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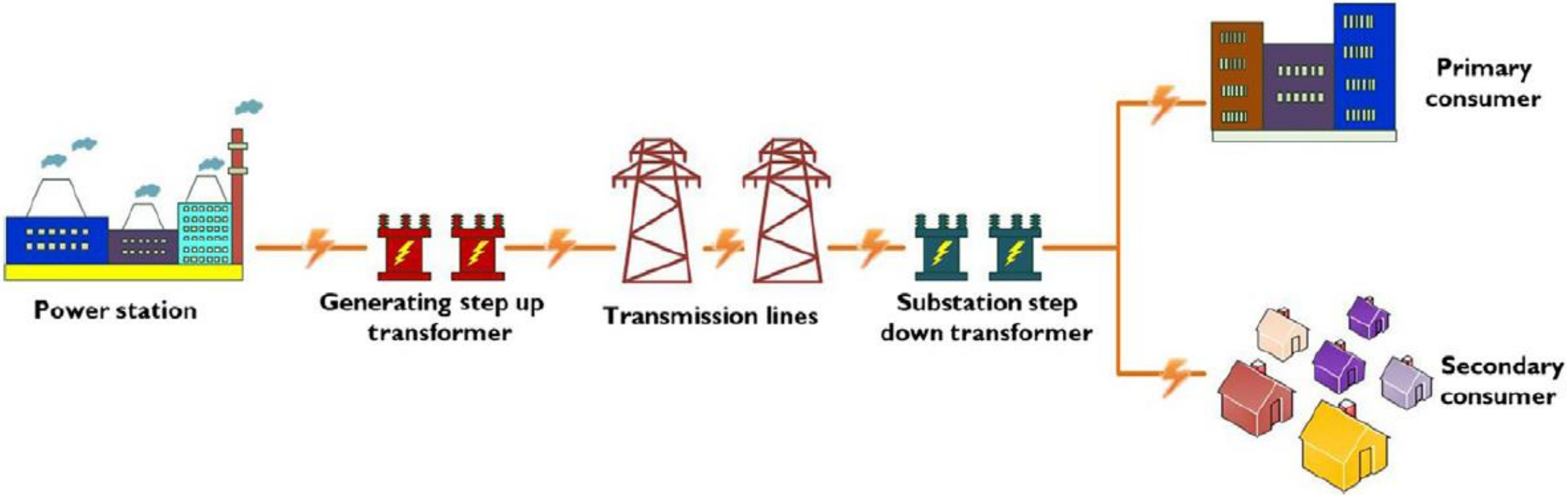
ENEE4403 - POWER SYSTEMS

By

Dr. Jaser Sa’ed

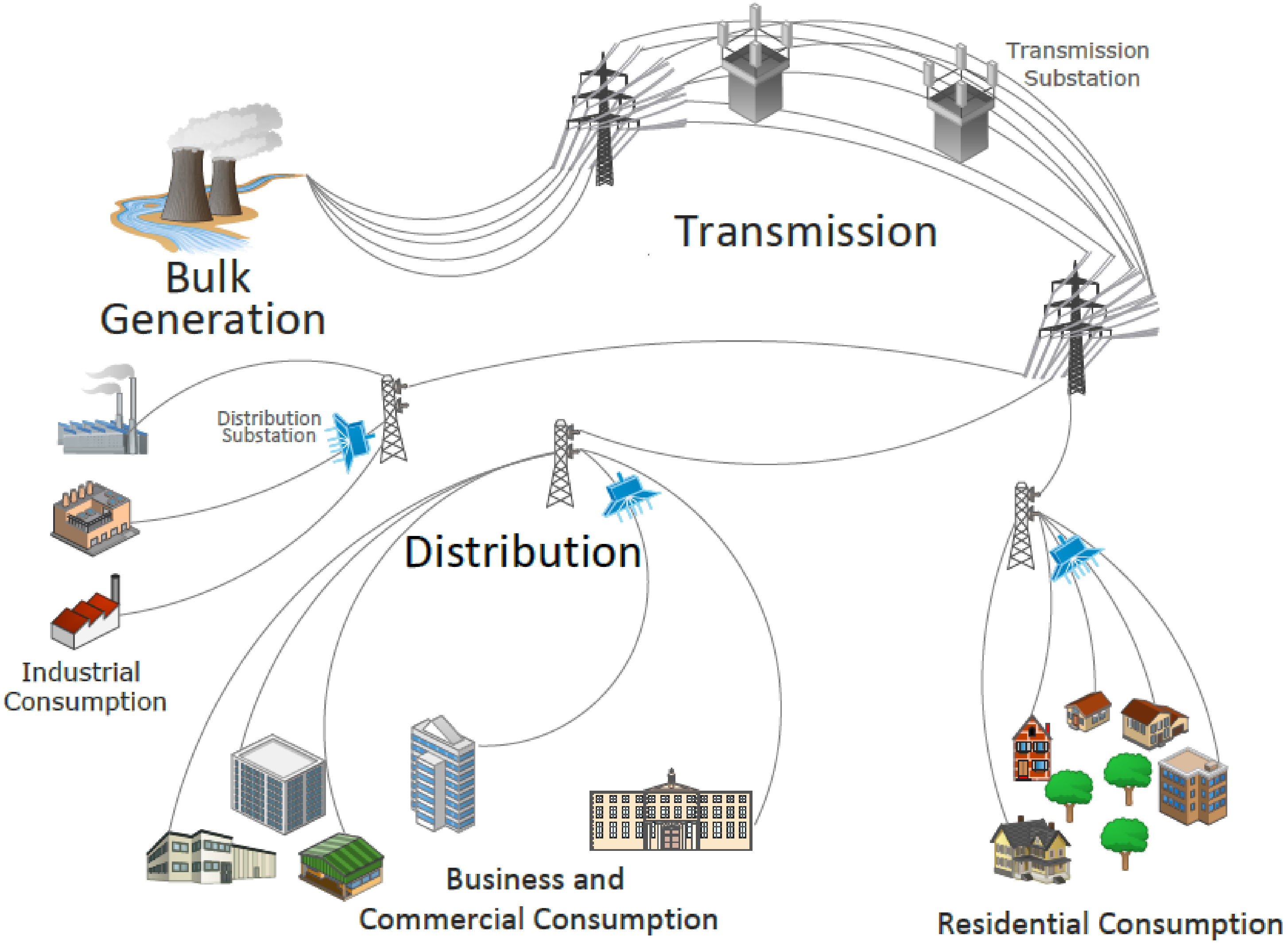
Department of Electrical and Computer Engineering

Power Systems - General Overview



Traditional electricity delivery system

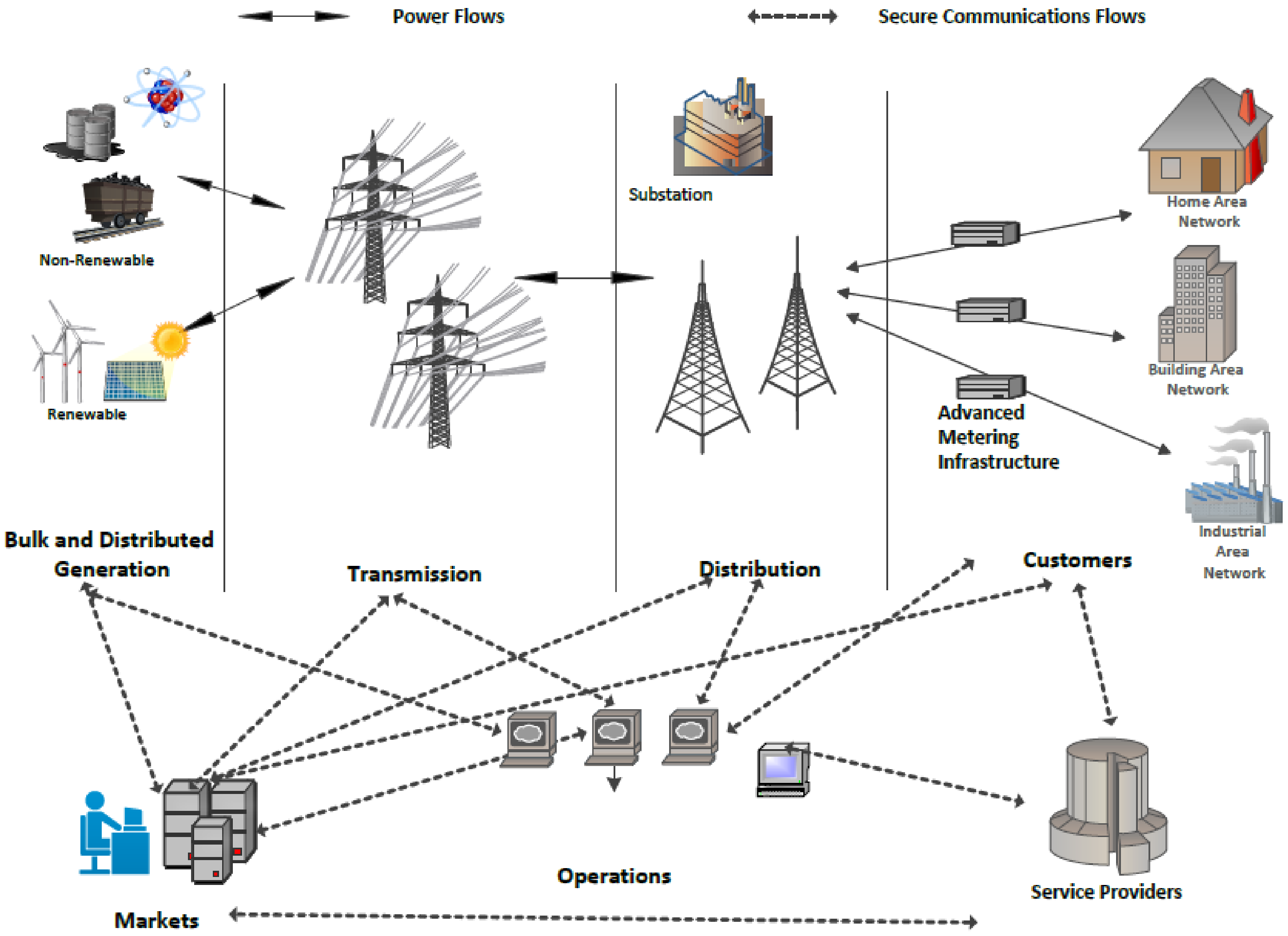
Power Systems - General Overview



A high-level structure of the current power grid

Reference : [1]

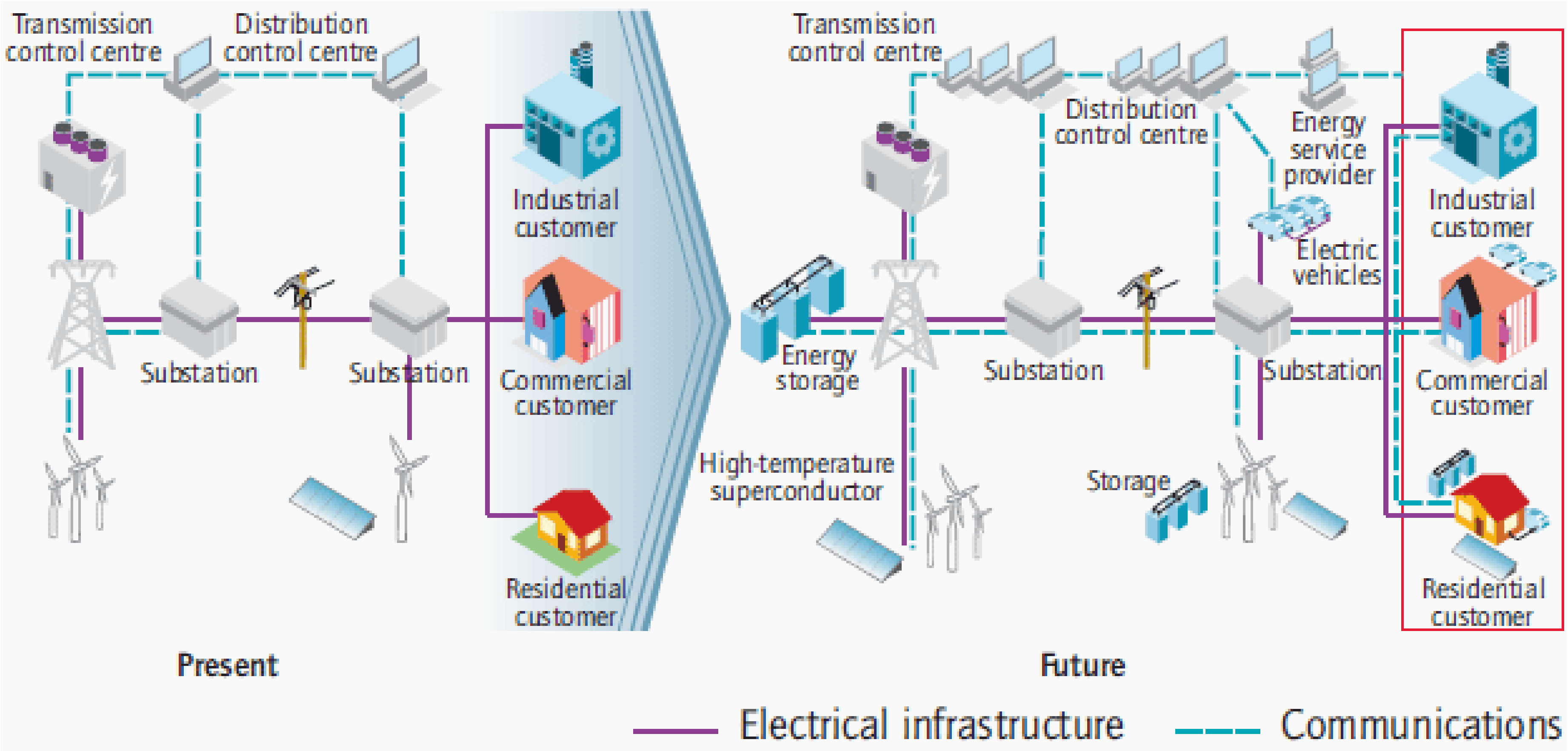
Smart Grids Technology - General Overview



Smart Grid Conceptual Model

Reference : [1]

From Conventional Grids to Smart Grids



Source: https://electrical-engineering-portal.com/

References

1. Book by: Hussein t. Mouftah and Melike Erol-Kantarci, “Smart Grid: Networking, Data Management, and Business, Models”, CRC Press, 2016, Ch. 6: pp. 117-156.
2. Book by: Akın Tascıkaraogˇlu and Ozan Erdinc, “Pathways to a Smarter Power System”, Elsevier: Academic Press, 2019, Ch. 1: pp. 1-27.

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Overview of Electrical Energy in Palestine

Dr. Jaser A. Sa’ed

General Overview

- The energy sector situation in Palestine is highly different compared to other countries in the Middle East due to many reasons: non availability of natural resources, unstable political conditions, financial crisis and high density population.

- Furthermore, Palestine depends on other countries for 100% of its fossil fuel imports and for 87% of its electricity imports.

- In addition high growth of population, increasing living standards and rapid growth of industrial have led to tremendous energy demand in Palestine in recent years.

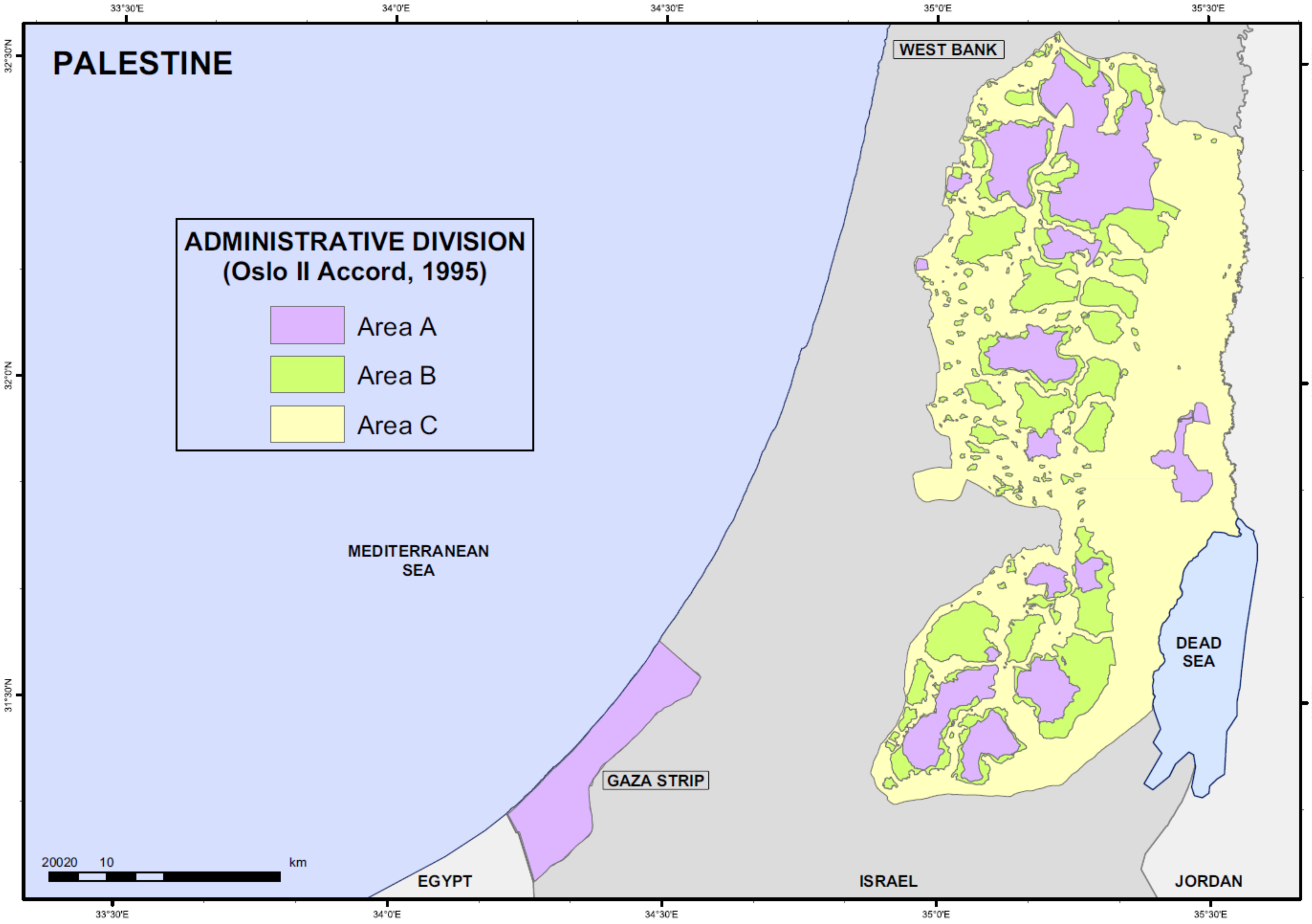
Palestinian population in 2014 by governorate

- Palestine is divided into two geographic areas: West Bank and Gaza Strip. In (2014), according to Palestinian Central Bureau of Statistics (PCBS) the population of Palestine is 4,550,368 in habitants for an area of 6020 km2, being the population density 756 people/km2, distributed as follows: West Bank 494 people/km2 , and Gaza Strip 4822 people/km2, one of the highest population density in the world.

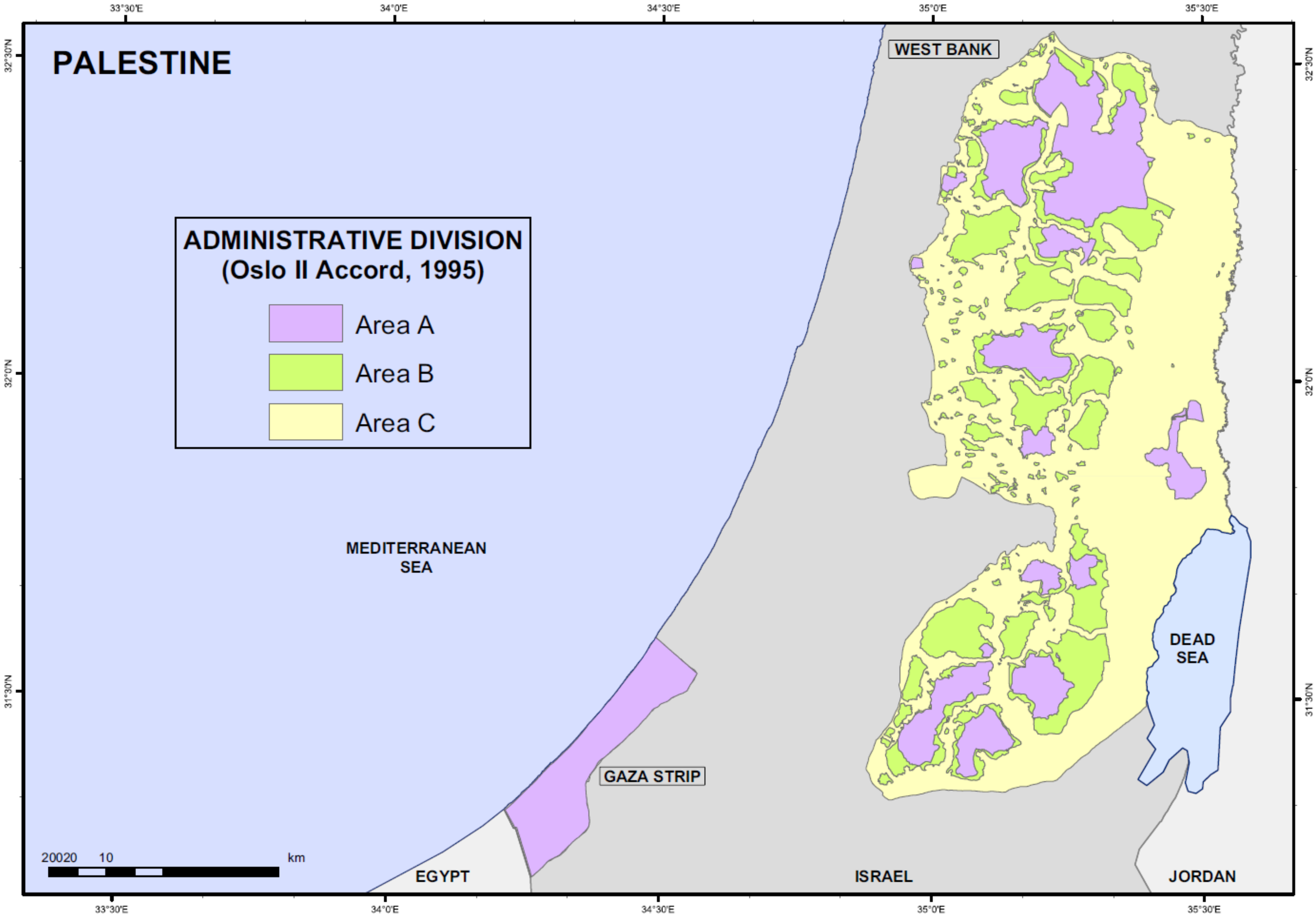
Administrative Divisions: Areas A, B and C

- The complex geographical and administrative situation of Palestine can be seen in its administrative divisions made by the Oslo II Accord in 1995, that divided West Bank into three administrative divisions: the Areas A, B and C.
- Area A indicates that full civil and security control belongs to the Palestine. Area B indicates that Palestine has civil control but security control is joint Israel and Palestine. Area C indicates that full civilian and security control is made by Israel.
- Approximately 60% of the land regions in the West Bank are classified as Area C. So, Israel control of these divisions therein severely hinders and affects the potential development of a traditional energy sector's infrastructure and regulations and policies, also hinders development initiatives

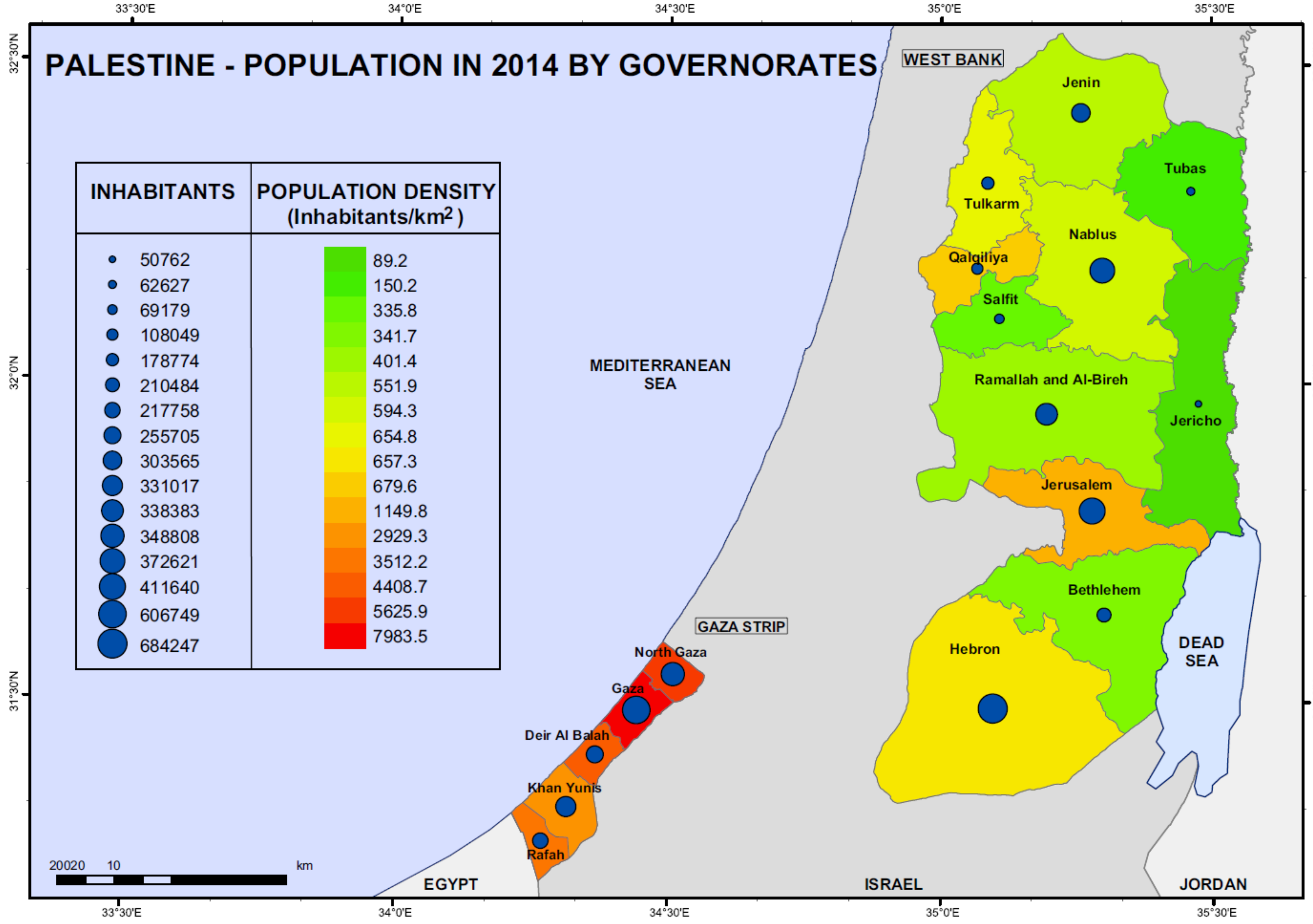
Administrative Divisions: Areas A, B and C



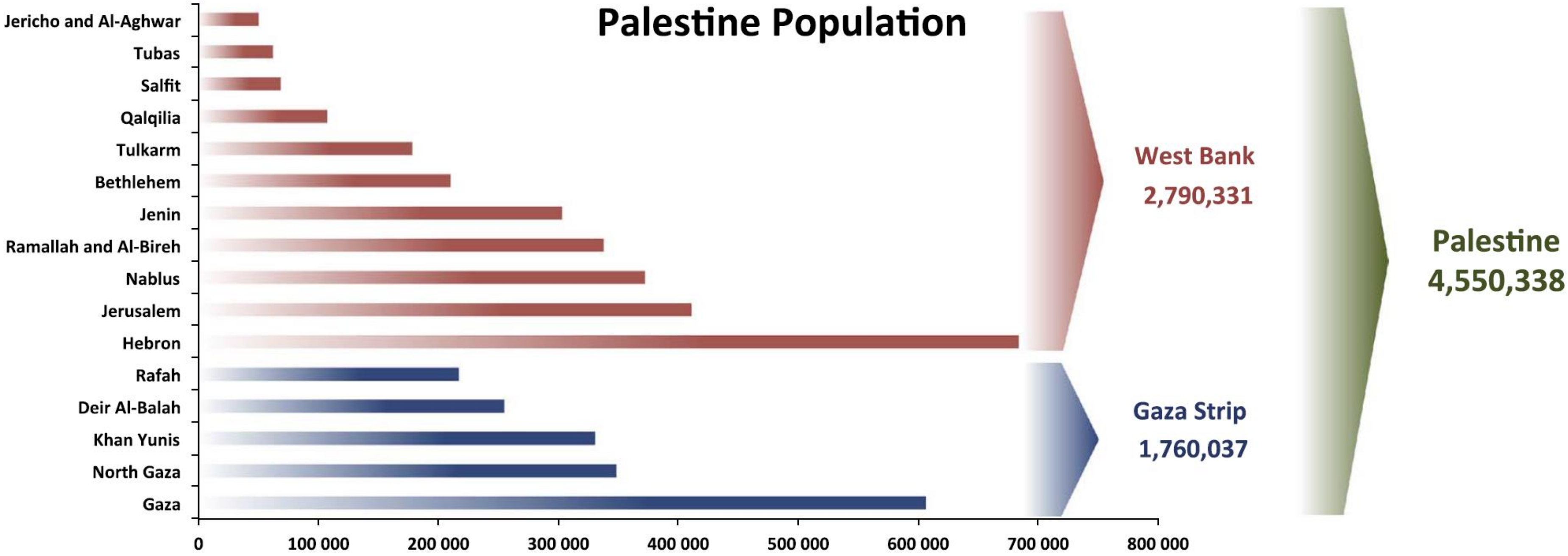
Administrative Divisions: Areas A, B and C



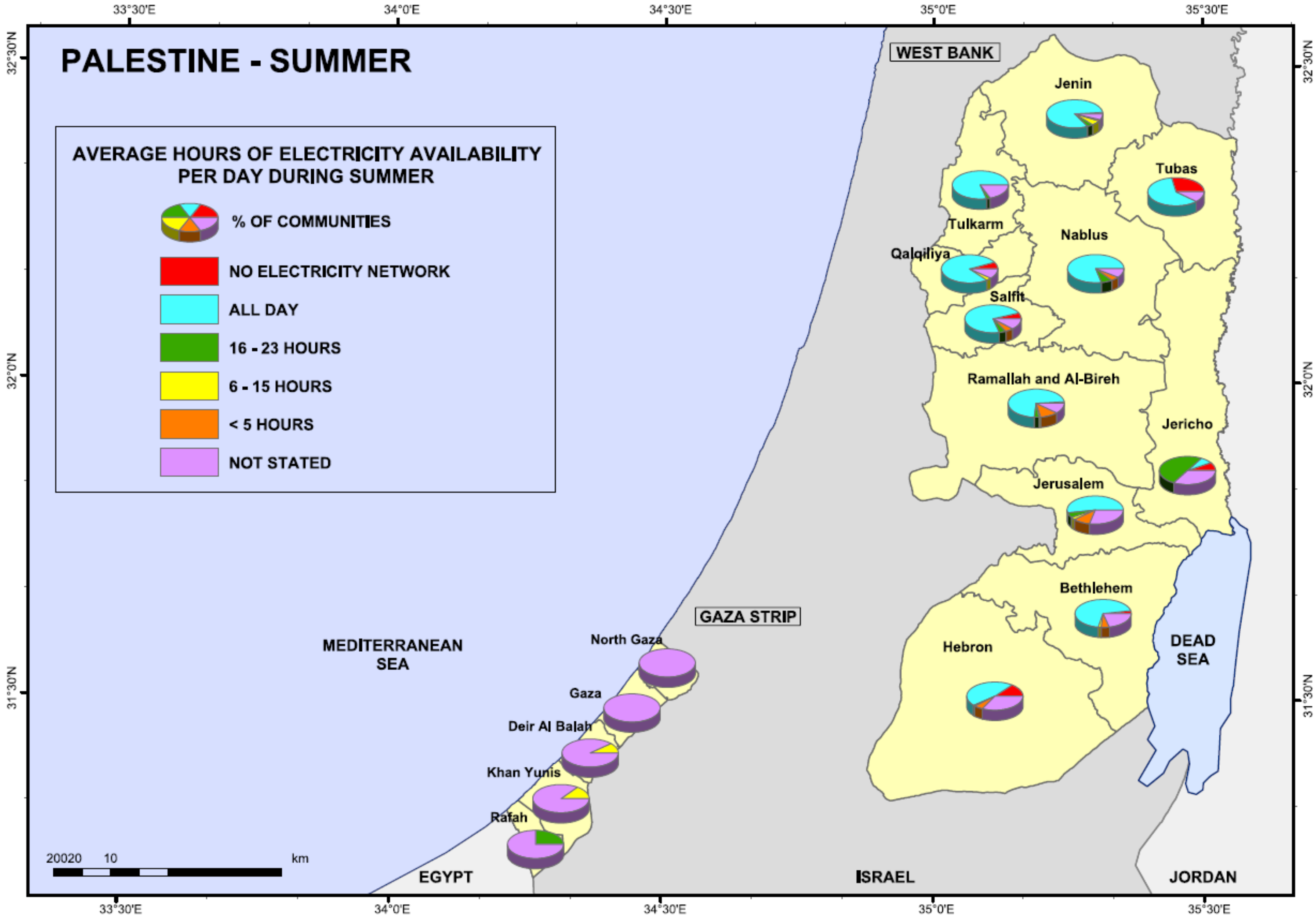
Population density in Palestine



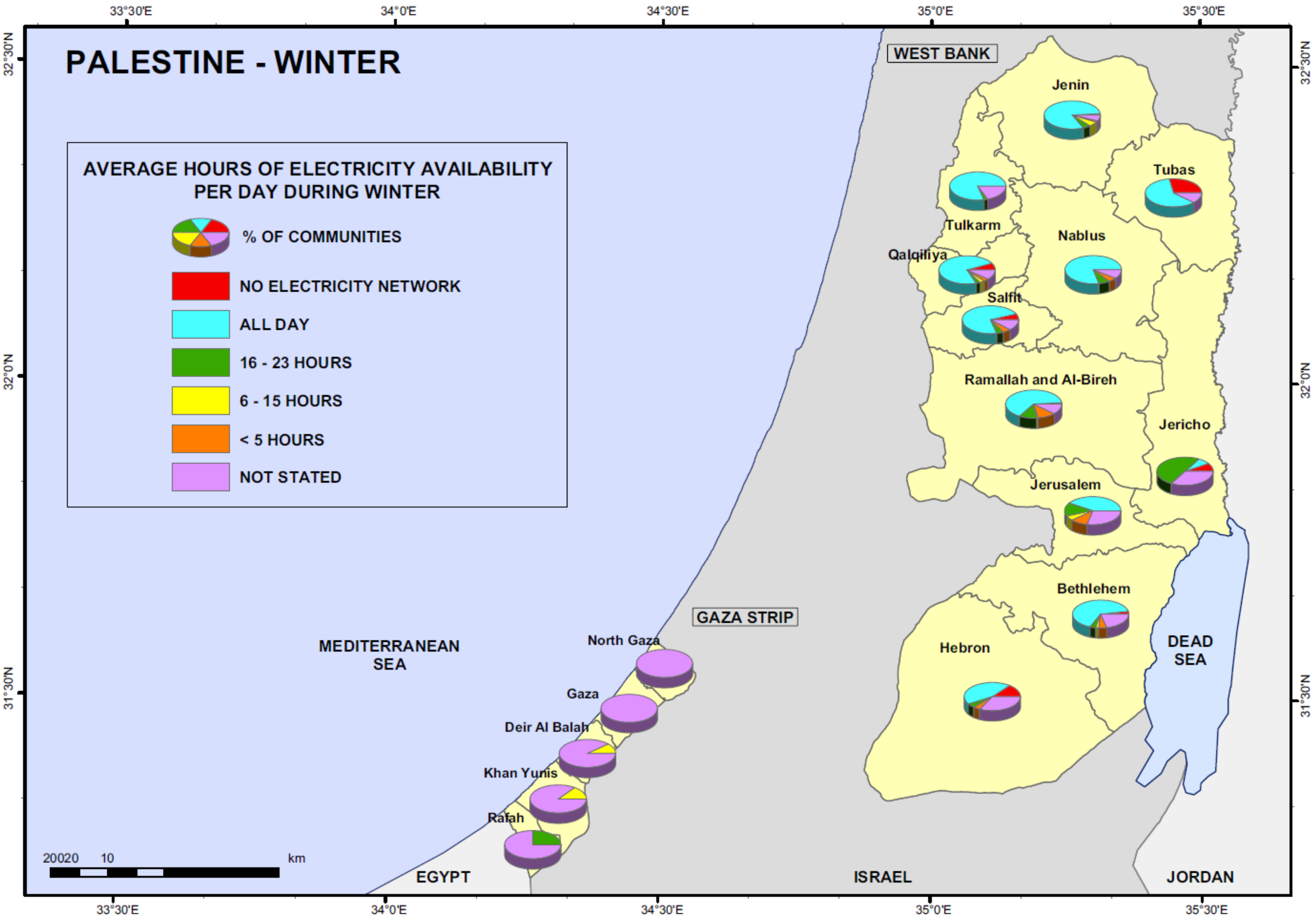
Palestinian population in 2014 by governorate



Average Hours of Electricity Availability per day in summer (2013).

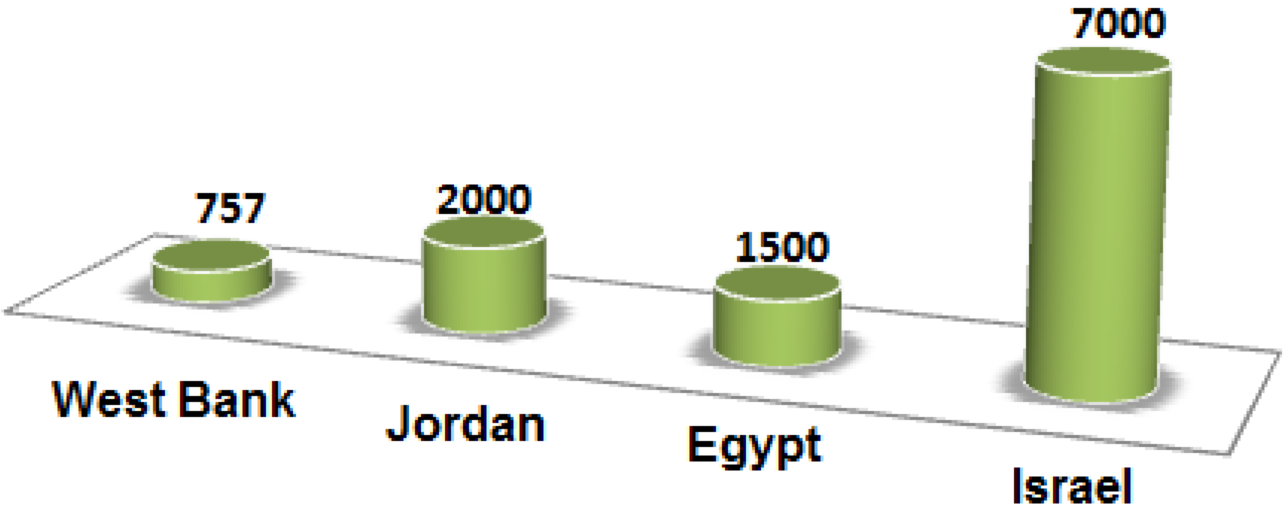


Average Hours of Electricity Availability per day in Winter (2013).



Energy Consumption

- The total energy consumption per habitant in Palestine is the lowest in the region (0.757 MW h/ inhabitant) and costs more than anywhere else in the Middle East countries.



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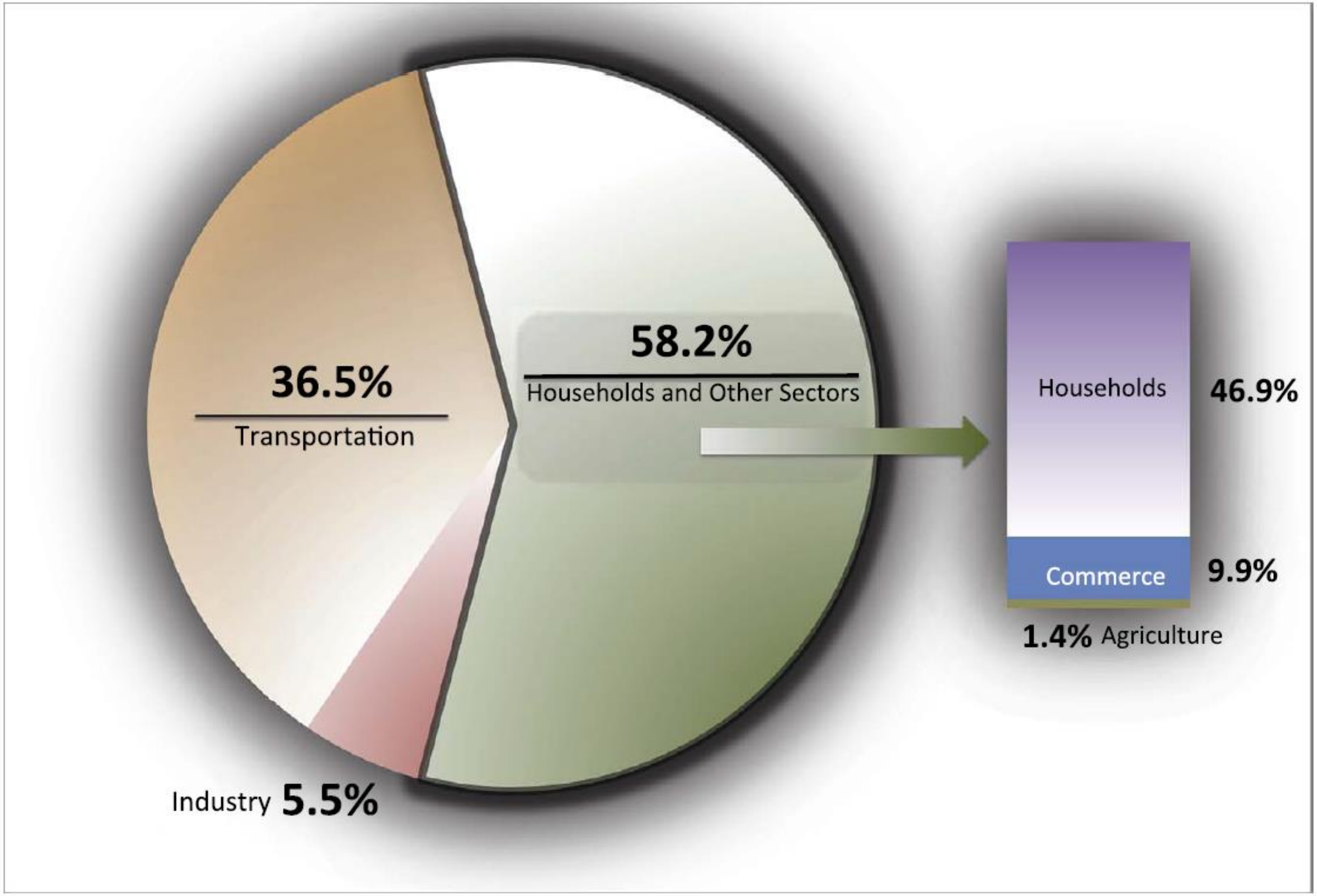
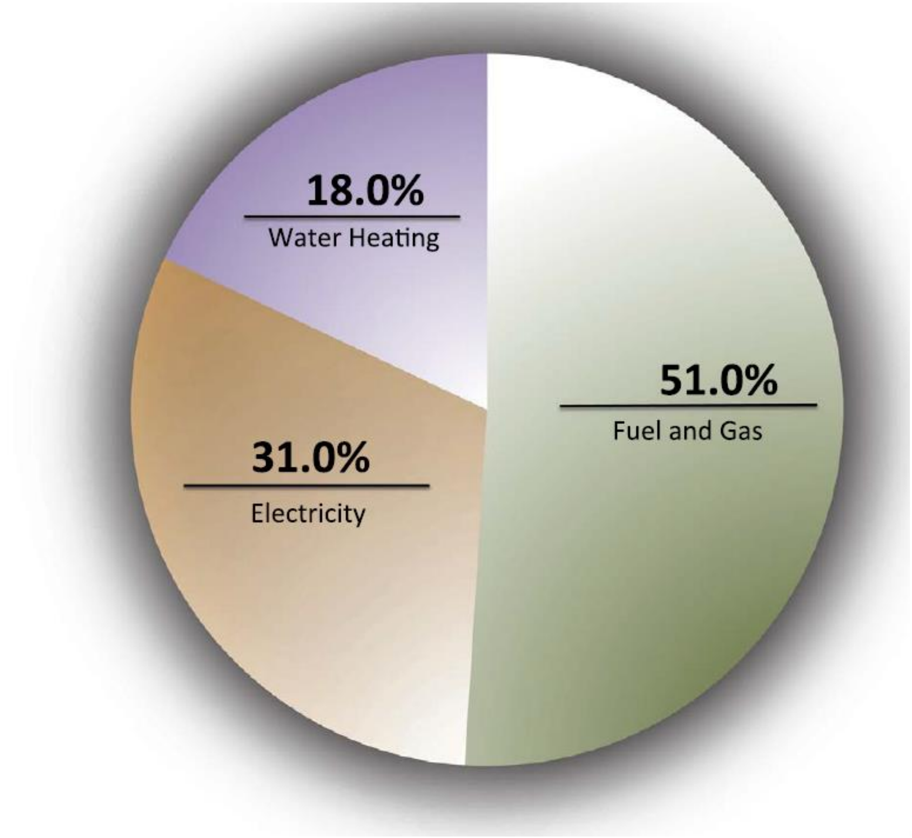
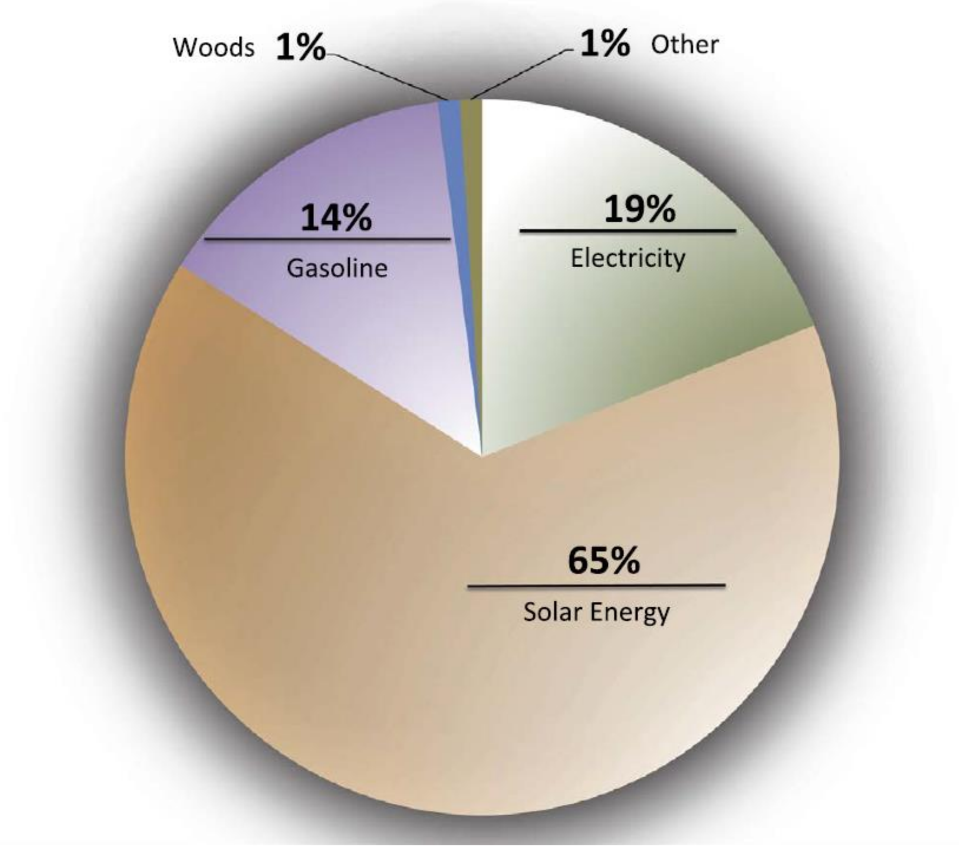
Energy consumption by sectors, 2013.

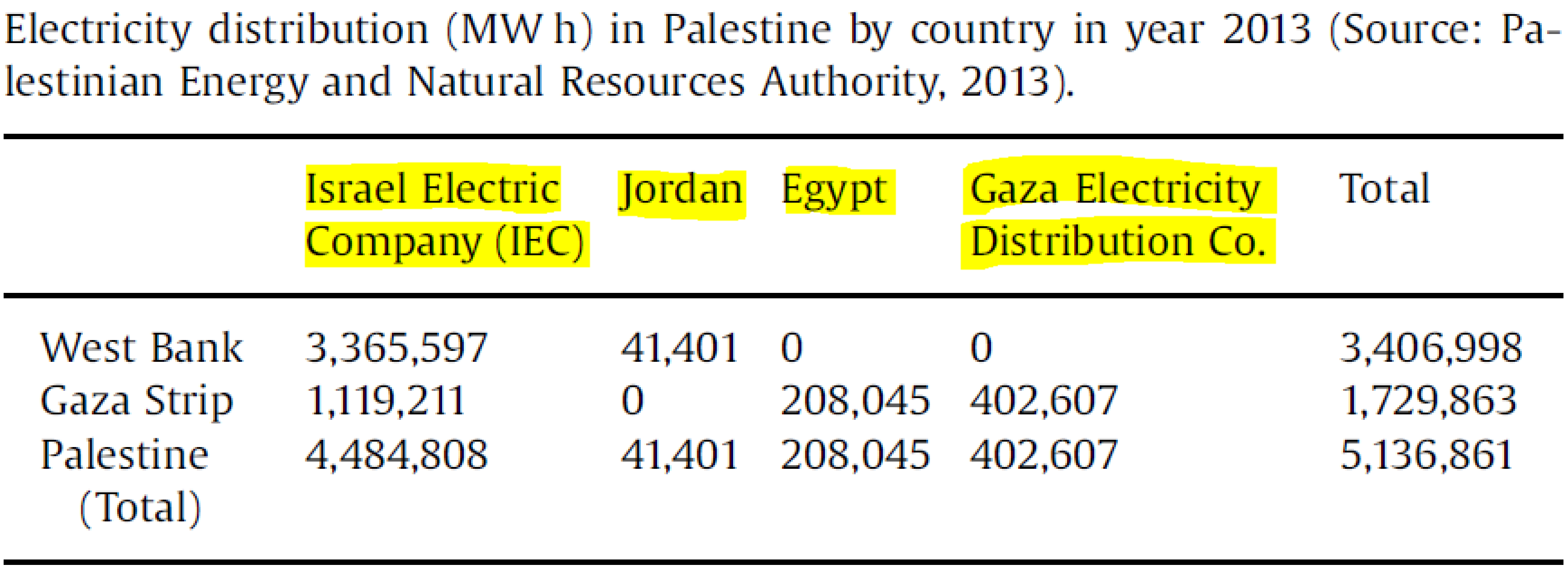




Total primary energy consumption in Palestine, 2013.

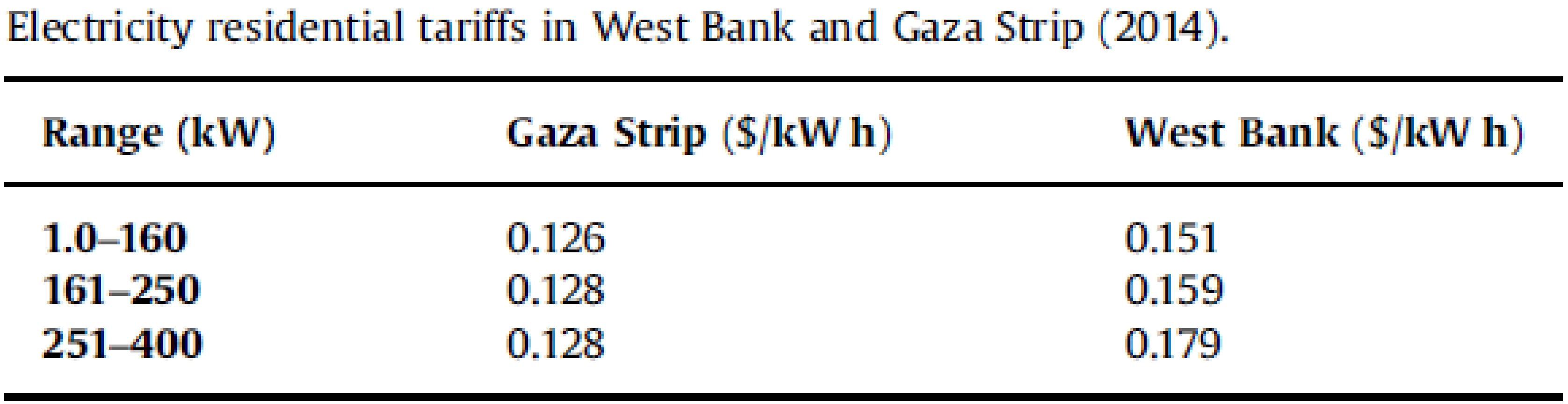


Distribution of energy consumption for water heating, 2013.



Electricity distribution (MW h) in Palestine by country in year 2013 (Source: Palestinian Energy and Natural Resources Authority, 2013).

	Israel Electric Company (IEC)	Jordan	Egypt	Gaza Electricity Distribution Co.	Total
West Bank	3,365,597	41,401	0	0	3,406,998
Gaza Strip	1,119,211	0	208,045	402,607	1,729,863
Palestine (Total)	4,484,808	41,401	208,045	402,607	5,136,861

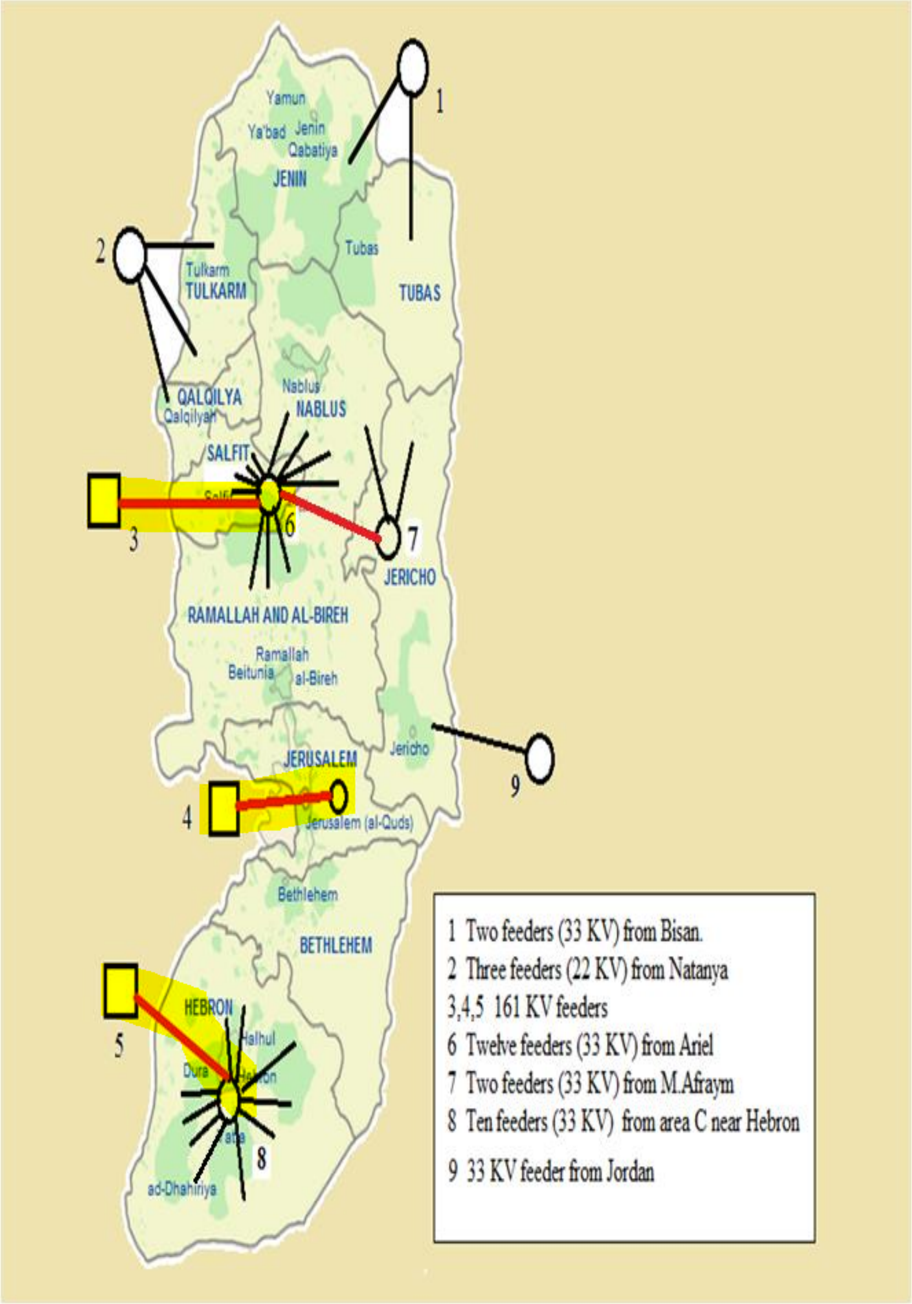


Electricity residential tariffs in West Bank and Gaza Strip (2014).

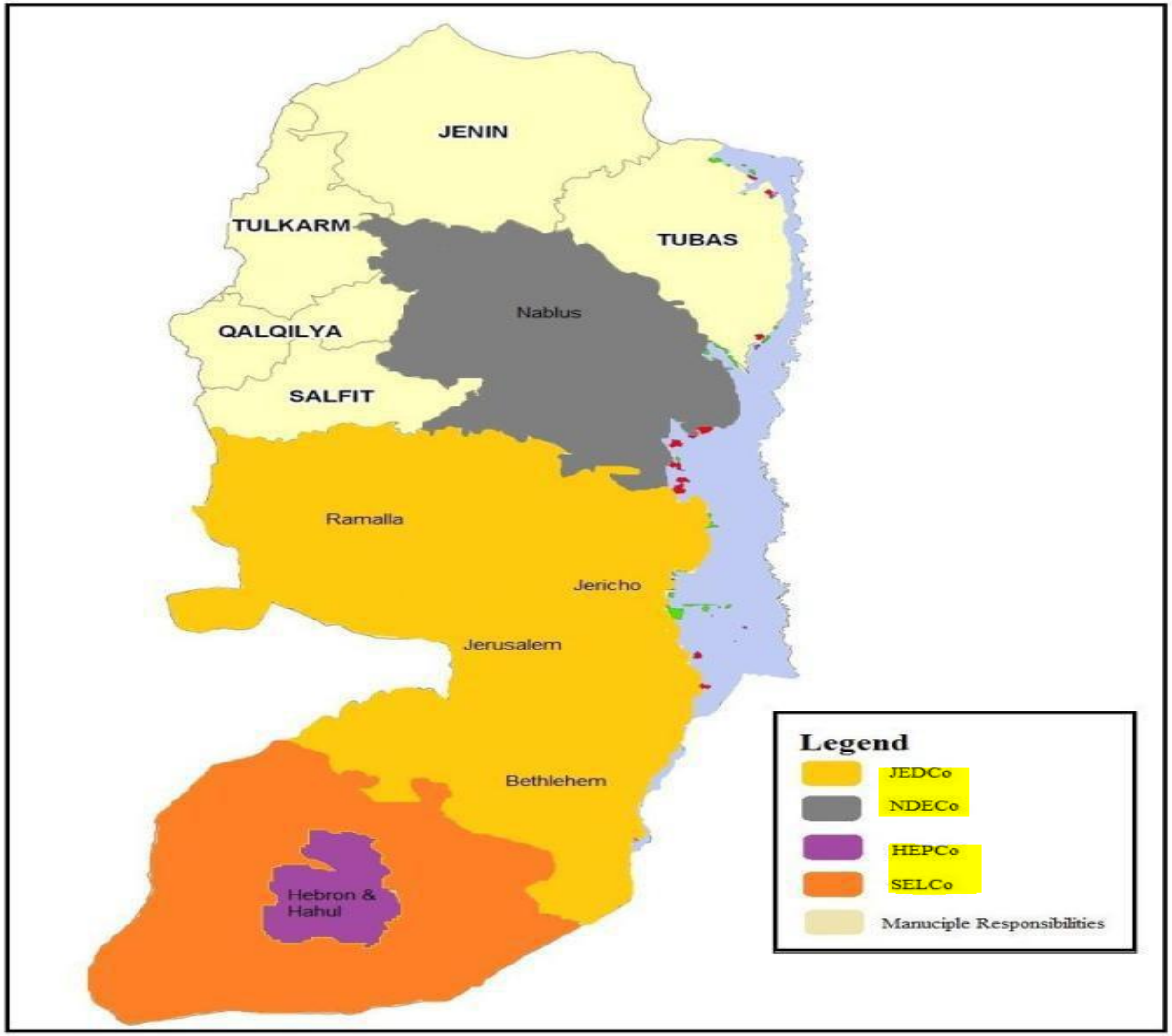
Range (kW)	**Gaza Strip ($/kW h)**	**West Bank ($/kW h)**
1.0–160	0.126	0.151
161–250	0.128	0.159
251–400	0.128	0.179

General Overview West Bank Electrical Network

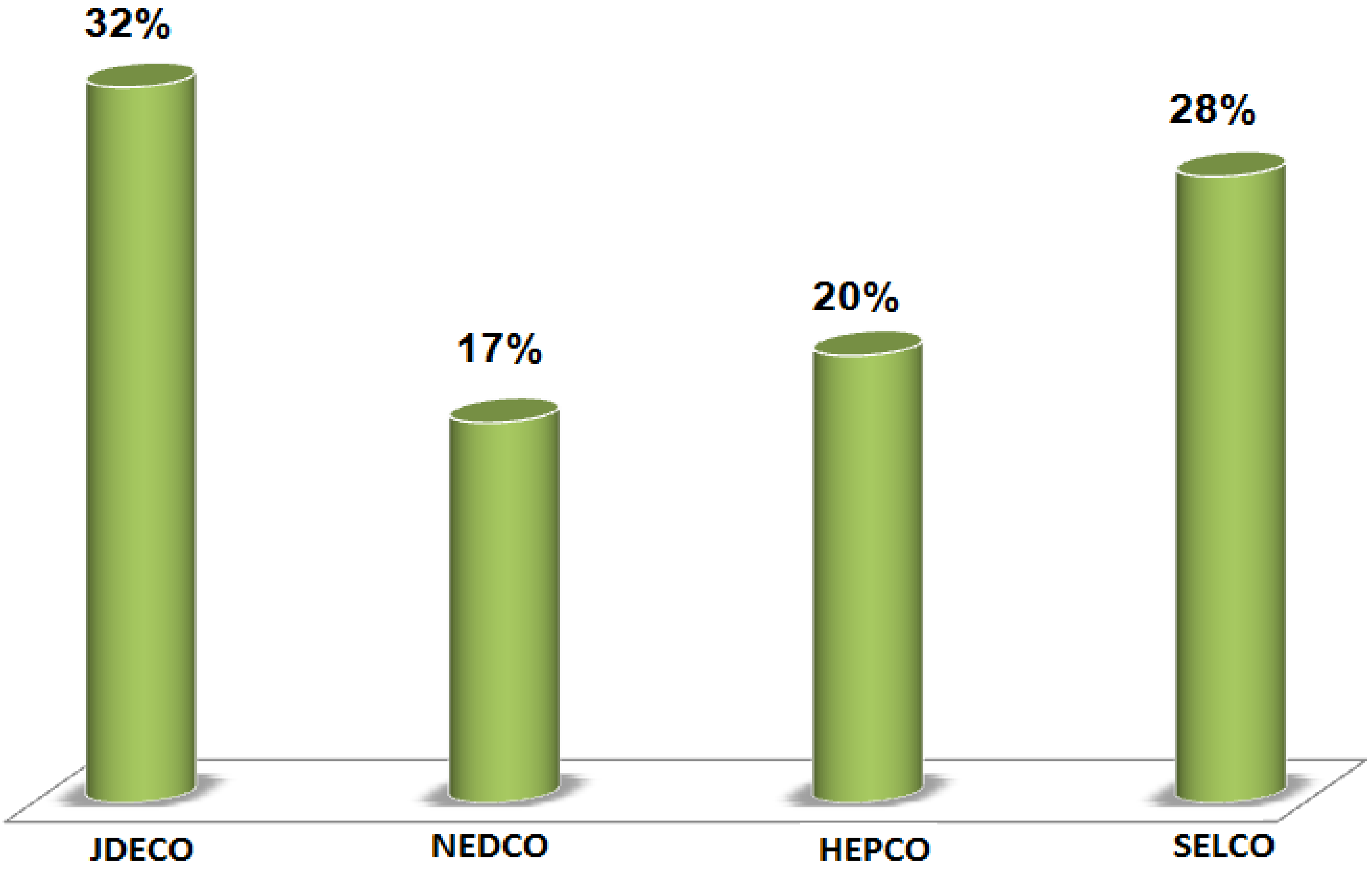
- The only main transmission lines constructed in the West Bank by IEC are three main 161 kV overhead lines feeding the three main substations: in Hebron, Qalandia (Atarot) and Salfiet (Ara'el).
- The ranges of voltage of West Bank networks are 400V, 6.6 kV, 11kv, 33 kV.



1 Two feeders (33 KV) from Bisan.
2 Three feeders (22 KV) from Natanya
3,4,5 161 KV feeders
6 Twelve feeders (33 KV) from Ariel
7 Two feeders (33 KV) from M.Afraym
8 Ten feeders (33 KV) from area C near Hebron
9 33 KV feeder from Jordan


JENIN
TULKARM
TUBAS
Nablus
QALQILYA
SALFIT
Ramalla
Jericho
Jerusalem
Bethlehem
Hebron &
Hahul
Legend
JEDCo
NDECo
HEPCo
SELCo
Manuciple Responsibilities

Power Losses



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Strengths and drawbacks of the current situation of RE in Palestine

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Strengths:

- High solar radiation.
- Palestine is geographically situated in an area with very good solar conditions. It has an average of solar irradiation of 5.4 kWh/m2/day.
- Awareness of the Palestinian government about renewable energies.
- Palestine government is in the way to develop the RE law and also creating a wind map.
- Local experience using RE.

Strengths and drawbacks of the current situation of RE in Palestine

- Solar thermal is widely used by around the country. About 70% of hot water is produced by solar thermal technology, which means people already know and rely on RE technology.
- Entrepreneurship character of the private sector.
- Significant potential contribution to cover the future energy demand increase-Electricity energy demand increases yearly for about 6%. RE can help to cover this annual increment.

Strengths and drawbacks of the current situation of RE in Palestine

Drawbacks

- No specific RE regulations defined. Since there are no regulation in the RE market, it is very difficult to create new companies and make investors establish their projects in the country.
- Energy dependency. Palestine depends on the energy imports mostly from Israel.
- Poor infrastructure. Currently the grid in Palestine it is divided into several isolated groups. It's being working for connect the different groups, and so have less points of connection with Israel and more managing capability of the energy in Palestine.

Strengths and drawbacks of the current situation of RE in Palestine

- Small of land surface availability. This is an issue for large scale RE installations. Palestine lacks of terrain, in most of its area it is not possible to build installations or it is needed for agriculture.
- Poor conditions to develop local industry. Due to the lack of energy it is difficult to develop industry.
- Government policy. Government does not have plans to solve the increasing demand of electricity problems neither to solve the short cuts problems.

Thanks For Your Attention

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BIRZEIT UNIVERSITY

Overview of Electrical Energy in West Bank

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Department of Electrical and Computer Engineering

Overview of Electrical Energy in West Bank

- There is no electrical power generation in West Bank.
- 96% of electrical energy consumed was imported from IEC.
- The remaining part was imported from Jordan.

Overview of Electrical Energy in West Bank

- The only main transmission lines constructed in the West Bank by IEC are three main 161 kV overhead lines feeding the three main substations: in Hebron, Qalandia (Atarot) and Salfiet (Ara’el).

Overview of Electrical Energy in West Bank

- These feeders supply West Bank by 800 MVA, 571 MVA which are supplied to the distribution companies and the remaining 229 MVA is supplied to municipalities.
- West Bank is fed from eight feeders by IEC and two feeders from Jordan.

Overview of Electrical Energy in West Bank

- The ranges of voltage of West Bank networks are 400V, 6.6 kV, 11kv, 33 kV.
- In Jerusalem Distribution Electric Company (JDECO), the voltage ranges are 400V, 11 kV and 33 kV.
- Northern Electricity Distribution Company (NEDCO) and Southern Electricity Company (SELCO) use 400V, 6.6 kV and 33 kV ranges.

Overview of Electrical Energy in West Bank

- In Hebron Electric Power Company (HEPCO) the ranges of voltage are 400V, 6.6 kV, 11 kV, 33 kV. Municipalities directly step down the voltage from 33 kV to 400 kV.

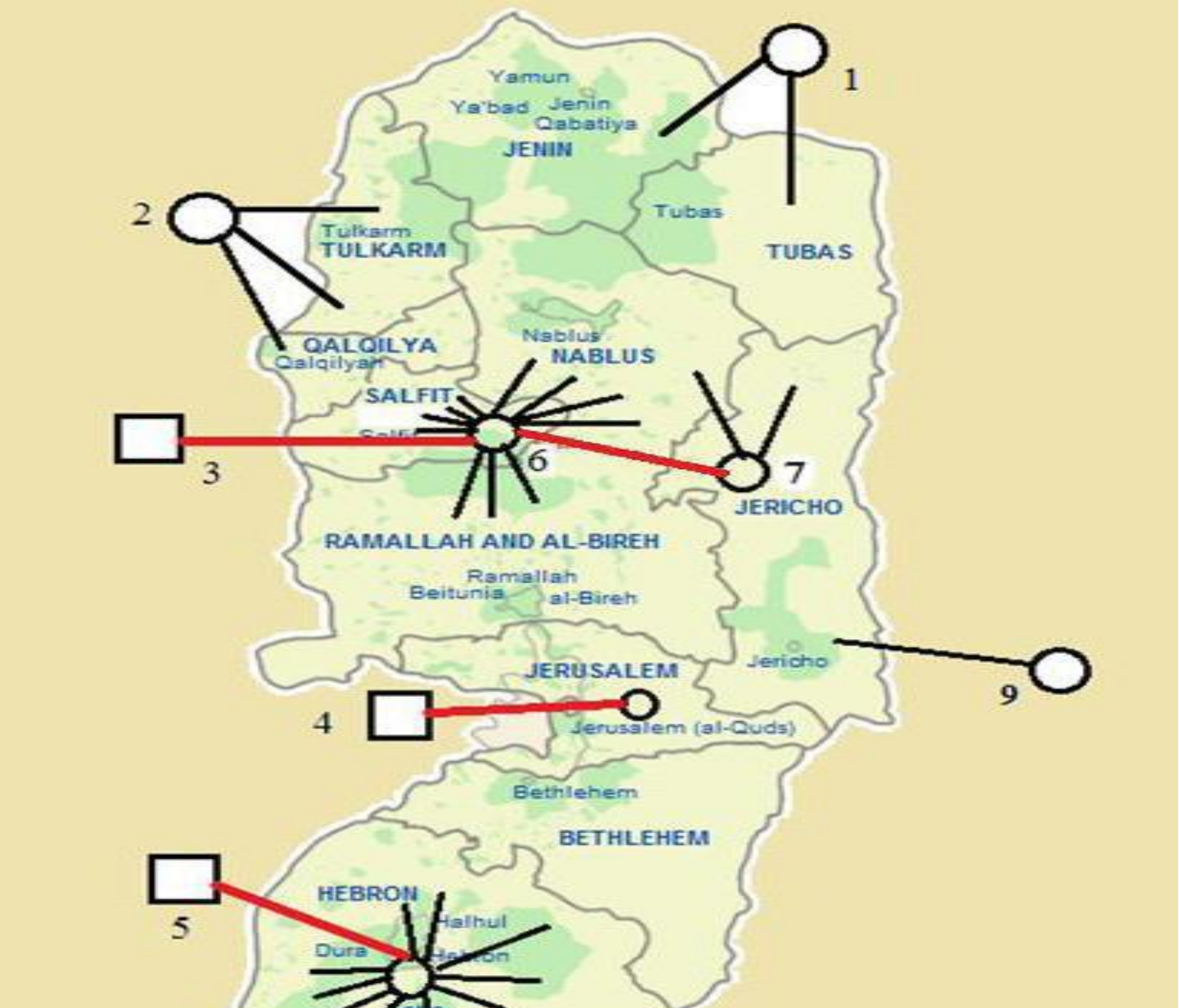
- These networks suffer from high transmission and distribution losses (technical and non technical) that varies from 17-32 %.

Overview of Electrical Energy in West Bank

- The maximum capacity of West Bank is nearly 800 MVA. 70% of the supply from Israel comes indirectly through three 161/33 kV substations; one in the south in area C close to Hebron, a second in the north in the Ariel settlement (area C) close to Nablus, and a third in Atarot industrial area (area C) near Jerusalem.

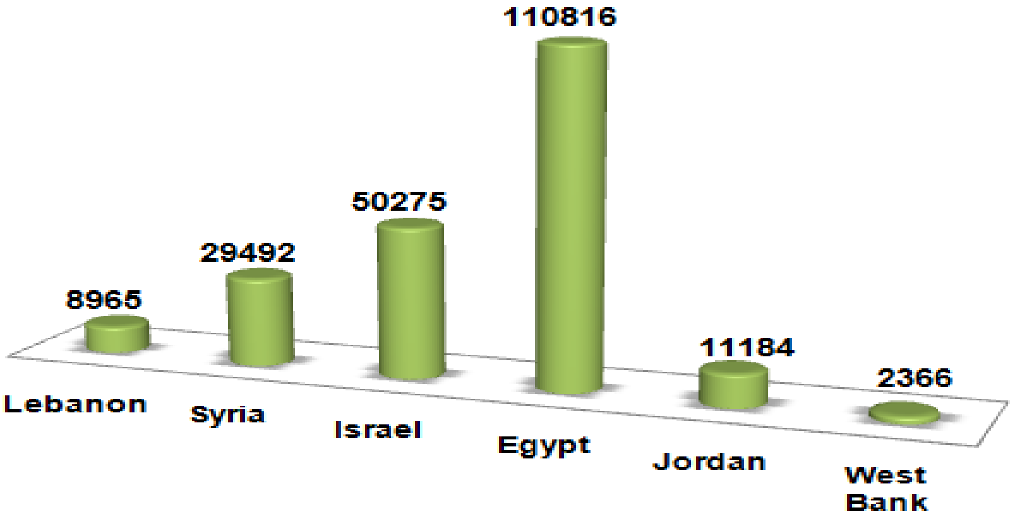
Overview of Electrical Energy in West Bank

- These feeders feed Hebron, Bethlehem, East Jerusalem, Ramallah, Jericho, Salfeet and Nablus.
- 30% comes directly through two 33 kV feeders from Beisan which feed both Jenin and Tubas. And three 22 kV feeders from Ntanya feed both Tulkarm and Qalqiliya . The supply from Jordan comes through 33 kV (can withstand 132 kV) overhead line (20MW) to supply only Jericho.
- The remaining power is generated by decentralized small diesel generators.

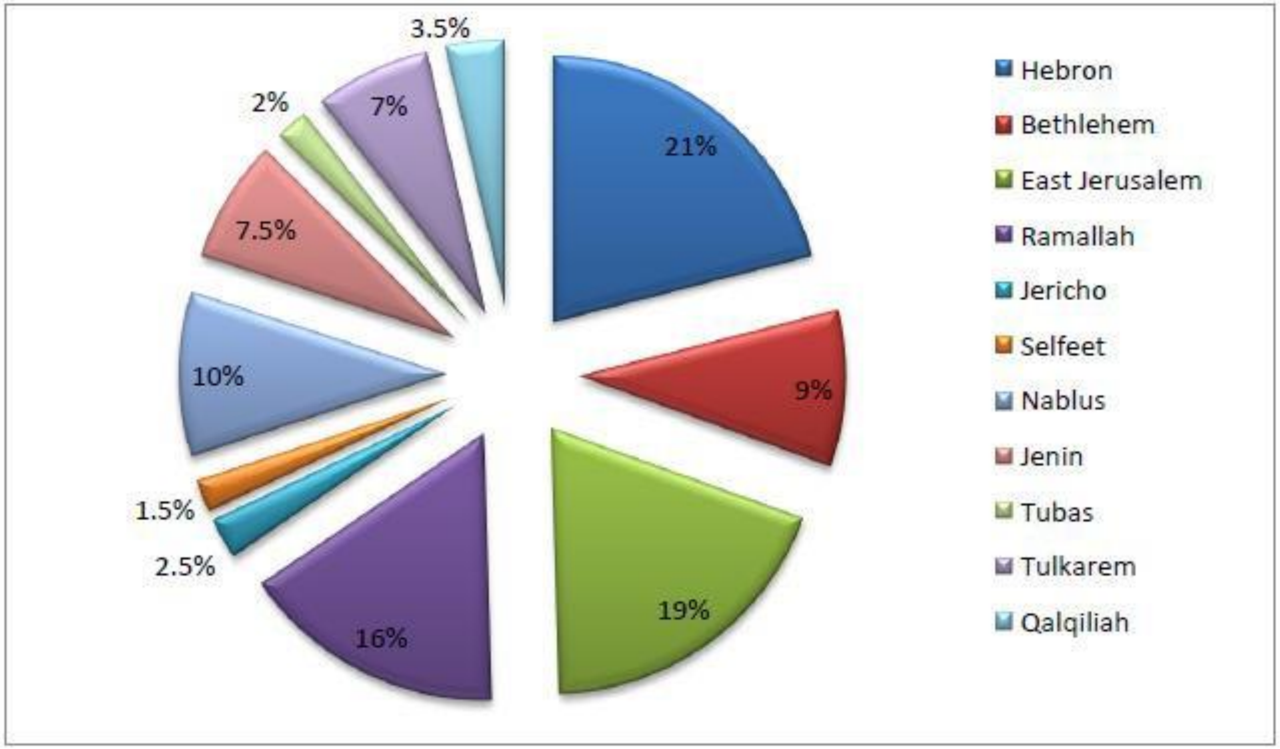

Yamun
Ya'bad
Jenin
Qabatiya
JENIN
1
2
Tubas
TUBAS
Tulkarm
TULKARM
QALQILYA
Qalqilyah
Nablus
NABLUS
SALFIT
Salfit
3
6
7
JERICHO
RAMALLAH AND AL-BIREH
Ramallah
Beitunia
al-Bireh
JERUSALEM
Jericho
9
4
Jerusalem (al-Quds)
Bethlehem
BETHLEHEM
HEBRON
Halhul
5
Dura
Hebron
Yatta
8
ad-Dhahiriya

Electrical Energy Consumption

- Total energy consumption in 2009 was 2366 GWh.
- The demand for electricity increases at a rate of 6.4%.

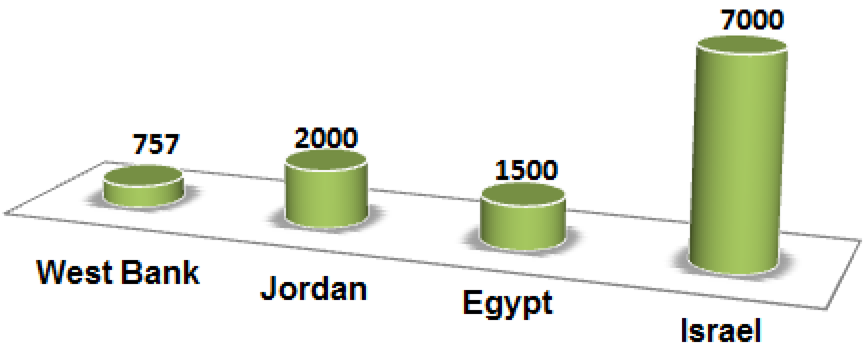


Electrical Energy Consumption



Consumption Per Capita

- Electricity consumption in West Bank is about 757 kWh per capita.
- This consumption is considered very low.



Electric Utilities in West Bank

- The electricity sector in West Bank is fragmented.
- Electricity is distributed by companies and municipalities.
- There are four utilities that distribute electricity in West Bank.

JDECO NEDCO HEPCO SELCO

Electric Utilities in West Bank

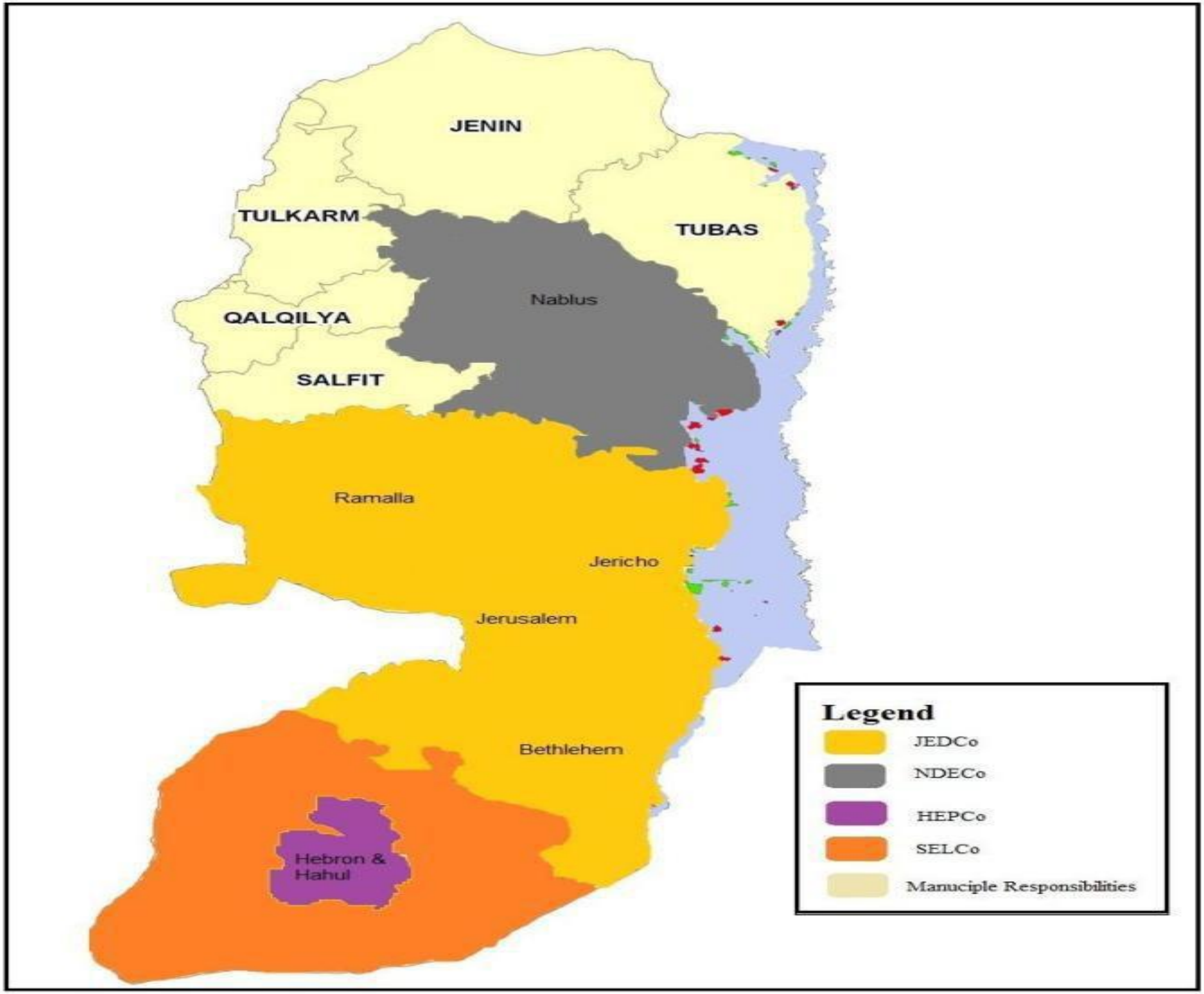
- Jerusalem District Electricity Company (JDECO), established in 1928, it is the largest distribution company in the West Bank covers approximately 25% of it. It serves Bethlehem, East Jerusalem, Ramallah and Jericho and connected to Atarot near Jerusalem and area C near to Hebron.
- Northern Electricity Distribution Company (NEDCO), established in 2008 to serve Nablus, Tulkarem, Jenin and other northern regions of the West Bank. But till now only Nabuls and Jenin city are under its responsibility. Connection point is in Areil settlement, at the north of Nablus

Electric Utilities in West Bank

- Southern Electricity Company (SELCO), established in 2002.It serves Dura, Yatta and Dahariah. Connection point is in area C near to Hebron.

- Hebron Electric Power Co. (HEPCO),established in 2000. It serves Hebron and Halhul. Connection point is in area C near to Hebron.

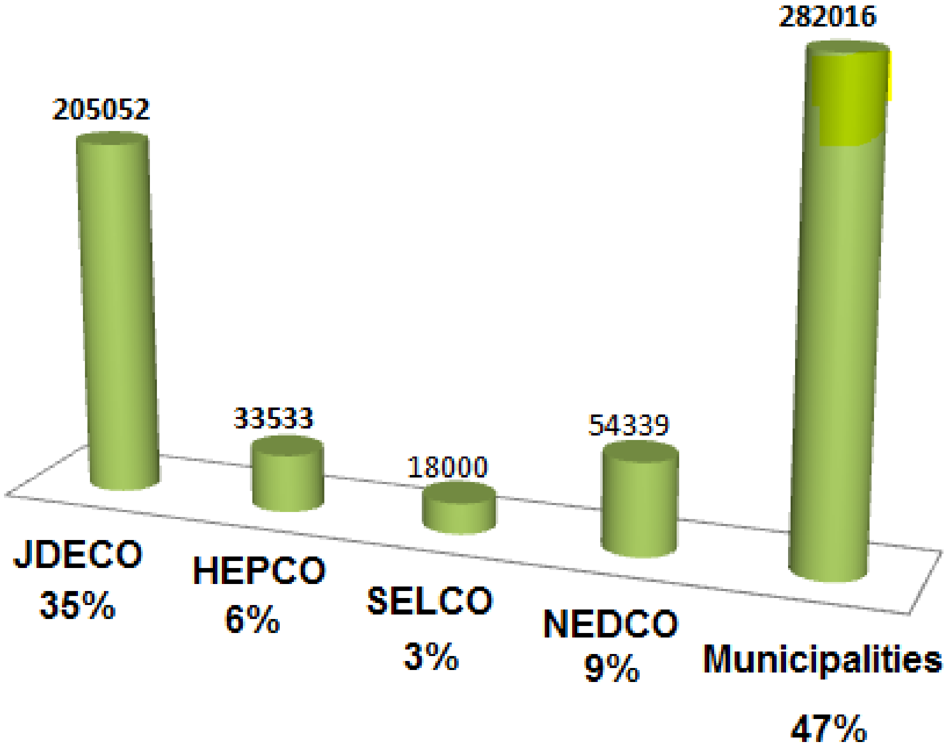
- The remaining areas of the West Bank are under municipal responsibility.


JENIN
TULKARM
TUBAS
Nablus
QALQILYA
SALFIT
Ramalla
Jericho
Jerusalem
Bethlehem
Hebron &
Hahul
Legend
JEDCo
NDECo
HEPCo
SELCo
Manuciple Responsibilities

Electricity Customers

- Number of electricity customers in the West Bank is approximately 592940.
- It increases at a rate of 4%.

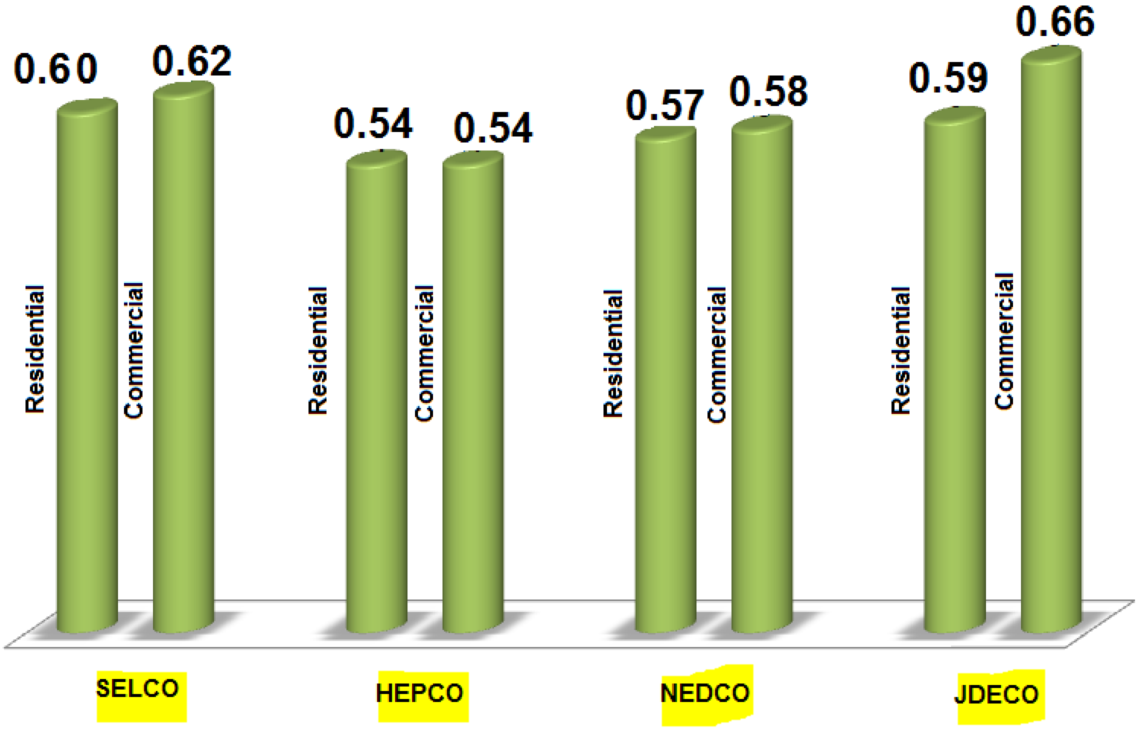
Electricity Customers in West Bank



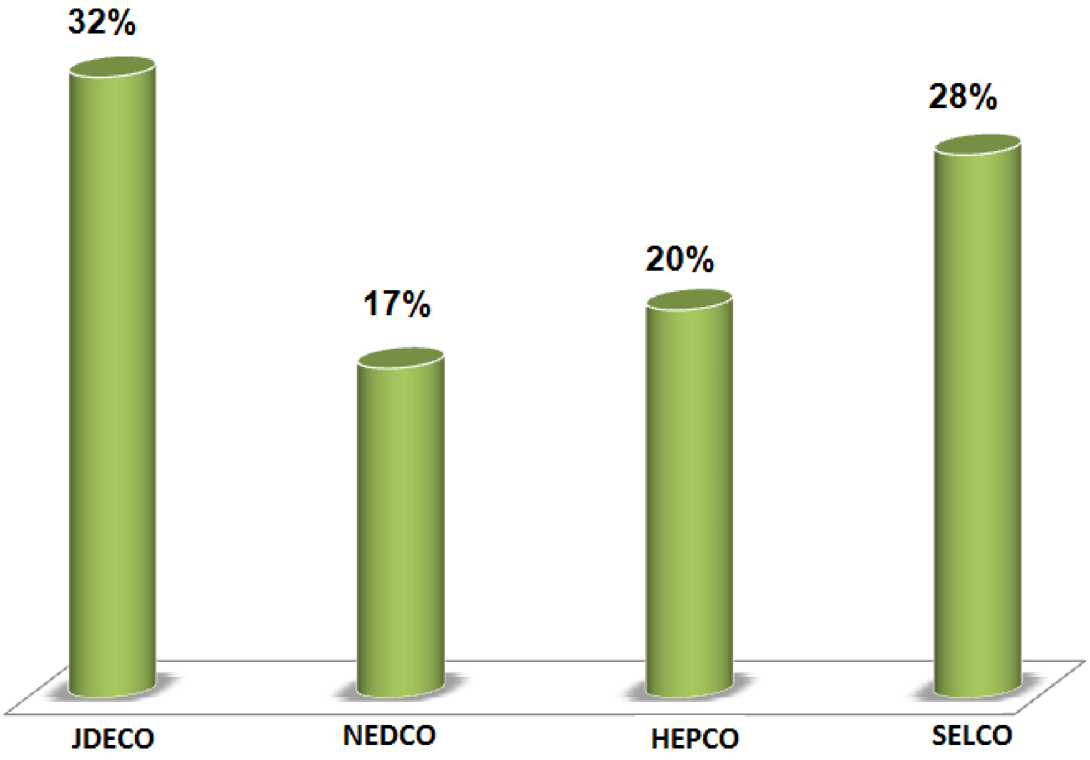
Tariff Structure

- The electricity price paid by consumers is somewhat high.
- Uniform tariff does not exist in West Bank.
- Distribution companies control the prices.
- Prices vary from one company to another.

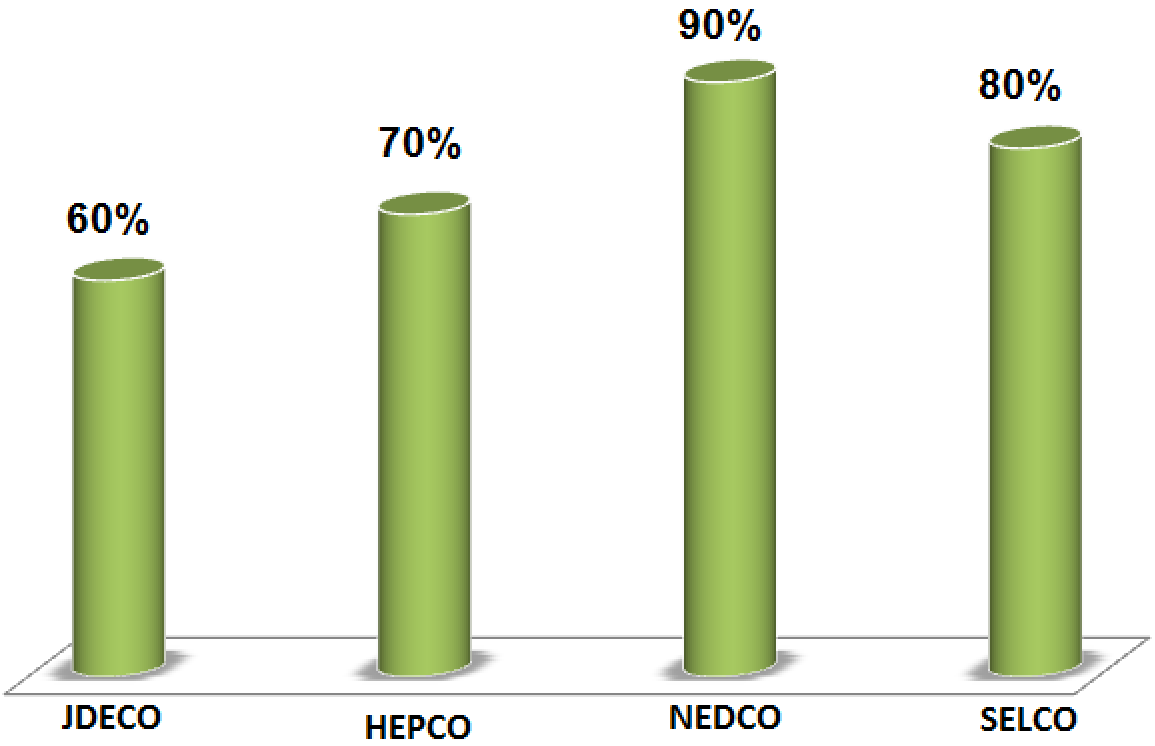
Prepay System



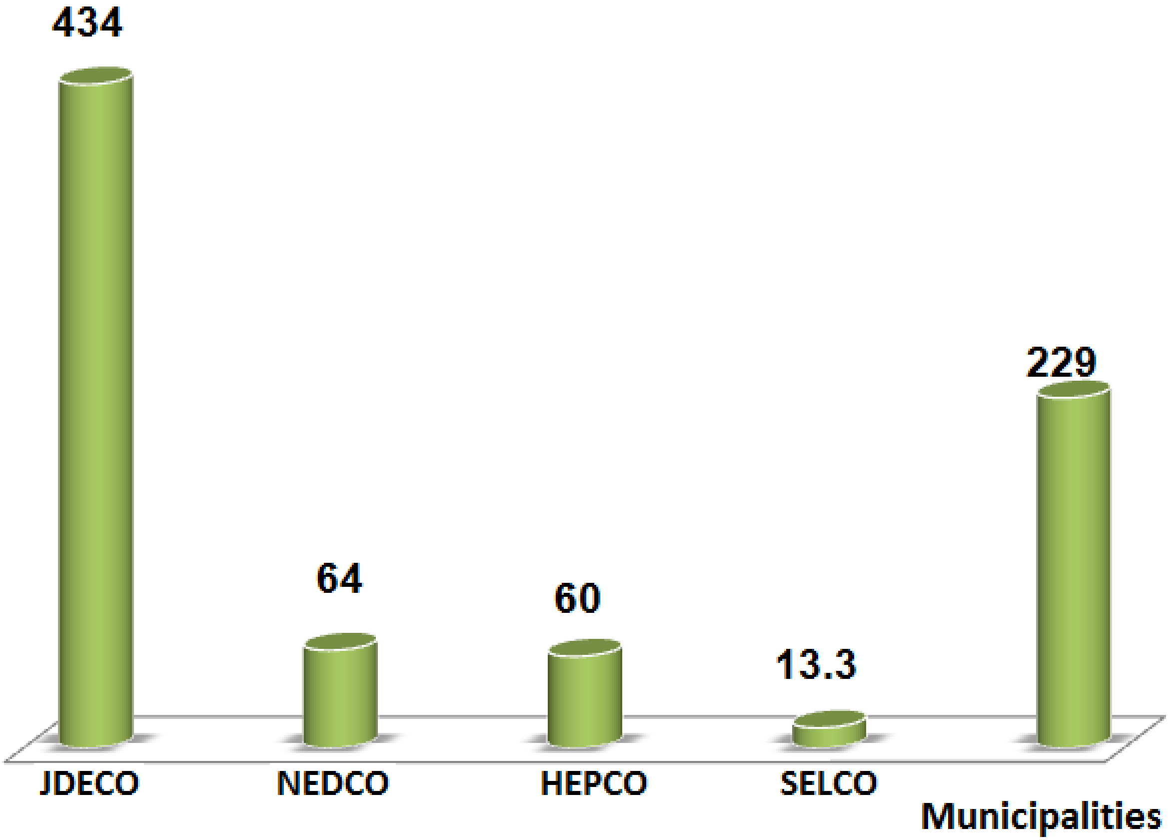
Losses



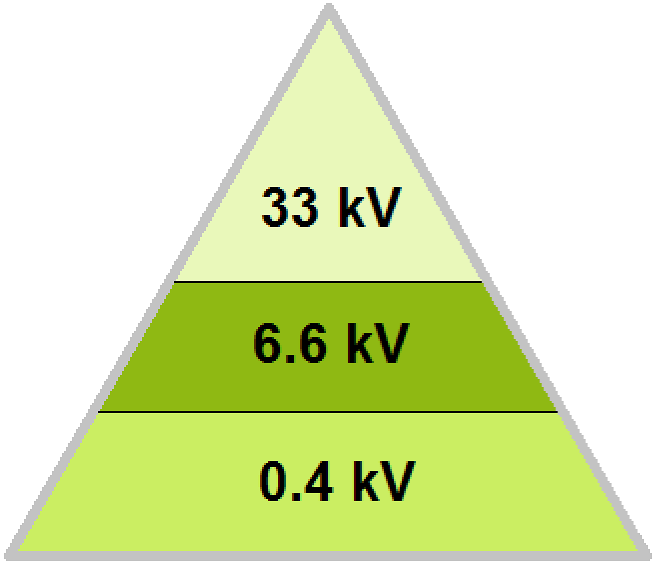
Load Factor

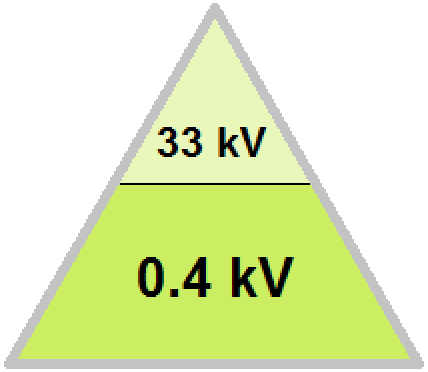


MVA Capacity

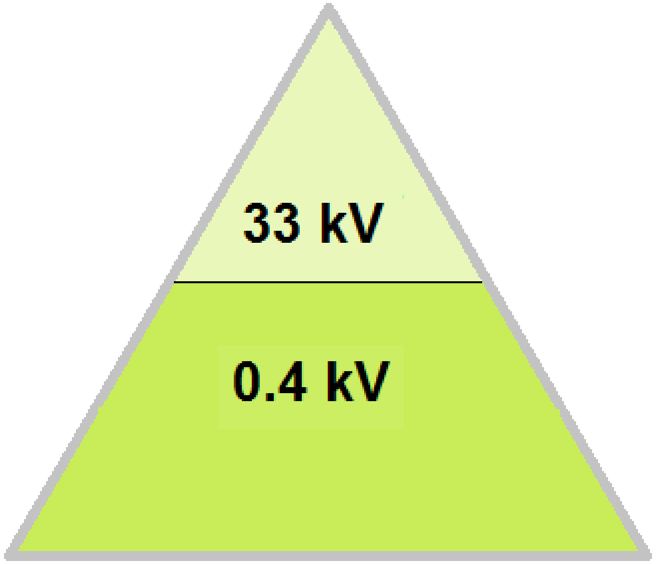


Distribution System in NEDCO & HEPCO





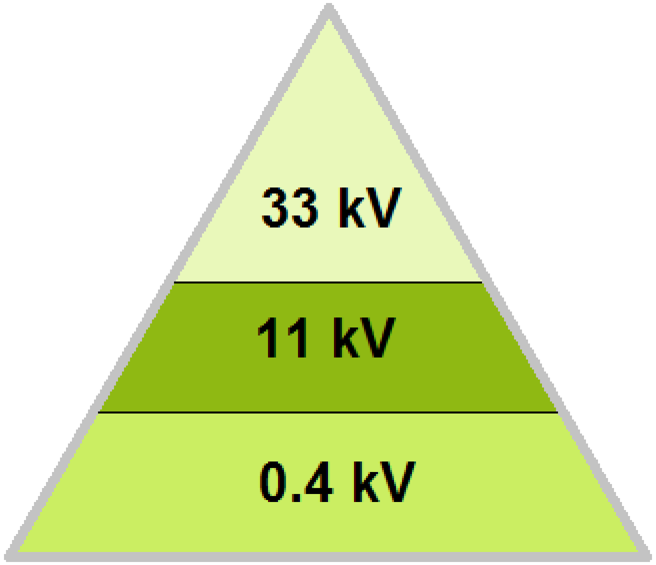
Distribution System in SELCO

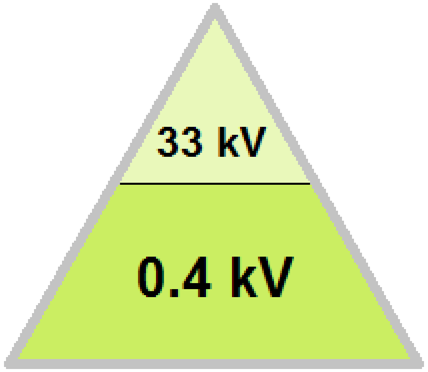


33 kV

0.4 kV

Distribution System in JDECO





Transmission Lines

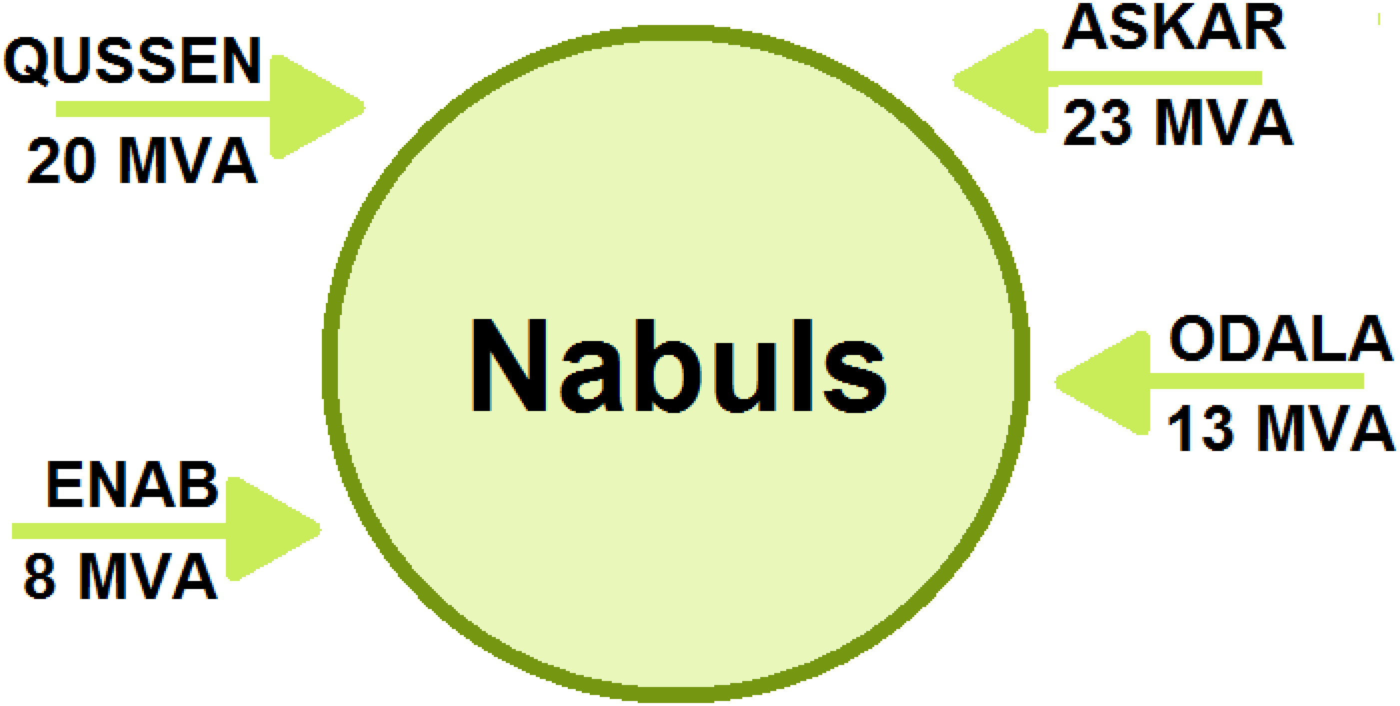
- **ACSR** transmission lines are used for 33kV,11kV and 6.6kV overhead lines.
- **ABC** transmission lines are used for 0.4kV overhead lines.
- **XLPE** transmission lines are use for 33kv,11kV,6.6kV for underground cables.

Transformers

- Dy11 Step down distribution transformers are used.

High Voltage Transformers	Low Voltage Transformers
15 MVA	1000 kVA
10 MVA	630 kVA
7.5 MVA	500 kVA
5 MVA	400 kVA
3 MVA	250 kVA
2.5 MVA	160 kVA &100 kVA

Example: Nablus Distribution System

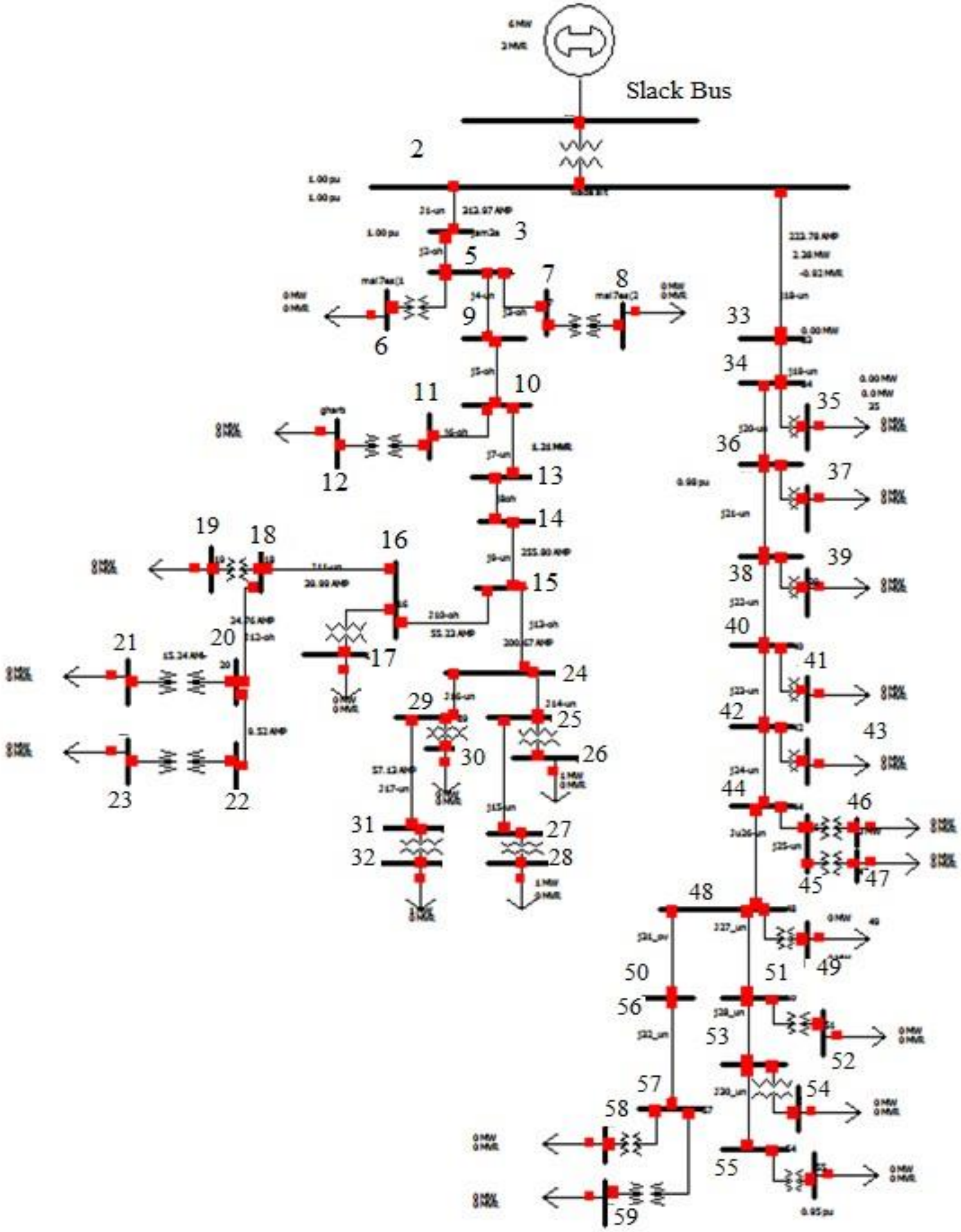


Example: Nablus Distribution System

Substation	Capacity (MVA)	Fed from	No. of Transformers (10MVA)
Askar	13	Odala	1
Central	22	Askar	2
Mujeer Aldeen	17	Qussen	2
Wadi Al-tufah	7	Qussen	1

Example: Wadi Altufah S/S

- Single line diagram consists of 59 buses and 25 transformers.
- Transformers are loaded to 40% of rated capacity and 0.92 power factor.


6 MW
2 MVR
Slack Bus
2
1.00 pu
1.00 pu
J1-un
212.97 AMP
1.00 pu
3
J2-oh
5
6
7
8
9
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11
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28
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31
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222.78 AMP
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-0.82 MVR
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Cont.

- Per unit values for transmission line per phase:

Type	Voltage (kV)	Resistance Pu/ km	Reactance Pu/ km
XLPE(120mm^2)	6.6	0.746	0.285
ACSR(95/15)	6.6	0.85	0.641
ACSR(50/8)	6.6	1.515	0.682

Cont.

- Per unit values for transformer per phase:

Capacity (MVA)	Z_{base}	R(Ω) Per unit	X(Ω) Per unit
0.25	0.4356	1.579798	0.672635
0.4		1.085859	0.46281
0.63		0.654729	0.277778
1		0.579431	0.247934
10	10.89	0.3434	0.135904

Simulation Results

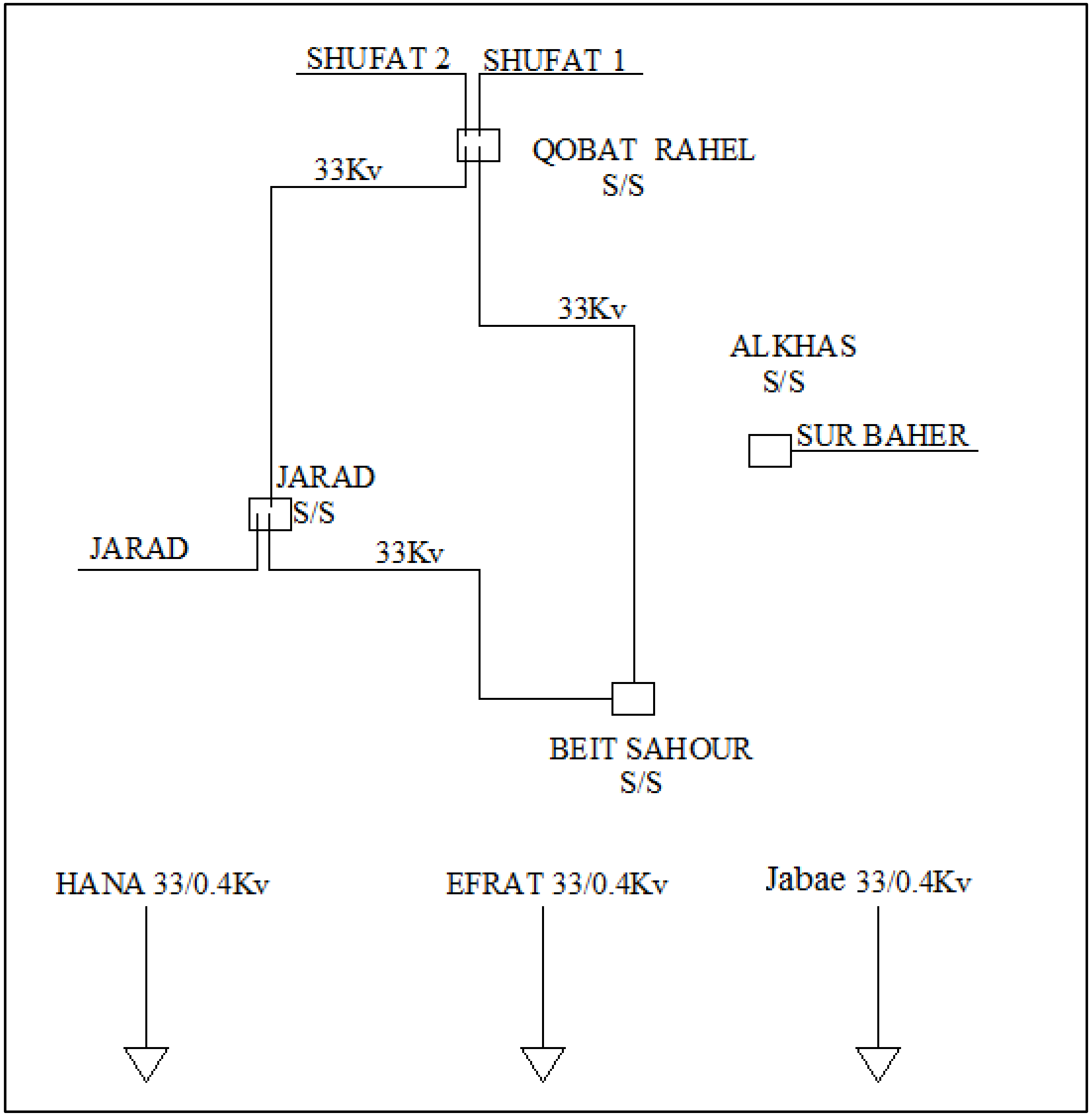
- The capacity of Wadi Altufah substation is **5.7** MW, **2.7** Mvar with **0.90** PF.
- A **5.2** MW, **2.4** Mvar is consumed by the load, with **0.89** PF as an average.
- The losses in the 6.6kV lines is **9%.**
- The maximum voltage drop on 6.6 kV was **10.3%.**

Example: Bethlehm Distribution System

- Bethlehm is fed from seven 33kV feeders.
- Four main substations.
- The rated capacity is 94.6 MVA.
- Consumed power about 211 GWh.

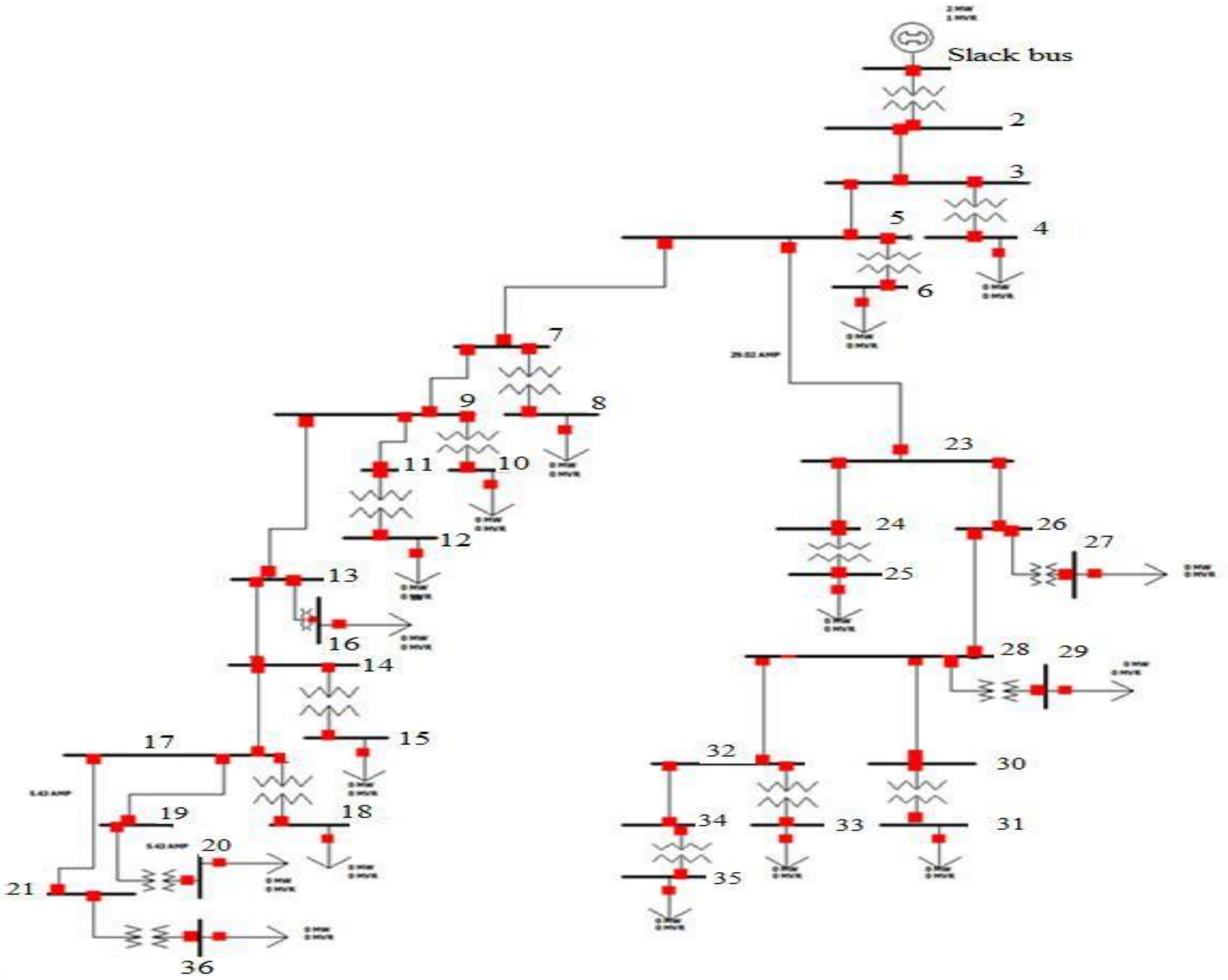
Substation	Transformers (33/11) kV
Qobat Rahel	2X15 MVA
Beit Sahour	10 MVA 7.5 MVA
Jarad	2X10MVA
Alkhas	5 MVA

Shufat1	20 MVA
Shufat2	20 MVA
Hana	20 MVA
Efrat	6 MVA
Jarad	20 MVA
Sur Baher	8.1 MVA
Jabae	0.5 MVA
Total	94.6 MVA



Example: Alkhas S/S

- Single line diagram consists of 36 buses and 16 transformers.
- Transformers are loaded to 40% of rated capacity and 0.92 power factor.


2 MW
1 MVR
Slack bus
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
23
24
25
26
27
28
29
30
31
32
33
34
35
36
29.02 AMP
5.43 AMP
5.43 AMP
0 MW
0 MVR

Simulation Results

- The capacity of Alkhas substation is **1.7** MW, **0.73** Mvar with **0.92** PF.
- A **1.65** MW, **0.7** Mvar is consumed by the load, with **0.91** PF as an average.
- The losses in the 11kV lines is **3.5%.**
- The maximum voltage drop on 11 kV was **4%.**

Electrical Energy Problems

- Absence in generating in West Bank.
- Absence of integrated electrical network.
- Lack of supply capacity of electrical energy to meet present and future needs.

Cont.

- Energy prices are very high.
- High transmission and distribution losses.

Future Plans in West Bank

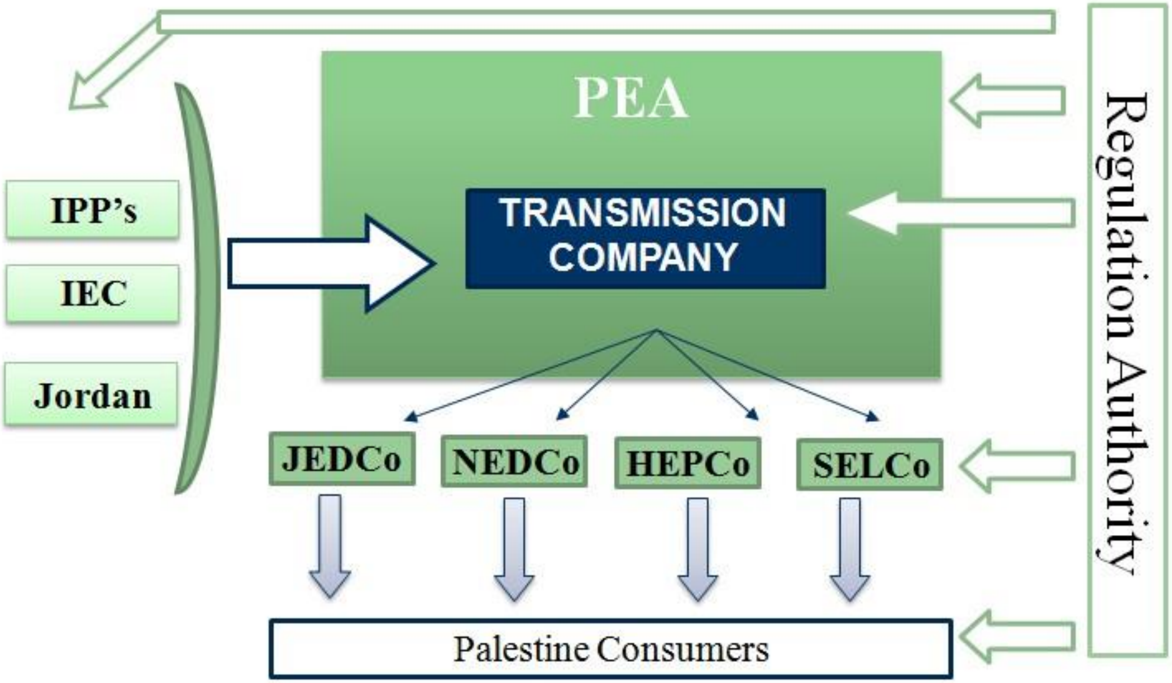
- A project is in its way to be implemented to install four new 161/33 kV transmission substations across West Bank.
- Palestine Energy Transmission Company Ltd. (PETL).
- Connection to seven Arab country grid.

Cont.

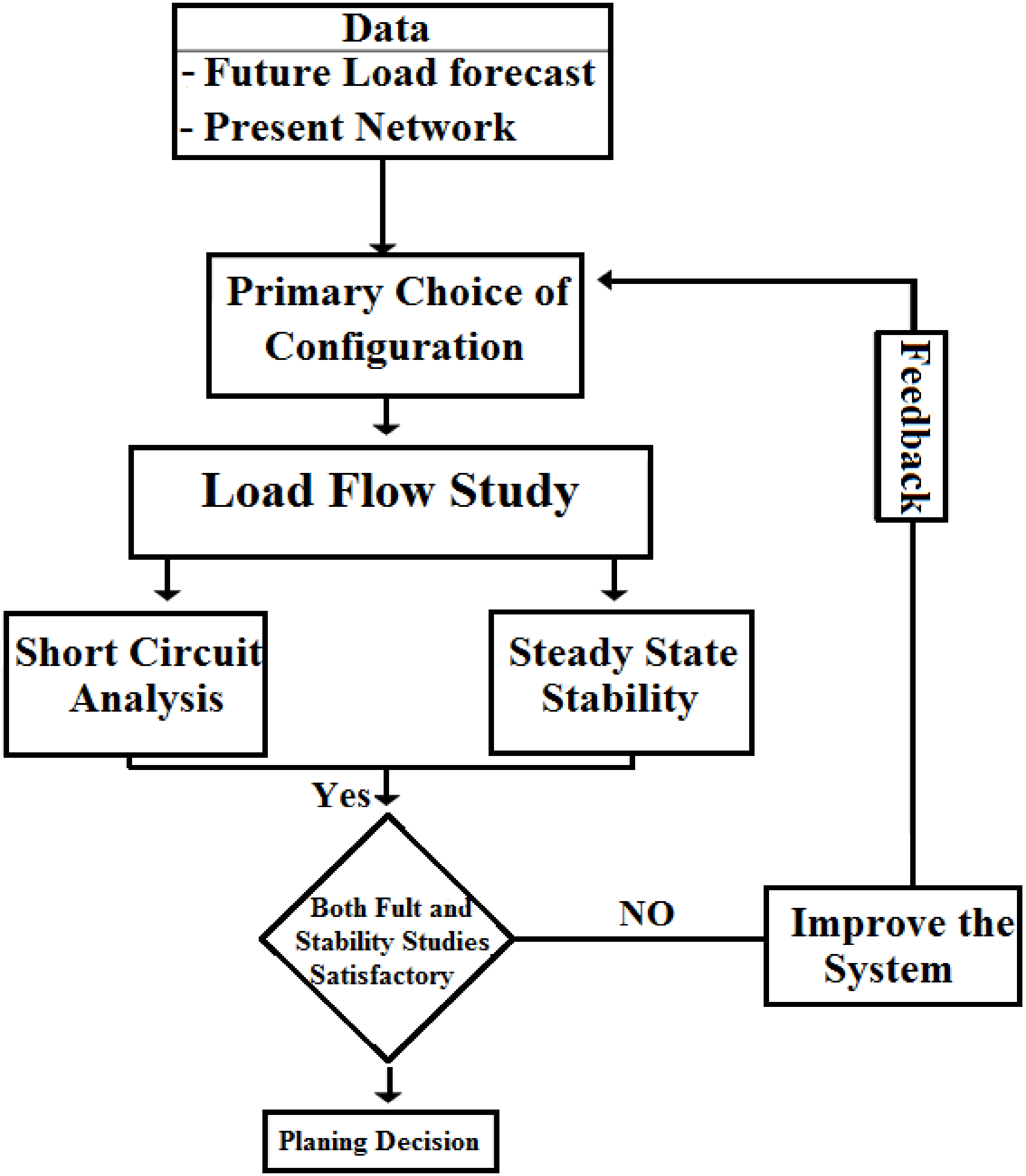
- Two new power plants in West Bank will be constructed, which are:

1) Jayyus Power Plant in the north, near Qalqiliya.
2) Turqumia Power Plant in the south, west of Hebron.

Future Organization of the Power Sector



PEA	Palestinian Energy Authority
IEC	Israeli Electric Corporation
IPP	Independent Power Producers



Data

- Future Load forecast
- Present Network

Primary Choice of Configuration

Load Flow Study

Short Circuit Analysis

Steady State Stability

Yes

Both Fult and Stability Studies Satisfactory

NO

Improve the System

Feedback

Planing Decision

Economic Voltage

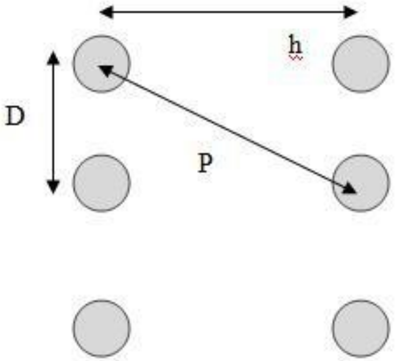
INCREASING IN VOLTAGE

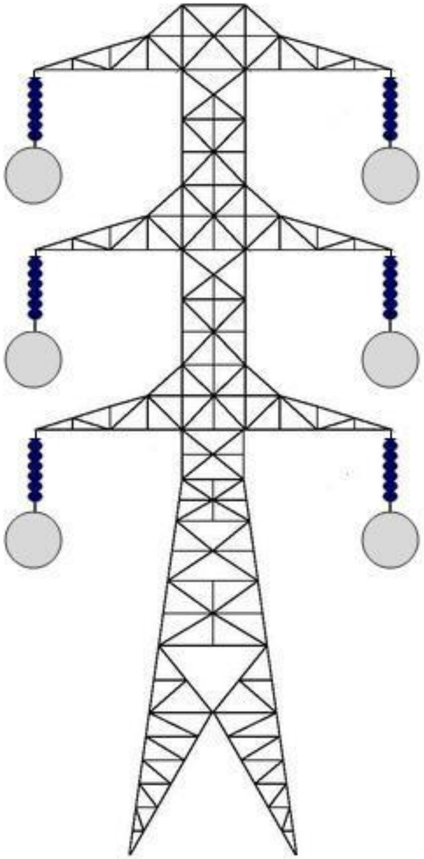
Cost of conductor material ↓

Cost of insulators ↑ Cost of Switchgears ↑

Cost of transformers ↑

Selection of Transmission Lines, Tower Example





$$X_L = 4\pi f * 10^{-7} * \ln\frac{Deq}{r}$$

$$B = 2\pi f\left(\frac{2\pi\varepsilon}{\ln\frac{Deq}{D/2}}\right)$$

Type of tower	Vertical spacing [m]	Horizontal spacing [m]
132 kV: Double circuit	**3.96**	**7.32**
230 kV: Double circuit	**6.70**	**12.6**

Transmission Lines Parameters

- >> Introduction to transmission Lines (T.L)
- >> Types of Overhead Line Conductors.
- >> Resistance Calculation.
- >> Inductance Calculation.
- >> Capacitance Calculation.

☑ Overhead transmission System

1. Although underground AC transmission would present a solution to some of environmental and aesthetic (جمالي) problems in overhead transmission Lines, there are technical and economic reasons that make the use of underground ac transmission not preferable.

2. The overhead transmission System is mostly used at high voltage level mainly because it is much cheaper Compared to underground system.

3. The selection of an economical voltage level for the T.L is based on the amount of power and the distance of transmission.

$\Longrightarrow$ The economical voltage between Lines in 3ø is given by :-

$$V = 5.5\sqrt{0.62\,L + \frac{P}{100}}$$, where

V = Line voltage in kV.

L = Length of T.L in km.

P = Peak real power in kW.

4. Standard transmission voltages are established

- → HV (30 – 230) kV
- → EHV (230 – 765) kV
- → UHV (765 – 1500) kV

Types of overhead line conductors

based on
- Conducting material
- the strength

[1] The material to be chosen for conduction of power should be such that it has the lowest resistance. This would reduce the transmission losses.

*
(a)

1) Silver resistivity	$1.6 \mu\Omega cm$
2) Copper resistivity	$1.7 \mu\Omega cm$
3) gold resistivity	$2.35 \mu\Omega cm$
4) aluminium resistivity	$2.65 \mu\Omega cm$

Problems : cost, theft, supply is quit limitted

*
(b)

The weight of material (density)

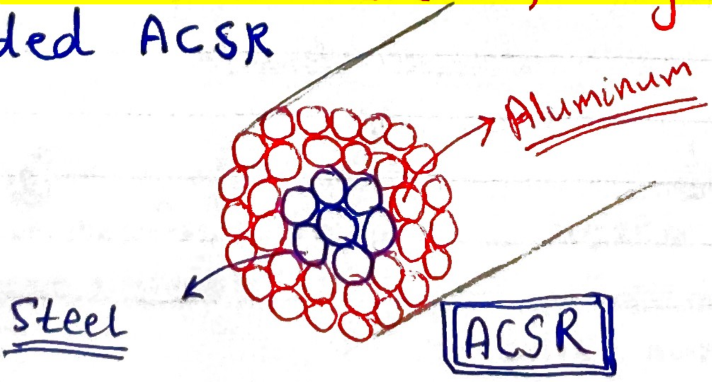
1) aluminium
2) Copper
3) Silver
4) gold

note : The weight of the aluminium conductors having the same resistivity as that of copper, is roughly 60 % less than of copper.

[2] In the early days of the transmission of electric power, conductors where usually copper, but aluminum conductors have completly replaced copper for overhead lines because of the much lower cost and lighter weight of an aluminum conductor compared with a copper conductor of the same resistance.

[3] The most commonly used conductors for high voltage transmission lines are :-

- * AAC ALL-Aluminum Conductors
- * AAAC AlL-Aluminum-Alloy Conductors (سبائك الألمنيوم)
- * ACSR Aluminum Conductor, Steel-Reinforced (معزز، مقوى)
- * ACAR Aluminum Conductor, Alloy-Reinforced.
- * Expanded ACSR



» Aluminum-alloy conductors have higher tensile strength (قوة الشد) than the ordinary aluminum.

» ACSR consists of a central core of steel strands surrounded by layers of aluminum strands.

» ACAR has a central core of higher-strength aluminum surrounded by layers of aluminum.

» Expanded ACSR has a filler such as (paper, fiber) separating the inner steel strands from the outer aluminum strands. The filler gives a larger diameter (and hence, lower Corona) for a given conductivity and tensile strength. Expanded ACSR is used for some extra-high voltage lines.

Stranded Conductors

» To increase the area stranded conductors are used. This increase the flexibility and the ability of the wire or cable to be bent.

» Generally the circular conductors of the same size are used for spiralling.

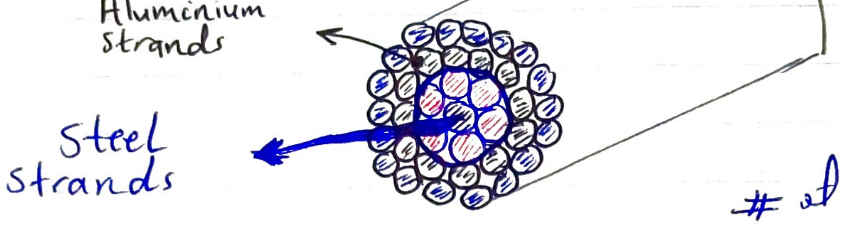
» Each layer of strands is spiraled in the opposite direction of its adjacent layer. This spiraling holds th strands in place (can't open up easily)

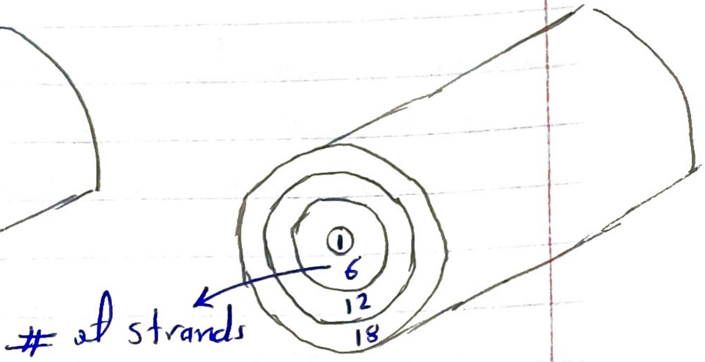
Stranded Conductors

ⓐ easier manufacturing (Larger sizes)

ⓑ better mech. strength, as well as better handling much more flexible.

Stranded Conductors





Total # of strands → 1, 7, 19, 37, 61, 91

▣ Line Resistance :-

R_{ac} 

R_{dc} 

» The dc resistance of a solid round conductor at a specifie temperature is given by :-

$$\boxed{R_{dc} = \frac{\rho^{T} l}{A} \ \Omega} \quad (*)$$

where

- $\rho \equiv$ conductor resistivity at temp T (°C)
- $l \equiv$ conductor length (m)
- $A \equiv$ Conductor cross-sectional area (m²)

» Conductor resistance depends on the following factors :-
☐1 Temperature ☐2 Spiraling ☐3 Frequency

☐1 Temperature

Resistivity of conductor metals varies linearly over normal operating temperatures according to

$$\rho^{T_2} = \rho^{T_1} \left(\frac{T_2 + T}{T_1 + T} \right)$$

⇒ The conductor resistance increase as temp increases.

$$\overset{R^{T_2} \xleftarrow{or}}{R_2} = \underset{R^{T_1} \swarrow or}{R_1} \left(\frac{T_2 + T}{T_1 + T} \right)$$

T ≙ temperat constant that d on the conductor material.

⊙ For Aluminum
T ≅ 228

2 Spiraling

- » Since a stranded conductor is spiraled, each strand is longer than the finished conductor. This results in a slightly higher resistance than the value calculated using equation (*).
- » The spiralling increase the resistivity of the conductors to an extent about 2% for the first layer on the centre conductor, about 4% for the second layer, and so on.

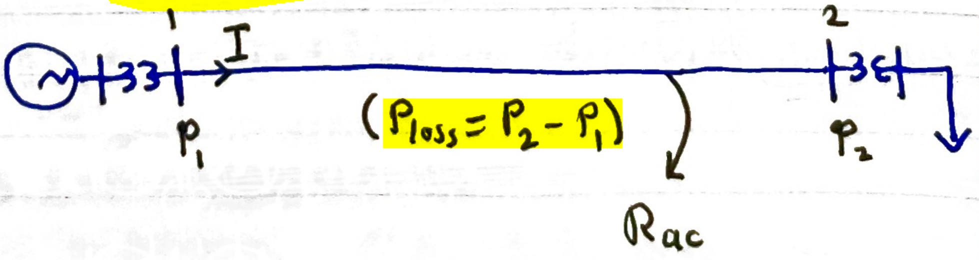
3 Frequency "skin effect"

- » When ac flows in a conductor, the current distribution is not uniform over the conductor cross-sectional area and the current density is greatest at the surface of the conductor. This causes the ac resistance to be somewhat higher than the dc resistance. This behavior is known as skin effect.
- » This uneven distribution does not assume large proportion at 50 Hz up to a thickness of about 10 mm.
- » At (50-60) Hz, the ac resistance is about 2 percent higher than the dc resistance.

Note:-

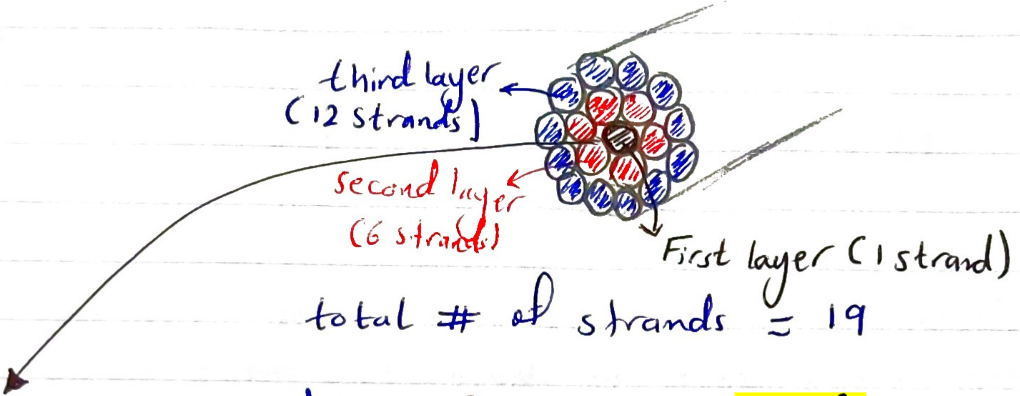
The ac resistance or effective resistance of a conductor is

$$\boxed{R_{ac} = \frac{P_{loss}}{I^2} \ \Omega}$$



example A copper Cable of 19 strands, each strand 2.032 mm in a diameter is laid over a length of 1km. The temperature rise was found to be 40. Find the value of total R for this cable.

Solution



total # of strands = 19

For 1 strand 

$$A_{1s} = \frac{\pi d^2}{4} = \frac{\pi (0.2032)^2}{4}$$

$$= 0.03243 \text{ cm}^2$$

at 20°C

$$R_{1s} = \frac{\rho l}{A} = \frac{1.7 * 10^{-6} * 100000}{0.03243}$$

$$= 5.24 \, \Omega$$

?!

$$R_{total} = \frac{5.24}{19} = 0.2758 \, \Omega$$

① Spiraling effect

First layer $R_{1con} = 5.24$

second layer $R_{6con} = \frac{5.24}{6} = 0.8733 \, \Omega \xrightarrow{\text{Spir. eff}} R_{6con} = 0.8733 * 1.02$

$$= 0.8908 \, \Omega$$

third layer $R_{12con} = \frac{5.24}{12} = 0.4367 \, \Omega \xrightarrow{\text{Spir. eff}} R_{12con} = 0.4367 * 1.04$

$$= 0.4541 \, \Omega$$

$R_{cable} = 5.24 \| 0.8908 \| 0.4541$

$R_{total} = 0.2844 \, \Omega$ 《3.1 % higher when we consider spiraling effect》

2 Temperature effect

for copper

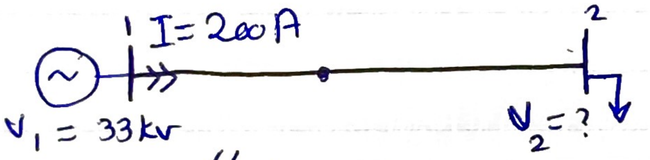
$$R_2 = R_1\left(\frac{T+T_2}{T+T_1}\right) = 0.2844\left(\frac{234.5+60}{234.5+20}\right)$$

the resistance at new temp.

$$= 0.329\ \Omega$$

$R = 0.2758\Omega$ ← Compared with (19.3 %)

note :- If the cable was carrying a current 200A, the drop from one end to the other end would be about 65.8 volts due to resistance.



$$((V_1 - V_2 = \text{Voltage drop}))$$

3 frequency effect

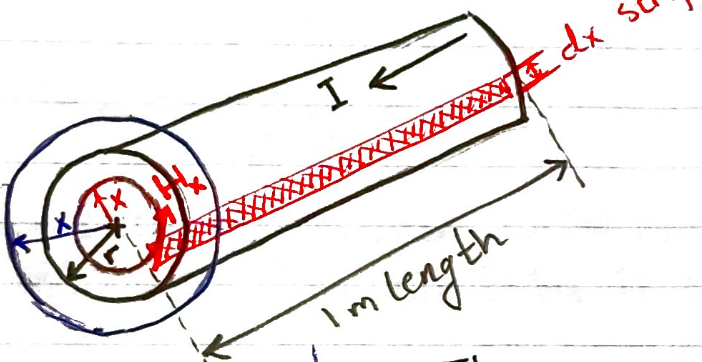
At freq 50 Hz the skin depth in a copper is of the order of 10 mm and hence would not have any significant effect as far as this problem is concerned.

Inductance

» For Calculating Inductance we need to go to four steps:

1. Magnetic Field Intensity H, from Ampere's Law
2. Magnetic Flux Density B, ($B = \mu H$)
3. Flux Linkages, (λ)
4. Inductance From Flux Linkages per ampere. ($L = \lambda / I$)

▦ Solid Cylindrical Conductor



[A] Internal Flux Linkage [B] External Flux Linkage

» The magnetic field intensity H_x, around a circle of radius x, is constant and tangent to the circle. The Ampere's Law relating H_x to the current I_x is given by:

$$\oint H_{tan}\, dl = I_{enclosed}$$

(محيط الدائرة)

$$\int_0^{2\pi x} H_x \cdot dl = I_x$$

$$\boxed{H_x = \frac{I_x}{2\pi x}} \quad \ldots \quad (1)$$

I_x → is the Current enclosed at radius x.

$\boxed{A}$ Internal Inductance

» A simple expression can be obtained for the internal flux linkage by neglecting the skin effect and assuming uniform current density throughout the conductor cross section, i.e.

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \Rightarrow I_x = \left(\frac{x}{r}\right)^2 I$$

• uniform current density

from (1) $$H_x = \frac{I_x}{2\pi x}$$

$$H_x = \frac{I}{2\pi r^2} x$$

» For a nonmagnetic conductor with constant permeability μ_o, the magnetic flux density is given by:

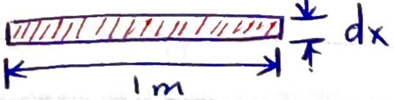
$$B_x = \mu_o H_x$$

$$B_x = \mu_o \left[\frac{I}{2\pi r^2} x\right]$$

$\mu_o \equiv$ permeability free space $\equiv 4\pi * 10^{-7}$ H/

» The differential flux $d\phi$ for a small region of thickness dx and one meter length of the conductor is

$$d\phi_x = B_x \underbrace{dx.1}_{\text{area of strip}}$$



⊙ The flux $d\phi_x$ links only the fraction of the conductor from the center to radius x.
Thus, on the assumption of uniform current density, only the fraction $\frac{\pi x^2}{\pi r^2}$ of the total current is linked by the flux, i.e.,

$$d\lambda_x = \left(\frac{x^2}{r^2}\right) d\phi_x$$

- $B_x = \mu_0 \left[\frac{I x}{2\pi r^2}\right]$
- $d\phi_x = B_x \, dx$

$$d\lambda_x = \left(\frac{x^2}{r^2}\right) d\phi_x$$

$$= \left(\frac{x^2}{r^2}\right)\left[B_x \, dx\right]$$

$$= \frac{x^2}{r^2}\left[\frac{\mu_0 I x}{2\pi r^2}\right] dx$$

$$d\lambda_x = \frac{\mu_0 I x^3}{2\pi r^4} dx$$

>> The total flux linkage

$$\lambda_{int} = \int_0^r d\lambda = \frac{\mu_0 I}{2\pi r^4}\int_0^r x^3 \, dx$$

$$= \frac{\mu_0 I}{8\pi} \text{ Wb/m}$$

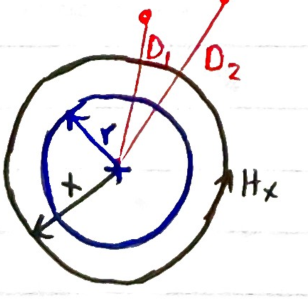
By defⁿ, for nonmagnetic material, the inductance L is the ratio of its total magnetic flux linkage to the current I, given by $L = \lambda / I$.

The Inductance due to the internal flux linkage is

$$L_{int} = \frac{\mu_0}{8\pi} = \frac{1}{2} * 10^{-7} \quad \text{H/m}$$

Note that L_{int} is independent of the conductor radius r.

$\boxed{B}$ Inductance due to external flux linkage



$$\oint H_{tan} \, dl = I_{enclosed}$$

$$\int_0^{2\pi x} H_x \, dl = I$$

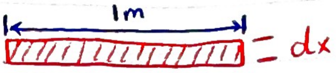
>> $H_x (2\pi x) = I$

$$H_x = \frac{I}{2\pi x} \text{ A/m} \qquad x > r$$

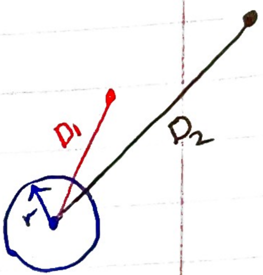
>> $B_x = \mu_0 H_x = 4\pi * 10^{-7}\left[\frac{I}{2\pi x}\right]$

$= 2 * 10^{-7}\frac{I}{x}$

$d\phi = B_x \cdot dx \cdot 1 = 2 * 10^{-7}\frac{I}{x}\,dx$



>> Total Flux Linkages between any two points



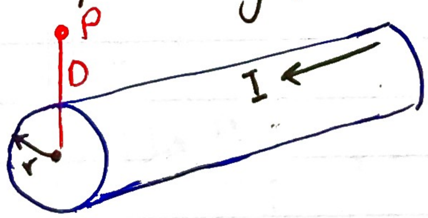
$$\lambda_{12} = \int_{D_1}^{D_2} d\lambda = 2 * 10^{-7} I \int_{D_1}^{D_2} \frac{1}{x}\,dx.$$

$$\lambda_{12} = \lambda_{ext} = 2 * 10^{-7} I \ln\frac{D_2}{D_1}$$

>> The inductance between two points external to a Conductor is

$$\boxed{L_{ext} = 2 * 10^{-7} \ln\frac{D_2}{D_1} \quad H/m}$$

▦ Total Flux Linkage up to any point P for this Conductor Carrying Current I.



note:-
ln(a*b) = ln(a)+ln(b)

$$\lambda_P = \underbrace{\frac{1}{2} * 10^{-7} I}_{\text{internal F.L.}} + \underbrace{2 * 10^{-7} I \ln\frac{D}{r}}_{\text{external F.L.}}$$

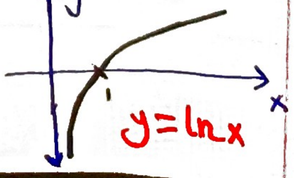
using $\frac{1}{2} =. 2 \ln e^{\frac{1}{4}}$

$$\lambda_P = 2 * 10^{-7} I \left(\ln e^{\frac{1}{4}} + \ln\frac{D}{r}\right) = 2 * 10^{-7} I \ln\frac{D}{e^{-\frac{1}{4}} r}$$

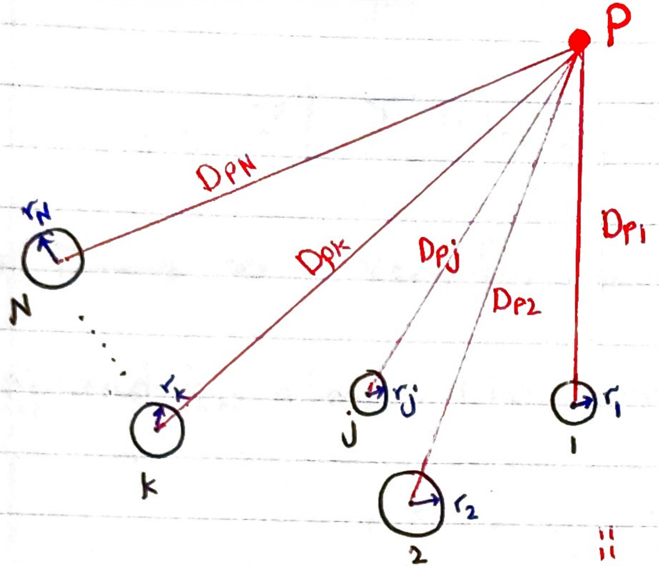
$$= 2 * 10^{-7} I \ln\frac{D}{r'}$$

where $r' = e^{-\frac{1}{4}} r = 0.7788r$ ≙ effective radius due to internal flux

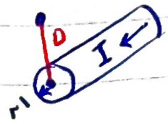
$$\boxed{L_P = \frac{\lambda_P}{I} = 2 * 10^{-7} \ln\left(\frac{D}{r'}\right) \quad H/m}$$



Composite Conductor :-



note :- $\lambda_p = 2*10^{-7} I \ln \frac{D}{r'}$



$$I_1 + I_2 + I_3 + \cdots + I_N = 0$$

$$\sum_{j=1}^{N} I_j = 0$$

$$\lambda_{kpk} = 2*10^{-7} I_k \ln \frac{D_{pk}}{r'_k}$$

$$\lambda_{kp1} = 2*10^{-7} I_1 \ln \frac{D_{p1}}{D_{k1}}$$

$$\lambda_{kpk} = 2*10^{-7} I_k \ln \frac{D_{pk}}{D_{kk}} \text{ ; where } D_{kk} = r'_k$$

↳ Flux Linkages for the conductor k up to a point p due to the current flowing in conductor k.

λ_{kp} → Flux linkages for the conductor k up to a point p due to the currents flowing in conductors 1, 2, ... N.

$$\lambda_{kp} = \lambda_{kp1} + \lambda_{kp2} + \cdots\cdots + \lambda_{kpN}$$

$$= 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{D_{pj}}{D_{kj}} \text{ , where } D_{kk} = r'_k$$

$$= 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}} + 2*10^{-7} \underbrace{\sum_{j=1}^{N} I_j \ln D_{pj}}_{j=1 \rightarrow j=N-1 \quad j=N}$$

$$= 2*10^{-7} \left[\sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}} + \underbrace{\sum_{j=1}^{N-1} I_j \ln D_{pj}}_{j=1 \rightarrow j=N-1} + \underbrace{I_N \ln D_{pN}}_{j=N} \right]$$

where

$$I_N = -(I_1 + I_2 + \cdots + I_{N-1}) = -\sum_{j=1}^{N-1} I_j$$

$$\lambda_{kP} = 2*10^{-7}\left[\sum_{j=1}^{N} I_j \ln\frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln D_{Pj} - \overset{I_N}{\left(\sum_{j=1}^{N-1} I_j\right)} \ln D_{PN}\right]$$

$$= 2*10^{-7}\left[\sum_{j=1}^{N} I_j \ln\frac{1}{D_{kj}} + \sum_{j=1}^{N-1} I_j \ln\frac{D_{Pj}}{D_{PN}}\right]$$

As $P \longrightarrow \infty$ very far away

D_{Pj} and D_{PN} almost the same $(D_{Pj} = D_{PN}) \Longrightarrow \left[\ln\frac{D_{Pj}}{D_{PN}} = \ln 1 = 0\right]$

$$\boxed{\lambda_k = 2*10^{-7}\sum_{j=1}^{N} I_j \ln\frac{1}{D_{kj}}}$$ ***

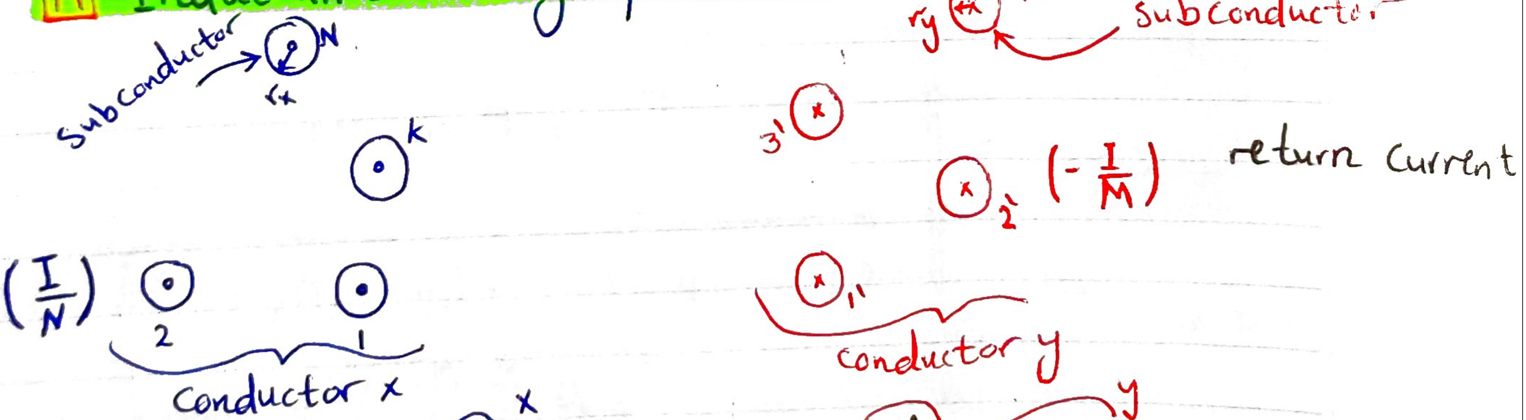
↳ Total Flux Linkages for the Conductor k.

Inductance

Inductance of Single-Phase Lines A

Inductance of 3 ϕ T.L B

A Inductance of Single-phase Lines



$$\phi_k = 2*10^{-7}\left[\overbrace{\frac{I}{N}\sum_{m=1}^{N}\ln\frac{1}{D_{km}}}^{x} - \overbrace{\frac{I}{M}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}}^{y}\right]$$ using **

↳ The total flux for any subconductor k in conductor x.

$$\Phi_k = 2*10^{-7}\left[\frac{I}{N}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{I}{M}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

Since only the fraction $\frac{1}{N}$ of the total conductor current I is linked by this flux, the flux linkage (λ_k) of sub conductor k is

$$\lambda_k = \frac{\phi_k}{N} = 2*10^{-7} I\left[\frac{1}{N^2}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

The total flux linkage of conductor x is :-

$$\lambda_x = \sum_{k=1}^{N}\lambda_k$$

$$= 2*10^{-7} I \sum_{k=1}^{N}\left[\frac{1}{N^2}\sum_{m=1}^{N}\ln\frac{1}{D_{km}} - \frac{1}{NM}\sum_{m=1'}^{M}\ln\frac{1}{D_{km}}\right]$$

$$= 2*10^{-7} I \ln \prod_{k=1}^{N}\frac{\left(\prod_{m=1'}^{M} D_{km}\right)^{\frac{1}{NM}}}{\left(\prod_{m=1}^{N} D_{km}\right)^{\frac{1}{N^2}}}$$

⋙ $L_x = 2*10^{-7}\ln\frac{D_{xy}}{D_{xx}}$ H/m/Conductor

⋙ $L_y = 2*10^{-7}\ln\frac{D_{xy}}{D_{yy}}$ H/m/conductor

where:

Geometric Mean Distance between x and y

$$D_{xy} = GMD_{x,y} = \sqrt[MN]{\prod_{k=1}^{N}\prod_{m=1'}^{M} D_{KM}}$$

$$= \sqrt[MN]{(D_{11'} D_{12'} D_{13'} \cdots D_{1M}) \cdots\cdots (D_{N1'} D_{N2'} \cdots D_{NM})}$$

$$D_{xx} = GMR_x = \sqrt[N^2]{\prod_{K=1}^{N}\prod_{m=1}^{N} D_{km}}$$

$$= \sqrt[N^2]{(D_{11} D_{12} D_{13} \cdots D_{1n}) \cdots (D_{N1} D_{N2} \cdots D_{NN})}$$

Geometric Mean Radius of Conductor x

$$D_{yy} = GMR_y = \sqrt[M^2]{\prod_{k=1'}^{M}\prod_{M=1'}^{M} D_{KM}}$$

$$= \sqrt[N^2]{(D_{1'1'} D_{1'2'} \cdots D_{1'M}) \cdots (D_{M1'} D_{M2'} \cdots D_{MM})}$$

Geometric Mean Radius of Conductor y.

Note:

- $\frac{1}{N^2}\left(\ln\frac{1}{a}+\ln\frac{1}{D}+\ln\frac{1}{c}\right)-\frac{1}{NM}\left(\ln\frac{1}{x}+\ln + \ln\frac{1}{z}\right)$

$$= \frac{1}{N^2}\left[\ln\frac{1}{abc}\right] - \frac{1}{NM}\left(\ln\frac{1}{xyz}\right)$$

$$= \ln\frac{1}{(abc)^{\frac{1}{N^2}}} - \ln\frac{1}{(xyz)^{\frac{1}{NM}}}$$

$$= \ln\frac{\frac{1}{(abc)^{\frac{1}{N^2}}}}{\frac{1}{(xyz)^{\frac{1}{NM}}}}$$

$$= \ln\frac{(xyz)^{\frac{1}{NM}}}{(abc)^{\frac{1}{N^2}}}$$

- $\ln A^{\alpha} = \alpha \ln A$

- $\sum \ln A_k = \ln \prod A_k$

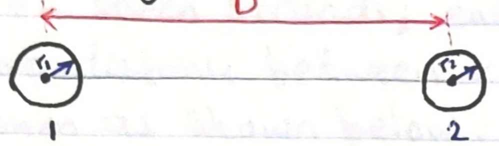
note that :-

$D_{11} = D_{22} = D_{33} = \cdots = D_{NN} = r'_x$

$D_{1'1'} = D_{2'2'} = D_{3'3'} = \cdots = D_{MM} = r'_y$

⋙ $L = L_x + L_y$ H/m/circuit

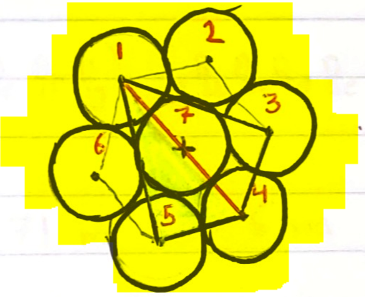
» if we have Single-phase two-wire line



$L_1 = 2*10^{-7} \ln \frac{D}{r_1'}$ H/m $\qquad$ $L_2 = 2*10^{-7} \ln \frac{D}{r_2'}$ H/m

$r_1' = 0.7788 r_1$ $\qquad$ $r_2' = 0.7788 r_2'$

Example A stranded conductor consists of seven identical strands each strand having a radius r as shown in Figure below, determine the GMR of the conductor interms of r.



$D_{12} = D_{16} = D_{17} = 2r$

$D_{14} = 4r$

$D_{13} = D_{15} = \sqrt{D_{14}^2 - D_{45}^2}$

$= \sqrt{16r^2 - 4r^2}$

$= \sqrt{12r^2}$

$= 2\sqrt{3}\, r$

$$GMR = \sqrt[N^2]{(D_{11} D_{12} D_{13} D_{14} D_{15} D_{16} D_{17})(D_{21} D_{22} D_{23} \cdots D_{27}) \cdots\cdots (D_{71} \cdots)}$$

$$= \sqrt[49]{\underbrace{(r' \cdot 2r \cdot 2\sqrt{3}r \cdot 4r \cdot 2\sqrt{3}r \cdot 2r \cdot 2r)^6}_{1,2,3,4,5,6} \underbrace{(r')(2r)^6}_{7}}$$

$= 2.1767\, r$

» With large number of strands the calculation of GMR can become very tedious. (مُمل، مُضجر)

» Usually these are available in the manufacturer's data. (Tables)

» The design of a power line requires the value of resistance and reactance to find out the active and reactive power, and the voltage drop in the process of power transfer over the transmission line.

» Power losses should be limitted to around (5-10)% of the total power transfered.

TABLE A.4 Characteristics of aluminum cable, steel, reinforced (Aluminum Company of America)—ACSR

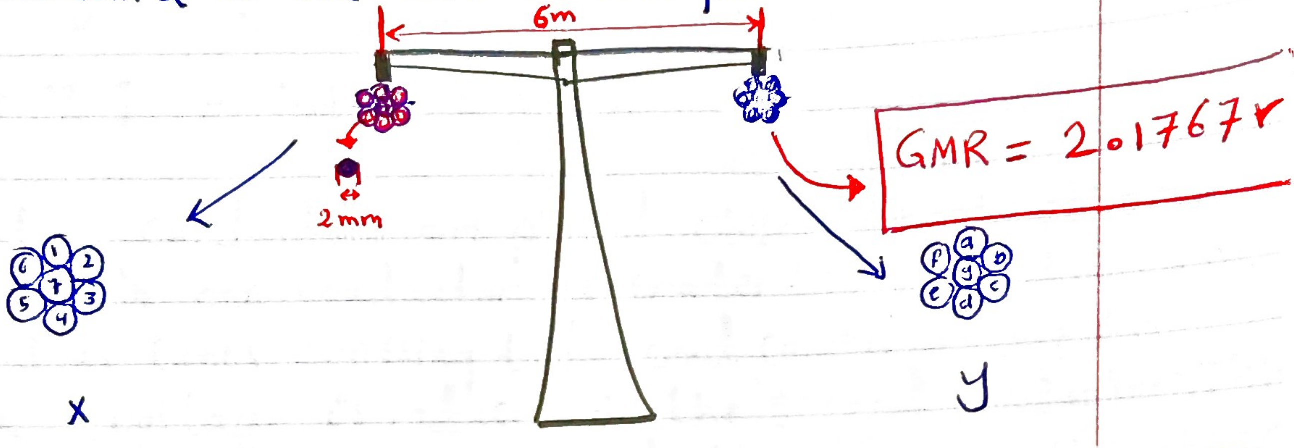
Code Word	Circular Mils Aluminum	Aluminum			Steel		Outside Diameter (inches)	Copper Equivalent* Circular Mils or A W G	Ultimate Strength (pounds)	Weight (pounds per mile)	Geometric Mean Radius at 60 Hz (feet)	Approx. Current Carrying Capacity† (amps)	r_a Resistance (Ohms per Conductor per Mile)								x_a Inductive Reactance (ohms per conductor per mile at 1 ft spacing all currents)	x'_a Shunt Capacitive Reactance (megohms per conductor per mile at 1 ft spacing)
				Strand Diameter (inches)		Strand Diameter (inches)							25°C (77°F) Small Currents				50°C (122°F) Current Approx. 75% Capacity‡					
													dc	25 Hz	50 Hz	60 Hz	dc	25 Hz	50 Hz	60 Hz	60 Hz	60 Hz
Joree	2 515 000	76		0.1819	19	0.0849	1.880		61 700		0.0621									0.0450	0.337	0.0755
Thrasher	2 312 000	76		0.1744	19	0.0814	1.802		57 300		0.0595									0.0482	0.342	0.0767
Kiwi	2 167 000	72	4	0.1735	7	0.1157	1.735		49 800		0.0570									0.0511	0.348	0.0778
Bluebird	2 156 000	84	4	0.1602	19	0.0961	1.762		60 300		0.0588									0.0505	0.344	0.0774
Chukar	1 781 000	84	4	0.1456	19	0.0874	1.602		51 000		0.0534									0.0598	0.355	0.0802
Falcon	1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	0.359	0.0814
Parrot	1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	0.362	0.0821
Plover	1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	0.365	0.0830
Martin	1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	0.369	0.0838
Pheasant	1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	0.372	0.0847
Grackle	1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	0.376	0.0857
Finch	1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 200	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	0.380	0.0867
Curlew	1 033 500	54	3	0.1384	7	0.1384	1.246	650 000	37 100	7 019	0.0420	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	0.385	0.0878
Cardinal	954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	0.390	0.0890
Canary	900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	0.393	0.0898
Crane	874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	0.395	0.0903
Condor	795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378	0.401	0.0917
Drake	795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.399	0.0912
Mallard	795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.393	0.0904
Crow	715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	0.407	0.0932
Starling	715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.405	0.0928
Redwing	715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.399	0.0920
Flamingo	666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601	0.412	0.0943
Rook	636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688	0.414	0.0950
Grosbeak	636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.412	0.0946
Egret	636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.406	0.0937
Peacock	605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	0.417	0.0957
Squab	605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720	0.415	0.0953
Dove	556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.420	0.0965
Eagle	556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.415	0.0957
Hawk	477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216				0.430	0.0988
Hen	477 000	30	2	0.1261	7	0.1261	0.883	300 000	23 300	3 933	0.0304	670	0.196	0.196	0.196	0.196	0.216				0.424	0.0980
Ibis	397 500	26	2	0.1236	7	0.0961	0.783	250 000	16 190	2 885	0.0265	590	0.235				0.259				0.441	0.1015
Lark	397 500	30	2	0.1151	7	0.1151	0.806	250 000	19 980	3 277	0.0278	600	0.235		Same as dc		0.259		Same as dc		0.435	0.1006
Linnet	336 400	26	2	0.1138	7	0.0855	0.721	4/0	14 050	2 442	0.0244	530	0.278				0.306				0.451	0.1039
Oriole	336 400	30	2	0.1059	7	0.1059	0.741	4/0	17 040	2 774	0.0255	530	0.278				0.306				0.445	0.1032
Ostrich	300 000	26	2	0.1074	7	0.0835	0.680	188 700	12 650	2 178	0.0230	490	0.311				0.342				0.458	0.1057
Piper	300 000	30	2	0.1000	7	0.1000	0.700	188 700	15 430	2 473	0.0241	500	0.311				0.342				0.462	0.1049
Partridge	266 800	26	2	0.1013	7	0.0788	0.642	3/0	11 250	1 936	0.0217	460	0.350				0.385				0.465	0.1074

*Based on copper 97%, aluminum 61% conductivity

†For conductor at 75°C, air at 25°C, wind 1.4 miles per hour (2 ft/sec), frequency = 60 Hz

‡ "Current Approx. 75% Capacity" is 75% of the "Approx. Current Carrying Capacity in Amps" and is approximately the current which will produce 50°C conductor temp. (25°C rise) with 25°C air temp., wind 1.4 miles per hour.

example Power is transmitted over the live stranded conductor with seven strands; each strand 2mm in diameter. The distance between the live and neutral wires is 6mm as shown below. Calculate the inductance and reactance of the line in mH per km.



GMR = 2.1767r

$$GMD_{xy} = \sqrt[49]{(D_{1a}\,D_{1b}\,D_{1c}\,D_{1d}\,D_{1e}\,D_{1f}\,D_{1g})(D_{2a}\,D_{2b}\,D_{2c}\,D_{2d}\,D_{2e}\,D_{2f}\,D_{2g})\cdots\cdots(D_{7a}\,D_{7b}\,D_{7c}\,D_{7d}\,D_{7e}\,D_{7f}\,D_{7g})}$$

$$= 5.99999971 \text{ m} \cong 6 \text{ m}$$

$$GMR_x = GMR_y = 2.1767r = (2.1767)(0.001)$$
$$= 0.0021767$$

$$L_x = 2*10^{-7}\ln\frac{D_{xy}}{D_{xx}} = 2*10^{-7}\ln\frac{6}{0.002177} \text{ H/m}$$

$$= 1.584*10^{-6} \text{ H/m}$$ per Conducto

$$L = L_x + L_y = 3.168*10^{-6} \text{ H/m}$$

$$X_L = wL = 2\pi f L$$ ≙ Reactance per meter length of Conductor

$$= 2\pi(50)(L)$$

$$= 9.954*10^{-4}\ \Omega/\text{m}$$

$$= 0.9954\ \Omega/\text{km}$$

Notes

>> The flux Linkage $\lambda = L . I$

>> The voltage drop due to this flux Linkage is

$$V = Z I = j\omega L I = j\omega\lambda$$

>> When two conductors are placed close to each other, current in one conductor generates the magnetic flux These flux Lines crossing the second conductor due to which a voltage is induced in the second conductor. This process af current in one conductor affecting the other conductor is the mutual inductance.

>> If we define the two conductors as 1 and 2, then

$$M_{12} = \frac{\lambda_{12}}{I_2}$$

where
- M_{12} is the mutual inductance between conductors 1 and 2.
- λ_{12} is the flux Linkage between Conductors 1 and 2.
- I_2 is the current in conductor 2.

This in turn introduces the voltage drop in the first conductor which is defined by:

$$\boxed{V_1 = j\omega M_{12} I}$$

Inductance

Inductance of Single ϕ **A** | Inductance of 3 ϕ T.L. **B**

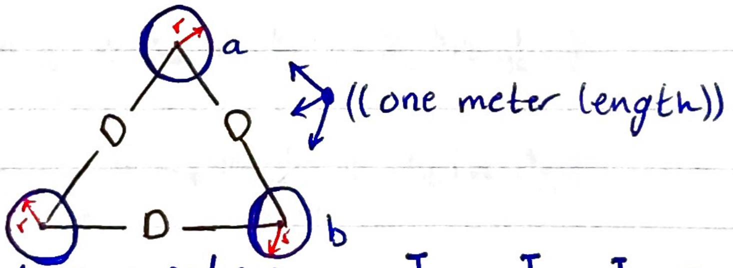
B Inductance of 3 ϕ T.L.

a) Symmetrical Spacing (Equilateral Spacing).
b) Asymmetrical Spacing.
c) Transposition.
d) Bundled Conductor.

Composite Conductor :-

$$\lambda_k = 2*10^{-7} \sum_{j=1}^{N} I_j \ln \frac{1}{D_{kj}}$$

a) Three phase line with equilateral spacing.



Assuming Balanced 3ϕ currents :- $I_a + I_b + I_c = 0$

⇒ The total flux linkage of phase a conductor is :-

$$\lambda_a = 2*10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D}\right)$$

$$= 2*10^{-7}\left(I_a \ln\frac{1}{r'} + (I_b + I_c)\ln\frac{1}{D}\right)$$

$$= 2*10^{-7}\left(I_a \ln\frac{1}{r'} - I_a \ln\frac{1}{D}\right) = 2*10^{-7} I_a \ln\frac{D}{r'}$$

$$L_a = \frac{\lambda_a}{I_a} = 2*10^{-7} \ln\frac{D}{r'} \ \text{H/m} = 0.2 \ln\frac{D}{D_s} \ \text{mH/km}$$

r' GMR

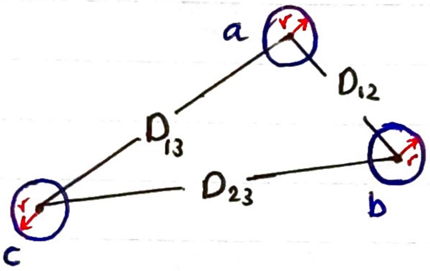
$$\lambda_a = \lambda_b = \lambda_c \Rightarrow L_a = L_b = L_c$$

** This means that the inductance per phase for 3ϕ circuit with equilateral spacing is the same as for one conductor of single phase circuit.

b)) Asymmetrical Spacing :-

» Practical transmission lines cannot maintain symmetrical spacing of conductors because of construction considerations.

» With asymmetrical spacing, even with balanced currents, the voltage drop due to line inductance will be unbalanced.



$$\lambda_a = 2*10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{13}}\right)$$

$$\lambda_b = 2*10^{-7}\left(I_a \ln\frac{1}{D_{12}} + I_b \ln\frac{1}{r'} + I_c \ln\frac{1}{D_{23}}\right)$$

$$\lambda_c = 2*10^{-7}\left(I_a \ln\frac{1}{D_{13}} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{r'}\right)$$

Or in matrix form $\boldsymbol{\lambda} = \mathbf{L}\,\mathbf{I}$

where the symmetrical inductance matrix $\mathbf{L}$ is given by:

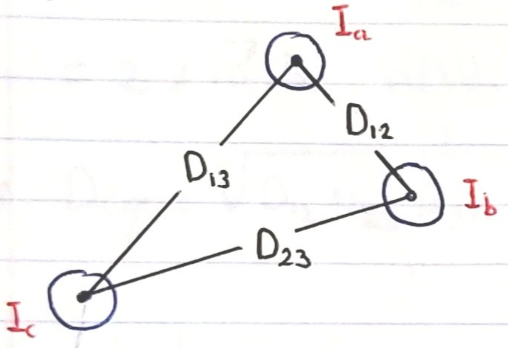
$$\mathbf{L} = 2*10^{-7}\begin{bmatrix} \ln\frac{1}{r'} & \ln\frac{1}{D_{12}} & \ln\frac{1}{D_{13}} \\ \ln\frac{1}{D_{12}} & \ln\frac{1}{r} & \ln\frac{1}{D_{23}} \\ \ln\frac{1}{D_{13}} & \ln\frac{1}{D_{23}} & \ln\frac{1}{r'} \end{bmatrix}$$

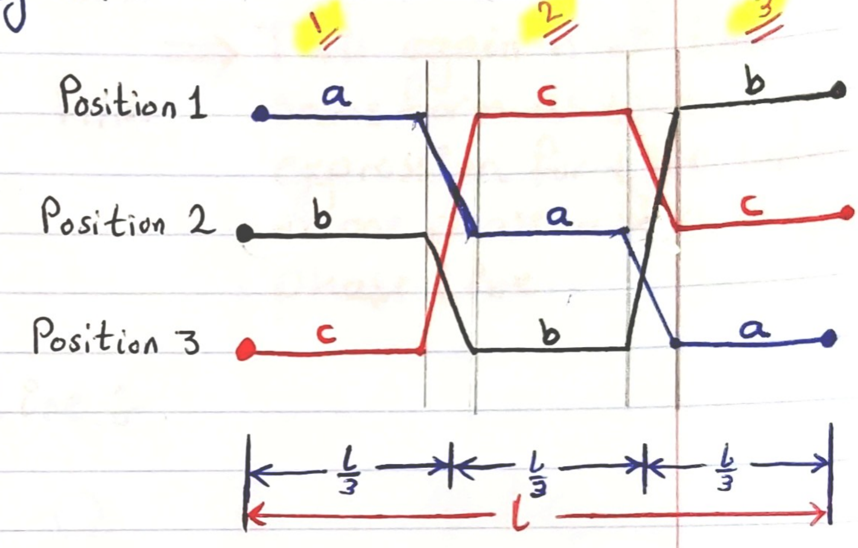
⇒ The phase inductances are not equal

c)) Three phase transposed Line :

» One way to regain symmetry and obtain per-phase model is Consider transposition.

» The transposition consists of interchanging the phase configuration every one-third the length.





$$\lambda_{a1} = 2*10^{-7}\left[I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{31}}\right]$$

r' ↔ GMR

$$\lambda_{a2} = 2*10^{-7}\left[I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{D_{12}}\right]$$

$$\lambda_{a3} = 2*10^{-7}\left[I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{31}} + I_c \ln\frac{1}{D_{23}}\right]$$

$$\lambda_a = \frac{\lambda_{a1}\left(\frac{l}{3}\right) + \lambda_{a2}\left(\frac{l}{3}\right) + \lambda_{a3}\left(\frac{l}{3}\right)}{l} = \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3}$$

$$= \frac{2*10^{-7}}{3}\left[3 I_a \ln\frac{1}{D_s} + I_b \ln\frac{1}{D_{12}D_{23}D_{31}} + I_c \ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

$$= \frac{2*10^{-7}}{3}\left[3 I_a \ln\frac{1}{D_s} - I_a \ln\frac{1}{D_{12}D_{23}D_{31}}\right]$$

$$\lambda_a = 2*10^{-7} I_a \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s}$$

$$L_a = \frac{\lambda_a}{I_a} = 2*10^{-7} \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s} \quad \text{H/m per phase}$$

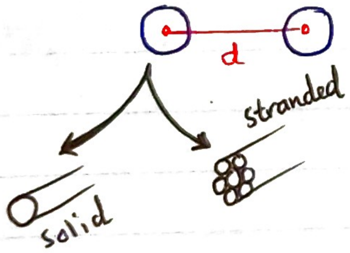
$$L_a = 2*10^{-7} \ln \frac{D_{eq}}{D_s} \quad \text{H/m}$$

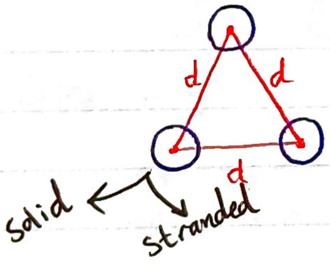
$$= 2*10^{-7} \ln \frac{GMD}{D_s} \quad \text{H/m}$$

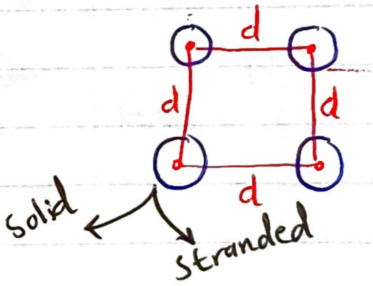
where $D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}}$

⇒ This again is of the same form as the expression for the induc of one phase of a single phase Line.

d)) Bundled Conductor line :-







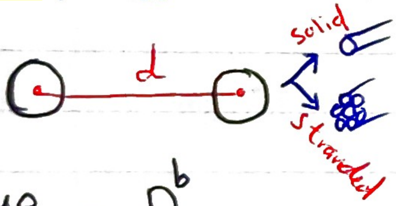
» Extra-high voltage transmission lines are usually constructed with bundled conductors. ⊛ Bundling reduces the line reactance, which improves the line performance and increase the power capability of the line. ⊛ Bundling also reduces the voltage surface gradient, which in turn reduces Corona loss, radio interference, and surge impedance. $\left(\sqrt{\frac{L}{C}}\right)$

» Typically, bundled conductors Consists of two, three, or four subconducters symmetrically arranged in configuration as shown in Figure above.

» The subconductors within abundle are separated at frequent intervals by spacer-dampers. spacer-dampers prevent clashing, provide damping, and connect the subconductors in parallel.

Bundling

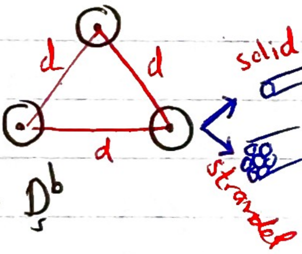
- Reduces Electric Field Strength on Conductor Surface
  - ↓ Reduces Corona
- Increases Effective Radius (GMR)
  - ↓ Reduces Inductance



$$GMR_b = D_s^b$$

$$= \sqrt[4]{(r'.d)^2}$$

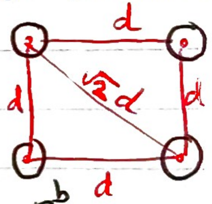
$$= \sqrt[2]{r'.d}$$



$$GMR_b = D_s^b$$

$$= \sqrt[9]{(r'.d.d)^3}$$

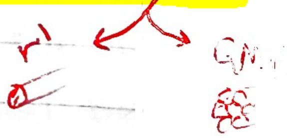
$$= \sqrt[3]{r'\ d^2}$$



$$GMR_b = D_s^b$$

$$= \sqrt[16]{(r'.d.d.d\sqrt{2})^4}$$

$$= 1.091\sqrt[4]{r'\ d^3}$$

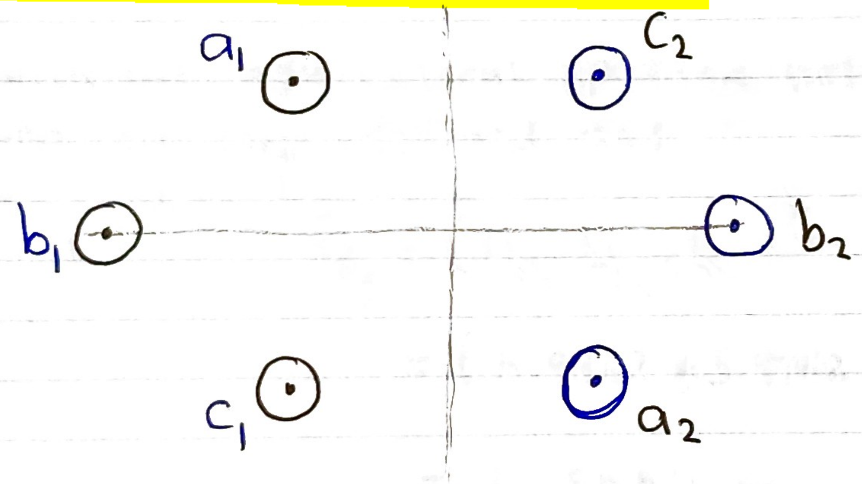


$$\boxed{L_a = 2*10^{-7}\ln\frac{D_{eq}}{D_s^b}\quad H/m}$$

» Three-phase Lines - Parallel Circuits.

» Three-phase Double-Circuit Lines.

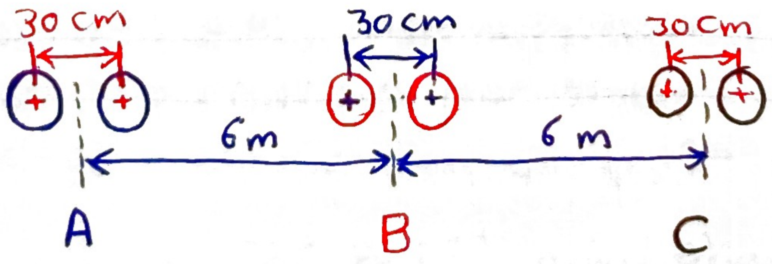
A three-phase double-Circuit line consists of two identical 3ϕ Circuits. The circuits are operated with abc, cba in parallel. Because of geometrical differences between Conductors, voltage drop due to line inductance will be unbalanced. To achieve balance, each phase conductor must be transposed within its group and with respect to parallel 3ϕ line.



example The conductor configuration of a completely transposed 3-ϕ overhead transmission line with bundled conductor is shown below. All the Conductors have a radius of 0.74 cm with a 30 cm bundle spacing.

a)) Determine the inductance per-phase in mH/km and in mH/m.

b)) Find the inductive line reactance per phase in Ω/m at $f = 50$ Hz.



$$D_{ab} = \sqrt[4]{d_{13}\; d_{14}\; d_{23}\; d_{24}}$$

$$= (6 * 6.3 * 5.7 * 6)^{1/4} = 5.9962 \text{ m}$$

Similarly,

$$D_{bc} = 5.9962 \text{ m}$$

$$D_{ca} = \sqrt[4]{d_{15}\; d_{16}\; d_{25}\; d_{26}}$$

$$= (12 * 12.3 * 11.7 * 12)^{\frac{1}{4}} = 11.9981 \text{ m}$$

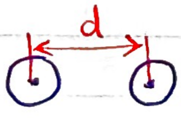
The equivalent equilateral spacing between the phases is given by Deq defined as :-

$$D_{eq} = (D_{ab} \cdot D_{bc} \cdot D_{ca})^{\frac{1}{3}}$$

$$= (5.9962 * 5.9962 * 11.9981)^{\frac{1}{3}}$$

$$= 7.5559 \text{ m}$$

$$D_s^b = \sqrt[2]{r' d}$$



$$= (0.7788 * r * 30)^{\frac{1}{2}} = 4.1580 \text{ cm}$$

a)) Inductance per phase for the given system is :-

$$L = 2 * 10^{-7} \ln \frac{D_{eq}}{D_s^b} \quad \text{H/m/phase}$$

$$= 1.04049 * 10^{-6} \quad \text{H/m/phase}$$

$$= 1.04049 * 10^{-3} \quad \text{mH/m/phase} = 1.04049 \text{ mH/km/phase}$$

b)) The inductive line reactance per phase

$$X_L = 2\pi f L = 2\pi (50)(1.04049) * 10^{-6} \quad \Omega\text{/m/phase}$$

$$= 3.270 * 10^{-4} \quad \Omega\text{/m/phase}$$

Transmission Lines Parameters

- T.L Resistance
- T.L Inductance
- T.L Capacitance

Transmission Line Capactance :

» Capacitance of transmission line is the result of the potential difference between the conductors, it causes them to be charged in the same manner as the plates of a capacitor, when there is a potential difference between them the capacitance between conductors is the charge per unit of the potential difference.

1)) Electric Field and Voltage Calculation
2)) Transmission Line Capacitance for:-
  - [A] Single-phase Line.
  - [B] 3ϕ Lines with equal spacing.
  - [C] 3ϕ Lines, bundled conductor, and unequal spacing.

1)) **Gauss's Law** → Electric Field Strength (E) → Voltage between Conductors → Capacitance $C = q/V$

Gauss's Law :- Total electric flux leaving a closed surface = Total charge within the vollume enclosed by the closed surface.

⇓ Leads to

Normal Electric Flux density integrated over the closed surface = charge enclosed by this closed surface.

surface integral over closed surface ←

$$\oiint D_\perp \, ds = \oiint \varepsilon E_\perp \, ds = Q_{enclosed}$$

Where,

$\varepsilon \triangleq$ permittivity of the medium $= \varepsilon_r \varepsilon_o$

$\varepsilon_o = 8.854 * 10^{-12}$ F/m

$D_\perp \triangleq$ normal component of electric flux density.

$E_\perp \triangleq$ normal component of electric field strength.

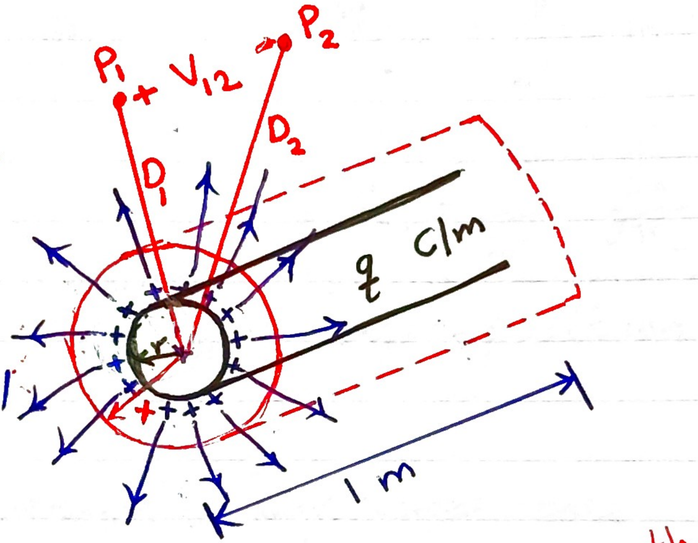
ds = the differential surface area.

Note :-

Inside the perfect conductor, Ohm's law give $E_{int} = 0$

⇓

That is, the internal electric field $E_{int} = 0$



1 m length

$$\oiint \varepsilon E_\perp \, ds = Q_{enclosed}$$

$$\varepsilon E_x (2\pi x)(1) = q \; (1)$$

$$E_x = \frac{q}{2\pi\varepsilon x} \quad \text{V/m}$$

$$V_{12} = \int_{D_1}^{D_2} E_x \, dx = \int_{D_1}^{D_2} \frac{q}{2\pi\varepsilon x} \, dx$$

$$V_{12} = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1} \quad \text{Volts}$$

$D_2 \xrightarrow{P_2} -$

$D_1 \xrightarrow{P_1} +$

where,

$$\varepsilon = \varepsilon_r \varepsilon_o$$

$$\varepsilon_o = 8.854 * 10^{-12} \text{ F/m}$$

note

⇓

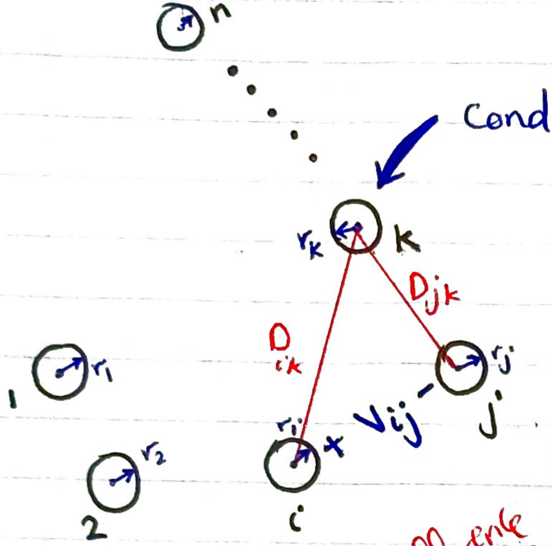
$-$ • P_2

V_{12}

$+$ • P_1

$$\boxed{V_{12} = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1} \text{ volts}}$$

Multi-Conductor System :



Conductor k has radius r_k and charge q_k ((per meter length of the conductor))

$$V_{ijk} = \frac{q_k}{2\pi\varepsilon} \ln \frac{D_{jk}}{D_{ik}} \quad \text{volts}$$

$$\boxed{V_{ij} = \sum_{k=1}^{n} \frac{q_k}{2\pi\varepsilon} \ln \frac{D_{jk}}{D_{ik}} \quad \text{volts}}$$

voltage difference due to charges in all conductors

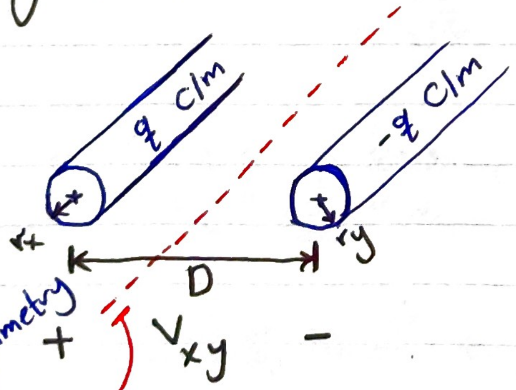
Super-position Theorem

Transmission Line Capacitance

Single-Phase Line $\boxed{A}$

Three-Phase Lines $\boxed{B}$

$\boxed{A}$ Single-Phase Line



due to Symmetry

↗ zero-voltage
↗ zero-potential
↗ potential of neutral

$$V_{xy} = \frac{1}{2\pi\varepsilon}\left[q \ln \frac{D_{yx}}{D_{xx}} - q \ln \frac{D_{yy}}{D_{xy}}\right]$$

$$= \frac{q}{2\pi\varepsilon} \ln \frac{D_{yx} D_{xy}}{D_{xx} D_{yy}}$$

$$= \frac{q}{\pi\varepsilon} \ln \frac{D}{\sqrt{r_x r_y}} \quad \text{Volts}$$

$$C_{xy} = \frac{q}{V_{xy}} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{\sqrt{r_x r_y}}\right)} \quad \text{F/m}$$

००० Notes ०००

» $V_{12}(q_1) = \frac{q_1}{2\pi\varepsilon} \ln \frac{D}{r}$

» $V_{12}(q_2) = \frac{q_2}{2\pi\varepsilon} \ln \frac{r}{D}$

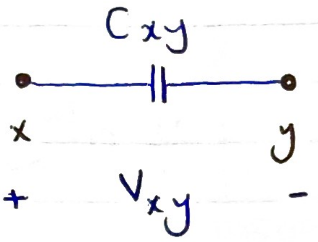
» $V_{21}(q_2) = \frac{q_2}{2\pi\varepsilon} \ln \frac{D}{r} = -V_{12}$

» $V_{12} = V_{12}(q_1) + V_{12}(q_2)$

$q_2 = -q_1$

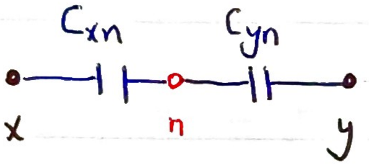
$$C_{xy} = \frac{\pi \varepsilon}{\ln\left(\frac{D}{\sqrt{r_x r_y}}\right)}$$ if $r_x = r_y$

$$C_{xy} = \frac{\pi \varepsilon}{\ln\left(\frac{D}{r}\right)}$$

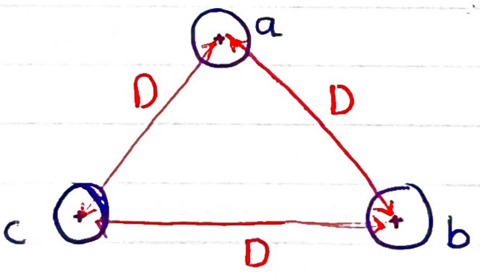


$$V_{xn} = V_{yn} = \frac{V_{xy}}{2}$$

$$C_n = C_{xn} = C_{yn} = \frac{q}{V_{xn}} = 2C_{xy} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \quad F/m$$



B Three-Phase Line with Equilateral Spacing:



$q_a + q_b + q_c = 0$

$$\Rightarrow V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{ba}}{D_{aa}} + q_b \ln\frac{D_{bb}}{D_{ab}} + q_c \ln\frac{D_{bc}}{D_{ac}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + q_c \cancelto{0}{\ln\frac{D}{D}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D}\right] \quad \text{Volts}$$

$$\Rightarrow V_{ac} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{ca}}{D_{aa}} + q_b \ln\frac{D_{cb}}{D_{ab}} + q_c \ln\frac{D_{cc}}{D_{ac}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{D}{D} + q_c \ln\frac{r}{D}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_c \ln\frac{r}{D}\right]$$

$V_{ab} + V_{ac}$

$$V_{ab}+V_{ac} = \left(\frac{1}{2\pi\varepsilon}\right)\left[2q_a \ln\frac{D}{r} + \overset{\ominus q_a}{\underbrace{(q_b+q_c)}} \ln\frac{r}{D}\right]$$

$$\boxed{V_{an} = \frac{1}{3}\left(V_{ab}+V_{ac}\right)}$$

$$\downarrow \quad = \frac{1}{3}\left(\frac{1}{2\pi\varepsilon}\right)\left[2q_a \ln\frac{D}{r} + q_a \ln\frac{D}{r}\right]$$

$$= \frac{q_a}{2\pi\varepsilon} \ln\frac{D}{r}$$

$C_{an} = \dfrac{2\pi\varepsilon}{\ln\frac{D}{r}}$ F/m Line to neutral

Notes:

$$V_{ab} = \sqrt{3}\,V_{an}\angle{+30} = \sqrt{3}\,V_{an}\left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right]$$

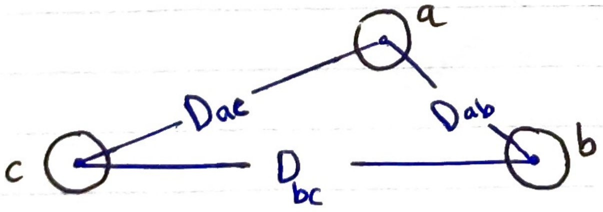
$$V_{ac} = -V_{ca} = \sqrt{3}\,V_{an}\angle{-30} = \sqrt{3}\,V_{an}\left[\frac{\sqrt{3}}{2} - j\frac{1}{2}\right]$$

+

$$V_{ab}+V_{ac} = 3V_{an}$$

$$\uparrow \quad \boxed{V_{an} = \frac{1}{3}\left(V_{ab}+V_{ac}\right)}$$

C 3ϕ with asymmetrical Spacing



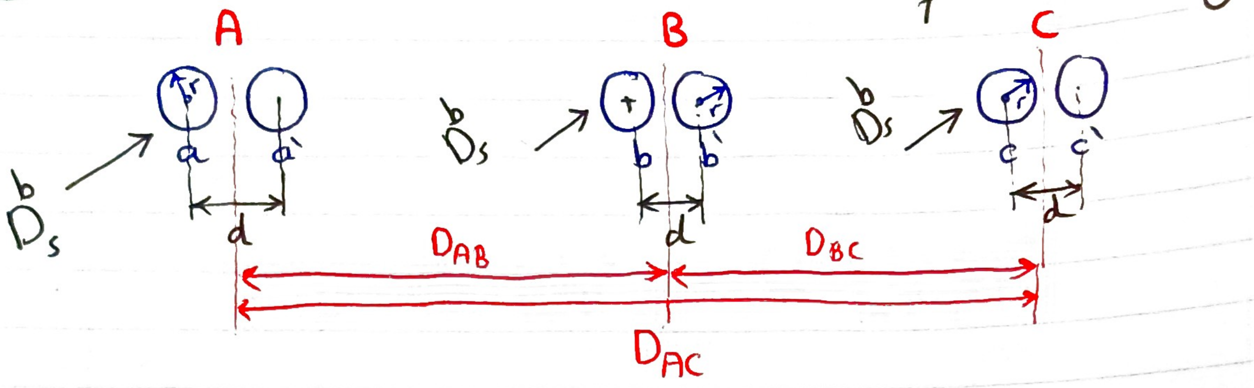
$$C_{an} = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{r}\right)}, \quad D_{eq} = \sqrt[3]{D_{ab}\,D_{ac}\,D_{bc}}$$

(r) solid

$\left(\dfrac{\text{outside diameter}}{2}\right)$

stranded

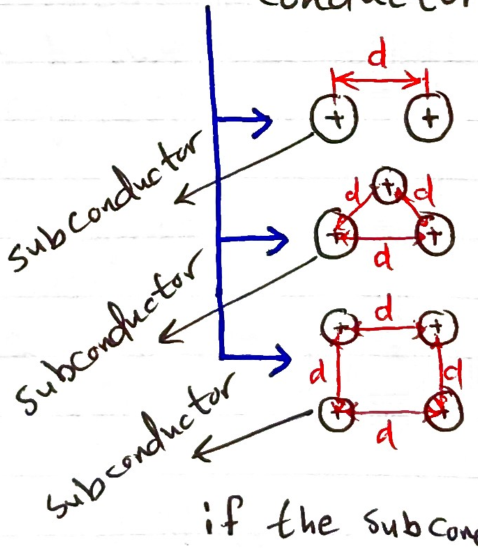
D 3ϕ Bundled Conductor with unequal spacing



$D_{AB} = GMD_{A,B}$, $D_{BC} = GMD_{B,C}$, $D_{AC} = GMD_{A,C}$

$$C_{an} = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{D_s^b}\right)}$$

$$D_{eq} = \sqrt[3]{D_{AB}\ D_{BC}\ D_{AC}}$$, $D_s^b \triangleq$ GMR for the bundled conductor



$D_s^b = \sqrt[2]{r\,d}$

$D_s^b = \sqrt[3]{r\,d^2}$

$D_s^b = 1.091\sqrt[4]{r\,d^3}$

if the subconductor is stranded

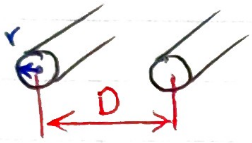
$r \longrightarrow \left[\frac{\text{outside diameter}}{2}\right]$

from manufacture's data (Tables)

$$C \rightarrow 1\phi, \quad 3\phi, \quad 3\phi \text{ Bundel}$$

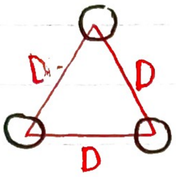
1 φ

$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{r}}$$

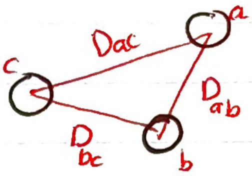


3 φ

$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{r}}$$

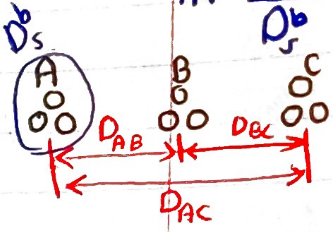


$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D_{eq}}{r}}$$



3φ Bundel

$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D_{eq}}{D_s^b}}$$



[if stranded conductor exist instead of solid
$r \longrightarrow \frac{\text{outside diameter}}{2}$]

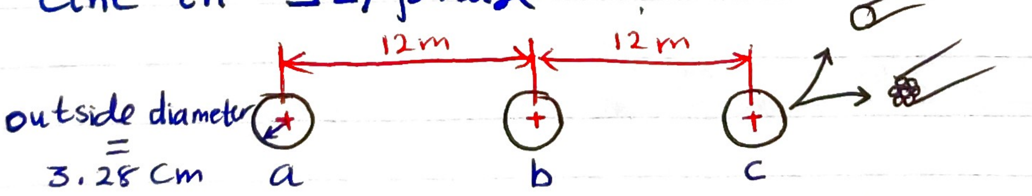
example

A three-phase, 400 kV, 50 Hz, 350 km overhead T.L. has flat horizontal spacing with three identical conductors. The conductors have an outside diameter of 3.28 cm with 12 m between adjacent conductors.

>> Determine the capacitive reactance-to-neutral in Ω/m/phase

>> Determine the capacitive reactance for the Line in Ω/phase

Solution



$$D_{eq} = \sqrt[3]{D_{ab}\,D_{ac}\,D_{bc}} = \sqrt[3]{(12)(24)(12)} = 15.119 \text{ m}$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{r}\right)} = 8.163 * 10^{-6} \mu F/m$$

note: $Z_c = \frac{1}{j\omega C}$

$Y_c = \omega C$

$$Y_n = 2\pi * 50 * C_n = 2.565 * 10^{-9} \text{ ℧/m/phase}$$

Length = 350 km

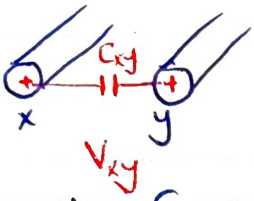
$$Y_n = 8.978 * 10^{-4} \text{ ℧/phase}$$

$$\text{Reactance} = X_n = \frac{1}{Y_n} = 1.1138 * 10^{-3} \text{ Ω/phase}$$

Line charging current:-

The current supplied to the transmission line capacitance is called charging current.

■ For a single-phase circuit operating at line-to-line voltage $V_{xy} = V_{xy} \angle 0^\circ$.



» The charging current is

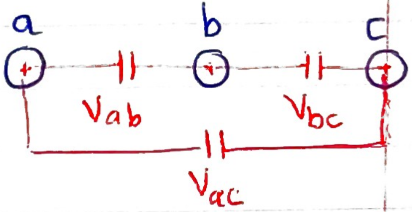
$$I_{chg} = Y_{xy} V_{xy} = j\omega C_{xy} V_{xy} \quad \text{Amp}$$

» The capacitor delivers reactive power, the reactive power delivered by this line-to-line capacitance is

$$Q_c = \frac{V_{xy}^2}{X_c} = Y_{xy} V_{xy}^2$$

$$= \omega C_{xy} V_{xy}^2 \quad \text{var}$$

■ For a completely transposed 3ϕ line that has $V_{an} = V_{LN} \angle 0^\circ$



» The phase a charging current i

$$I_{chg} = Y_{an} V_{an} = j\omega C_{an} V_{LN}$$

» The reactive power delivered by phase a is

$$Q_{C1\phi} = Y_{an} V_{an}^2 = \omega C_{an} V_{LN}^2 \quad \text{Va}$$

» The total reactive power supplied by the 3ϕ line is

$$Q_{C3\phi} = 3 Q_{C1\phi} = 3\omega C_{an} V_{LN}^2$$

$$= \sqrt{3}\sqrt{3}\, \omega C_{an} V_{LN} V_{LN}$$

$$Q_{C3\phi} = \omega C_{an} V_{LL}^2 \quad \text{Var}$$

Transmission Line Modeling

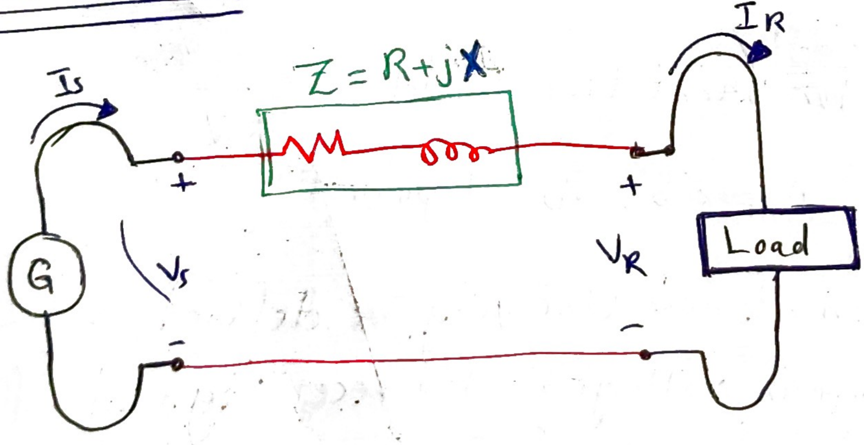
- Short Line Model (Less than 80 km)
- Medium Line Model (80km < L < 250 km)
- Long Line Model (L ≥ 250 km)

» Lumped parameter system.

» Distributed parameter system.

- we use Lumped parameters which give good accuracy for short lines and for lines of medium length.
- If an overhead line is classified as short, shunt capacitance is so small that it can be omitted entirely with little loss of accuracy, and we need to consider only the series resistance R and the series inductance L for the total length of the Line.

1| Short Line Model :-



» line length < 80 km

» Generally MV/LV Lin

» Capacitance cab be neglected

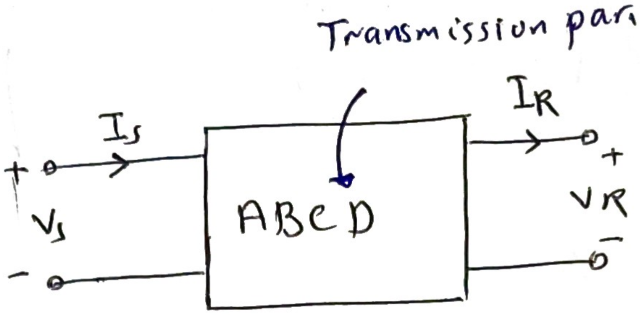
$$Z = (r + j\omega L)\, l$$
$$= R + jX$$

where r and L are the per-phase resistance and inductance per unit length, respectively, and l is the line length.

The phase voltage at the sending end is

$$V_s = V_R + Z I_R \quad \text{———} \quad ①$$

$$I_s = I_R$$



Two-port representation of a T.L

$$V_s = A V_R + B I_R$$
$$I_s = C V_R + D I_R \implies \begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

ثنائي

Since we are dealing with a linear passive, bilateral two-port network, the ~~de[illegible] raidt~~ of the transmission ~~matrix~~ is unity :-

$$AD - BC = 1$$

$$\implies \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

- According to ① for short Line model

$A = 1$ per unit, $B = Z\,\Omega$, $C = 0\,S$, $D = 1$ per unit

- Voltage regulation of the line may be defined as the percentage change in voltage at the receiving end of the line (expressed as percent of full load voltage) in going from no-load to full load.

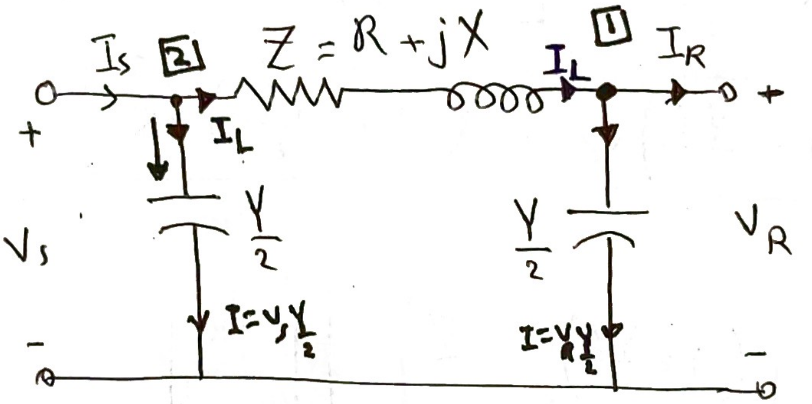
$$\boxed{\text{Percent } VR = \frac{|V_{R(NL)}| - |V_{R(FL)}|}{|V_{R(FL)}|} * 100}$$

Voltage regulation is a measure of line voltage drop.

At no Load $I_R = 0 \implies V_{R(NL)} = \frac{V_s}{A}$ ↙ A=1 for Short Line.

Medium Line Model

- 80km < Length < 250km.
- As the length of line increases, the line charging current becomes appreciable and the shunt capacitance must be considered.
- For medium length lines, half of the shunt capacitance may be considered to be lumped at each end of the line. This is referred to as the nominal π model and is shown in Figure below :-



$\boxed{I_L = I_R + V_R \frac{Y}{2}}$

E_R

in terms of

$V_s (V_R, I_R)$

$I_s (V_R, I_R)$

Z ≡ total series impedance of the Line.

Y ≡ total shunt admittance of the Line.

$Y = (g + j\omega c) l$

Under normal conditions, the shunt conductance per unit length, which represents the leakage current over the insulators and due to corona, is negligible and g is assumed to be Zero. C is the line to neutral capacitance per km, and l is the line length.

1. $$V_s = V_R + Z I_L$$

$$= V_R + Z\left(\overbrace{I_R + V_R \cdot \frac{Y}{2}}^{I_L}\right)$$

$$\boxed{V_s = \left(1 + \frac{YZ}{2}\right) V_R + Z I_R}$$

$$\boxed{\begin{aligned} V_s &= A V_R + B I_R \\ I_s &= C V_R + D I_R \end{aligned}}$$

2. $$I_s = I_L + V_s \cdot \frac{Y}{2}$$

$$= \left(\overbrace{I_R + V_R \cdot \frac{Y}{2}}^{I_L}\right) + \frac{\boxed{V_s}\, Y}{2}$$

$$= I_R + \boxed{\frac{V_R Y}{2}} + \left[\left(1 + \frac{YZ}{2}\right) V_R + Z I_R\right] \frac{Y}{2}$$

$$\boxed{I_s = Y\left(1 + \frac{YZ}{4}\right) V_R + \left(1 + \frac{YZ}{2}\right) I_R}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & Z \\ Y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

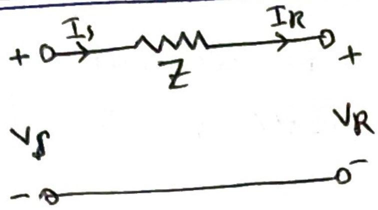
$A = D = 1 + \frac{YZ}{2}$ per unit

$B = Z\ \Omega$

$C = Y\left(1 + \frac{YZ}{4}\right) S$

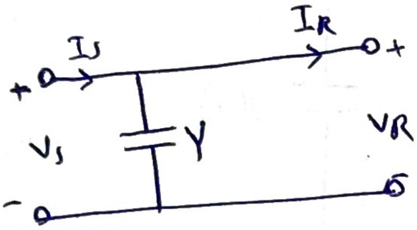
since the π model is a symmetrical two-port networt (A = D)

ABCD Matrix



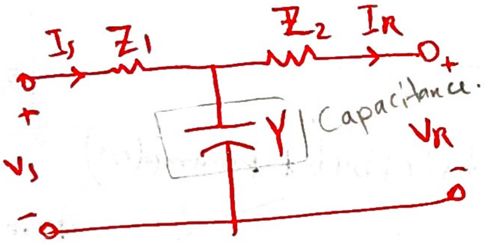
$$\left[\begin{array}{c|c} 1 & Z \\ \hline 0 & 1 \end{array}\right]$$

Short line



$$\left[\begin{array}{c|c} 1 & 0 \\ \hline Y & 1 \end{array}\right]$$

Compensat for Reactive power.

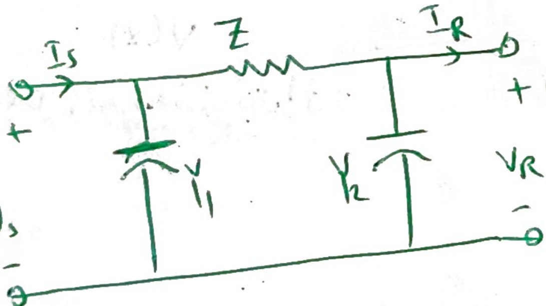


T- circuit

$$\left[\begin{array}{c|c} (1+YZ_1) & (Z_1+Z_2+YZ_1Z_2) \\ \hline Y & (1+YZ_2) \end{array}\right]$$

$$\boxed{AD-BC=1}$$

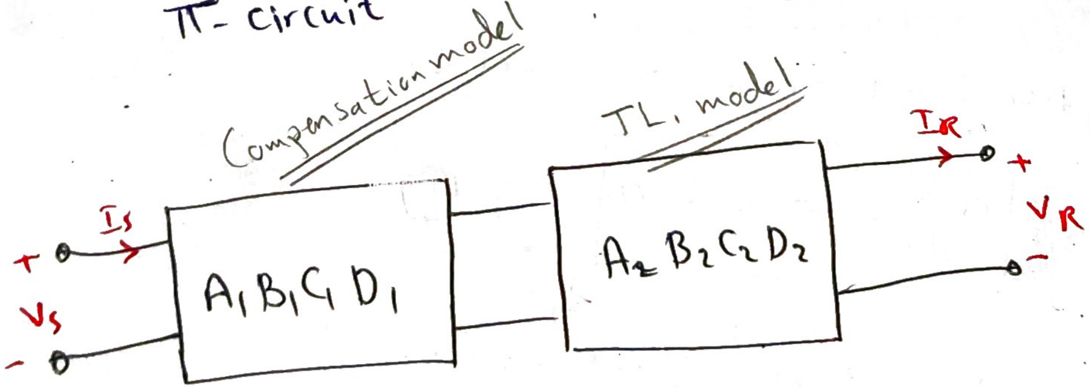
we can [illegible] it instead of using [illegible]



π- circuit

$$\left[\begin{array}{c|c} (1+Y_2Z) & Z \\ \hline (Y_1+Y_2+Y_1Y_2Z) & (1+Y_1Z) \end{array}\right]$$

$$\boxed{AD-BC=1}$$

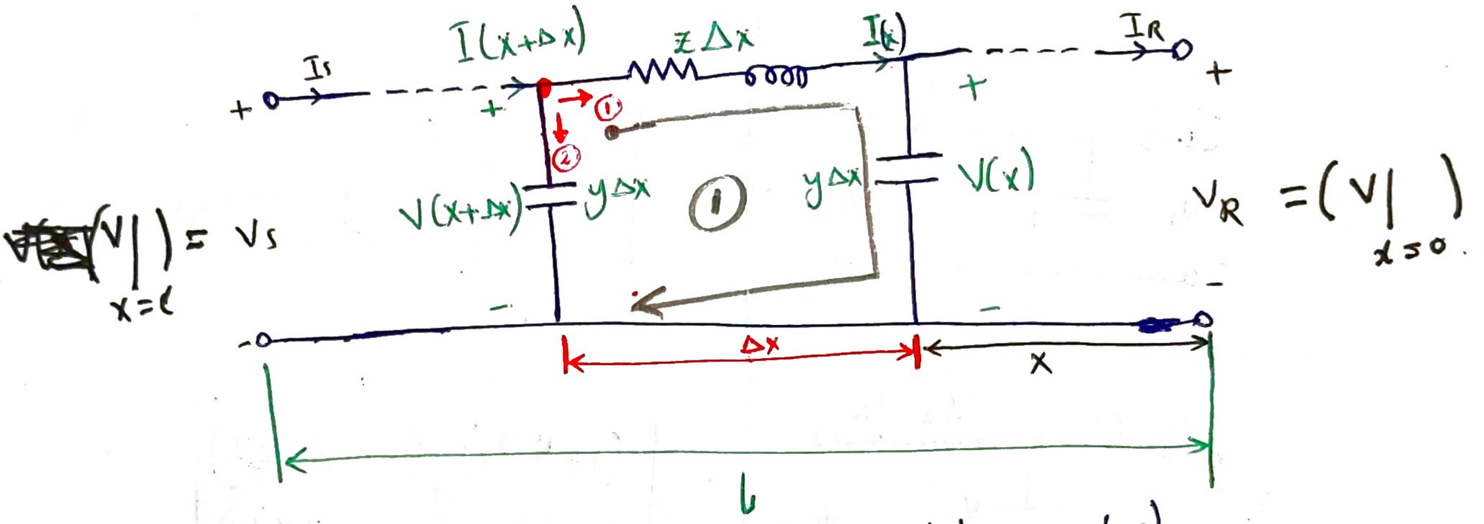


Cascaded networks

$$\left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_2 \end{array}\right]\left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right]=\left[\begin{array}{c|c} (A_1A_2+B_1C_2) & (A_1B_2+B_1D_2) \\ \hline (C_1A_2+D_1C_2 & (C_1B_2+D_1D_2) \end{array}\right]$$

3 Long Line Model :-

* For the short and medium length Lines ~~reasonably~~ app. accurate models were obtained by assuming the Line parameters to be Lumped. For lines 250km and longer and for amore accurate solution the exact effect of the distributed parameters must be considered.



(Long-line with distributed parameters)

target

V(x) I(x)

بدلالة par العروضية بالدرأة

- $z = r + j\omega L$
- $y = g + j\omega C$

① KVL $\boxed{1}$ $V(x+\Delta x) = z\Delta x\, I(x) + V(x)$

$\boxed{1}$ ← $\frac{V(x+\Delta x) - V(x)}{\Delta x} = z\, I(x)$

Taking the limit as $\Delta x \to 0$, we have

$$\boxed{\frac{dV(x)}{dx} = z\, I(x)} \text{ —— } ①$$

②↓→① KCL $\boxed{2}$ $I(x+\Delta x) = \overset{①}{I(x)} + y\Delta x \overset{②}{V}(x+\Delta x)$

•→① ↓ ②

$\frac{I(x+\Delta x) - I(x)}{\Delta x} = y\, V(x+\Delta x)$

lim $\Delta x \to 0$

$\frac{dI(x)}{dx} = y\, V(x)$ → ② and from ① ⟹

نرجع للمعادلة رقم ①

$$\frac{dV(x)}{dx} = z\,I(x) \quad \text{— ①}$$ from ① return to 1

نشتق

$$\boxed{\frac{d^2V(x)}{dx^2} = z\,\frac{dI(x)}{dx}}$$

substituting

$$\Rightarrow \frac{dI(x)}{dx} = y\,V(x) \quad \xrightarrow{\text{من معادلة ②}} ②$$ from ② return to 2

$$\frac{d^2V(x)}{dx^2} = z\,\underbrace{\boxed{\frac{dI(x)}{dx}}}_{yV_x}$$

$$\frac{d^2V(x)}{dx^2} = \boxed{z\,y}\,V(x)$$

$$zy = \gamma^2 \,,$$

$$\boxed{\frac{d^2V(x)}{dx^2} - \gamma^2 V(x) = 0}$$

✓ $V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$

where $\gamma \equiv$ propagation constant $= \sqrt{zy} = \alpha + j\beta$

attenuation constant (α)   phase constant. (β)

$$= \sqrt{\underbrace{(r + j\omega L)}_{z}\,\underbrace{(g + j\omega C)}_{y}}$$

$$\therefore \quad V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

we want to find I_x

$$I(x) = \frac{1}{z}\,\frac{dV(x)}{dx} \equiv \text{from — ①}$$

$$= \frac{\gamma}{z}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right)$$

$$= \sqrt{\frac{y}{z}}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right)$$

$$= \frac{1}{Z_c}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right), \quad Z_c \equiv \text{characteristic impedance}$$

$$Z_c = \sqrt{z/y}$$

$\boxed{V_x \; I_x}$

V(x) ✓   I(x) ?

$$\boxed{\begin{aligned} V(x) &= A_1 e^{\gamma x} + A_2 e^{-\gamma x} \\ I(x) &= \frac{1}{Z_c}\left(A_1 e^{\gamma x} - A_2 e^{-\gamma x}\right) \end{aligned}}$$

***

$A_1 = ?!$, $A_2 = ?!$

Two boundary conditions:-

$\boxed{\text{at} \quad x = 0}$

$x = 0$

① $V(x) = V_R \quad \Longrightarrow \quad V_R = A_1 + A_2$

② $I(x) = I_R \qquad\qquad\quad I_R = \dfrac{A_1 - A_2}{Z_c}$

$$\Longrightarrow \quad A_1 = \frac{V_R + Z_c I_R}{2}$$

$$A_2 = \frac{V_R - Z_c I_R}{2}$$

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$I(x) = \frac{\frac{V_R}{Z_c} + I_R}{2} e^{\gamma x} \mp \frac{\frac{V_R}{Z_c} - I_R}{2} e^{-\gamma x}$$

Rearranged

$V(x) = \square \, V_R + \square \, I_R$

$I(x) = \square \, V_R + \square \, I_R$

$$\boxed{\begin{aligned} V(x) &= \square \, V_R + \square \, I_R \\ I(x) &= \square \, V_R + \square \, I_R \end{aligned}}$$

$$V(x) = \overbrace{\frac{e^{\gamma x} + e^{-\gamma x}}{2}}^{\cosh \gamma x} V_R + Z_c \overbrace{\frac{e^{\gamma x} - e^{-\gamma x}}{2}}^{\sinh \gamma x} I_R$$

$$I(x) = \frac{1}{Z_c} \underbrace{\frac{e^{\gamma x} - e^{-\gamma x}}{2}}_{\sinh \gamma x} V_R + \underbrace{\frac{e^{\gamma x} + e^{-\gamma x}}{2}}_{\cosh \gamma x} I_R$$

$$V(x) = \cosh \gamma x \, V_R + Z_c \sinh \gamma x \, I_R$$

$$\Longrightarrow \quad I(x) = \frac{1}{Z_c} \sinh \gamma x \, V_R + \cosh \gamma x \, I_R$$

We are particularly interested in the relation between the sending end and the receiving end of the line.

Setting $x = l$

$V(l) = V_s$

$I(l) = I_s$

$$\Longrightarrow \boxed{\begin{aligned} V_s &= \cosh\gamma l \, V_R + Z_c \sinh\gamma l \, I_R \\ I_s &= \frac{1}{Z_c}\sinh\gamma l \, V_R + \cosh\gamma l \, I_R \end{aligned}} \quad \text{———} \; ①$$

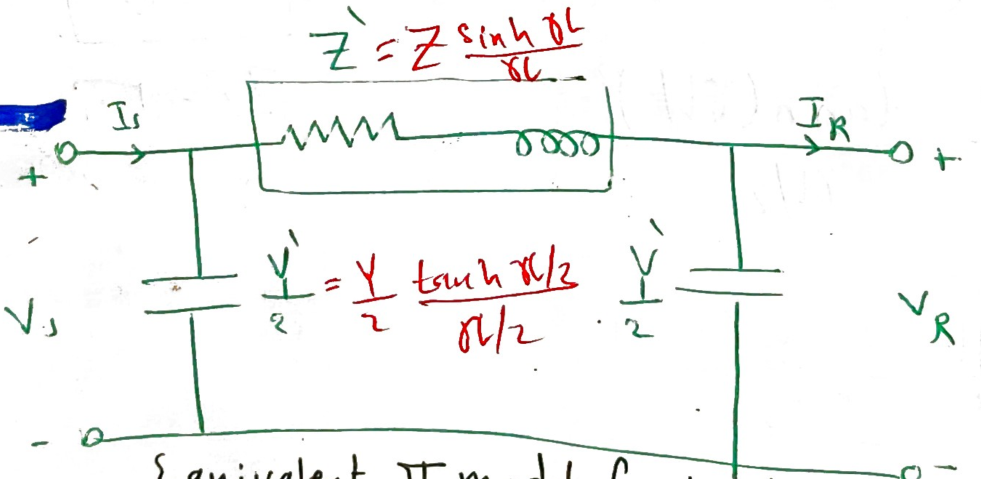
$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh\gamma l & Z_c \sinh\gamma l \\ \frac{1}{Z_c}\sinh\gamma l & \cosh\gamma l \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

(ABCD matrix)

before

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} (1+\frac{YZ}{2}) & Z \\ Y(1+\frac{YZ}{4}) & (1+\frac{YZ}{2}) \end{bmatrix}$$

note that, as before, $A = D$ and $AD - BC = 1$.



Equivalent π model for long length Line.

$$\boxed{\begin{aligned} V_s &= \Big(1 + \underbrace{\frac{Z'Y'}{2}\Big)}_{\parallel ②} V_R + \underbrace{Z'}_{\parallel ①} I_R \\ I_s &= Y'\Big(1 + \frac{Z'Y'}{4}\Big) V_R + \Big(1 + \frac{Z'Y'}{2}\Big) I_R \end{aligned}} \quad \text{———} \; ②$$

$Z_c \sinh\gamma l$

$\cosh\gamma l$

Comparing ① with ②

① $Z' = Z_c \sinh \gamma l$

$$= \sqrt{\frac{z}{y}} \sinh \gamma l$$

$$= zl \frac{\sinh \gamma l}{\sqrt{zy}\, l} = Z \frac{\sinh \gamma l}{\gamma l}$$

② $\cosh \gamma l = 1 + \frac{\overset{\swarrow}{Z'} Y'}{2}$

$$\cosh \gamma l = 1 + \frac{(Z_c \sinh \gamma l\, Y')}{2} = 1 + Z_c \sinh \gamma l \cdot \frac{Y'}{2} = \cosh \gamma l$$

$$\frac{Y'}{2} = \frac{1}{Z_c} \cdot \boxed{\frac{\cosh \gamma l - 1}{\sinh \gamma l}} \leftarrow \tanh \frac{\gamma l}{2}$$

$$= \frac{1}{Z_c} \tanh \frac{\gamma l}{2}$$

$$\boxed{Y = yl}$$

$$= \frac{Y}{2} \frac{\tanh(\gamma l/2)}{\gamma l/2}$$

$$Z_c = \sqrt{z/y}$$

$$= \frac{yl}{2} \frac{\tanh(\gamma l/2)}{\frac{\sqrt{zy}\, l}{2}}$$

Note:-

$\cosh(\gamma l) = \cosh(\alpha l) . \cos(\beta l) + j \sinh(\alpha l) . \sin(\beta l)$

$\sinh(\gamma l) = \sinh(\alpha l) \cdot \cos(\beta l) + j \cosh(\alpha l) \cdot \sin(\beta l)$

Loss Less Line :-

→ A, B, C, D par.
→ Z', $\frac{Y'}{2}$ (model)

loss-less
- صعب cosh
- بدون approx. idea
- good idea in power flow analysis.

$Z = j\omega L \quad \Omega/m \qquad (r = 0)$

$y = j\omega C \quad S/m \qquad (g = 0)$

$Z_s = \sqrt{\frac{Z}{y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$ ≡ Surge Impedance

↙ purely resistive.

lossless Line
- A B C D par
- Z' $\frac{Y'}{2}$ model
- wave length λ
- SIL

$\underbrace{\gamma}_{\text{real} \quad \text{Imag}} = \sqrt{Zy} = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = j\beta \ m^{-1}$

↙ purely imag.

real α attenuation constant

Imag β phase constant

$\beta = \omega\sqrt{LC}$ = phase constant ; $\alpha = 0$ since there is no loss in the line.

ABCD Parameters (Lossless Line) :-

* $A(x) = D(x) = \cosh(\gamma x) = \cosh(\underline{\underline{j\beta}}\, x) = \frac{e^{j\beta x} + e^{-j\beta x}}{2} = \cos(\beta x)$ perunit

↓ not hyp. Function

note
$\sinh(\gamma x) = \sinh(j\beta x) = j\frac{e^{j\beta x} - e^{-j\beta x}}{2j} = j\sin(\beta x)$ per unit

↙ $\left(\overset{\text{not}}{X}\right)$ hyp function

* $B(x) = Z_c \sinh(\gamma x) = j Z_c \sin(\beta x)$

$= j\sqrt{\frac{L}{C}} \cdot \sin(\beta x) \ \Omega$

* $C(x) = \frac{\sinh(\gamma x)}{Z_c} = \frac{j\sin(\beta x)}{\sqrt{\frac{L}{C}}} \ S$

π - model for Loss-less line →

model :-

- $Z' = Z_c \sinh \overset{j \sin(\beta l)}{\gamma l}$

$= j \underbrace{Z_c \sin(\beta l)}$

$= j X'$

- $\frac{Y'}{2} = \frac{Y}{2} \frac{\tanh \frac{\overset{\gamma = j\beta}{\gamma l}}{2}}{\gamma l/2} = \frac{Y}{2} \frac{\tanh(j \beta l/2)}{j \beta l/2}$

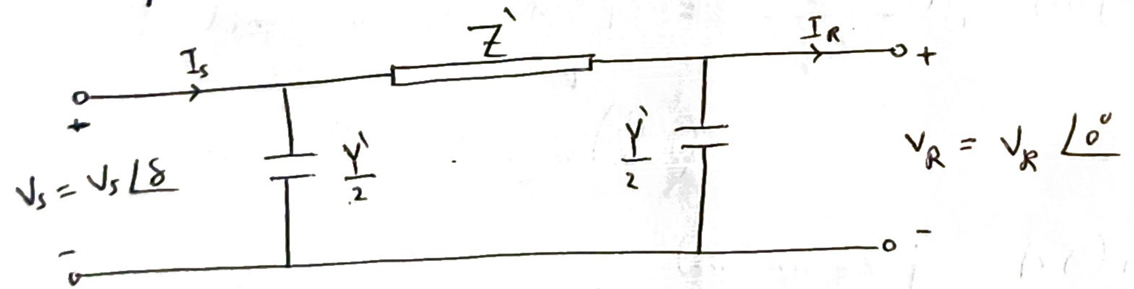
$= \frac{Y}{2} \frac{\sinh(j \beta l/2)}{(j \frac{\beta l}{2}) \cosh(\frac{j \beta l}{2})}$

$= \left(\frac{j w C l}{2}\right) \frac{\cancel{j} \sin(\beta l/2)}{(\cancel{j} \frac{\beta l}{2}) \cos(\beta l/2)}$

$= \frac{j w C l}{2} \frac{\tan(\beta l/2)}{\beta l/2}$

$= \frac{j w C' l}{2}$

π- Equivalent Circuit ((Loss Less Line)) :-



$Z' = (j w L l)\left(\frac{\sin \beta l}{\beta l}\right) = j X' \ \Omega$

$\frac{Y'}{2} = \left(\frac{j w C l}{2}\right) \frac{\tan(\beta l/2)}{(\beta l/2)} = \frac{j w C' l}{2} \ S$

For a lossless line :-

$V(x) = A(x) V_R + B(x) I_R$
$= \cos(\beta x) V_R + j Z_c \sin(\beta x) I_R$ || $I(x) = C(x) V_R + D(x) I_R$
$= j \frac{\sin(\beta x)}{Z_c} V_R + \cos(\beta x) I_x$

Wave Length ((Loss Less Line)) :- A wavelength is the distance required to change the phase of the Voltage or current by 2π radians or $360°$.

$$v = \frac{w}{\beta} = \frac{2\pi f}{\beta}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{w\sqrt{LC}} = \frac{1}{f\sqrt{LC}} \quad m$$

* The expression for the inductance per unit length L and capacitance per unit length C of a transmission line were derived in previous chapter. When the internal flux linkage of a conductor is neglected $GMR_L = GMR_C$

$$r' = r$$

$$\lambda \cong \frac{1}{f\sqrt{\mu_0 \varepsilon_0}}$$

$$\mu_0 = 4\pi * 10^{-7} \qquad \Rightarrow \lambda = 6000\,km, \text{ for } 50\,Hz$$

$$\varepsilon_0 = 8.85 * 10^{-12} \qquad \Rightarrow f\lambda = v = \frac{1}{\sqrt{LC}} \cong \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \cong 3 * 10^{8}\, m/sec.$$

$\equiv$ Velocity of propagation of voltage and current waves on loss-less Line

Surge Impedance Loading :- (SIL) is the power delivered by a lossless line to a load resistance equal to the surge impedance Z_c.

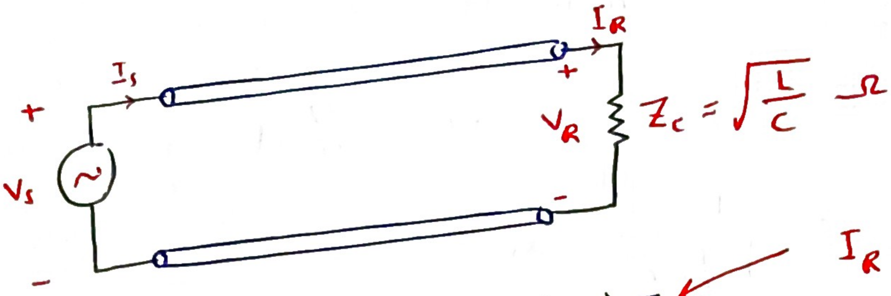
$$\boxed{Z_c = \sqrt{L/C}}$$

* $V(x) = A(x) V_R + B(x) I_R$

$$= \cos(\beta x) V_R + j Z_c \sin(\beta x) I_R$$

* $I(x) = C(x) V_R + D(x) I_R$

$$= j \frac{\sin(Bx)}{Z_c} V_R + \cos(\beta x) I_R$$



$I_R = \frac{V_R}{Z_c}$

① $V(x) = \cos(\beta x)\, V_R + j\, Z_c \sin(\beta x)\, I_R$

$= \cos(\beta x)\, V_R + j\, Z_c \sin(\beta x) \left(\frac{V_R}{Z_c}\right)$

$= \underbrace{\left[\cos(\beta x) + j \sin(\beta x)\right]}_{e^{j\beta x}} V_R$

$= e^{j\beta x}\, V_R$ volts.

$|V(x)| = |V_R|$ volts ; Voltage is constant along the line.

② $I(x) = j\, \frac{\sin(\beta x)}{Z_c} V_R + \cos(\beta x) \frac{V_R}{Z_c}$

$= \left[\cos \beta x + j \sin \beta x\right] \frac{V_R}{Z_c}$

$= \left[e^{j\beta x}\right] \frac{V_R}{Z_c}$ A.

$S(x) = P(x) + j\, Q(x) = V(x)\, I^*(x)$

$= \left[e^{j\beta x}\, V_R\right]\left[\frac{e^{j\beta x}\, V_R}{Z_c}\right]^*$

$= \frac{|V_R|^2}{Z_c}$; Real power along the line is constant and reactive power flow is zero.

⊙ at rated line voltage

$SIL = \frac{V^2_{rated}}{Z_c}$

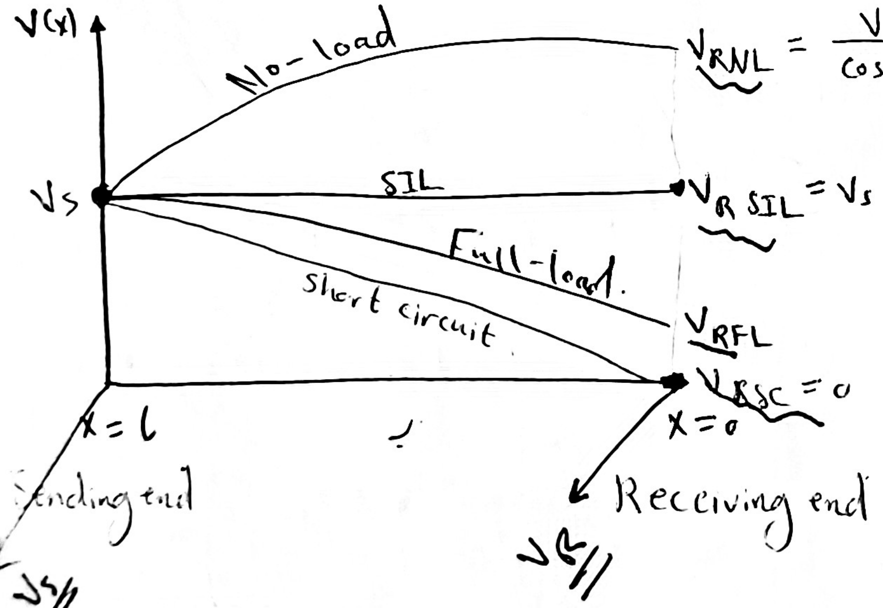
$\boxed{L\text{-}L}$

Vrated (kV)	$Z_c = \sqrt{L/C}$	SIL = V^2_{rated}/Z_c (MW)
230	380	140
345	285	420
500	250	1000
765	257	2280

Voltage Profiles:-

$$V_{NL}(x) = [\cos(\beta x)]\, V_{RNL}$$

$$V_{SC}(x) = \bar{Z}_c \sin\beta x \; I_{RSC}$$



, $I_{R\,NL} = 0$

$V_{NL}(x) = (\cos\beta x)\, V_{RNL}$

Voltage profiles of an uncompensated lossless line with fixed sending end voltage.

$\boxed{\text{example } 5.5 \quad 160}$

$$\boxed{V(x) = \cos(\beta x)\, V_R + j Z_c \sin(\beta x)\, I_R}$$

① no-load, $I_{RNL} = 0$

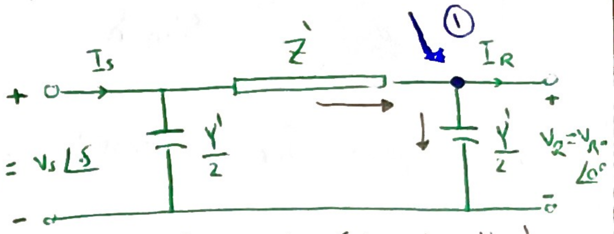
$V_{NL}(x) = \cos(\beta x)\, V_{RNL}$

$V_{RNL} = \frac{V_s}{\cos(\beta l)}$

② for short circuit at the load $V_{RSC} = 0$

$V_{SC}(x) = (Z_c \sin\beta x)\, I_{RSC}$

Steady-State Stability Limit :



$Z' = jX'$ (loss-less line)

$\frac{Y'}{2} = j\frac{w c' l}{2}$

KCL at node ①

$$I_R = \frac{V_s - V_R}{Z'} - \frac{Y'}{2} V_R$$

$$= \frac{V_s e^{j\delta} - V_R}{jX'} - j\frac{w c' l}{2} V_R$$

complex power at the receiving end

$$S_R = V_R I_R^* = V_R \left(\frac{V_s e^{j\delta} - V_R}{jX}\right)^* + j\frac{w c' l}{2} V_R^2$$

$$= V_R \frac{j}{j}\left(\frac{V_s e^{-j\delta} - V_R}{-jX'}\right) + j\frac{w c' l}{2} V_R^2$$

$$\boxed{A e^{j\theta} = A\cos\theta + jA\sin\theta}$$

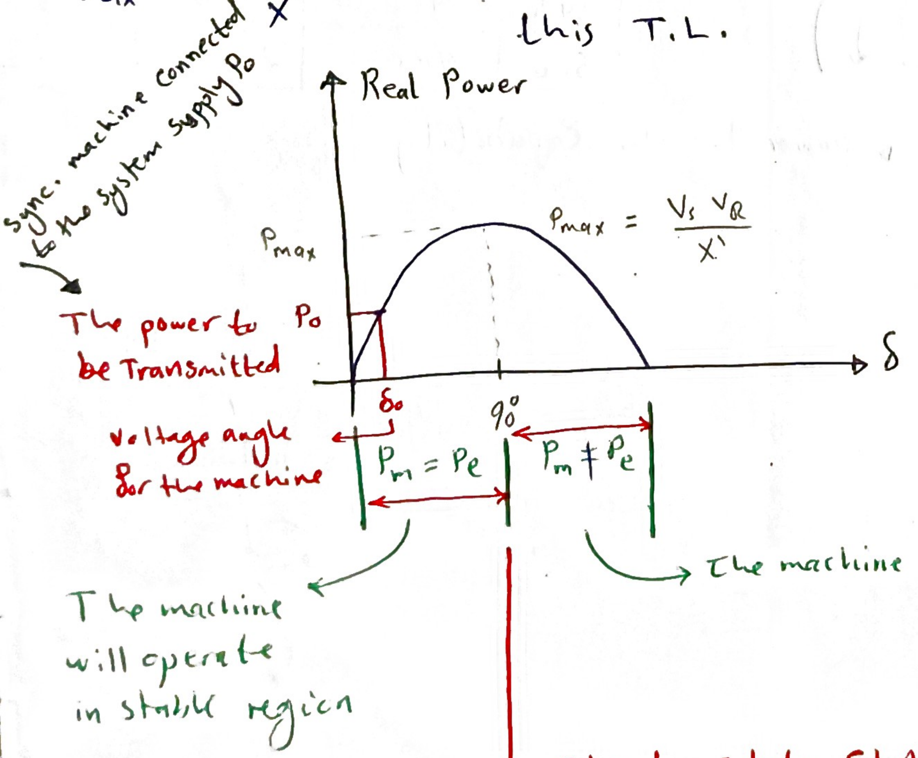
$$= \frac{j V_R V_s \cos\delta + V_R V_s \sin\delta - j V_R^2}{X'} + j\frac{w c' l}{2} V_R^2$$

real power

$$\underset{\text{real part}}{P} = \underbrace{P_s = P_R}_{\text{loss-less line}} = Re(S_R) = \frac{V_R V_s}{X'} \underline{\sin\delta} \ W$$

→ max when $\delta = 90°$

$P_{max} = \frac{V_R V_s}{X'}$ W , max power that can be transmitted over this T.L.



Steady-state Stability limit

if an attempt were made to exceed this limit, then the machine would loss synchron

notes :-

① $P_{max} = \frac{V_s V_R}{X'}$ ← ثابت ← نتحكم

$V_s \cong V_R \cong 1$ per unit

o o
o o bundel

GMR↑, L↓, X↓, P_{max}↑

allow you to Transmitt more power on the T.L.

② 

input ↑, δ ↑, P_0 ↑

In tearms of SIL

$$P = \frac{V_R\ V_s}{X'} \sin\delta$$

real power for loss-less line.

$$\odot\ Z' = Z_c \sinh \gamma l$$
$$= j Z_c \sin(\beta l)$$
$$= j X'$$

$$\odot\ \lambda = \frac{2\pi}{\beta} \Rightarrow \beta = \frac{2\pi}{\lambda}$$

$$= \frac{V_s\ V_R\ \sin\delta}{Z_c \sin\beta l}$$

$$= \left(\frac{V_s . V_R}{Z_c}\right) \cdot \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

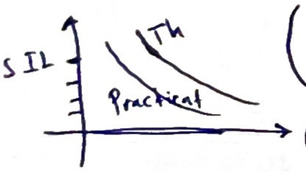
$$= \left(\frac{V_s}{V_{rated}}\right)\left(\frac{V_R}{V_{rated}}\right) \cdot \left(\frac{V^2_{rated}}{Z_c}\right) \cdot \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

$$= \left(V_{s.p.u}\right)\left(V_{R.p.u}\right)\left(SIL\right) . \frac{\sin\delta}{\sin\left(\frac{2\pi l}{\lambda}\right)} \quad W$$

$$P_{max} = \frac{V_{s.p.u}\ V_{R.p.u}\ SIL}{\sin\left(\frac{2\pi l}{\lambda}\right)}$$

of t
Line.

Voltage kV	SIL (MW)	Typical Thermal Rating (MW)
230	150	400
345	400	1200
500	900	2600



$$(l \uparrow\ ,\ P_{max} \downarrow)$$

power transfer capability

Maximum Power Flow (Lossy Line):-

$$\overset{\text{real}}{\underset{\text{img}}{}} A = \cosh(\gamma l) = A\angle\theta_A$$

$$\overset{\text{real}}{\underset{\text{img}}{}} B = Z' = Z'\angle\theta_Z$$

$$I_R = \frac{V_s - A V_R}{B} = \frac{V_s e^{j\delta} - A V_R e^{j\theta_A}}{Z' e^{j\theta_Z}}$$

$$S_R = P_R + jQ_R = V_R^* I_R^* = V_R \left[\frac{V_s e^{j(\delta-\theta_Z)} - A V_R e^{j(\theta_A - \theta_Z)}}{Z'} \right]^*$$

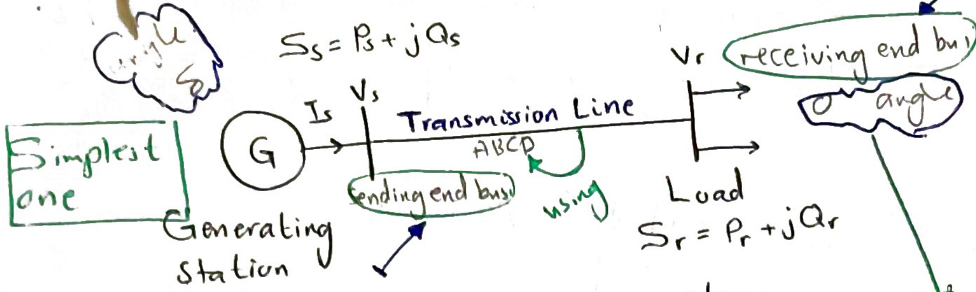
$$= \frac{V_R V_s}{Z'} e^{j(\theta_Z - \delta)} - \frac{A V_R^2}{Z'} e^{j(\theta_Z - \theta_A)}$$

$$P_R = \mathrm{Re}(S_R) = \underbrace{\frac{V_R V_s}{Z'} \cos(\theta_Z - \delta)}_{\text{same as previous}} - \underbrace{\frac{A V_R^2}{Z'} \cos(\theta_Z - \theta_A)}_{\text{راح نشلها هذا الجزء}}$$

Two component

$$P_{max}\Big|_{\theta_Z = \delta}$$

Transmission Line Steady State Operation :-



A two bus power system.

as refrence.

when we talk about the S.S.O on T.L what we really mean is how the Line is perform when we want to Transmit certain ~~[illegible]~~ amount of power throug it.

Power flow on transmission Lines :-

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

① $V_s = A V_R + B I_R$

$\frac{1}{B} V_s = \frac{A}{B} V_R + I_r$

$I_r = \frac{1}{B} V_s - \frac{A}{B} V_r$

② $I_s = C V_R + D I_r$

$I_s = C V_R + \frac{D}{B} V_s - \frac{DA}{B} V_r$

$= \frac{D}{B} V_s + \left(\frac{CB}{B} - \frac{DA}{B}\right) V_R$

$= \frac{D}{B} V_s - \frac{1}{B} V_R$

rec end current → $I_r = \frac{1}{B} V_s - \frac{A}{B} V_r$ ------ ①

sending end current $I_s = \frac{D}{B} V_s - \frac{1}{B} V_r = \frac{A}{B} V_s - \frac{1}{B} V_r$ --- ②

we know that $\boxed{A = D}$

Let $V_r = |V_r| \angle 0$ (as a refrense phaser.)

$V_s = |V_s| \angle \delta$, , ↑ Vs leads Vr by δ

δ : the angle ~~by~~ which the Vs leads Vr.

complex number ← $D = A = |A| \angle \alpha$

complex number ← $B = |B| \angle \beta$

Then, from ① and ②

$$I_r = \frac{|V_s|}{|B|} \angle(\delta - \beta) - \frac{|A||V_r|}{|B|} \angle(\alpha - \beta)$$

$$I_s = \frac{|A||V_s|}{|B|} \angle(\alpha + \delta - \beta) - \frac{|V_r|}{|B|} \angle -\beta$$

The Conjugates of I_r and I_s are:-

$$I_r^* = \frac{|V_s|}{|B|} \angle(\beta - \delta) - \frac{|A||V_r|}{|B|} \angle(\beta - \alpha)$$

$$I_s^* = \frac{|A||V_s|}{|B|} \angle(\beta - \alpha - \delta) - \frac{|V_r|}{|B|} \angle \beta$$

Complex Power

≫ ⓐ $S_r = P_r + jQ_r = V_r I_r^*$

$$= |V_r| \angle 0 \left[\frac{|V_s|}{|B|} \angle(\beta-\delta) - \frac{|A||V_r|}{|B|} \angle(\beta-\alpha) \right]$$

$$= \frac{|V_s||V_r|}{|B|} \angle(\beta-\delta) - \frac{|A||V_r|^2}{|B|} \angle(\beta-\alpha)$$

≫ ⓑ $S_s = P_s + jQ_s = V_s I_s^*$

$$= |V_s| \angle \delta \left[\frac{|A||V_s|}{|B|} \angle(\beta-\alpha-\delta) - \frac{|V_r|}{|B|} \angle \beta \right]$$

$$= \frac{|A||V_s|^2}{|B|} \angle(\beta-\alpha) - \frac{|V_r||V_s|}{|B|} \angle(\beta+\delta)$$

real and reactive power

ⓐ

real power

$$P_r = \frac{|V_s||V_r|}{|B|} \cos(\beta - \boxed{\delta}) - \frac{|A||V_r|^2}{|B|} \cos(\beta-\alpha)$$

power angle or voltage angle at the sending end phase

reactive power

$$Q_r = \frac{|V_s||V_r|}{|B|} \sin(\beta-\delta) - \frac{|A||V_r|^2}{|B|} \sin(\beta-\alpha)$$

ⓑ

real power

$$P_s = \frac{|A||V_s|^2}{|B|} \cos(\beta-\alpha) - \frac{|V_r||V_s|}{|B|} \cos(\beta+\delta)$$

$$Q_s = \frac{|A||V_s|^2}{|B|} \sin(\beta-\alpha) - \frac{|V_r||V_s|}{|B|} \sin(\beta+\delta)$$

δ: 1 paranter variable δ note ① ②

notes

① For agiven system voltage level V_s and V_r will be very near to the system voltage and they don't not chang much. (33kV, ...)

$(\beta=\delta)$ $$P_{r\,max} = \frac{|V_s||V_r|}{|B|} - \frac{|A||V_r|^2}{|B|} \cos(\beta-\alpha)$$

② β, α T.L paranters and they already there so, they are fixed.

$$Q_r = -\frac{|A||V_r|^2}{|B|} \sin(\beta-\alpha)$$

max power which can be transmitted or recieved.

i.e [illegible]

First Short Line :- $\Longrightarrow$ $A = D = 1\angle 0^\circ$
$B = Z\angle\theta$

(حتى نفهم ال Conspet اساسي
for power flow

Simpler

$$P_r = \frac{|V_s||V_r|}{|Z|}\cos(\theta - \delta) - \frac{|V_r|^2}{|Z|}\cos\theta$$

$$Q_r = \frac{|V_s||V_r|}{|Z|}\sin(\theta - \delta) - \frac{|V_r|^2}{|Z|}\sin\theta$$

$$P_s = \frac{|V_s|^2}{|Z|}\cos\theta - \frac{|V_s||V_r|}{|Z|}\cos(\theta + \delta)$$

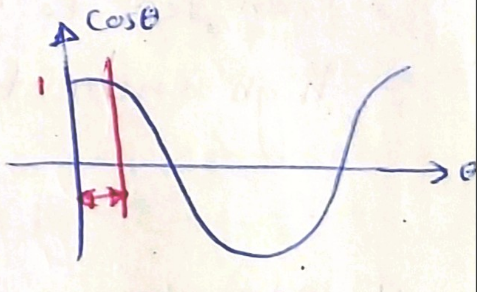
$$Q_s = \frac{|V_s|^2}{|Z|}\sin\theta - \frac{|V_s||V_r|}{|Z|}\sin(\theta + \delta)$$

$(Z = R + jX)$

As $R \ll X$, $|Z| \approx X$ and $\theta = 90^\circ$, substituting these values in the above equations

$$\boxed{P_r = \frac{|V_s||V_r|}{X}\sin\delta}$$

$$Q_r = \frac{|V_s||V_r|}{X}\cos\delta - \frac{|V_r|^2}{X}$$



As δ is normally small ; $\cos\delta \cong 1$

$$Q_r = \frac{|V_s||V_r|}{X} - \frac{|V_r|^2}{X}$$

cos15 = 0.966
cos 20 = 0.94.

$$\boxed{Q_r = \frac{|V_r|}{X}(|V_s| - |V_r|)}$$

- From these relasnips we can conclode the following points

1. For fixed values of V_s, V_r and X the real power depends on angle δ the phase angle by which V_s leads V_r. This angle δ is called power angle. When δ = 90° P is maximum. For system stability (considerations δ has to be kept well below 90°.

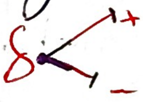
لانها هي اللي تحدد كمية البور اللي هننقلها من خلال T.L

range (20°-30°) ↑↑ distarbances will → 90°

2. Power can be transferred over line even when $|V_s| \leq |V_r|$.

The phase difference δ between V_r and V_s causes the flow of power in the line. Power systems are operated with almost the same voltage magnitudes (i.e., 1pu) at important busses by using methods of Voltage Control.

because this provides a much better operating conditions for the system



V_s leads V_r $\left[V_s \xrightarrow{\text{power flow}} V_r\right.$

V_s lags V_r $\left.V_r \xrightarrow{\text{power flow}} V_s\right]$

direction of power flow (δ) not |V|

3. The maximum real power transferred over a line increases with increase in V_s and V_r.
An increase of 100% in V_r and V_s increases the power transfer to 400%. This is the reason for adopting high and extra high transmission voltages.

400% double the voltage

تبني
(تتخذ اسلوب او وجهة نظر معينة)

4. The maximum real power depends on the reactance X which is directly proportional to line inductance. A decrease in inductance increases the line capacity. The line inductance can be decreased by using bundled conductors.

>> Another method of reducing line inductance is by inserting capacitance in series with the line. This method is known as series compensation. The series capacitors are usually installed at the middle of the line.

(Positive Reactance L + negative Reactance C) → effective Reactance will be ↓
series

5. The reactive power transferred over a line is directly proportional to $(|V_s| - |V_r|)$ i.e., voltage drop along the line, and is independent of power angle. This means the voltage drop on the line is due to the transfer of reactive power over the line. To maintain a good voltage profile, reactive power control is necessary.

Voltage Control

Reactive Power Compensation equipment has the following effect:-

V_s, V_r ±5% (nominal value) → (10% loss in line)

1. Reduction in current. $S = P + jQ$, $Q\downarrow, S\downarrow, I\downarrow, V =$ constant = nominal value
   $S = VI^*$
2. Maint~~enance~~ain ~~xV~~ voltage profile within limits. ثبات
   $Q_r = \frac{|V_r|}{X}\underbrace{(|V_s| - |V_r|)}$
3. Reduction of losses in the system. $(I^2R)\downarrow$ since $I\downarrow$

4. Reduction in investment in the system per kW of load supplied. (الاسعار، اقل) $(Q\downarrow, I\downarrow, r\downarrow)$
5. Decrease in KVA loading of generators and lines. This decrease in KVA loading relieves overload condition or releases capacity for additional load growth.
   يزيح، يخفف
   يبعث، يطلق، نمو
6. Improvement in power factor of generators.

→ Reactive compensation of T.L. ←

1. Static Var Compensation.
2. Rotating Compensators (synchronous compensator)
3. Using Transformer. (Tap transformer)
4. Using Power Electronics (STAT COM)

Static Compensation

The performance of transmission lines, especially those of medium length and longer, can be improved by reactive compensation of a series or parallel type.

1. Series compensation consists of a capacitor bank placed in series with each phase conductor of the line. Series compensation reduces the series impedance of the line, which is the رئيسي principal cause of voltage drop and the most important factor in determining the maximum power which the line can transmit.

2. Shunt compensation refers to:
   - ⓐ The placement of inductors from each line to neutral to reduce partially or completely the shunt susceptance of a high-voltage line, which is particularly important at light loads when the voltage at the receiving end may otherwise become very high. ((Shunt Reactors))
   - ⓑ Shunt Capacitors are used for lagging Power factor circuits created by heavy loads. The effect is to supply the requisite reactive power to maintain the receiving end voltage at satisfactory level.

example A 50 Hz, 138 kV, 3-phase transmission Line is 200 km long. The distributed line parameters are

$R = 0.1 \ \Omega / \text{km}$

$L = 1.2 \ \text{mH/km}$

$C = 0.01 \ \mu\text{F/km}$

$G = 0$

The transmission line delivers $\overset{\overset{3\phi \text{ power}}{P_r}}{\underline{40 \text{ MW}}}$ at $\overset{V_r}{\underline{132 \text{ KV}}}$ with 0.95 power factor lagging. Find the sending end voltage and current, and also the transmission line efficiency.

يوزع

Solution: For the given values of R, L and C, we have for $w = 2\pi(50)$,

$$z = 0.1 + j \overset{(wL)}{0.377} = 0.39 \angle 75.14^\circ \ \Omega/\text{km}.$$

$$y = j \underset{(jwC)}{3.14} * 10^{-6} = 3.14 * 10^{-6} \angle 90 \ \mho/\text{km}.$$

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sin$$

$$I_1 = I_2 \cosh \gamma l + \left(\frac{V_2}{Z_c}\right) \sinh$$

From the above values

$$Z_c = \sqrt{(z/y)} = 352.42 \angle -7.43^\circ \ \Omega$$

$$\gamma l = 200\sqrt{zy} = 0.2213 \angle 82.57^\circ = 0.0286 + j\,0.2194$$

⇨ ◉ $\sinh \gamma l = \dfrac{e^{\gamma l} - e^{-\gamma l}}{2} = 0.2195 \angle 82.67^\circ$

⇒ ◉ $\cosh \gamma l = \dfrac{e^{\gamma l} + e^{-\gamma l}}{2} = 0.975 \angle 0.37^\circ$

The values of power and voltage specified in the problem refers to 3-phase and line-to-line quantities.

$\boxed{V_{LN}}$ $$|V_2| = 132/\sqrt{3} = 76.2 \text{ kV}$$

also, using V_2 as reference; $\angle V_2 = 0^\circ$, we get

$$V_2 = 76.2 \angle 0^\circ \ \text{kV}$$

Note:

$$\cosh(\gamma l) = \cosh(\alpha l) * \cos(\beta l) + j \sinh(\alpha l) * \sin(\beta l)$$
$$\sinh(\gamma l) = \sinh(\alpha l) * \cos(\beta l) + j \cosh(\alpha l) * \sin(\beta l)$$

Now, per phase power supplied to the load.

$$P_{load} = \frac{40}{3} = 13.33 \text{ MW.}$$

Given the value of power factor = 0.95, we can find I_2

$$P_{load} = \underbrace{0.95}\,|V_2| \cdot |I_2|$$

Thus, $|I_2| = 184.1$

Also, since I_2 lags V_2 by $\cos^{-1} 0.95 = 18.195^\circ$,

$$I_2 = 184.1 \angle{-18.195^\circ}$$

Finally, we have:-

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$$

$$V_1 = 82.96 \angle 8.6 \text{ kV}$$

Sending end voltage.

⇒ L-to-N voltage at the sending end.

Similarly,

$$I_1 = I_2 \cosh \gamma l + \left(\frac{V_2}{Z_c}\right) \sinh \gamma l$$

$$= 179.46 \angle 17.79$$

Sending end current.

* Power P_s

We now calculate the efficiency of transmission.

Perphase input power, $P_{in} = Re(V_1 I_1^*)$

$$= 14.69 \text{ MW}$$

Hence, $\eta = \frac{13.33}{14.69} = 0.907.$

That is, the efficiency of transmission is 90.7%.

example

A 3 phase 132 kV overhead line delivers $\overset{S_r}{60\text{ MVA}}$ at $\overset{V_r, V_2}{132\text{ kV}}$ and power $\overset{PF_r}{\text{factor}}$ 0.8 lagging at its receiving end. The Constants of the line are $A = 0.98 \angle 3^\circ$ and $B = 100 \angle 75^\circ$ ohms per phase. Find

(a) sending end voltage and power angle.

(b) sending end active and reactive power.

(c) line losses and vars absorbed by the line.

(d) and (e)

Solution :-

((a))

V_{LN} (phase voltage) ← refrence voltage

$$V_r = \frac{132000}{\sqrt{3}} = 76210 \angle 0^\circ$$

$$I_r = \left[\frac{60 * 10^6}{3}\right] / \left[\frac{132000}{\sqrt{3}}\right]$$

$\frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$S_r = V_r I_r^*$

$$\boxed{I_r = 262.43 \angle -36.87^\circ}$$ ← $-\cos^{-1}$ p.F

$$V_s = A \cdot V_r + B \cdot I_r$$

$$= (0.98 \angle 3^\circ)(76210 \angle 0) + (100 \angle 75^\circ)(262.43 \angle -36.87^\circ)$$

$$= 97.33 * 10^3 \angle 11.92^\circ \quad V$$

L-L

* Sending end Line voltage $= (\sqrt{3})(97.33)$ kV

$= 168.58$

* Power angle $(\delta) = 11.92^\circ$

(d) capacity of static compensation equipment at the receiving end to reduce the sending end voltage to 145 KV for the same Load conditions. part (a) $V_s \downarrow$ (we need to reduce)

Load
→ Power
→ Voltage
→ P.f.

(e) The unity power factor load which can be supplied at the receiving end with 132 KV as the line Voltage at both the ends.

We have 3 phase power given as

$$S_s = |A||V_s|^2 |B|^{-1} \angle(B-\alpha) - |V_r||V_s||B|^{-1} \angle(\beta+\delta)$$

L-L

L-L

$$= \frac{(0.98) * (168.58)^2}{(100)} \angle(75-3^\circ) - \frac{(132)(168.58)}{(100)} \angle(75^\circ + 11.92^\circ)$$

$$= 278.49 \angle 72^\circ - 222.53 \angle(86.92^\circ)$$

notes :-

3-ϕ power
$\Rightarrow$ Vs and Vr
L-L voltages

1-ϕ power
$\Rightarrow$ Vs and Vr
L-N voltage

$\Rightarrow$ Sending end active power

$$P_s = 278.49 \cos(72^\circ) - 222.53 \cos(86.92^\circ)$$

$$= 86.06 - 11.96 = 74.10 \text{ MW}$$

$\Rightarrow$ Sending end reactive power

$$Q_s = 278.49 \sin 72^\circ - 222.53 \sin 86.92^\circ$$

$$= 264.89 - 222.21$$

$$= 42.65 \text{ MVar Lagging}$$

((c)) * Line Losses $= P_s - P_r$

$$= 74.10 - \underbrace{60 * 0.8}_{48 \text{ MW}}$$

$$= 26.10 \text{ MW}$$

* MVar absorbed by line $= Q_s - Q_r$

$$\sin\theta \quad \theta = \cos^{-1} p.f$$

$$= 42.65 - \underbrace{60 * 0.6}_{36 \text{ MVar}}$$

$$= 6.65 \text{ MVar.}$$

ⓓ $P_r = 60 * 0.8 = 48$ MW

$|V_s| = 145$ KV

$|V_r| = 132$ KV

* $P_r = |V_s||V_r||B|^{-1}\cos(\beta-\overset{?}{\delta}) - |A||V_r|^2|B|^{-1}\cos(\beta-\alpha)$

? for this particular operating conditions.

$48 = \frac{(145)(132)}{100}\cos(\beta-\delta) - \frac{(0.98)(132)^2}{(100)}\cos(75-3^\circ)$

$48 = 191.4\cos(\beta-\delta) - 170.75\cos(72)$

$\cos(\beta-\delta) = 0.5275$

β ✓
δ ? ✓

$\angle(\beta-\delta) = \cos^{-1}(0.5275) = 58.16^\circ$

* $Q_r = |V_s||V_r||B|^{-1}\sin(\beta-\delta) - |A||V_r|^2|B|^{-1}\sin(\beta-\alpha)$

$= \frac{(145)(132)}{(100)}\sin(58.16) - \frac{(0.98)(132)^2}{(100)}\sin(72^\circ)$

$= 162.60 - 162.40$

$= 0.20$ MVar

$Q_c = V_{rms}.I_{rms}\sin(\underset{-90}{\theta_v-\theta}$

$= V_{rms}\,I_{rms}\,(-1)$

$\boxed{Q_c = -V_{rm}\,I_{rms}}$

$= -V_{rms}[\underbrace{wC}_{Y_c}V_{rms}]$

$= -V_{rms}^2 * wC$

Thus for $V_s = 145$ kV, $V_r = 132$ kV and $P_r = 48$ MW, a lagging MVar of 0.2 will be supplied from the line along with the real power of 48 MW. Since the load requires 36 MVar lagging, the static compensation
(60 * sin θ)
equipment must deliver 36 − 0.2, i.e, 35.8 MVar lagging (or must absorb 35.8 MVar leading). The capacity of static capacitors is, therefore, 35.8 MVar.

$\boxed{Q_c = -w\,C\,V_{rms}^2}$

$|V_s| = |V_r| = 132 \text{ kV}$, $Q_r = 0$

$$Q_r = |V_s||V_r||B|^{-1} \sin(\beta - \delta) - |A||V_r|^2 |B|^{-1} \sin(\beta - \alpha)$$

$$0 = \frac{(132)(132)}{(100)} \sin(\beta - \delta) - \frac{(0.98)(132)^2}{(100)} \sin(75^\circ - 3^\circ)$$

$$\angle(\beta - \delta) = 68.75^\circ$$

e)

$$P_r = |V_s||V_r|\,|B|^{-1} \cos(\beta - \delta) - |A||V_r|^2 |B|^{-1} \cos(\beta - \alpha)$$

$$= \frac{(132)(132)}{(100)}\left[\cos(68.75)\right] - \frac{(0.98)(132)^2}{(100)} \cos(72^\circ)$$

$$= 63.13 - 52.77$$

$$= 10.36 \text{ MW}$$