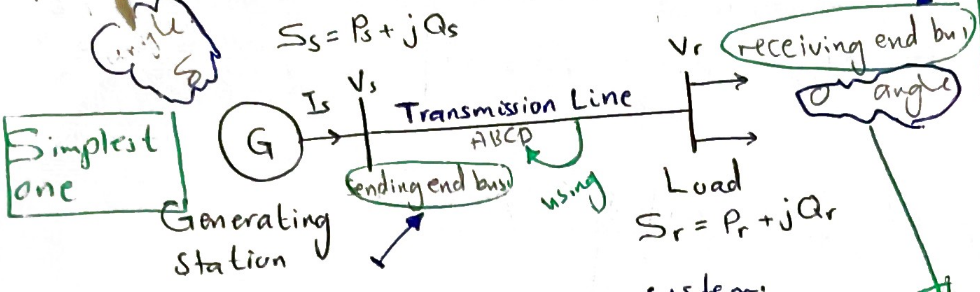
Two component

same as previous — راح نقللها
هذا الجزء

$$P_{max} \Big|_{\theta_Z = \delta}$$

Transmission Line Steady State Operation :-

[when we talk about the S.S.O on T.L what we really mean is how the Line is perform when we want to Transmit certain amount of power throug it.]



A two bus power system.

as refrence.

Power flow on transmission Lines :-

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

① $V_s = AV_R + BI_R$

$\frac{1}{B} V_s = \frac{A}{B} V_R + I_r$

$I_r = \frac{1}{B} V_s - \frac{A}{B} V_r$

② $I_s = CV_R + DI_r$

$I_s = CV_R + \frac{D}{B} V_s - \frac{DA}{B} V_r$

$= \frac{D}{B} V_s + \left(\frac{CB}{B} - \frac{DA}{B}\right) V_R$

$= \frac{D}{B} V_s - \frac{1}{B} V_R$

rec. end current $I_r = \frac{1}{B} V_s - \frac{A}{B} V_r$ ------ ①

sending end current $I_s = \frac{D}{B} V_s - \frac{1}{B} V_r = \frac{A}{B} V_s - \frac{1}{B} V_r$ ---- ②

we know that $\boxed{A = D}$

Let $V_r = |V_r| \angle 0$ (as a refrense phaser.)

$V_s = |V_s| \angle \delta$, , V_s leads V_r by δ

[δ: the angle by which the V_s leads V_r.]

complex number $D = A = |A| \angle \alpha$

complex number $B = |B| \angle \beta$

Then, from ① and ②

$$I_r = \frac{|V_s|}{|B|} \angle(\delta - \beta) - \frac{|A||V_r|}{|B|} \angle(\alpha - \beta)$$

$$I_s = \frac{|A||V_s|}{|B|} \angle(\alpha + \delta - \beta) - \frac{|V_r|}{|B|} \angle -\beta$$

The conjugates of I_r and I_s are:-

$$I_r^* = \frac{|V_s|}{|B|} \angle(\beta - \delta) - \frac{|A||V_r|}{|B|} \angle(\beta - \alpha)$$

$$I_s^* = \frac{|A||V_s|}{|B|} \angle(\beta - \alpha - \delta) - \frac{|V_r|}{|B|} \angle \beta$$

Complex Power

(a) $S_r = P_r + jQ_r = V_r I_r^*$

$$= |V_r|\angle 0\left[\frac{|V_s|}{|B|}\angle(\beta-\delta) - \frac{|A||V_r|}{|B|}\angle(\beta-\alpha)\right]$$

$$= \frac{|V_s||V_r|}{|B|}\angle(\beta-\delta) - \frac{|A||V_r|^2}{|B|}\angle(\beta-\alpha)$$

(b) $S_s = P_s + jQ_s = V_s I_s^*$

$$= |V_s|\angle\delta\left[\frac{|A||V_s|}{|B|}\angle(\beta-\alpha-\delta) - \frac{|V_r|}{|B|}\angle\beta\right]$$

$$= \frac{|A||V_s|^2}{|B|}\angle(\beta-\alpha) - \frac{|V_r||V_s|}{|B|}\angle(\beta+\delta)$$

real and reactive power

(a)

real power $P_r = \frac{|V_s||V_r|}{|B|}\cos(\beta-\delta) - \frac{|A||V_r|^2}{|B|}\cos(\beta-\alpha)$

power angle or voltage angle at the sending end / voltage phase

reactive power $Q_r = \frac{|V_s||V_r|}{|B|}\sin(\beta-\delta) - \frac{|A||V_r|^2}{|B|}\sin(\beta-\alpha)$

δ: 1 paramter variable δ, note (1) (2).

(b)

real power $P_s = \frac{|A||V_s|^2}{|B|}\cos(\beta-\alpha) - \frac{|V_r||V_s|}{|B|}\cos(\beta+\delta)$

$Q_s = \frac{|A||V_s|^2}{|B|}\sin(\beta-\alpha) - \frac{|V_r||V_s|}{|B|}\sin(\beta+\delta)$

notes

(1) for a given system voltage level V_s and V_r will be very near to the system voltage and they don't change much. (33kV, ...)

$(\beta=\delta)$ $P_{r\,max} = \frac{|V_s||V_r|}{|B|} - \frac{|A||V_r|^2}{|B|}\cos(\beta-\alpha)$

(2) β, α T.L paramters and they already there so, they are fixed.

max power which can be transmitted or recieved.

$Q_r = -\frac{|A||V_r|^2}{|B|}\sin(\beta-\alpha)$

i.e. [illegible]

For Short Line :- $\Longrightarrow$ $A = D = 1\angle 0^\circ$
$B = Z\angle\theta$

(حتى نفهم ال Conspet أوضح for power flow

Simpler)

$$P_r = \frac{|V_s||V_r|}{|Z|}\cos(\theta - \delta) - \frac{|V_r|^2}{|Z|}\cos\theta$$

$$Q_r = \frac{|V_s||V_r|}{|Z|}\sin(\theta - \delta) - \frac{|V_r|^2}{|Z|}\sin\theta$$

$$P_s = \frac{|V_s|^2}{|Z|}\cos\theta - \frac{|V_s||V_r|}{|Z|}\cos(\theta + \delta)$$

$$Q_s = \frac{|V_s|^2}{|Z|}\sin\theta - \frac{|V_s||V_r|}{|Z|}\sin(\theta + \delta)$$

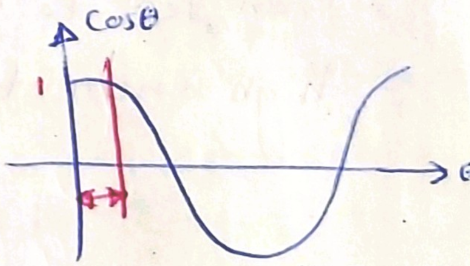
$(Z = R + jX)$

As $R \ll X$, $|Z| \approx X$ and $\theta = 90^\circ$, substituting these values in the above equations

$$\boxed{P_r = \frac{|V_s||V_r|}{X}\sin\delta}$$

$$Q_r = \frac{|V_s||V_r|}{X}\cos\delta - \frac{|V_r|^2}{X}$$

As δ is normally small ; $\cos\delta \cong 1$



$\cos 15 = 0.966$
$\cos 20 = 0.94.$

$$Q_r = \frac{|V_s||V_r|}{X} - \frac{|V_r|^2}{X}$$

$$\boxed{Q_r = \frac{|V_r|}{X}\left(|V_s| - |V_r|\right)}$$

- From these relaships we can conclude the following points

1. For fixed values of V_s, V_r and X the real power depends on angle δ the phase angle by which V_s leads V_r. This angle δ is called power angle. When $\delta = 90^\circ$ P is maximum. For system stability considerations δ has to be kept well below 90°.

لأنها هي اللي تحدد قيمة البور اللي سأنقلها من خلال T.L

range $(20^\circ - 30^\circ)$ ↑↑ disturbances will $\rightarrow 90^\circ$

2. Power can be transferred over line even when $|V_s| \lesssim |V_r|$.

The phase difference δ between V_r and V_s causes the flow of power in the line. Power systems are operated with almost the same voltage magnitudes (i.e., 1pu) at important busses by using methods of Voltage Control.

because this provides a much better operating conditions for the system

δ + V_s leads V_r $\left[V_s \xrightarrow{\text{power flow}} V_r \right.$

δ − V_s lags V_r $\left. V_r \xrightarrow{\text{power flow}} V_s \right]$

direction of power flow (δ) not $|V|$

3. The maximum real power transferred over a line increases with increase in V_s and V_r.

100% double the voltage

An increase of 100% in V_r and V_s increases the power transfer to 400%. This is the reason for adopting high and extra high transmission voltages.

تبنّي (تتخذ أسلوب أو وجهة نظر معينة)

4. The maximum real power depends on the reactance X which is directly proportional to line inductance. A decrease in inductance increases the line capacity. The line inductance can be decreased by using bundled conductors.

>> Another method of reducing line inductance is by inserting capacitance in series with the line. This method is known as series compensation. The series capacitors are usually installed at the middle of the line.

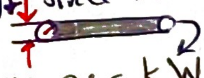
(Positive Reactance L + negative Reactance C) → effective Reactance will be ↓
series

5. The reactive power transferred over a line is directly proportional to $(|V_s| - |V_r|)$ i.e., voltage drop along the line, and is independent of power angle. This means the voltage drop on the line is due to the transfer of reactive power over the line. To maintain a good voltage profile, reactive power control is necessary.

Voltage Control

Reactive Power compensation equipment has the following effect:-

V_s, V_r ±5% (nominal value) → (10% loss in line)

1. Reduction in current. $S = P + jQ$, $Q\downarrow$, $S\downarrow$, $I\downarrow$, V = constant = nominal value
   $S = VI^*$
2. Maint~~enance~~ain ~~xy~~ voltage profile within limits.
   $Q_r = \frac{|V_r|}{X}(|V_s| - |V_r|)$
3. Reduction of losses in the system. $(I^2R)\downarrow$ since $I\downarrow$
   
   d
4. Reduction in investment (اموال، استثمار) in the system per kW of load supplied. $(Q\downarrow, \downarrow I, r\downarrow)$
5. Decrease in kVA loading of generators and lines. This decrease in kVA loading relieves (يريح، يخفف) overload condition or releases capacity (يعتق، يطلق، يحرر) for additional load growth.
6. Improvement in power factor of generators.

→ Reactive compensation of T.L. ←

1. Static Var Compensation.
2. Rotating Compensators (synchronous compensator)
3. Using Transformer. (Tap transformer)
4. Using Power Electronics (STAT COM)

Static Compensation

The performance of transmission lines, especially those of medium length and longer, can be improved by reactive compensation of a series or parallel type.

[1] Series compensation consists of a capacitor bank placed in series with each phase conductor of the line. Series compensation reduces the series impedance of the line, which is the principal cause of voltage drop and the most important factor in determining the maximum power which the line can transmit.

رئيسي

[2] Shunt compensation refers to:

(a) The placement of inductors from each line to neutral to reduce partially or completely the shunt susceptance of a high-voltage line, which is particularly important at light loads when the voltage at the receiving end may otherwise become very high. ((Shunt Reactors))

(b) Shunt Capacitors are used for lagging power factor circuits created by heavy loads. The effect is to supply the requisite reactive power to maintain the receiving end voltage at satisfactory level.

example A 50 Hz, 138 kV, 3-phase transmission Line is 200 km long
The distributed line parameters are

$R = 0.1\ \Omega/km$

$L = 1.2\ mH/km$

$C = 0.01\ \mu F/km$

$G = 0$

3φ power

P_r V_r

The transmission line delivers 40 MW at 132 KV with 0.95 power factor lagging. Find the sending end voltage and current, and also the transmission line efficiency.

Solution: For the given values of R, L and C, we have for $\omega = 2\pi(50)$,

$$z = 0.1 + j\underset{(\omega L)}{0.377} = 0.39\angle 75.14^\circ\ \Omega/km.$$

$$y = j\underset{(j\omega C)}{3.14} * 10^{-6} = 3.14 * 10^{-6}\angle 90\ \mho/km.$$

$$V_1 = V_2 \cosh\gamma l + Z_c I_2 \sinh\gamma l$$

$$I_1 = I_2 \cosh\gamma l + \left(\frac{V_2}{Z_c}\right)\sinh\gamma l$$

From the above values

$$Z_c = \sqrt{(z/y)} = 352.42\angle -7.43^\circ\ \Omega$$

$$\gamma l = 200\sqrt{zy} = 0.2213\angle 82.57^\circ = 0.0286 + j\,0.2194$$

$$\sinh\gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = 0.2195\angle 82.67^\circ$$

$$\cosh\gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = 0.975\angle 0.37^\circ$$

The values of power and voltage specified in the problem refers to 3-phase and line-to-line quantities.

$\boxed{V_{LN}}$
$$|V_2| = 132/\sqrt{3} = 76.2\ kV$$

also, using V_2 as reference; $\angle V_2 = 0^\circ$, we get

$$V_2 = 76.2\angle 0^\circ\ kV$$

Note:

$$\cosh(\gamma l) = \cosh(\alpha l) * \cos(\beta l) + j\sinh(\alpha l) * \sin(\beta l)$$
$$\sinh(\gamma l) = \sinh(\alpha l) * \cos(\beta l) + j\cosh(\alpha l) * \sin(\beta l)$$

Now, per phase power supplied to the load.

$$P_{load} = \frac{40}{3} = 13.33 \text{ MW}.$$

Given the value of power factor = 0.95, we can find I_2

$$P_{load} = 0.95 \, |V_2| \cdot |I_2|$$

Thus, $|I_2| = 184.1$

Also, since I_2 lags V_2 by $\cos^{-1} 0.95 = 18.195^\circ$,

$$I_2 = 184.1 \angle -18.195^\circ$$

Finally, we have:-

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$$

$$V_1 = 82.96 \angle 8.6 \text{ kV}$$

Sending end voltage

L-to-N voltage at the sending end.

Similarly,

$$I_1 = I_2 \cosh \gamma l + \left(\frac{V_2}{Z_c}\right) \sinh \gamma l$$

$$= 179.46 \angle 17.79$$

Sending end current

*Power P_s

We now calculate the efficiency of transmission.

Perphase input power, $P_{in} = Re(V_1 \overset{*}{I_1})$

$$= 14.69 \text{ MW}$$

Hence, $\eta = \frac{13.33}{14.69} = 0.907.$

That is, the efficiency of transmission is 90.7%.

example

A 3 phase 132 kV overhead line delivers 60 MVA (S_r) at 132 kV (V_r, V_2) and power factor (PF_r) 0.8 lagging at its receiving end. The constants of the line are $A = 0.98\angle 3^\circ$ and $B = 100\angle 75^\circ$ ohms per phase. Find

(a) sending end voltage and power angle.

(b) sending end active and reactive power.

(c) line losses and vars absorbed by the line.

(d) and (e)

Solution :- (a)

V_{LN} (phase voltage) — refrence voltage.

$$V_r = \frac{132000}{\sqrt{3}} = 76210\angle 0^\circ$$

$$I_r = \left[\frac{60 \times 10^6}{3}\right] \Big/ \left[\frac{132000}{\sqrt{3}}\right]$$

$S_r = V_r I_r^*$

$$\boxed{I_r = 262.43\angle -36.87^\circ} \leftarrow -\cos^{-1} p.F$$

$$V_s = A \cdot V_r + B \cdot I_r$$

$$= (0.98\angle 3^\circ)(76210\angle 0) + (100\angle 75^\circ)(262.43\angle -36.87^\circ)$$

$$= 97.33 \times 10^3 \angle 11.92^\circ \ \text{V}$$

L-L

* Sending end Line voltage $= (\sqrt{3})(97.33)$ kV $= 168.58$

* Power angle $(\delta) = 11.92^\circ$

(d) capacity of static compensation equipment at the receiving end to reduce the sending end voltage to 145 kV for the same load conditions. part (a) $V_s \downarrow$ (we need to reduce)

$= 145$ kV ——— 132 kV

(e) The unity power factor load which can be supplied at the receiving end with 132 kV as the line voltage at both the ends. 132 kV ——— 132 k } purely resistive load.

Load
→ Power
→ voltage
→ p.f.

We have 3 phase power given as

$$S_s = \frac{|A||V_s|^2}{|B|} \angle(\beta - \alpha) - \frac{|V_r||V_s|}{|B|} \angle(\beta + \delta)$$

L-L

$$= \frac{(0.98) * (168.58)^2}{(100)} \angle(75 - 3^\circ) - \frac{(132)(168.58)}{(100)} \angle(75^\circ + 11.92^\circ)$$

L-L

$$= 278.49 \angle 72^\circ - 222.53 \angle(86.92^\circ)$$

notes :-

$\boxed{\text{3-}\phi \text{ power} \Rightarrow V_s \text{ and } V_r \text{ L-L voltages}}$

$\Rightarrow$ Sending end active power

$$P_s = 278.49 \cos(72^\circ) - 222.53 \cos(86.92^\circ)$$

$$= 86.06 - 11.96 = 74.10 \text{ MW}$$

$\boxed{\text{1-}\phi \text{ power} \Rightarrow V_s \text{ and } V_r \text{ L-N voltage}}$

$\Rightarrow$ Sending end reactive power

$$Q_s = 278.49 \sin 72^\circ - 222.53 \sin 86.92^\circ$$

$$= 264.89 - 222.21$$

$$= 42.65 \text{ MVar Lagging.}$$

((c)) * Line Losses $= P_s - P_r$

$$= 74.10 - \underbrace{60 * 0.8}_{48 \text{ MW}}$$

$$= 26.10 \text{ MW}$$

* MVar absorbed by line $= Q_s - Q_r$

$\sin\theta$; $\theta = \cos^{-1} \text{pf}$

$$= 42.65 - \underbrace{60 * 0.6}_{36 \text{ MVar}}$$

$$= 6.65 \text{ MVar.}$$

(d) $P_r = 60 * 0.8 = 48$ MW

$|V_s| = 145$ KV

$|V_r| = 132$ KV

$$* \quad P_r = |V_s||V_r||B|^{-1}\cos(\beta-\delta) - |A||V_r|^2|B|^{-1}\cos(\beta-\alpha)$$

? for this particular operating conditions.

$$48 = \frac{(145)(132)}{100}\cos(\beta-\delta) - \frac{(0.98)(132)^2}{(100)}\cos(75-3^\circ)$$

$$48 = 191.4\cos(\beta-\delta) - 170.75\cos(72)$$

$$\cos(\beta-\delta) = 0.5275$$

β ✓

δ ?

$$\angle(\beta-\delta) = \cos^{-1}(0.5275) = 58.16^\circ$$

$$* \quad Q_r = |V_s||V_r||B|^{-1}\sin(\beta-\delta) - |A||V_r|^2|B|^{-1}\sin(\beta-\alpha)$$

$$= \frac{(145)(132)}{(100)}\sin(58.16) - \frac{(0.98)(132)^2}{(100)}\sin(72^\circ)$$

$$= 162.60 - 162.40$$

$$= 0.20 \text{ MVar}$$

-90

$$Q_c = V_{rms} \cdot I_{rms}\sin(\theta_v-\theta_i)$$

$$= V_{rms}\, I_{rms}\,(-1)$$

$$\boxed{Q_c = -V_{rm}\, I_{rms}}$$

Y_c

$$= -V_{rms}\left[\omega C\, V_{rms}\right]$$

$$= -V_{rms}^2\,\omega C$$

Thus for $V_s = 145$ kV, $V_r = 132$ kV and $P_r = 48$ MW, a lagging MVar of 0.2 will be supplied from the line along with the real power of 48 MW. Since the load requires 36 MVar lagging, the static compensation

(60 * sin θ)

equipment must deliver 36 − 0.2, i.e, 35.8 MVar lagging (or must absorb 35.8 MVar leading). The capacity of static capacitors is, therefore, 35.8 MVar.

$$\boxed{Q_c = -\omega C\, V_{rms}^2}$$

$|V_s| = |V_r| = 132\ kV$, $Q_r = 0$

$$Q_r = |V_s||V_r||B|^{-1}\sin(\beta-\delta) - |A||V_r|^2|B|^{-1}\sin(\beta-\alpha)$$

$$0 = \frac{(132)(132)}{(100)}\sin(\beta-\delta) - \frac{(0.98)(132)^2}{(100)}\sin(75^\circ-3^\circ)$$

$$\angle(\beta-\delta) = 68.75^\circ$$

e)

$$P_r = |V_s||V_r||B|^{-1}\cos(\beta-\delta) - |A||V_r|^2|B|^{-1}\cos(\beta-\alpha)$$

$$= \frac{(132)(132)}{(100)}[\cos(68.75)] - \frac{(0.98)(132)^2}{(100)}\cos(72^\circ)$$

$$= 63.13 - 52.77$$

$$= 10.36\ MW$$