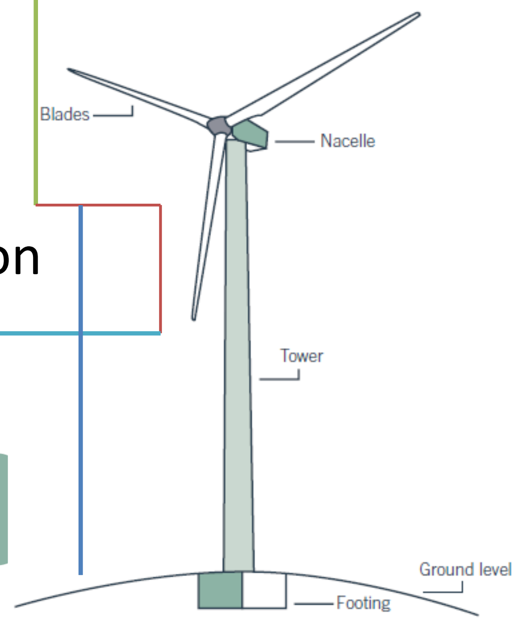
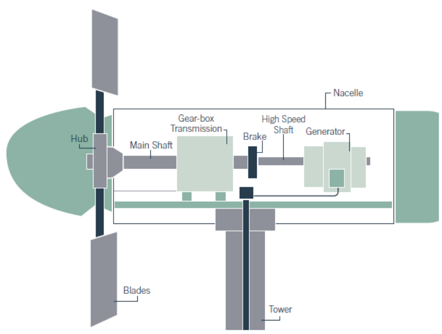
WE System Components &Power Calculation





Summary of previous lecture

POWER IN THE WIND

$$P_w = \frac{1}{2}\rho A v^3$$

- ρ- is the air density (kg/m3) (at 15◦C and 1 atm, ρ = 1.225 kg/m^3);
- A - is the cross-sectional area through which the wind passes (m^2)
- v = wind speed normal to A (m/s) (a useful conversion: 1 m/s = 2.237 mph).

POWER IN THE WIND

$$P_w = \frac{1}{2}\rho A v^3$$

- Notice that the power in the wind increases as the *cube* of wind-speed.
- Later we will see that most wind turbines aren't even turned on in low-speed winds,
- Wind power is proportional to the swept area of the turbine rotor.
- For a conventional horizontal axis turbine,
- the **area** ***A*** **is obviously just**

$A = (\pi/4)D$^2,

Windspeed (m/s)	Windspeed (mph)	Power (W/m^2)
0	0	0
1	2.24	1
2	4.47	5
3	6.71	17
4	8.95	39
5	11.19	77
6	13.42	132
7	15.66	210
8	17.90	314
9	20.13	447
10	22.37	613
11	24.61	815
12	26.84	1058
13	29.08	1346
14	31.32	1681
15	33.56	2067

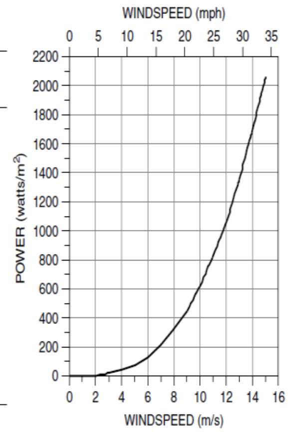
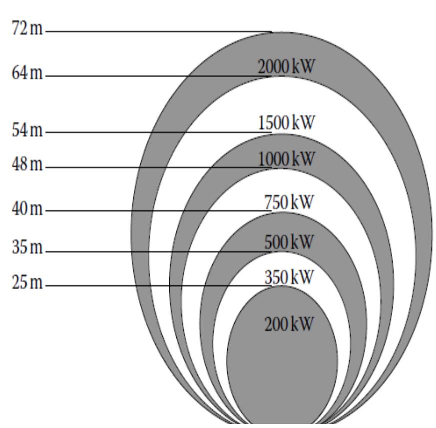


Figure 6.5 Power in the wind, per square meter of cross section, at 15°C and 1 atm.

POWER For different rotor diameters



Turbine output power for different wind turbine diameters.

Average wind speed

- Previous Example dramatically illustrates the inaccuracy associated with using average wind speeds.
- While both of the wind regimes had the same *average* wind speed, the combination of 9-m/s and 3-m/s winds (average 6 m/s) produces 75% more energy than winds blowing a steady 6 m/s.
- Later we will see that, under certain common assumptions about wind speed probability distributions , energy in the wind is typically almost twice the amount that would be found by using the average wind speed.

$$P_w = \frac{1}{2}\rho A v^3$$

IMPACT OF TOWER HEIGHT (important)

$$\left(\frac{v}{v_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}$$

Friction Coefficient

Terrain Characteristics	Friction Coefficient (α)
Smooth hard ground, calm weather	0.10
Tall grass on level ground	0.15
High cops, hedges, and shrubs	0.20
Wooded countryside, many trees	0.25
Small town with trees and shrubs	0.30
Large city with tall buildings	0.40

$$\left(\frac{v}{v_0}\right) = \frac{\ln(H/z)}{\ln(H_0/z)}$$

TABLE 6.4 Roughness Classifications for Use in (6.16)

Roughness Class	Description	Roughness Length $z(m)$
0	Water surface	0.0002
1	Open areas with a few windbreaks	0.03
2	Farm land with some windbreaks more than 1 km apart	0.1
3	Urban districts and farm land with many windbreaks	0.4
4	Dense urban or forest	1.6

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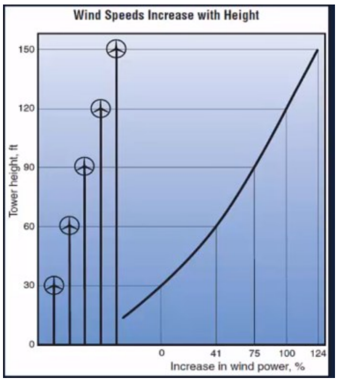
Impact of friction coefficient on windspeed



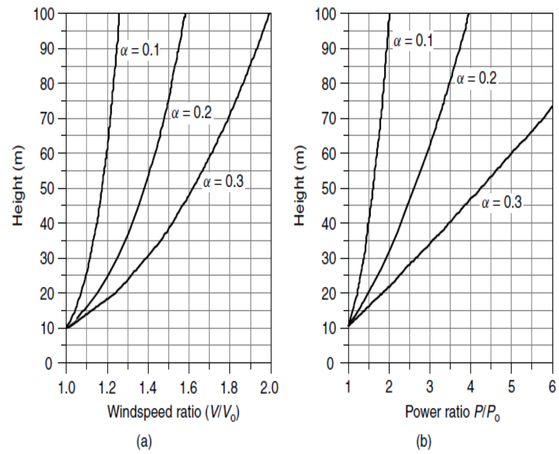
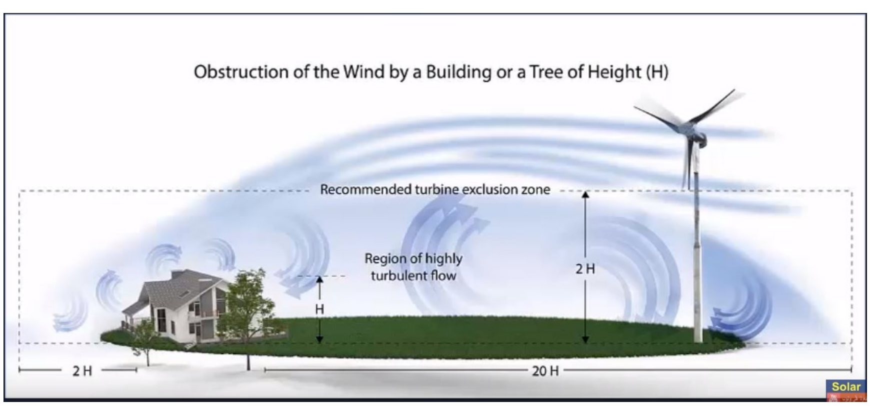


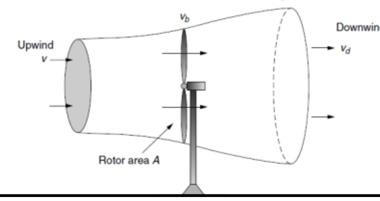
Figure 6.8 Increasing (a) windspeed and (b) power ratios with height for various friction coefficients α using a reference height of 10 m. For $\alpha = 0.2$ (hedges and crops) at 50 m, windspeed increases by a factor of almost 1.4 and wind power increases by about 2.6.

Placement of wind Turbines



System Components

- The original derivation for the maximum power that a turbine can extract from the wind is credited to a German physicist, **Albert Betz**, who first formulated the relationship in 1919. The analysis begins by imagining what must happen to the wind as it passes through a wind turbine.
- **As can be seen, wind approaching from the left is slowed down as a portion of its kinetic energy is extracted by the turbine.**
- The wind leaving the turbine has a lower velocity and its pressure is reduced, causing the air to expand downwind of the machine.
- An envelope drawn around the air mass that passes through the turbine forms what is called a *stream tube*, as suggested in the figure.



Maximum Rotor Effeciency

- It is interesting to note that a number of energy technologies have certain fundamental constraints that restrict the maximum possible conversion efficiency from one form of energy to another.
- For PV, it is the **band gap of the material** that limits the conversion efficiency from sunlight into electrical energy.
- And now, we will explore the constraint that limits the ability of a wind turbine to convert kinetic energy in the wind to mechanical power.

System Components

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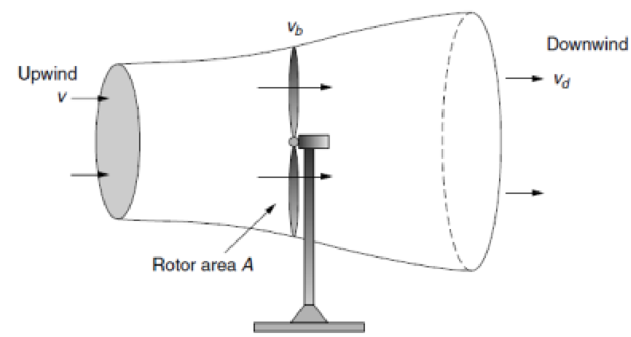


Mass Flow Rate

(for info only)

[redacted]

Power Extracted from wind

(for info only)

[redacted]

(for info only)

Power Extracted from wind

[redacted]

Maximum Power

(for info only)

- To find the maximum possible rotor efficiency, we simply take the derivative of Pb with respect to λ and set it equal to zero:

$$\frac{dC_p}{d\lambda} = \frac{1}{2}[(1+\lambda)(-2\lambda) + (1-\lambda^2)] = 0$$
$$= \frac{1}{2}[(1+\lambda)(-2\lambda) + (1+\lambda)(1-\lambda)] = \frac{1}{2}(1+\lambda)(1-3\lambda) = 0$$

- Which has a solution:

$$\lambda = \frac{v_d}{v} = \frac{1}{3}$$

Blade Efficiency

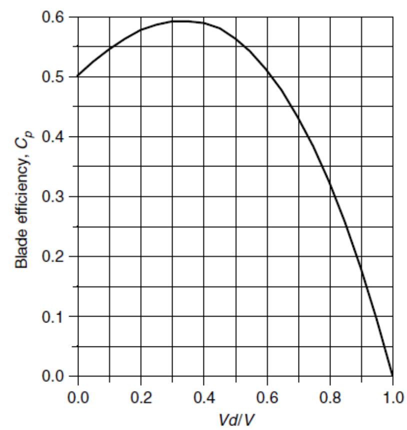
- If we now substitute $\lambda = 1/3$ into the equation for rotor efficiency Cp (we find that the theoretical maximum blade efficiency is

$$\text{Maximum rotor efficiency} = \frac{1}{2}\left(1+\frac{1}{3}\right)\left(1-\frac{1}{3^2}\right) = \frac{16}{27} = 0.593 = 59.3\%$$

- This conclusion, that the maximum theoretical efficiency of a rotor is 59.3%, is called the *Betz efficiency* or, sometimes, *Betz' law*. A plot, showing this maximum occurring when the wind is slowed to one-third its upstream rate, is shown

Blade Efficiency

- The blade efficiency reaches a maximum when the wind is slowed to one-third of its upstream value.



Modern wind Turbine Blades

- **The obvious question is, how close to the Betz limit for rotor efficiency of 59.3 percent are modern wind turbine blades?**
- Under the best operating conditions, they can approach 80 percent of that limit, which puts them in the range of about 45 to 50 percent efficiency in converting the power in the wind into the power of a rotating generator shaft.
- For a given wind speed, rotor efficiency is a function of the rate at which the rotor turns.
- If the rotor turns too slowly, the efficiency drops off since the blades are letting too much wind pass by unaffected. If the rotor turns too fast, efficiency is reduced as the turbulence caused by one blade increasingly affects the blade that follows.

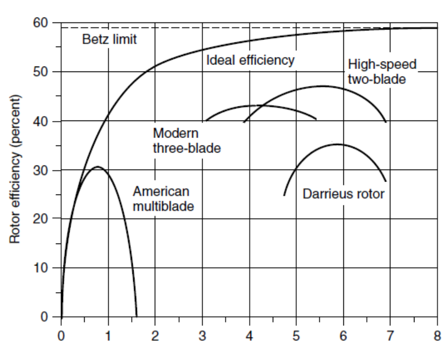
- **The usual way to illustrate rotor efficiency is to present it as a function of its *tip-speed ratio* (TSR).**
- **The tip-speed-ratio is the speed at which the outer tip of the blade is moving divided by the wind speed:**

$$\text{Tip-Speed-Ratio (TSR)} = \frac{\text{Rotor tip speed}}{\text{Wind speed}} = \frac{\text{rpm} \times \pi D}{60\ v}$$

- where rpm is the rotor speed, revolutions per minute; *D* is the rotor diameter (m); and *v* is the wind speed (m/s) upwind of the turbine.

Tip speed Ratio (TSR)

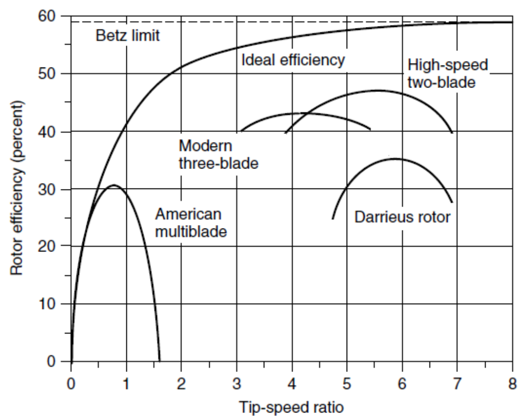
- A plot of typical efficiency for various rotor types versus TSR is given in Figure
- The American multi-blade spins relatively slowly, with an optimal TSR of less than 1 and maximum efficiency just over 30%.
- The two- and three-blade rotors spin much faster, with optimum TSR in the 4–6 range and maximum efficiencies of roughly 40–50%.



Rotors with fewer blades reach their optimum efficiency at higher rotational speeds.

Tip speed Ratio (TSR)

- Also shown is a line corresponding to an "ideal efficiency," which approaches the Betz limit as the rotor speed increases.
- The curvature in the maximum efficiency line reflects the fact that a slowly turning rotor does not intercept all of the wind, which reduces the maximum possible efficiency to something below the Betz limit.



Example: Important

- **Example 6.7 How Fast Does a Big Wind Turbine Turn?**
- A 40-m, three bladed wind turbine produces 600 kW at a wind speed of 14 m/s. Air density is the standard 1.225 kg/m3. Under these conditions,
- a. At what rpm does the rotor turn when it operates with a TSR of 4.0?
- b. What is the tip speed of the rotor?
- c. If the generator needs to turn at 1800 rpm, what gear ratio is needed to match the rotor speed to the generator speed?
- d. What is the efficiency of the complete wind turbine (blades, gear box, generator) under these conditions

Example

a) Using 6.27

$$rpm = \frac{TSR \text{ x } 60 \text{ v}}{\pi D} = \frac{4\text{x}60 \text{ s/min x}14 \text{ m/s}}{40\pi \text{ m/rev}} = 26.7 \text{ rev/min}$$

This is about 2.2 seconds per revolution ....which seems slow

b) The tip of the blade is moving at

$$\text{Tip Speed} = \frac{26.7 \text{ rev/min x } 40\pi \text{ m/rev}}{60 \text{ s/min}} = 55.9 \text{ m/sec}$$

Even though 2.2 s/rev sounds slow, but the tip of rotor is rotating at 55 m/sec or 55.9 x60 sec /min x60 min/hour = 20124 m/h = 201.24 km/h or 125.77 mph

c) the generator needs to spin at 1800 rpm, then thegear box must increase the rotor shaft speed by a factor equal to gear ratio

$$\text{Gear Ratio} = \frac{1800}{26.7} = 67.4$$

Example

d) Power in the wind

$$P_W = \frac{1}{2}\rho A v_w{}^3$$

$$= \frac{1}{2}(1.225)\text{x}\frac{\pi}{4}\text{x}40^2\text{x}14^3$$

$$= 2112 \text{ kW}$$

So the overall effeciency of the turbine, from wind to electricity is :

$$\text{Overall Efficiency} = \frac{600 \text{ kW}}{2112 \text{ kW}} = 0.284 \text{ or } 28.4\%$$

The rotor itself is about 43% efficient, $\xrightarrow{\text{then}}$

$$\text{efficieny of gear box} = \frac{28.4}{43} \cong 66\ \%$$

Blade Efficiency

- The answers derived in the above example are fairly typical for large wind turbines.
- That is, a large turbine will spin at about 20–30 rpm; the gear box will speed that up by roughly a factor of 50–70; and the overall efficiency of the machine is usually in the vicinity of 25–30%.
- In later presentations, we will explore these factors more carefully.

AVERAGE POWER IN THE WIND

- Having presented the equations for *power* in the wind and described the essential components of a wind turbine system, it is time to put the two together to determine how much *energy* might be expected from a wind turbine in various wind regimes,
- The cubic relationship between power in the wind and wind velocity tells us that we cannot determine the average power in the wind by simply substituting average wind speed into (6.4).
- We can begin to explore this important nonlinear characteristic of wind by rewriting (6.4) in terms of average values:

$$P_{avg} = (\tfrac{1}{2}\rho A v^3)_{avg} = \tfrac{1}{2}\rho A (v^3)_{avg}$$

- In other words, we need to find the average value of the cube of velocity.
- To do so will require that we introduce some statistics.

- Suppose, for example, that during a 10-h period, there were 3 h of no wind, 3 h at 5 mph, and 4 h at 10 mph.
- The average wind speed would be

$$v_{\text{avg}} = \frac{\text{Miles of wind}}{\text{Total hours}} = \frac{3\text{ h}\cdot 0\text{ mile/hr} + 3\text{ h}\cdot 5\text{ mile/h} + 4\text{ h}\cdot 10\text{ mile/h}}{3+3+4\text{ h}}$$

$$= \frac{55\text{ mile}}{10\text{ h}} = 5.5\text{ mph}$$

- By regrouping some of the terms above, we could also think of this as having no wind 30% of the time, 5 mph for 30% of the time, and 10 mph 40% of the time:

$$v_{\text{avg}} = \left(\frac{3\text{ h}}{10\text{ h}}\right)\times 0\text{ mph} + \left(\frac{3\text{ h}}{10\text{ h}}\right)\times 5\text{ mph} + \left(\frac{4\text{ h}}{10\text{ h}}\right)\times 10\text{ mph} = 5.5\text{ mph}$$

$$v_{\text{avg}} = \frac{\sum_i [v_i \cdot (\text{hours @ } v_i)]}{\sum \text{hours}} = \sum_i [v_i \cdot (\text{fraction of hours @ } v_i)]$$

$$v_{\text{avg}} = \sum_i [v_i \cdot \text{probability}(v = v_i)]$$

- **We know that the quantity of interest in determining average power in the wind is not the average value of *v*, but the average value of *v*^3.**
- The averaging process yields the following:

$$(v^3)_{avg} = \frac{\sum_i [v_i{}^3 \cdot (\text{hours} \ @ \ v_i)]}{\sum \text{hours}} = \sum_i [v_i{}^3 \cdot (\text{fraction of hours} \ @ \ v_i)]$$

Or, in probabilistic terms,

$$(v^3)_{avg} = \sum_i [v_i{}^3 \cdot \text{probability}(v = v_i)]$$

v (m/s)	Hrs/yr
0	24
1	276
2	527
3	729
4	869
5	941
6	946
7	896
8	805
9	690
10	565
11	444
12	335
13	243
14	170
15	114
16	74
17	46
18	28
19	16
20	9
21	5
22	3
23	1
24	1
25	0
Total hrs	8,760

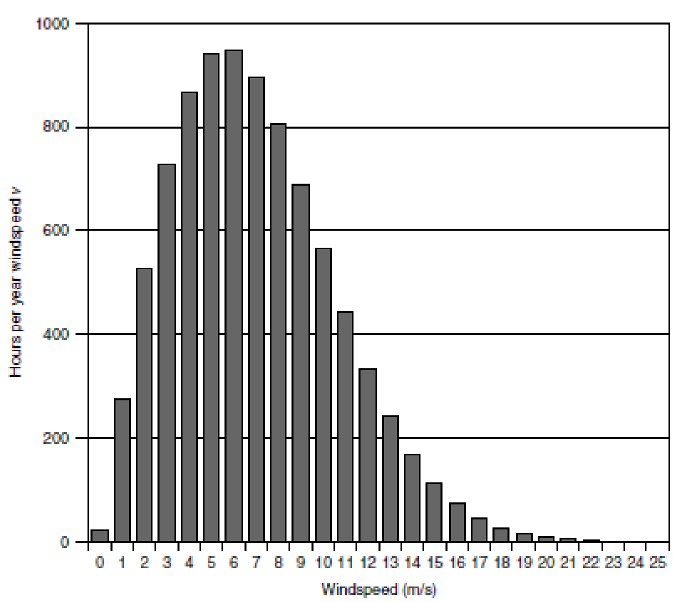


Figure 6.22 An example of site data and the resulting wind histogram showing hours that the wind blows at each windspeed.

Example 6.9 Average Power in the Wind. Using the data given in Fig. 6.22, find the average windspeed and the average power in the wind (W/m^2). Assume the standard air density of 1.225 kg/m^3. Compare the result with that which would be obtained if the average power were miscalculated using just the average windspeed.

Solution. We need to set up a spreadsheet to determine average wind speed v and the average value of v^3. Let's do a sample calculation of one line of a spreadsheet using the 805 h/yr at 8 m/s:

$$\text{Fraction of annual hours at 8 m/s} = \frac{805\text{ h/yr}}{24\text{ h/d} \times 365\text{ d/yr}} = 0.0919$$

$$v_8 \cdot \text{Fraction of hours at 8 m/s} = 8\text{ m/s} \times 0.0919 = 0.735$$

$$(v_8)^3 \cdot \text{Fraction of hours at 8 m/s} = 8^3 \times 0.0919 = 47.05$$

The rest of the spreadsheet to determine average wind power using (6.29) is as follows:

Wind Speed v_i (m/s)	Hours @ v_i per year	Fraction of Hours @ v_i	$v_i \times$ Fraction Hours @ v_i	$(v_i)^3$	$(v_i)^3 \times$ fraction Hours @ v_i
0	24	0.0027	0.000	0	0.00
1	276	0.0315	0.032	1	0.03
2	527	0.0602	0.120	8	0.48
3	729	0.0832	0.250	27	2.25
4	869	0.0992	0.397	64	6.35
5	941	0.1074	0.537	125	13.43
6	946	0.1080	0.648	216	23.33
7	896	0.1023	0.716	343	35.08
8	805	0.0919	0.735	512	47.05
9	690	0.0788	0.709	729	57.42
10	565	0.0645	0.645	1,000	64.50
11	444	0.0507	0.558	1,331	67.46
12	335	0.0382	0.459	1,728	66.08
13	243	0.0277	0.361	2,197	60.94
14	170	0.0194	0.272	2,744	53.25
15	114	0.0130	0.195	3,375	43.92
16	74	0.0084	0.135	4,096	34.60
17	46	0.0053	0.089	4,913	25.80
18	28	0.0032	0.058	5,832	18.64
19	16	0.0018	0.035	6,859	12.53
20	9	0.0010	0.021	8,000	8.22
21	5	0.0006	0.012	9,261	5.29
22	3	0.0003	0.008	10,648	3.65
23	1	0.0001	0.003	12,167	1.39
24	1	0.0001	0.003	13,824	1.58
25	0	0.0000	0.000	15,625	0.00
Totals:	8760	1.000	7.0		653.24

The average windspeed is

$$v_{avg} = \sum_i [v_i \cdot (\text{Fraction of hours @ } v_i)] = 7.0 \text{ m/s}$$

The average value of v^3 is

$$(v^3)_{avg} = \sum_i [v_i{}^3 \cdot (\text{Fraction of hours @ } v_i)] = 653.24$$

The average power in the wind is

$$P_{avg} = \tfrac{1}{2}\rho(v^3)_{avg} = 0.5 \times 1.225 \times 653.24 = 400 \text{ W/m}^2$$

If we had miscalculated average power in the wind using the 7 m/s *average* windspeed, we would have found:

$$P_{average}(\text{WRONG}) = \tfrac{1}{2}\rho(v_{avg})^3 = 0.5 \times 1.225 \times 7.0^3 = 210 \text{ W/m}^2$$

In the above example, the ratio of the average wind power calculated correctly using $(v^3)_{avg}$ to that found when the average velocity is (mis)used is $400/210 = 1.9$. That is, the correct answer is nearly twice as large as the power found when average windspeed is substituted into the fundamental wind power equation $P = \frac{1}{2}\rho A v^3$. In the next section we will see that this conclusion is always the case when certain probability characteristics for the wind are assumed.

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- Rayleigh pdf

$$f(v) = \frac{2v}{c^2}\exp\left[-\left(\frac{v}{c}\right)^2\right] \quad \text{Rayleigh p.d.f.}$$

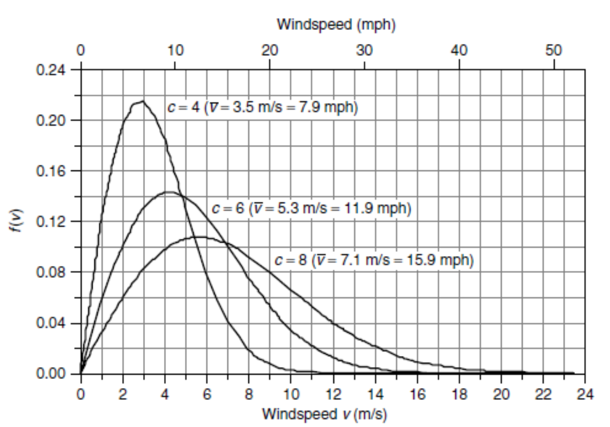

Figure 6.25 The Rayleigh probability density function with varying scale parameter c. Higher scaling parameters correspond to higher average windspeeds.

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Important

$$\overline{P} = \frac{6}{\pi} \cdot \frac{1}{2}\rho A\overline{v}^3 \qquad \text{(Rayleigh assumptions)} \qquad (6.48)$$

That is, with Rayleigh statistics, *the average power in the wind is equal to the power found at the average windspeed multiplied by 6/π or 1.91.*

Example 6.10 Average Power in the Wind. Estimate the average power in the wind at a height of 50 m when the windspeed at 10 m averages 6 m/s. Assume Rayleigh statistics, a standard friction coefficient $\alpha = 1/7$, and standard air density $\rho = 1.225\ \text{kg/m}^3$.

Solution. We first adjust the winds at 10 m to those expected at 50 m using

$$\overline{v}_{50} = \overline{v}_{10}\left(\frac{H_{50}}{H_{10}}\right)^{\alpha} = 6 \cdot \left(\frac{50}{10}\right)^{1/7} = 7.55\ \text{m/s}$$

So, using (6.48), the average wind power density would be

$$\overline{P}_{50} = \frac{6}{\pi} \cdot \frac{1}{2}\rho\overline{v}^3 = \frac{6}{\pi} \cdot \frac{1}{2} \cdot 1.225 \cdot (7.55)^3 = 504\ \text{W/m}^2$$

Example 6.11 Annual Energy Delivered by a Wind Turbine. Suppose that a NEG Micon 750/48 (750-kW generator, 48-m rotor) wind turbine is mounted on a 50-m tower in an area with 5-m/s average winds at 10-m height. Assuming standard air density, Rayleigh statistics, Class 1 surface roughness, and an overall efficiency of 30%, estimate the annual energy (kWh/yr) delivered.

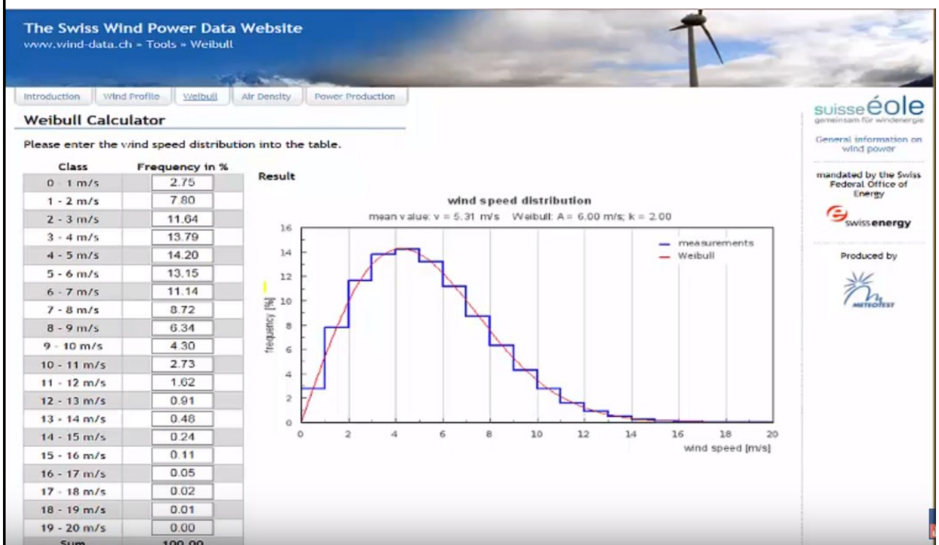
Solution. We need to find the average power in the wind at 50 m. Since "surface roughness class" is given rather than the friction coefficient α, we need to use (6.16) to estimate wind speed at 50 m. From Table 6.4, we find the roughness length z for Class 1 to be 0.03 m. The average windspeed at 50 m is thus

$$v_{50} = v_{10}\frac{\ln(H_{50}/z)}{\ln(H_{10}/z)} = 5\ \text{m/s} \cdot \frac{\ln(50/0.03)}{\ln(10/0.03)} = 6.39\ \text{m/s}$$

Average power in the wind at 50 m is therefore (6.48)

$$\overline{P}_{50} = \frac{6}{\pi} \cdot \frac{1}{2}\rho\overline{v}^3 = 1.91 \times 0.5 \times 1.225 \times (6.39)^3 = 304.5\ \text{W/m}^2$$

Websites for wind data



Since this 48-m machine collects 30% of that, then, in a year with 8760 hours, the energy delivered would be

$$\text{Energy} = 0.3 \times 304.5\ \text{W/m}^2 \times \frac{\pi}{4}(48\ \text{m})^2 \times 8760\ \text{h/yr} \times \frac{1\ \text{kW}}{1000\ \text{W}}$$

$$= 1.45 \times 10^6\ \text{kWh/yr}$$

- Optimum Spacing

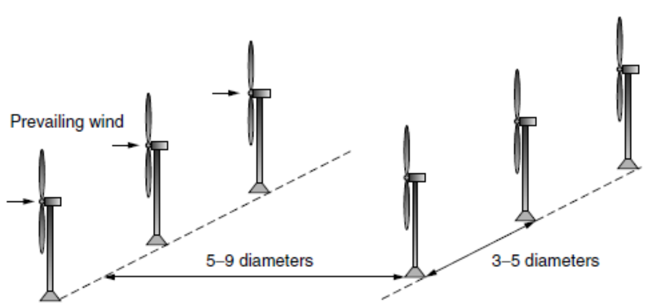


Figure 6.29 Optimum spacing of towers is estimated to be 3–5 rotor diameters between wind turbines within a row and 5–9 diameters between rows.

Power Curve

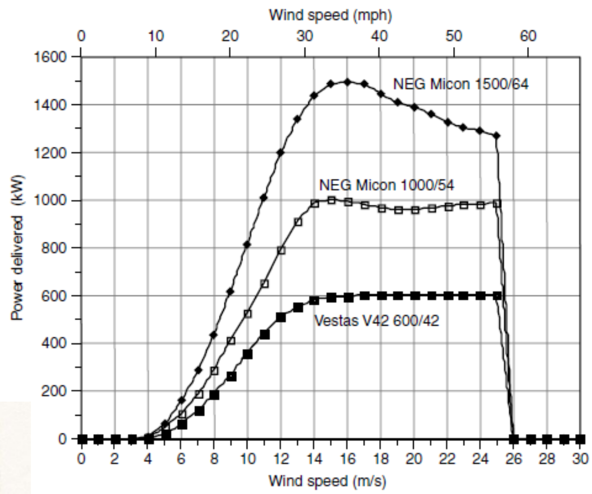


Figure 6.35 Power curves for three large wind turbines.

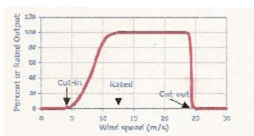


Figure 2-28: Typical WTG power curve.