Mechanical Design of Overhead Line
‘Sag and tension calculations’

Calculation of sag for catenary shape of conductor

- For earlier derivations, we have assumed that the sag is small compared to the span, hence conductor takes the shape of parabola.
- If the sag is comparable with the span, then the conductor takes the shape of catenary, and the expressions derived earlier are no longer valid for such a case.
- Consider a conductor, supported at the points A and B with O as the lowest point on the conductor.

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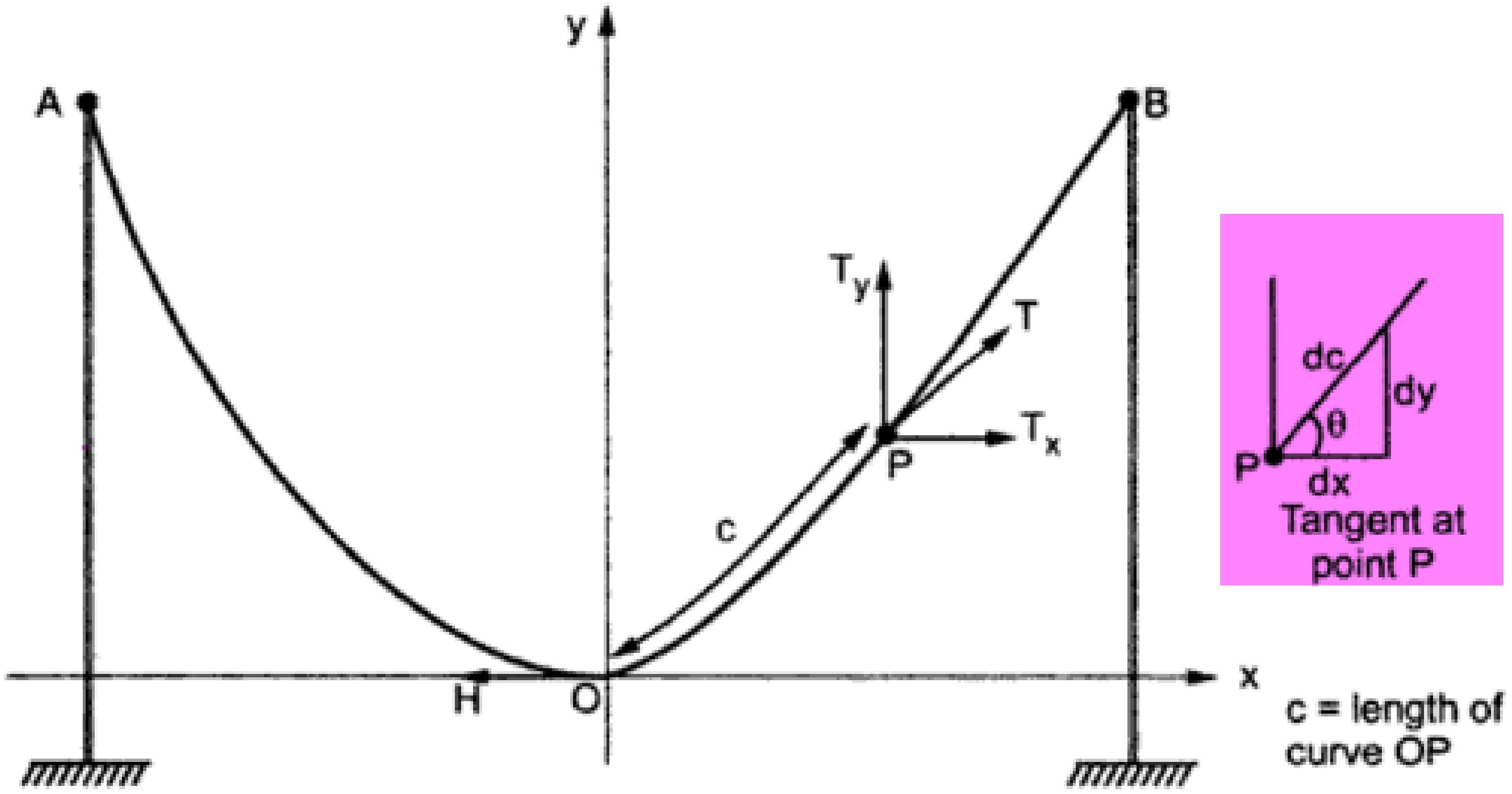
'Sag and tension calculations'



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Mechanical Design of Overhead Line
‘Sag and tension calculations’

- Consider a point P on the conductor such that $l(OP) = c$. Its coordinates are x and y, taking O as the origin. The tension T is acting tangentially at point P.
- The forces acting on the portion OP are:
  - ➢The weight wc of the conductor acting vertically downwards
  - ➢The horizontal tension H acting at O
  - ➢The horizontal and vertical components of tension T in the conductor (T_x and T_y acting at P)

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‘Sag and tension calculations’

- In the equilibrium condition:

$$T_x = H \qquad T_y = wc$$

$$\tan\theta = \frac{T_y}{T_x} = \frac{wc}{H} = \frac{dy}{dx}$$

Mechanical Design of Overhead Line
'Sag and tension calculations'

$$(dc)^2 = (dy)^2 + (dx)^2$$

$$\frac{dc}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(\frac{wc}{H}\right)^2} \Rightarrow dx = \frac{dc}{\sqrt{1+\left(\frac{wc}{H}\right)^2}}$$

$$x = \left(\frac{H}{w}\right)\sinh^{-1}\left(\frac{wc}{H}\right) + A$$

where A constant of integration

Mechanical Design of Overhead Line ‘Sag and tension calculations’

- At $x = 0$, $c = 0$, hence we get $A = 0$

$$x = \left(\frac{H}{w}\right)\sinh^{-1}\left(\frac{wc}{H}\right)$$

- Solve for c, we get $A = 0$

$$\boxed{c = \frac{H}{w}\sinh\left(\frac{wx}{H}\right)}$$

Mechanical Design of Overhead Line
'Sag and tension calculations'

$$\frac{dy}{dx} = \frac{wc}{H} = \frac{w}{H}\left[\frac{H}{w}\sinh\left(\frac{wx}{H}\right)\right] = \sinh\left(\frac{wx}{H}\right)$$

$$dy = \sinh\left(\frac{wx}{H}\right)dx$$

$$\boxed{y = \frac{H}{w}\cosh\left(\frac{wx}{H}\right) + B}$$

where B is the constant of integration

Mechanical Design of Overhead Line
'Sag and tension calculations'

- At $y = 0$, $x = 0$, hence we get $A = 0$

$$0 = \frac{H}{w} + B \Rightarrow B = -\frac{H}{w}$$

- In this case, y is given by:

$$y = \frac{H}{w}\left[\cosh\left(\frac{wx}{H}\right) - 1\right]$$

- Sag:

$$S = \frac{H}{w}\left[\cosh\left(\frac{wl}{2H}\right) - 1\right]$$

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Mechanical Design of Overhead Line 'Sag and tension calculations'

- The tension T is:

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{H^2 + (wc)^2}$$

$$= \sqrt{H^2 + w^2 \frac{H^2}{w^2} \sinh^2\left(\frac{wx}{H}\right)} = \sqrt{H^2\left[1 + \sinh^2\left(\frac{wx}{H}\right)\right]}$$

$$T = H \cosh\left(\frac{wx}{H}\right)$$

Mechanical Design of Overhead Line
‘Sag and tension calculations’

Example

A transmission line has a span of 240 m. Calculate the sag if the weight of the conductor per unit length 0.578 kg/m, ultimate tensile strength is 5200 kg, and factor of safety is 2.

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Mechanical Design of Overhead Line
‘Sag and tension calculations’

```matlab
H=150;                          %set starting value
nmax=25;                        %set max number of iterations
eps=1;                          %initialize error bound eps
Hvals=H;                        %initialize array of iterates
n=0;                            %initialize n (counts iterations)

w=0.578;
L=240;
BS=5200;
FOS=2;
T=BS/FOS;

while eps>=1e-5 && n<=nmax      %set while-conditions
    HH=H-((H*(cosh(w*L/(2*H)))-T)/((cosh(w*L/(2*H)))-(w*L/(2*H))*(sinh(w*L/(2*H))))); %compute next iterate
    Hvals=[Hvals;HH];           %write next iterate in array
    eps=abs(HH-H);              %compute error
    H=HH;n=n+1;                 %update x and n
end                             %end of while-loop

Hvals;
HH;
S=(HH/w)*((cosh(w*L/(2*HH)))-1);
```

Command Window

```
>> Hvals

Hvals =

   1.0e+03 *

    0.1500
    2.8928
    2.5991
    2.5991
    2.5991

>> HH

HH =

   2.5991e+03

>> S

S =

    1.6013
```

supports at unequal levels

Let,

l: span length

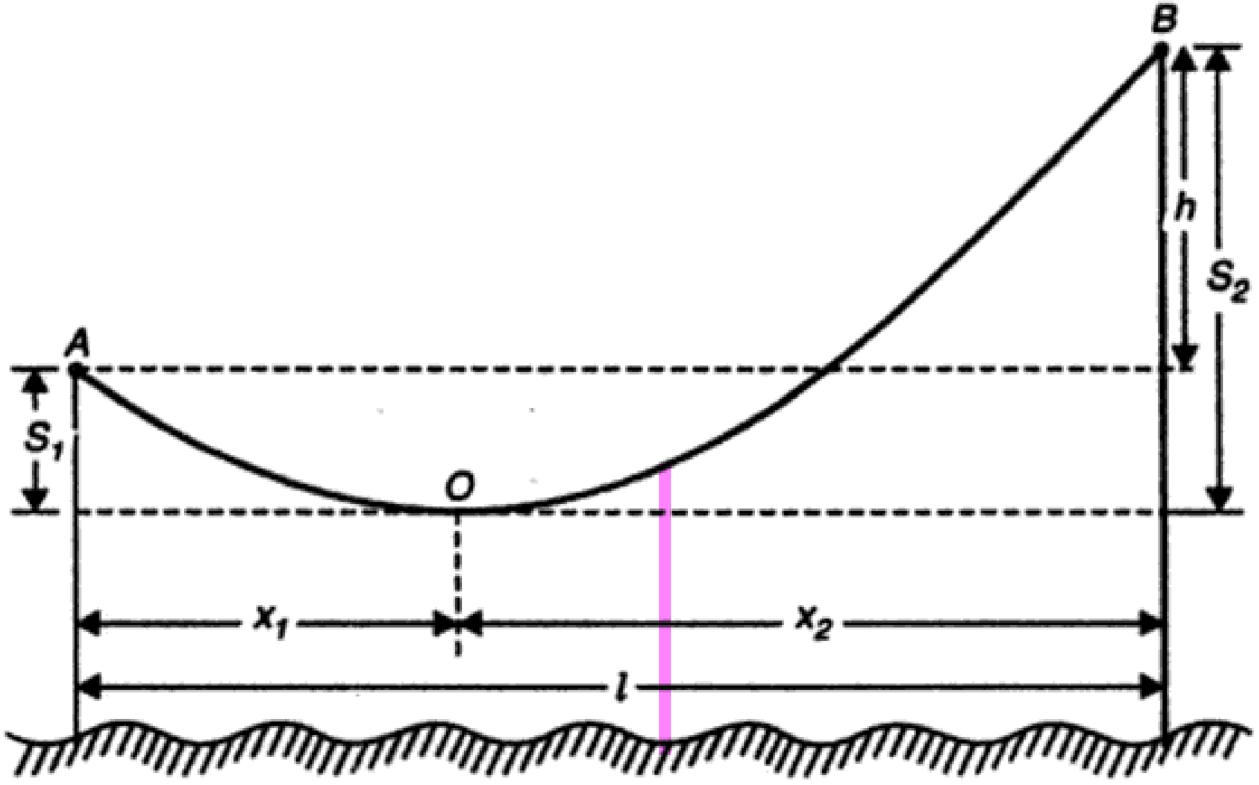
h: difference in levels between two supports

x_1 : distance of support *A* from *O*

x_2 : distance of support *B* from *O*

w: weight of conductor per unit length

T: tension in the conductor



Mechanical Design of Overhead Line
‘Sag and tension calculations’

- Now, Sag, $S_1 = \frac{H}{w}\left[\cosh\left(\frac{wx_1}{H}\right) - 1\right]$, $\quad S_2 = \frac{H}{w}\left[\cosh\left(\frac{wx_2}{H}\right) - 1\right]$

$$h = S_2 - S_1 = \frac{H}{w}\left[\cosh\left(\frac{wx_2}{H}\right) - 1\right] - \frac{H}{w}\left[\cosh\left(\frac{wx_1}{H}\right) - 1\right]$$

$$h = \frac{H}{w}\left[\cosh\left(\frac{wx_2}{H}\right) - \cosh\left(\frac{wx_1}{H}\right)\right]$$

- Also, $\boxed{l = x_1 + x_2}$

Mechanical Design of Overhead Line
'Sag and tension calculations'

- Solving for x_1 and x_2, we get
- and, after finding x_1 and x_2, The values of S_1 and S_2 can be calculated.

Mechanical Design of Overhead Line
‘Sag and tension at erection’

- We have seen that there is a major difference between the calculation of sag and tension in still air and the calculation of sag and tension under sever atmospheric condition like ice coating and wind pressure.
- The factor of safety is provided for a particular tension and sag occurring at a specific temperature and ice, wind conditions.
- These conditions are different than the conditions at the time of erection of the line.
- So if the values are know at lowest temperature of -5.5 ^{o}C, we must be able to calculate all values at the time of erection of the line at (-5.5 ^{o}C + t)

Mechanical Design of Overhead Line
'Sag and tension at erection'

- Let the values T_1, f_1, w_1, c_1, S_1 and t_1 are the known values of tension, stress, equivalent weight, half span conductor length, maximum sag and temperature respectively under sever conditions including ice and wind pressure at -5.5 °C.

$$T_1 = f_1 \times \text{area}$$

$$\text{where } f_1 = \text{stress in kg/m}^2$$

- While the values T_2, f_2, w_2, c_2, S_2 and t_2 are the values specified normal erection conditions without ice and wind pressure and at normal temperature t_2 which are to calculated.

Mechanical Design of Overhead Line ‘Sag and tension at erection’

- Recall the derived equation of *c*

$$c = \left(\frac{H}{w}\right)\sinh\left(\frac{wx}{H}\right)$$

- Conductor half span length, $c_{l/2}$ when $x = l/2$ is given by:

$$c_{l/2} = \left(\frac{H}{w}\right)\sinh\left(\frac{wl}{2H}\right)$$

Mechanical Design of Overhead Line
'Sag and tension at erection'

- The last equation can be expanded as:

$$c = \left(\frac{H}{w}\right)\sinh\left(\frac{wx}{H}\right) = \frac{H}{w}\left[\left(\frac{wx}{H}\right) + \frac{1}{3!}\left(\frac{wx}{H}\right)^3 + \cdots\right] \cong x + \frac{w^2x^3}{6H^2}$$

- In such approximation, we can assume $H = T$

$$\boxed{c = x + \frac{w^2x^3}{6T^2}}$$

- So the half span length of the conductor is given by:

$$c_{l/2} = \frac{l}{2} + \frac{w^2l^3}{48T^2} = \frac{l}{2}\left[1 + \frac{w^2l^2}{24T^2}\right]$$

Mechanical Design of Overhead Line

‘Sag and tension at erection’

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- When the temperature increases from t_1 to t_2, the wire gets elongated and the half span length of the conductor changes from c_1 to c_2
- The increase in length is given by:

$$\Delta c' = c_1(t_2 - t_1)\alpha \cong \frac{l}{2}(t_2 - t_1)\alpha$$

where α is temperature cofficient of conductor at t_1 °C

Mechanical Design of Overhead Line

‘Sag and tension at erection’

- When the temperature increases from t_1 to t_2, the stress decreases from f_1 to f_2
- Correspondingly, there is a reduction in the length of the wire which is given by:

$$\Delta c'' = c_1 (f_2 - f_1)\frac{1}{Y} = c_1 (T_2 - T_1)\frac{1}{aY} \cong \frac{l}{2}(f_2 - f_1)\frac{1}{Y}$$

where Y is Young's modulus

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Mechanical Design of Overhead Line
‘Sag and tension at erection’

- The new half span length c_2 at temperature t_2 is given by:

$$c_2 = c_1 + \Delta c' + \Delta c'' = c_1 + \left(t_2 - t_1\right)\frac{\alpha l}{2} + \left(T_2 - T_1\right)\frac{l}{2aY}$$

- Recall the half span length of the conductor

$$c_2 = \frac{l}{2} + \frac{w_2^2 l^3}{48T_2^2} = \frac{l}{2} + \frac{w_2^2 l^3}{48\, f_2^2 a^2}$$

Mechanical Design of Overhead Line ‘Sag and tension at erection’

- From the last two equations:

$$\frac{l}{2}+\frac{w_2^2 l^3}{48 f_2^2 a^2}=c_1+\left(t_2-t_1\right)\frac{\alpha l}{2}-\left(T_1-T_2\right)\frac{l}{2aY}$$

- Substituting the half span length of the conductor, c_1, we get:

$$\frac{l}{2}+\frac{w_2^2 l^3}{48 f_2^2 a^2}=\frac{l}{2}+\frac{w_1^2 l^3}{48 f_1^2 a^2}+\left(t_2-t_1\right)\frac{\alpha l}{2}-\left(T_1-T_2\right)\frac{l}{2aY}$$

$$\frac{w_2^2 l^3}{48 f_2^2 a^2}=\frac{w_1^2 l^3}{48 f_1^2 a^2}+\left(t_2-t_1\right)\frac{\alpha l}{2}-\left(T_1-T_2\right)\frac{l}{2aY}$$

Mechanical Design of Overhead Line
'Sag and tension at erection'

- Cancelling *l*:

$$\frac{w_2^2 l^2}{48T_2^2} = \frac{w_1^2 l^2}{48T_1^2} + (t_2 - t_1)\frac{\alpha}{2} - (T_1 - T_2)\frac{l}{2aY}$$

- Multiplying by 2*aY*:

$$\frac{w_2^2 l^2 aY}{24T_2^2} = \frac{w_1^2 l^2 aY}{24T_1^2} + (t_2 - t_1)\alpha\, aY - (T_1 - T_2)$$

Mechanical Design of Overhead Line ‘Sag and tension at erection’

- Multiplying by T_2^2, we get:

$$T_2^2\left[T_2-T_1+\frac{w_1^2l^2aY}{24T_1^2}+(t_2-t_1)\alpha\, aY\right]-\frac{w_2^2l^2aY}{24}=0$$

- Once T_2 is known, sag at erection time can be obtained as::

$$S_2=\frac{w_2l^2}{8T_2}$$

Mechanical Design of Overhead Line
‘Sag and tension at erection’

- Example

An overhead line has a conductor diameter of 1.6 cm and is erected across a span of 200 m on level supports. The radial thickness of ice under severe conditions is 1.25 cm and the dead weight of the conductor is 0.7 kg/m run. The ultimate stress of the conductor is 7000 kg, the modulus of elasticity is 7.5×10^{5} kg/cm^2 and its coefficient of linear expansion is $16.5\times10^{-6}/^{0}$C. Assume a wind pressure of 39 kg/m^2 and ice covering at temperature of -5.0^{0}C as the worst conditions, a safety factor of 2 being required under these conditions. The weight of ice is 913.5 kg/m^3. find the sag in still air at the time of erection when the temperature is 35^{0}C.

Mechanical Design of Overhead Line
‘Sag and tension at erection’

- Conductor diameter, $d = 1.6$ cm
- Span length, $l = 200$ m;
- Radial thickness, $t = 1.25$ cm
- Weight of conductor/m length, $w_2 = 0.7$ kg/m
- Safety factor, F.O.S = 2
- The ultimate stress of the conductor, = 7000 kg,
- Tension (worst conditions): $T_1 = 7000/2 = 3500$ kg

Mechanical Design of Overhead Line
‘Sag and tension at erection’

- Young’s modulus, $Y = 7.5 \times 10^5$ kg/cm^2
- Temperature coefficient, $\alpha = 16.5 \times 10^{-6}/^0C$
- Wind pressure, P = 39 kg/m^2.
- Ice density, = 913.5 kg/m^3.

Mechanical Design of Overhead Line
‘Sag and tension at erection’

- Total weight of 1 m length of conductor is (worst conditions):

$$w_1 = \sqrt{(w_c + w_i)^2 + w_w^2}$$

- Weight of ice for a 1 m length of conductor

$$w_i = \text{density of ice} \times \text{volume per unit length}$$

$$w_i = \text{density of ice} \times \frac{\pi}{4}\left[(d+2t)^2 - d^2\right] \times 1 = \text{density of ice} \times \pi t\,(d+t) \times 1$$

$$w_c = \left(913.5 \times 10^{-4}\right)\left(\pi \times 1.25(1.6+1.25) \times 1\right) = 1.022 \text{ kg/m}$$

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Mechanical Design of Overhead Line
'Sag and tension at erection'

- Wind force on 1 m length of conductor:

$$w_w = \text{wind pressure per unit area} \times \text{projected area per unit length}$$

$$w_w = \text{wind pressure} \times \left[(d + 2t) \times 1 \right]$$

$$w_w = 39 \times 10^{-2} \times \left[(1.6 + 2.5) \times 1 \right] = 1.599 \text{ kg/m}$$

- Total weight of 1 m length of conductor (worst conditions):

$$w_1 = \sqrt{(0.7 + 1.022)^2 + 1.599^2} = 2.35 \text{ kg/m}$$

Mechanical Design of Overhead Line
‘Sag and tension at erection’

- Cross section area of the conductor (still air):

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 1.6^2 = 2.0106 \text{ cm}^2$$

- Tension equation at time of erection:

$$T_2^2 \left[T_2 - T_1 + \frac{w_1^2 l^2 a Y}{24 T_1^2} + (t_2 - t_1) \alpha\, a Y \right] - \frac{w_2^2 l^2 a Y}{24} = 0$$

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Mechanical Design of Overhead Line
'Sag and tension at erection'

- Tension equation at time of erection:

$$T_2^2\left[T_2 - T_1 + \frac{w_1^2 l^2 aY}{24T_1^2} + (t_2 - t_1)\alpha aY\right] - \frac{w_2^2 l^2 aY}{24} = 0$$

$$T_2^2\left[\begin{array}{l} T_2 - 3500 + \dfrac{(2.35^2)(200^2)(2.0106)(7.5\times10^5)}{24(3500^2)} + \\ +(35-(-5))(16.5\times10^{-6})(2.0106)(7.5\times10^5) \end{array}\right] -$$

$$-\frac{(0.7^2)(200^2)(2.0106)(7.5\times10^5)}{24} = 0$$

Mechanical Design of Overhead Line
‘Sag and tension at erection’

- Tension equation at time of erection:

$$T_2^2\left[T_2 - T_1 + \frac{w_1^2 l^2 aY}{24T_1^2} + (t_2 - t_1)\alpha aY\right] - \frac{w_2^2 l^2 aY}{24} = 0$$

$$T_2^2[T_2 - 3500 + 1133.014 + 995.247] - 12314.925\times 10^5 = 0$$

$$T_2^2[T_2 - 1371.74] - 12314.925\times 10^5 = 0$$

$$T_2^3 - 1371.74T_2^2 - 12314.925\times 10^5 = 0$$

Mechanical Design of Overhead Line ‘Sag and tension at erection’

- Tartaglia’s cubic formula (1530, published by Cardano in 1545): Roots to

$$ax^3 + bx^2 + cx + d = 0$$

$$x = -\frac{1}{3a}\left(b + \Phi + \frac{\Delta_0}{\Phi}\right)$$

$$\Phi = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}},$$

$$\Delta_0 = b^2 - 3ac$$

$$\Delta_1 = 2b^3 - 9abc + 27a^2d$$

Mechanical Design of Overhead Line
‘Sag and tension at erection’

- Solution for T_2

$$T_2^3 - 1371.74 T_2^2 - 12314.925 \times 10^5 = 0$$

$$a = 1,\quad b = -1371.74, c = 0,\quad d = -12314.925 \times 10$$

$$\Delta_0 = b^2 - 3ac = 1881670.628$$

$$\Delta_1 = 2b^3 - 9abc + 27a^2 d = -1.8413 \times 10^{10}$$

$$\Phi = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}} = -558.526,$$

$$T_2 = -\frac{1}{3a}\left(b + \Phi + \frac{\Delta_0}{\Phi}\right) = \boxed{1766.42 \text{ kg}}$$

Mechanical Design of Overhead Line
'Sag and tension at erection'

- Sag at time of erection

$$S_2 = \frac{w_2 l^2}{8T_2} = \frac{0.7\left(200^2\right)}{8\left(1766.42\right)} = 1.98\ \text{m}$$

- Sag (calculate in worst conditions)

$$S_1 = \frac{w_1 l^2}{8T_1} = \frac{2.35\left(200^2\right)}{8\left(3500\right)} = 3.36\ \text{m}$$

Mechanical Design of Overhead Line
‘Stringing chart’

- We have seen that the tension T_2 at time of erection is given by

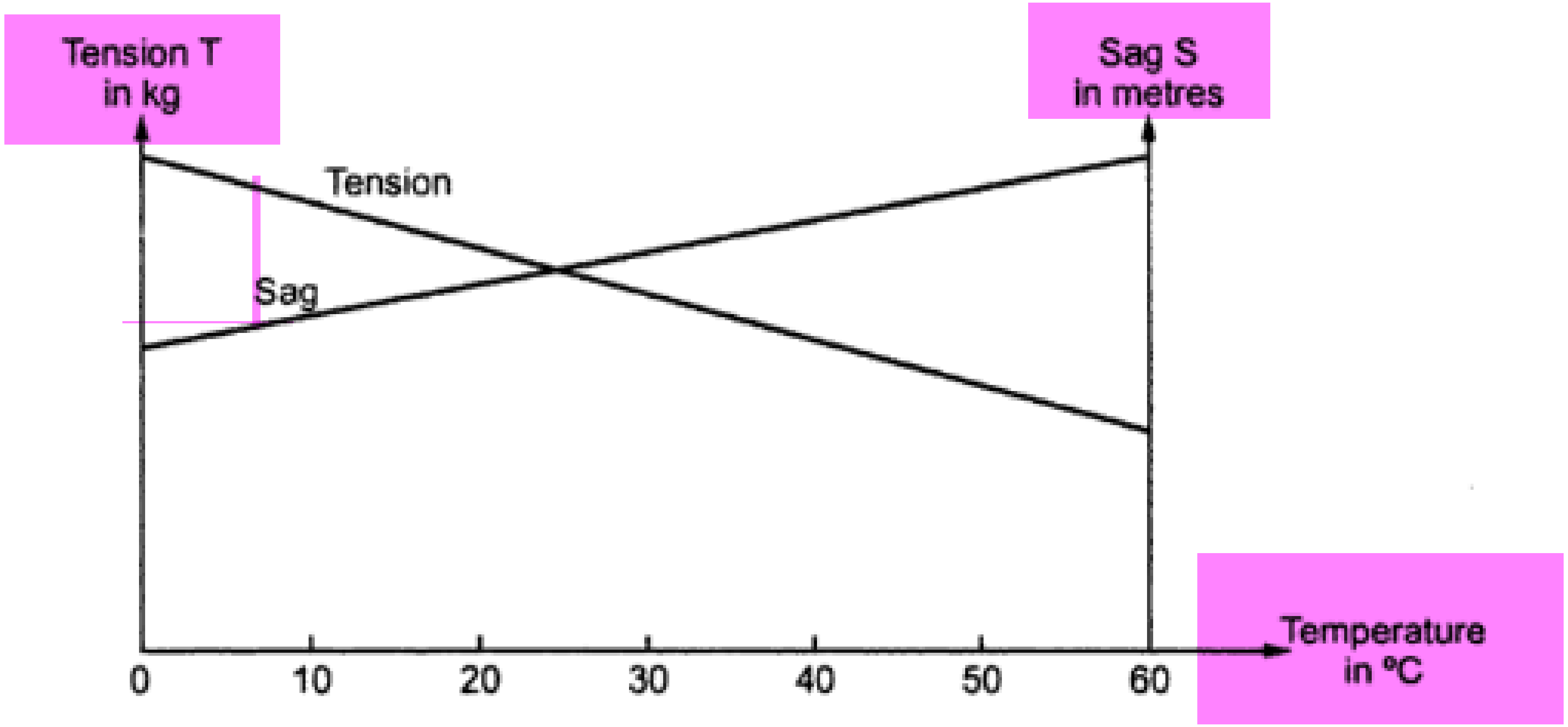
$$T_2^2\left[T_2 - T_1 + \frac{w_1^2 l^2 aY}{24T_1^2} + (t_2 - t_1)\alpha\, aY\right] - \frac{w_2^2 l^2 aY}{24} = 0$$

- It is a cubic equation and is very difficult and time consuming to solve.

Mechanical Design of Overhead Line
‘Stringing chart’

- So instead of solving the cubic equation, it is possible to obtain the graph of tension in kg against temperature ^{o}C and the graph of sag in meters against temperature ^{o}C.
- Such graphs is called *stringing charts*
- The stringing chart is very useful to find the tension and the sag at any temperature and the loading conditions when these values at any other temperature and the loading conditions are known to us.
- These charts are useful at erection of transmission line conductor for adjusting the sag and tension properly.

Mechanical Design of Overhead Line
‘Stringing chart’



Stringing chart

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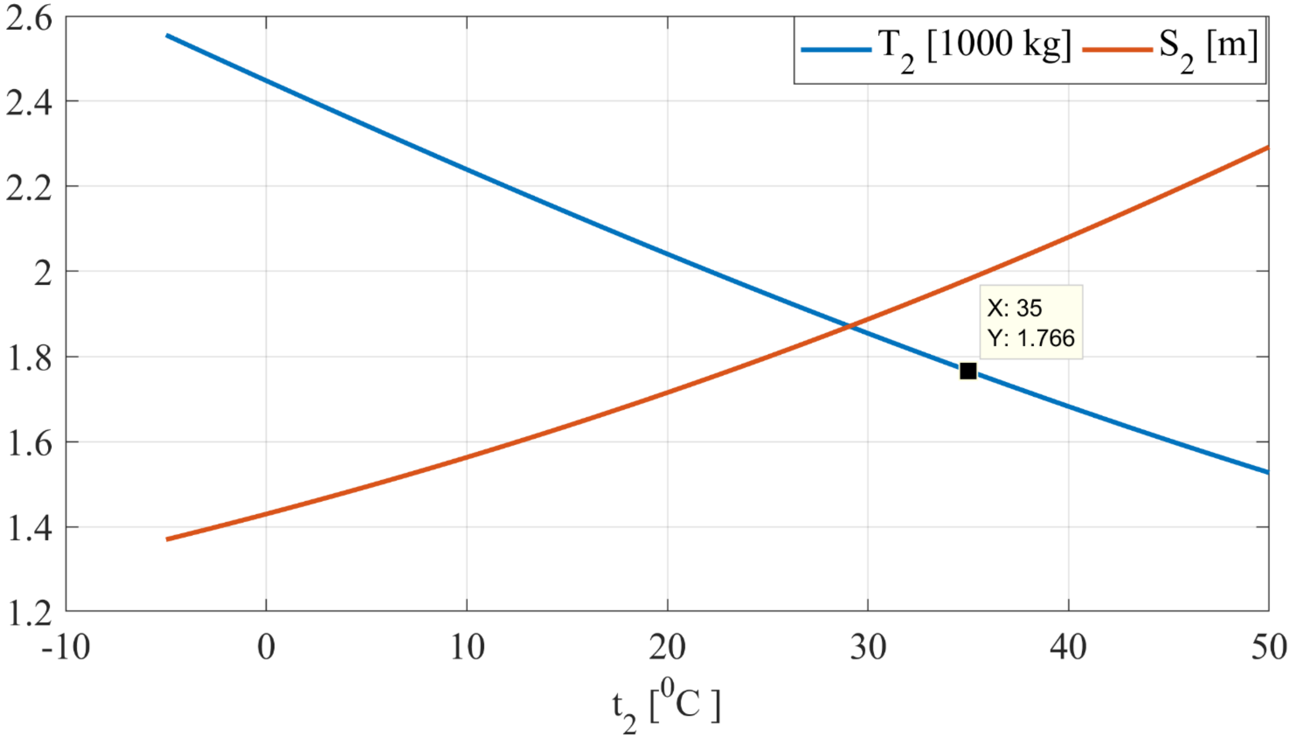
Mechanical Design of Overhead Line 'Stringing chart'



Stringing chart for the last example

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Untitled.m

```
t2=-5:1:60;
b=-3500+1133.014+24.881*(t2+5);
d=-12314.925e5;
D0=b.^2;
D1=2*(b.^3)+27.*d;
Phi=-abs((((sqrt((D1.^2)-(4.*D0.^3)))+D1)/2).^(1/3));
T2=-(b+Phi+(D0./Phi))/3;
S2=0.7*(200^2)./(8.*T2);
plot(t2,T2/1000)
hold on
plot(t2,S2)
```

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Mechanical Design of Overhead Line

‘Methods of checking the sag’

- *Sight method*
  - ➢Wooden stripes are used as targets
  - ➢The length of each of this target is equal to the minimum clearance needed from ground
  - ➢The wooden stripes are placed along the pole vertically at each end of the span so that the distance of the upper end of the targets and the conductors at the poles is equal to the sag.
  - ➢The conductor is correctly sagged when its lower point in the span is in line with the top of the targets when viewed from one end of the span to other.

Mechanical Design of Overhead Line
‘Methods of checking the sag’

- *Stop watch or return wave method*
  - ➢A sharp blow is given to the conductor.
  - ➢This will cause a wave to travel to the other pole from where the wave is reflected again back
  - ➢The wave will travel back and forth until it finally dies out by the absorption of energy by the conductor
  - ➢There exist a definite relation between the time in seconds taken by the wave to return to the first support and the amount of sag:

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Mechanical Design of Overhead Line
‘Methods of checking the sag’

- The amount of estimated sag is given by:

$$S = \frac{g}{8}\left(\frac{time}{2n}\right)^2$$

where

g is the gravitational acceleration (g = 9.8 m/s^2)

S is the sag in [m]

$time$ is the total time in [s]

n is the number of return waves counted.

Mechanical Design of Overhead Line
‘Methods of checking the sag’

- The return wave can be felt by a man on the first pole by keeping his finger lightly on the conductor
- The wave may be initiated in the conductor by
  - striking the conductor
  - throwing a light, dry, nonmetallic rope or cord over the conductor and pulling it down strongly and quickly releasing the conductor

Mechanical Design of Overhead Line
‘Methods of checking the sag’

- Example

In the previous example, a sharp blow is given to the conductor at (pole ‘A’) to cause a wave to travel to the other pole (pole ‘B’). The return wave is felt by a man on the first pole by keeping his finger lightly on the conductor. The man counted the number of return waves at pole A (12 return waves counted) before it finally dies out by the absorption of energy by the conductor. If the time of the stop watch at this moment was 30 sec. Check the sag at the time of erection?

Mechanical Design of Overhead Line
‘Methods of checking the sag’

- Solution

$$S = 1.227\left(\frac{t}{2n}\right)^2$$

$$S = 1.227\left(\frac{30}{2\times 12}\right)^2 = 1.92\text{ m}$$

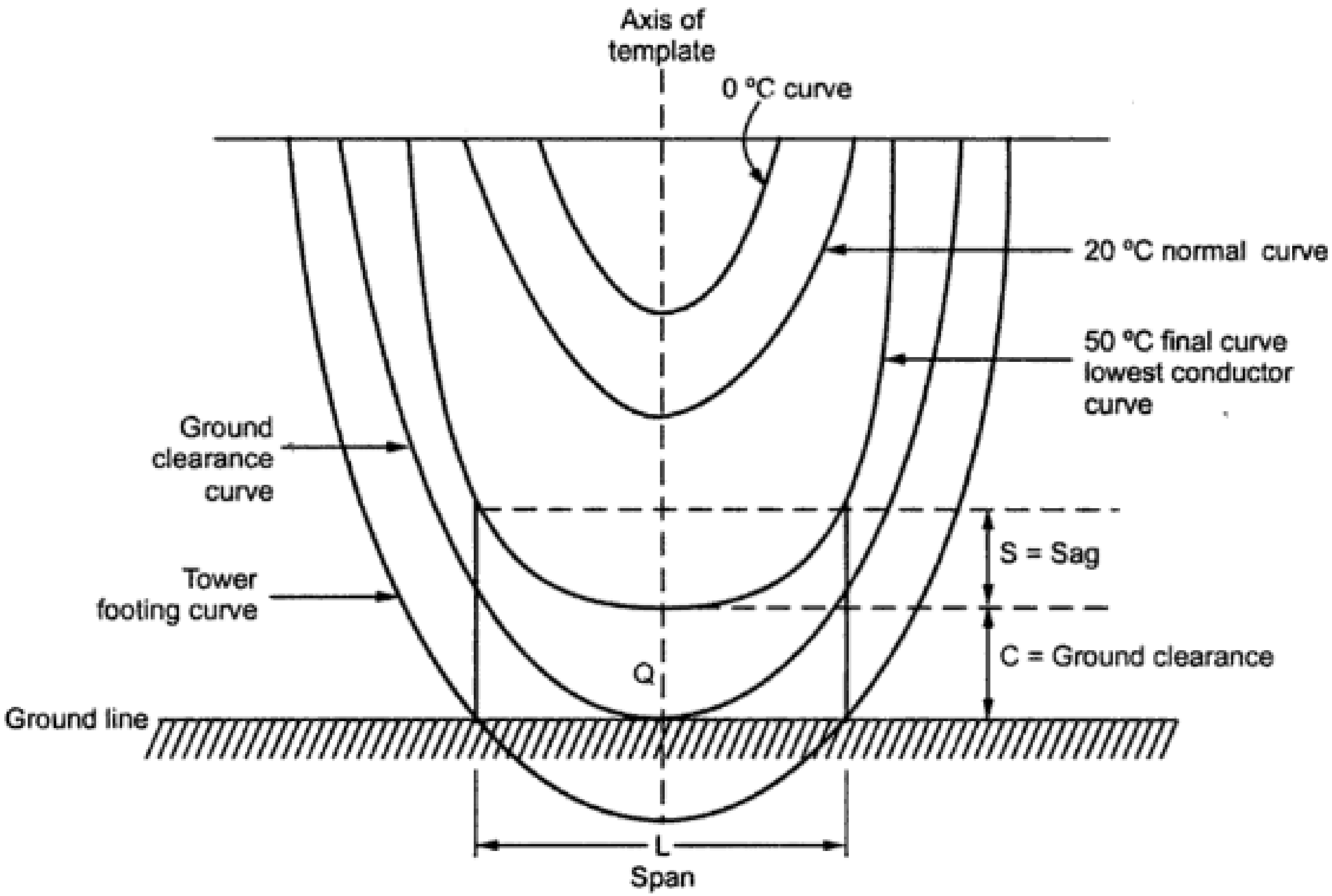
Mechanical Design of Overhead Line

‘Sag template’

- For locating the tower positions, the sag and the nature of conductor curve is calculated and plotted on a thin stiff plastic sheet. Such a graph is called *sag template*
- The minimum ground clearance curve is plotted parallel to the conductor curve to be tangential to the ground level at point Q.
- The tower footing line intersects the points on the ground where towers are suited. This line helps to fix up position of towers, with minimum clearance of C is maintained throughout.

Mechanical Design of Overhead Line
‘Sag template’

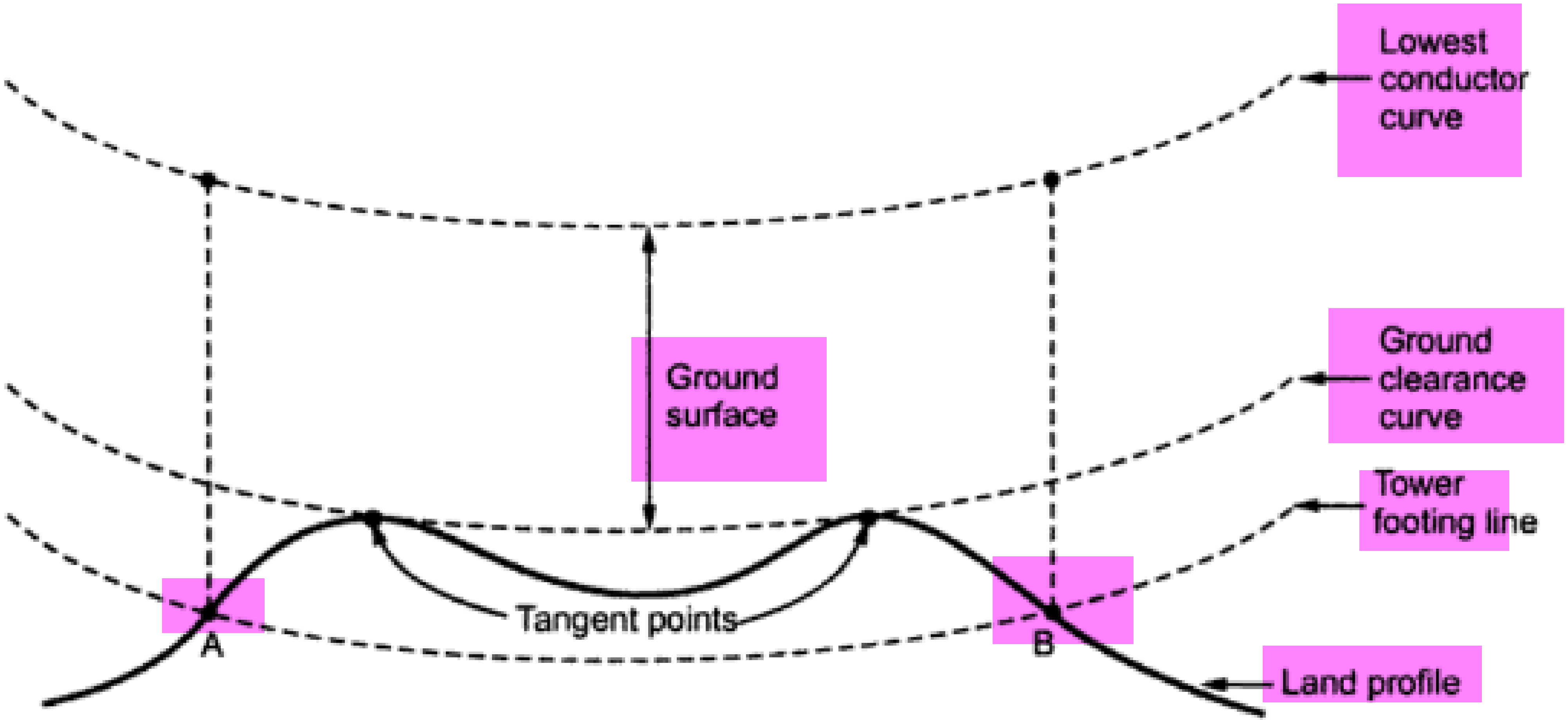
Sag template



Mechanical Design of Overhead Line ‘Sag template’

- While using the sag template, the profile of the land is drawn on the paper.
- Then the sag template is placed on the route drawn. It is then adjusted such that the ground clearance curve just touches the profile.
- At this position, the tower footing line gives the position of towers.
- The actual conductor curve is then represented by the lowest conductor curve on the template.
- The points A and B give the positions of the towers.

Mechanical Design of Overhead Line
‘Sag template’



Application of Sag template

Mechanical Design of Overhead Line
‘Sag template’

Voltage level (kV)	Span (m)	Minimum ground clearance (m)
0.4	80	4.6
11	100	4.6
33	150-200	5.2
66	200-300	6.3
132	350-360	6.3
220	360-380	7.0
400	400	8.8