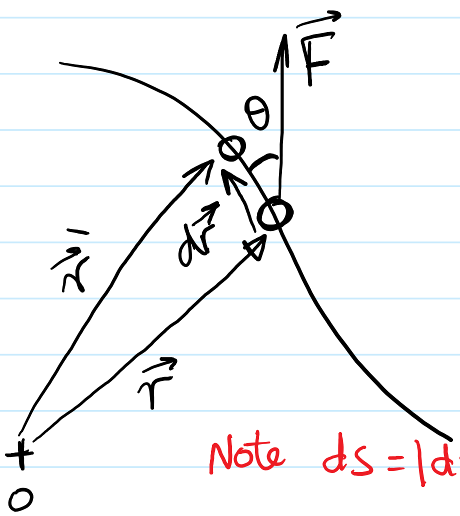
Chapter 14
Thursday, July 7, 2016 4:31 PM

Kinetics of a Particle: Work and Energy

14.1 The work of a Force:

A Force F does work on a particle only when the particle undergoes a displacement in the direction of the force.



Note $ds = |dr|$

$dU = |F||dr|\cos\theta$
$= |F|\cos\theta \, ds$ → scalar quantity

$dU = \vec{F} \cdot d\vec{r}$ [Joules] [lb-ft]

dot product

if $0° \leq \theta < 90°$ ⇒ work is +ve
(force component + disp. have the same sense)

if $90° < \theta \leq 180°$ ⇒ work is −ve
(force component + disp. have opposite sense)

if $\theta = 90°$ ⇒ No work is done.
or $dr = 0$

* Work of a Variable Force

$$U = \int^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int^{s_2} F\cos\theta \, ds$$

$$U = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int_{s_1} F\cos\theta \, ds$$

Area under $F\cos\theta$ _ s graph.

* Work of a weight

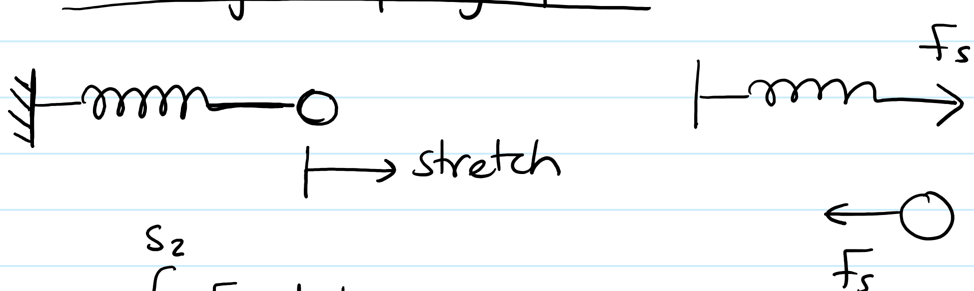
$$U = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int_{\vec{r}_1}^{\vec{r}_2} (-W\hat{j}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int_{\vec{r}_1}^{\vec{r}_2} -W dy = \int_{y_1}^{y_2} -W dy = -W(y_2 - y_1)$$

$$\boxed{U = -W \Delta y}$$

Note : $y_2 > y_1$ ↑ work −ve
$y_2 \leqslant y_1$ ↓ work +ve

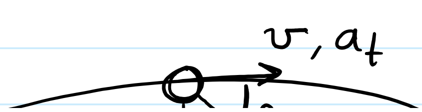
* Work of a spring force

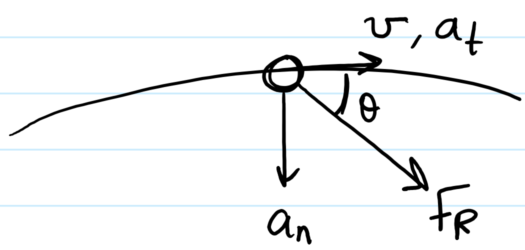


$$U = \int_{s_1}^{s_2} -F_s \, ds'$$

$$= \int_{s_1}^{s_2} -k s' \, ds' = -\frac{k s^2}{2} \Big|_{s_1}^{s_2} = -\frac{1}{2} k (s_2^2 - s_1^2)$$

14.2 Principle of Work & Energy.





$(a\,ds = v\,dv)$

$$F_t = F_R\cos\theta \Rightarrow F_t = ma_t$$

$$F_R\cos\theta = mv\frac{dv}{ds}$$

$$\int_{s_1}^{s_2} F_R\cos\theta\, ds = \int_{v_1}^{v_2} mv\, dv$$

$$\Sigma U_{1-2} = \frac{1}{2}mv^2 \Big|_{v_1}^{v_2}$$

[Joules]

Sum of the work of all forces acting on the body as it moves from $1 \rightarrow 2$

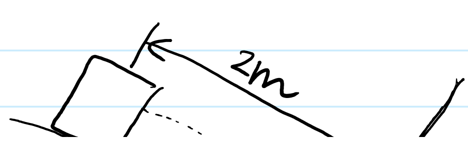
final and initial kinetic energy.

$$T = \frac{1}{2}mv^2$$

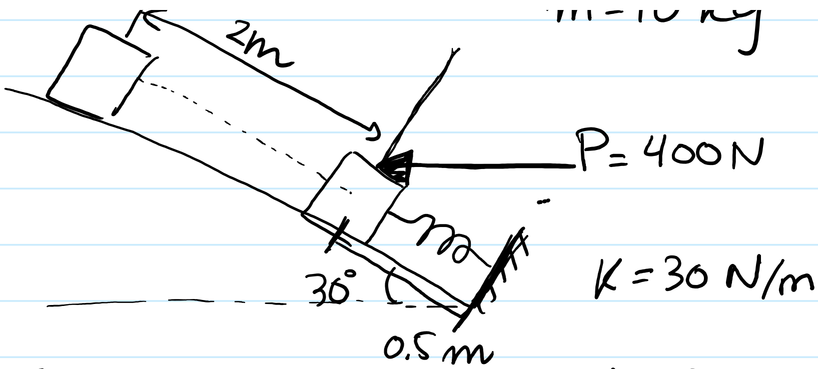
$$\Sigma U_{1\rightarrow 2} = T_2 - T_1$$

$$T_1 + \Sigma U_{1\rightarrow 2} = T_2$$

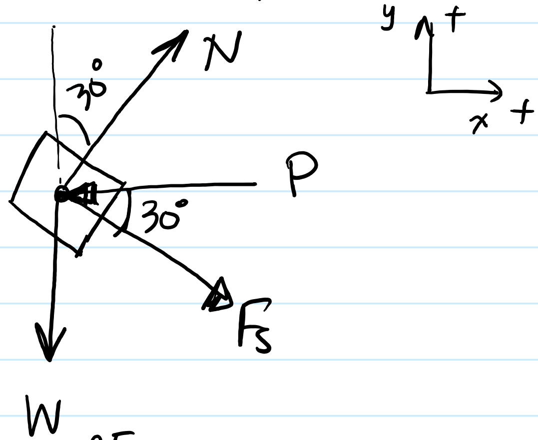
example 14.1



$m = 10\text{ kg}$



Find total work from all forces :



P is constant $\Rightarrow \int_{0.5}^{2.5} P\cos\theta \, ds$

$= 400 \cos 30 \int_{0.5}^{2.5} ds = 400 \cos 30 (2.5 - 0.5)$

$= 692.8 \text{ J}$

$F_s \Rightarrow U_s = -\frac{1}{2} k (s_2^2 - s_1^2)$

$= -\frac{1}{2}(30)(2.5^2 - 0.5^2) = -90 \text{ J}$

$W \Rightarrow U_W = -W \Delta y$

$y_1 = 0.5 \sin 30$

$y_2 = 2.5 \sin 30$

$U_W = -10 \times 9.81 (2) \sin 30 = -98.1 \text{ J}$

$U_W = -10 \times 9.81\,(2)\sin 30 = -98.1\ J$

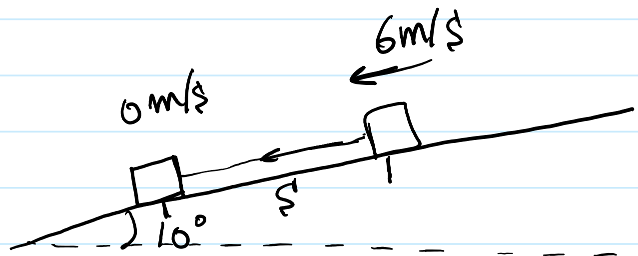
$U_N = 0$ $\quad N \perp$ displacement

$U_T = U_P + U_S + U_W + U_N = 505\ J.$

14.3 Principle of work & Energy for a system of particles

$$\Sigma T_1 + \Sigma U_{1\to 2} = \Sigma T_2$$

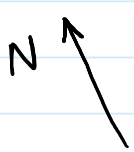
<u>example 14.2</u>



$W = 17.5 \times 10^3\ N$

slams on brake

$\mu_k = 0.5$



$N = W\cos 10^\circ = 17.5 \times 10^3 \times \cos 10 = 17.2 \times 10^3\ N$

$F_k = 8.6 \times 10^3\ N$

$$T_1 + \Sigma U_{1\to2} = T_2$$

$$\frac{1}{2}mv_1^2 + U_w + U_N + U_F = \frac{1}{2}mv_2^2$$

$$\frac{1}{2}m(6)^2 + U_W + 0 + U_F = 0$$

$$U_W = -W\Delta y = -W(-S\sin 10^\circ)$$

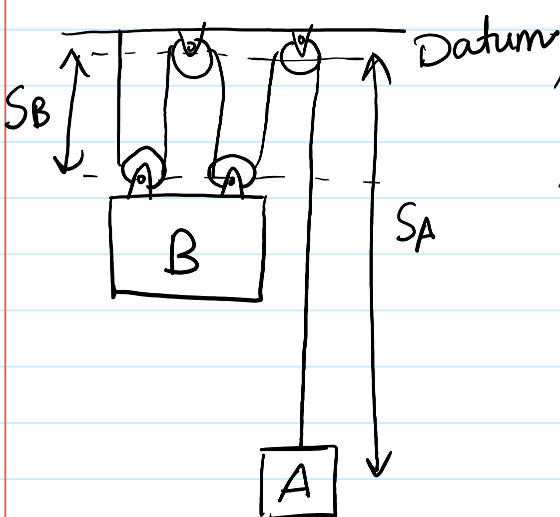
$$= WS' \sin 10^\circ$$

$$U_F = -F_k S'$$

$$\frac{1}{2}\frac{W}{g}(6)^2 = W\sin 10^\circ(S') - F_k(S) = 0$$

$$\boxed{S' = 5.75\ m}$$

example 14.6



$m_A = 10\ kg$

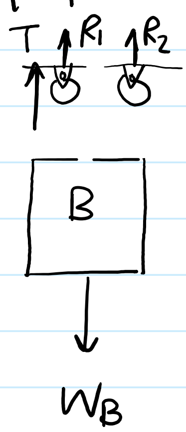
$m_B = 100\ kg$

S_B from when it is released from rest till $v_B = 2\ m/s$

$$\Sigma T_1 + \Sigma U_{1\to2} = \Sigma T_2$$

T ↑R1 ↑R2

$\Sigma T_1 + \Sigma U_{1\to 2} = \Sigma T_2$



Tension & Reactions do not do work (no displacement)

$$\frac{1}{2} m_A (v_A^2)_1 + \frac{1}{2} m_B (v_B^2)_1 + W_B (s_{B_2} - s_{B_1}) + W_A (s_{A_2} - s_{A_1})$$
$$= \frac{1}{2} m_A (v_A^2)_2 + \frac{1}{2} m_B (v_B^2)_2$$

$$4 s_B + s_A = L$$

$$\left.\begin{aligned} 4 s_{B_1} + s_{A_1} &= L \\ 4 s_{B_2} + s_{A_2} &= L \end{aligned}\right\} \quad 4(s_{B_2} - s_{B_1}) + (s_{A_2} - s_{A_1}) = 0$$

$$4 \Delta s_B + \Delta s_A = 0$$

$$4 v_B + v_A = 0$$

$$0 + 0 + 981 \, \Delta s_B + 98.1 \, \underset{(-4\Delta s_B)}{\Delta s_A} = \frac{1}{2}(10)\left((-4 v_B)^2\right)_2$$
$$+ \frac{1}{2}(100)(v_B^2)_2$$

$$\Delta s_B = 0.883 \text{ m} \downarrow$$

14.4 Power & Efficiency

$P = dU$ (amount of work done per unit

dt   of time)

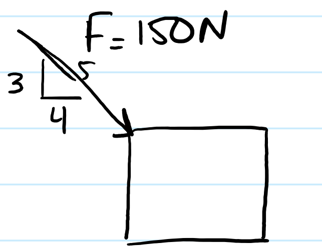
$U = \vec{F} \cdot d\vec{r}$

$$\frac{dU}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v} \quad \text{(scalar)}$$

[J/s , Watts]

$$\epsilon = \frac{\text{power output}}{\text{power input}} < 1$$

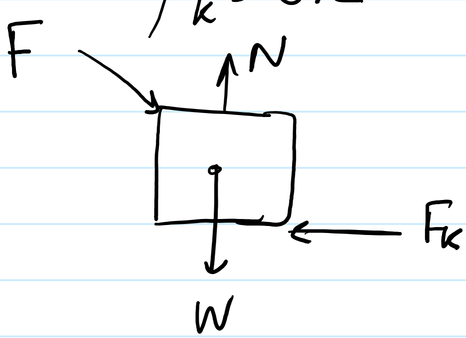
example 14.7

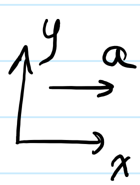


$m = 50\ kg$, initially @ rest

Find power @ $t = 4\ s$

$\mu_k = 0.2$





$$\Sigma F_x = F\left(\frac{4}{5}\right) - \mu_k N = m a_x$$

$$\Sigma F_y = N - W - F\left(\frac{3}{5}\right) = 0$$

$N = 580.5\ N$

$a_x = 0.078\ m/s^2$

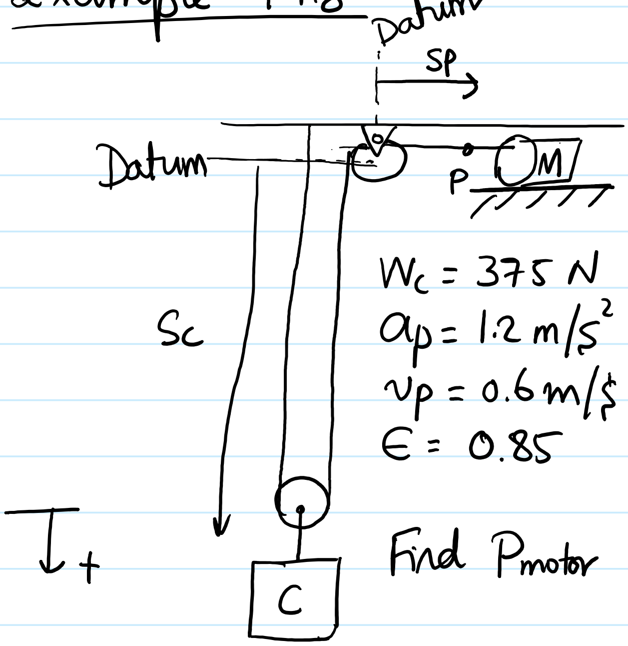
Forces are constant → a_c

$v_2 = v_1 + at$

$v_2 = 0 + 0.078(4) = 0.312\ m/s$

$P = F \cdot v = 150\left(\frac{4}{5}\right)(0.312) = 37.4\ W$

<u>example 14.8</u>



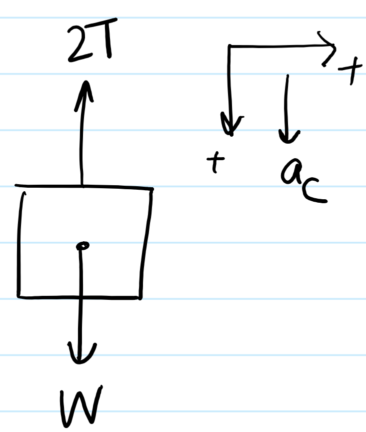
$W_c = 375\ N$

$a_p = 1.2\ m/s^2$

$v_p = 0.6\ m/s$

$\epsilon = 0.85$

Find P_{motor}



$\Sigma F_y = W_c - 2T = m_c a_c$

$2S_c + S_p = L$

$2v_c + v_p = 0$

$2a_c + a_p = 0 \qquad a_c = -0.6\ m/s^2$

$\boxed{T = 199\ N}$

$\boxed{T = 199\ N}$

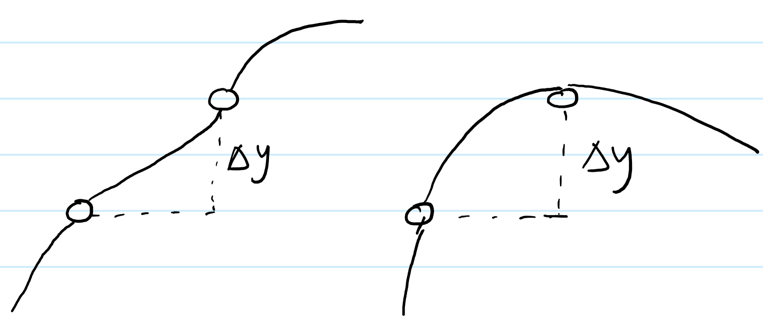
power output $P = \vec{T} \cdot \vec{v}_p = 199(0.6) = 119.4W$

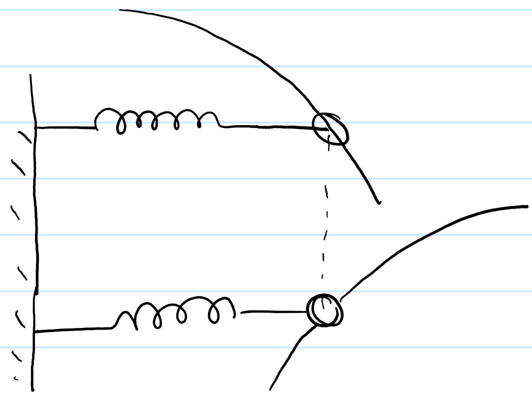
$\epsilon = \frac{P_{out}}{P_{in}} \Rightarrow P_{in} = \frac{P_{out}}{0.85} = 140.5W$

14.5 Conservative Forces & Potential energy:

When the work done by a force in moving a particle from one point to another is independent of the path followed by the particle, then this force is a conservative force. ex:

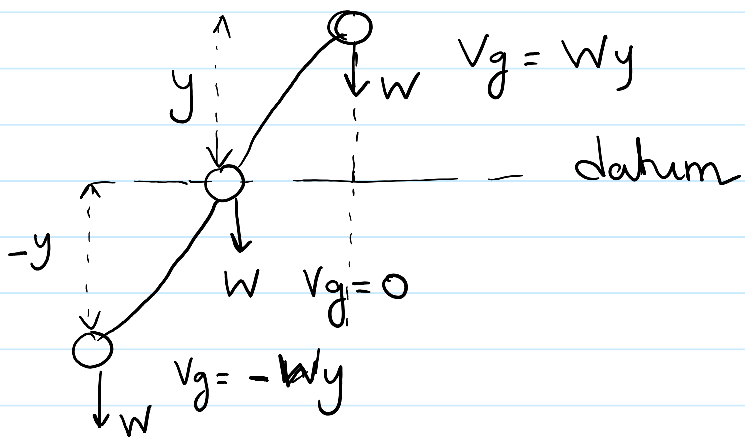
1) Weight → depends only on vertical displacement.
2) Spring force → depends only on spring deflection





Potential Energy: a measure of the work done by a conservative force. when it moves the particle from a given position to a reference datum

1) Gravitational Potential Energy $V_g = Wy$



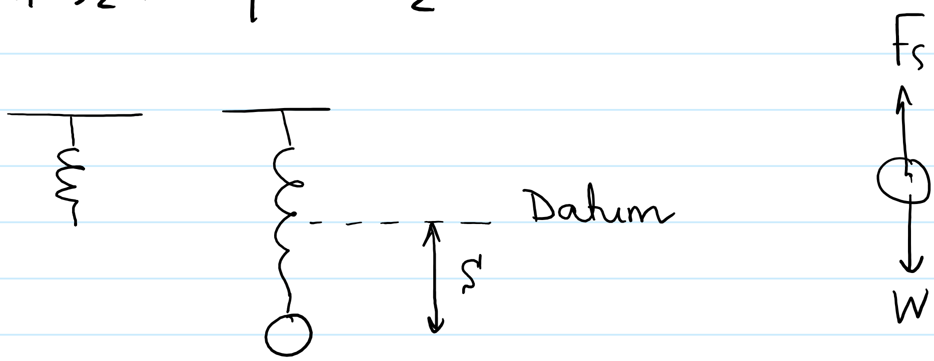
* Object above datum → V_g +ve
Object below datum → V_g -ve

2) Elastic Potential Energy $V_e = \frac{1}{2} k s^2$

* Always V_e +ve

Potential function : $V = V_g + V_e$

$U_{1\to 2} = V_1 - V_2$



$V_g = -Ws$ s is below the datum.

$$V_e = \frac{1}{2} k s^2$$

$$V = -Ws + \frac{1}{2} k s^2$$

if the particle moves from $s_1 \rightarrow s_2$

$$\left.\begin{aligned} V_1 &= -Ws_1 + \frac{1}{2} k s_1^2 \\ V_2 &= -Ws_2 + \frac{1}{2} k s_2^2 \end{aligned}\right\} \quad U_{1\rightarrow 2} = V_1 - V_2$$

$$U_{1\rightarrow 2} = -Ws_1 + \frac{1}{2}ks_1^2 + Ws_2 - \frac{1}{2} k s_2^2$$

$$= \underbrace{W(s_2 - s_1)}_{U_W} \underbrace{-\frac{1}{2} k (s_2^2 - s_1^2)}_{U_s}$$

* How do we know that a force is conservative?

it should satisfy $\boxed{\vec{F} = -\nabla V}$

$$\nabla \text{ (del)} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

example $V_g = Wy$

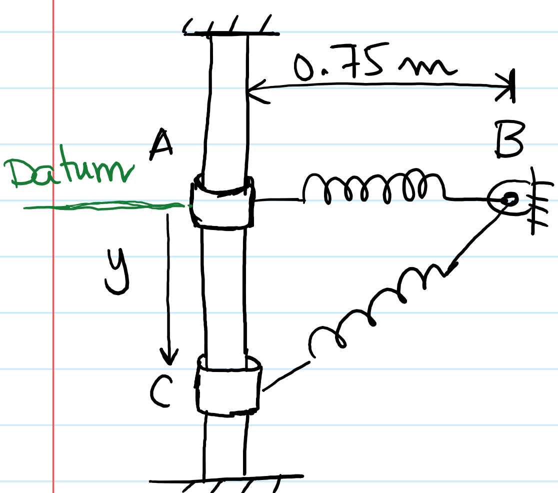
$$\left.\begin{aligned} \frac{\partial}{\partial x} V_g &= 0 \\ \frac{\partial}{\partial y} V_g &= \frac{\partial}{\partial y} Wy = W \\ \frac{\partial}{\partial z} V_g &= 0 \end{aligned}\right\} \quad \begin{aligned} \vec{F} &\overset{?}{=} -(0\hat{i} + W\hat{j} + 0\hat{k}) \\ \vec{F} &= -W\hat{j} \quad \checkmark \end{aligned}$$

14.6 Conservation of energy:

If <u>only</u> conservative forces are applied to a body ⇒
Work & Energy → Conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

example 14.2.



$m = 2$ kg, $k = 3$ N/m
@ A spring is unstretched.

@ $y = 1$ m Find v_C if

1) $v_A = 0$
2) $v_A = 2$ m/s ↑

Part (1) $v_A = 0$

Find spring elongation (s)

$$s_{BC} = \sqrt{0.75^2 + 1^2} = 1.25 \text{ m}$$

$$s = 1.25 - 0.75 = 0.5 \text{ m}$$

$$V_1 + T_1 = V_2 + T_2$$

$$V_{g_1} + V_{e_1} + T_1 = V_{g_2} + V_{e_2} + T_2$$

$$0 + 0 + 0 = -mgy + \frac{1}{2} k s^2 + \frac{1}{2} m v_c^2$$

$$\boxed{v_c = 4.39 \text{ m/s}}$$

$$v_c = 4.39 \text{ m/s}$$

Part 2

$$0 + 0 + \frac{1}{2} m v_A^2 = -mgy + \frac{1}{2} k s^2 + \frac{1}{2} m v_c^2$$

$$v_c = 4.82 \text{ m/s}$$