Chapter 13

Tuesday, April 12, 2016 5:09 PM

Kinetics of a Particle : Force & Acceleration

→ Forces causing motion

$\boxed{13.1}$ Newton's second law of motion

Newton's 2^{nd} Law: "When an <u>unbalanced</u> force acts on a particle, the particle will <u>accelerate</u> in the direction of the force with a magnitude that is proportional to the force".

$$\underset{[N]}{\vec{F}} = \underset{[kg]}{m}\,\underset{[m/s^2]}{\vec{a}}$$

m → constant of proportionality.

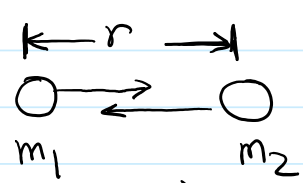
m: +ve scalar (mass of the particle): it provides a quantitative measure of the resistance of the particle to a change in its velocity.

$$\underset{[N]}{\vec{F}} = \underset{[slug]}{m}\,\underset{[ft/s^2]}{\vec{a}} \Rightarrow \text{equation of motion}$$

EOM.

<u>Newton's law of Gravitational Attraction</u> (mutual attraction)

$$F = G\,\frac{m_1 m_2}{r^2}$$



F: force of attraction between 2 particles (Newton)

G: universal constant of gravitation (experimental)

$G = 66.73 \times 10^{-12}\ m^3/kg\,s^2$

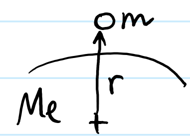
m_1, m_2 : masses of both particles (kg)

r : distance between 2 particles (m)

If a particle is located near the surface of the earth

The only gravitational force having any sizable magnitude is between the earth and the particle (W weight)

particle of mass (m)
earth of mass (M_e)
distance between the earth center and the particle (r)



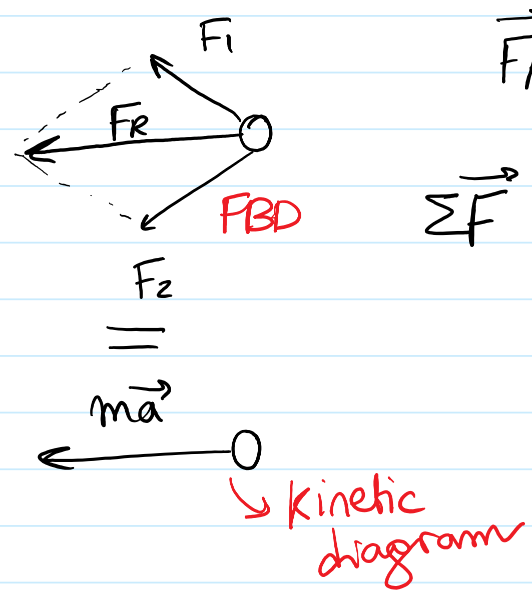
$$W = \frac{G\, m M_e}{r^2} = \frac{G M_e}{r^2}\, m \qquad \text{let } g = \frac{G M_e}{r^2}$$

$$W = m g$$ → acceleration due to gravity

On the surface of the earth at sea level and a latitude of 45° (standard location) → $g = 9.81\ m/s^2$
$= 32.2\ ft/s^2$

*   *   *   *

$\boxed{13.2}$ The equation of Motion



$\vec{F}_R = \Sigma \vec{F}$ (vector summation)

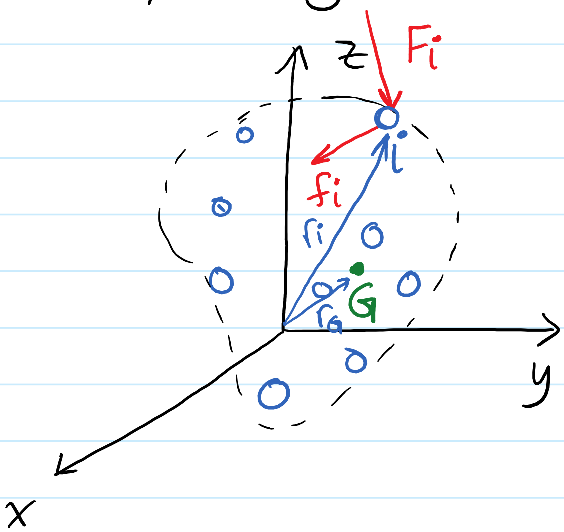
$\Sigma \vec{F} = m\vec{a}$ (EOM)

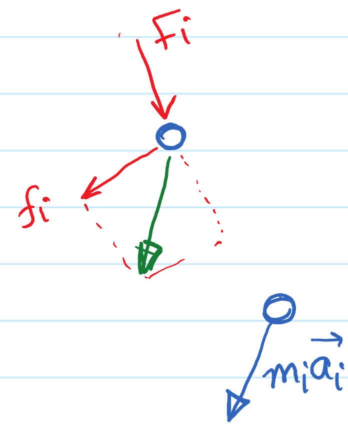
if $\Sigma \vec{F} = 0 \Rightarrow \vec{a} = 0$
static equilibrium
(Newton's first Law).

* To measure acceleration → Newtonian (Inertial) frame of reference : fixed or translates with constant

velocity.

13.3 EOM for a system of Particles





xyz inertial frame of reference

<u>EOM for particle i</u>

$$\vec{F}_i + \vec{f}_i = m_i \vec{a}_i$$

<u>EOM for all particles</u>

$$\sum \vec{F}_i + \sum \vec{f}_i = \sum m_i \vec{a}_i$$

→ zero, internal forces are equal in magnitude & opposite directions.

$$\sum \vec{F}_i = \sum m_i \vec{a}_i$$

<u>By definition</u> $m \vec{r}_G = \sum m_i \vec{r}_i$

Defferentiate twice $m \vec{a}_G = \sum m_i \vec{a}_i$

$$\therefore \boxed{\sum \vec{F}_i = m \vec{a}_G}$$

13.4 EOM: Rectangular Coordinates

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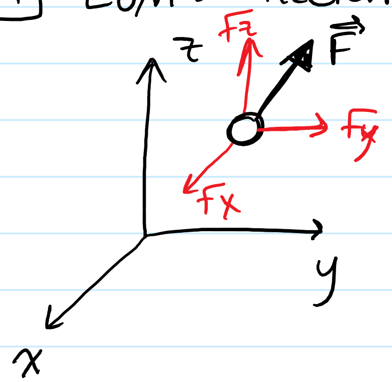


$$\Sigma F = m\vec{a}$$

$$\Sigma F_x \hat{i} + \Sigma F_y \hat{j} + \Sigma F_z \hat{k} = m(a_x \hat{i} + a_y \hat{j} + a_z \hat{k})$$

$$\Sigma F_x = m a_x$$
$$\Sigma F_y = m a_y$$
$$\Sigma F_z = m a_z$$

* Free Body Diagram (FBD)

1) Inertial Frame – One axis in the direction of Motion
2) Isolate object of interest – Draw outline of object.
3) Sketch all external forces (active or reactive)
   moves objects ← active; reactive → result of supports
4) Do not forget the weight unless it is neglected.
5) Label forces with magnitudes & Directions.
6) Direction of unknown forces is assumed.
7) Set direction of acceleration (if unknown assume positive)
8) Find unknown forces from FBD
9) Find unknown accelerations

* Friction

$F_f = \mu_s N$ (on the verge of motion)

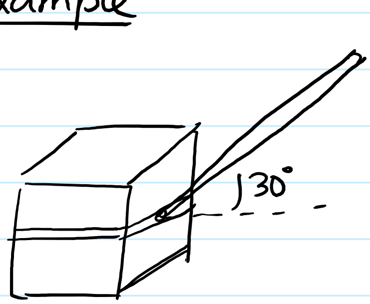
$F_f = \mu_k N$ (object is moving).

Direction is opposite to motion.

* <u>Springs</u>

$F_s = K\,\delta$ $\quad$ k : stiffness
$\delta = l - l_0$ (deformation)

<u>Example</u>

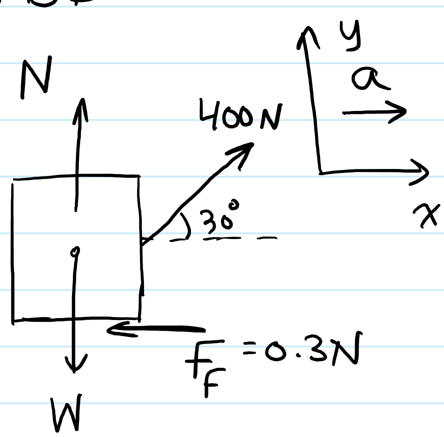


50 kg box rests on a surface
$\mu_k = 0.3$

Find velocity after 3 s of motion

$W = 50 \times 9.81$
$= 490.5\ N$

FBD



$\overset{+}{\rightarrow}\ \Sigma F_x = 400 \cos 30 - 0.3N = ma_x$

$\overset{+}{\uparrow}\ \Sigma F_y = N + 400 \sin 30 - W = 0$

$\therefore\ N = 290.5\ N$

$a_x = 5.185\ m/s^2$

Since P is constant $\rightarrow$ $\vec{a}$ is constant

$v = v_0 + at$

$v = 5.185(3) = 15.6\ m/s \rightarrow +$

$\boxed{13.6}$ EOM : Normal & Tangential Coordinates.

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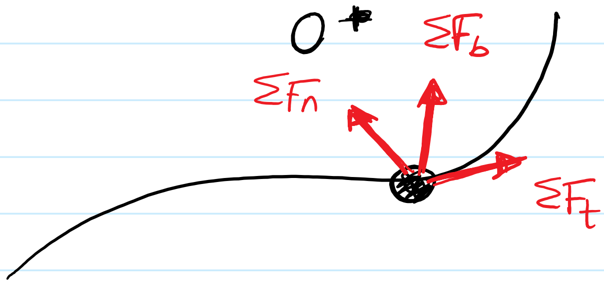


$$\Sigma \vec{F} = m\vec{a}$$

$$\Sigma F_t \vec{u}_t + \Sigma F_n \vec{u}_n + \Sigma F_b \vec{u}_b = m\left(a_t \vec{u}_t + a_n \vec{u}_n\right)$$

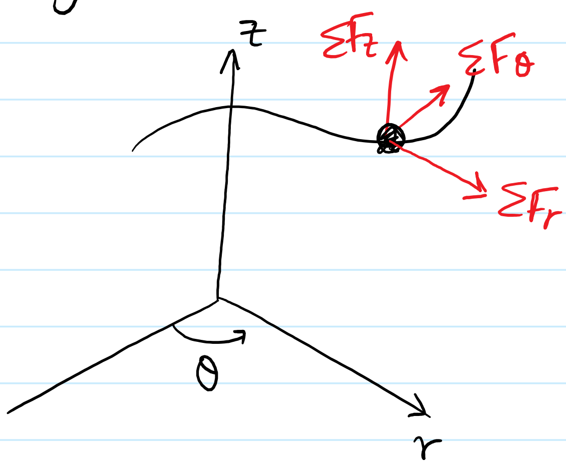
$$\Sigma \vec{F}_t = m\vec{a}_t$$

$$\Sigma \vec{F}_n = m\vec{a}_n \quad \text{(Centripital force)}$$

$$\Sigma \vec{F}_b = 0$$

Recall that $\vec{a}_t = \dfrac{dv}{dt}$ $\qquad \vec{a}_n = \dfrac{v^2}{\rho}$

13.7 EOM: Cylindrical Coordinates



$$\Sigma F = m\vec{a}$$

$$\Sigma F_r \vec{u}_r + \Sigma F_\theta \vec{u}_\theta + \Sigma F_z \vec{u}_z = m\left(a_r \vec{u}_r + a_\theta \vec{u}_\theta + a_z \vec{u}_z\right)$$

$$\Sigma \vec{F}_r = m\vec{a}_r$$

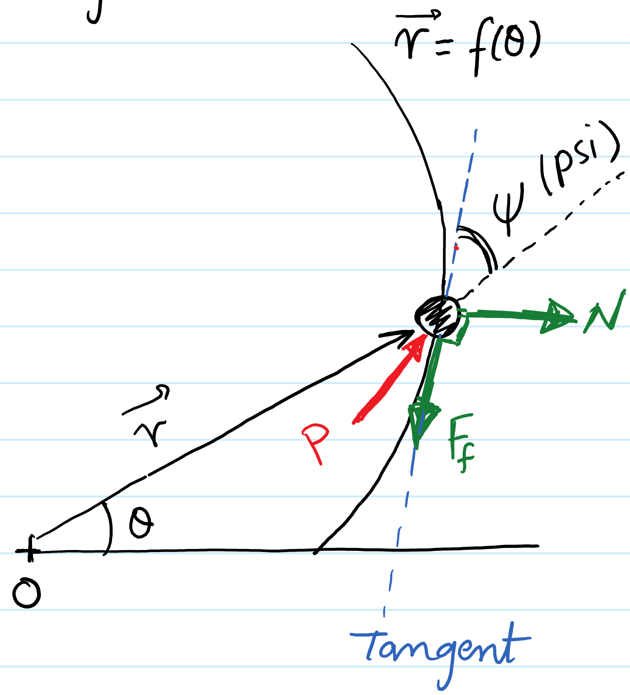
$$\Sigma F_\theta = m\vec{a}_\theta$$

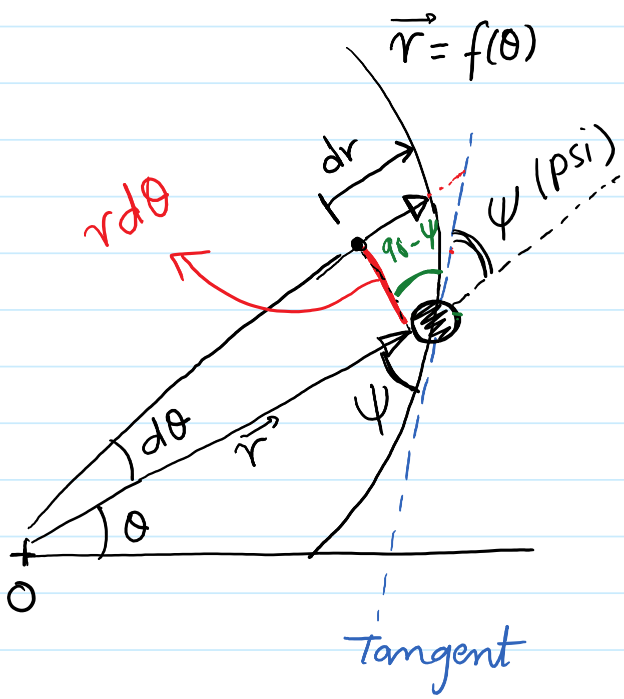
$$\Sigma F_z = m\vec{a}_z$$

<u>Note:</u> Curvilinear motion:

* Normal force is perpendicular to the tangent of the curve

* Friction force is along the tangent in the opposite direction of the motion





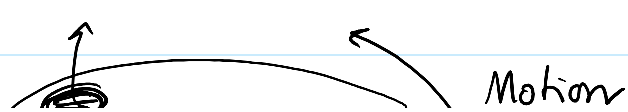
$$\tan(90-\psi) = \frac{dr}{r d\theta}$$

$$\therefore \tan\psi = \frac{r d\theta}{dr}$$

$$\boxed{\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)}}$$

<u>Example</u>: 13.7

disk D released from rest



$m_D = 3\,kg$

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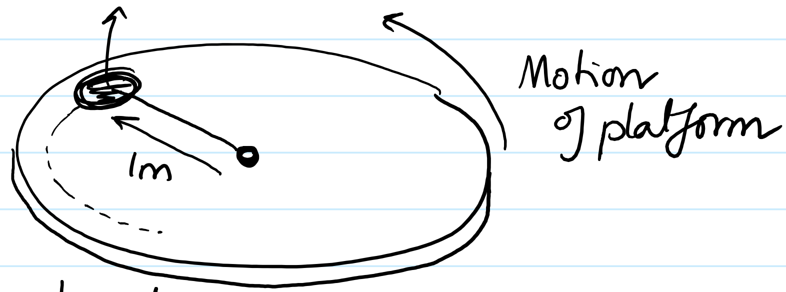


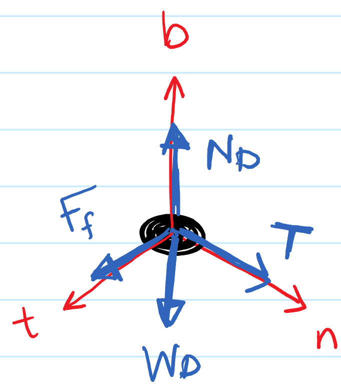
$v_{D_1} = 0$

time to break the chord

$T_{max} = 100\ N$

$\mu_k = 0.1$

FBD



$\Sigma F_t = f_k = m a_t$

$$\boxed{0.1\,N_D = 3\,a_t}$$

$\Sigma F_n = T = m a_n$

$$\boxed{100 = 3\,a_n}$$

$100 = 3\left(\frac{v^2}{\rho}\right)$

$\frac{100}{3} = v^2$

$$\boxed{v = 5.77\ m/s}$$

$\Sigma F_b = N_D - W_D = 0$

$$\boxed{N_D = 29.43\ N}$$

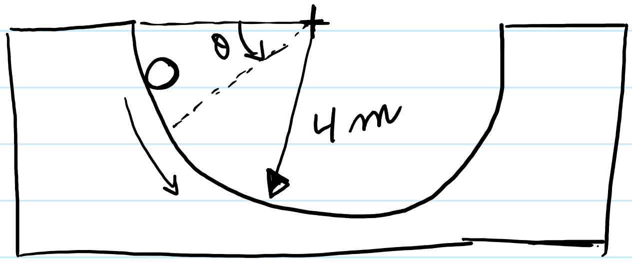
$$\boxed{a_t = 0.981\ m/s^2}$$

a_t is constant (F_t constant)

$v_{cr} = v_0 + at$

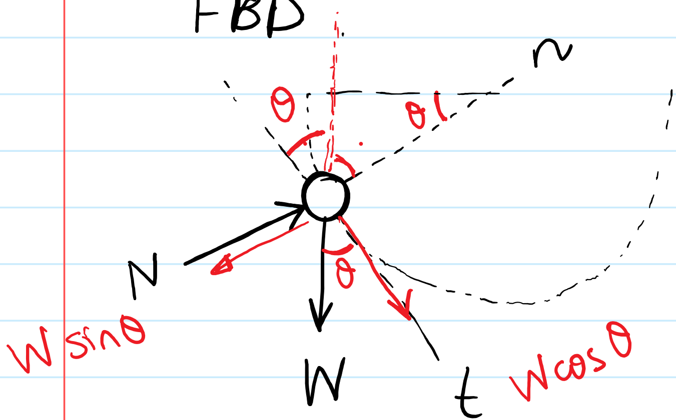
$5.77 = 0 + 0.981\,t \Rightarrow \boxed{t = 5.89\ s}$

Example 13.9



$m = 60$ kg
starts from rest
@ $\theta = 0$
Find N @ $\theta = 60°$

FBD



$$\Sigma F_n = N - W\sin\theta = m a_n$$

$$N - W\sin\theta = m \frac{v^2}{\rho}$$

$\therefore$ we need v @ $\theta = 60$

$$\Sigma F_t = W\cos\theta = m a_t$$

$$a_t = \frac{W}{m}\cos\theta$$

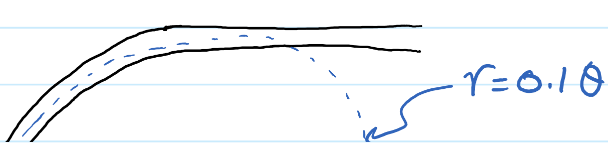
$$\int_0^v v\,dv = \int_0^{60} a\,ds \qquad ds = r\,d\theta = 4\,d\theta$$

$$\frac{v^2}{2} = \int_0^{60} 4g \cos\theta \, d\theta = 4g \sin\theta \Big|_0^{60}$$

$$\Rightarrow v^2 = 67.97 \ m^2/s^2$$

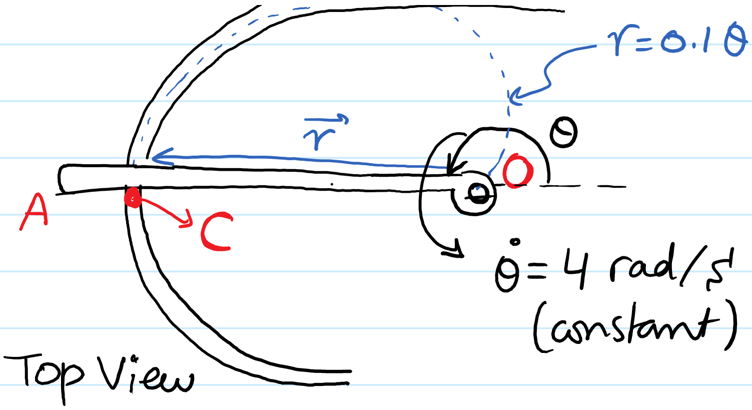
then $N = 1529.23$ N

Example 13.12

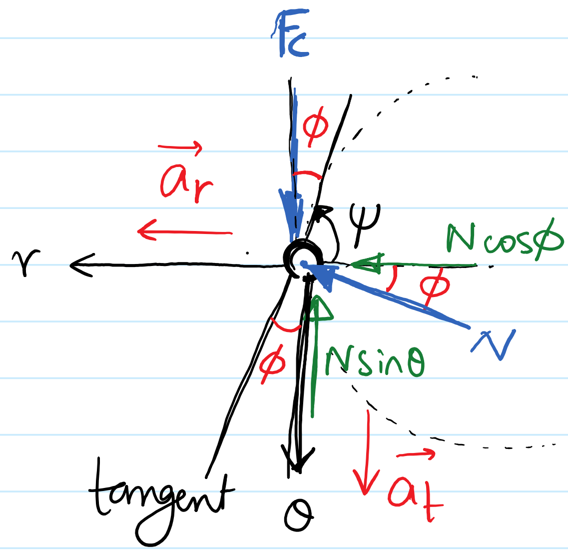


$m_C = 0.5$ kg

Find F @ $\theta = \pi$ rad.



Find F @ $\theta = \pi$ rad.



W inside the paper (no interest)

$$\psi = \tan^{-1}\frac{r}{\frac{dr}{d\theta}} = \tan^{-1}\theta$$

$$\psi = \tan^{-1}\pi = 72.3^\circ$$

$$\phi = 90 - \psi = 17.7^\circ$$

$$\Sigma F_r = N\cos\phi = m\,a_r$$

$$\Sigma F_\theta = F_c - N\sin\phi = m\,a_t$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$r = 0.1\pi$ m $\quad \theta = \pi$ rad
$\dot{r} = 0.1$ m/s $\quad \dot{\theta} = 4$ rad/s
$\ddot{r} = 0$ m/s^2 $\quad \ddot{\theta} = 0$ rad/s^2

$$a_r = -5.03 \text{ m/s}^2$$
$$a_\theta = 3.2 \text{ m/s}^2$$

$$F_c = 0.8 \text{ N}$$

$$N = -2.64 \text{ N}$$

Notes : rolls freely / rotates freely ⇒ no friction
rolls without slipping

rolls with slipping / slips / skids ⇒ friction