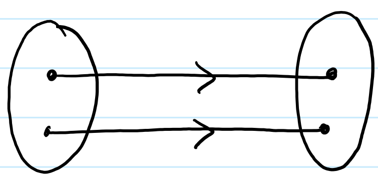
Chapter 16

Tuesday, July 19, 2016 10:17 AM

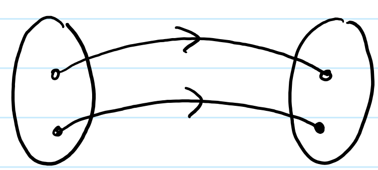
$\boxed{16.1}$ Planar Rigid Body motion.

1) Translation:

- Rectilinear Translation: paths of motion for any two points on the body are parallel lines.
- Curvilinear Translation: paths of motion for any two points are along curved lines which are equidistant.



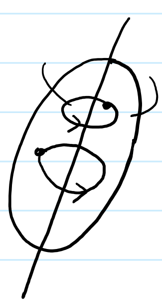
Path of Rectilinear Translation



Path of curvilinear translation.

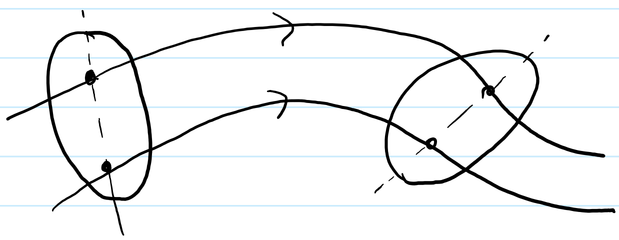
2) Rotation about a fixed axis:

All particles on the body except those lying on the axis of rotation move along circular paths.



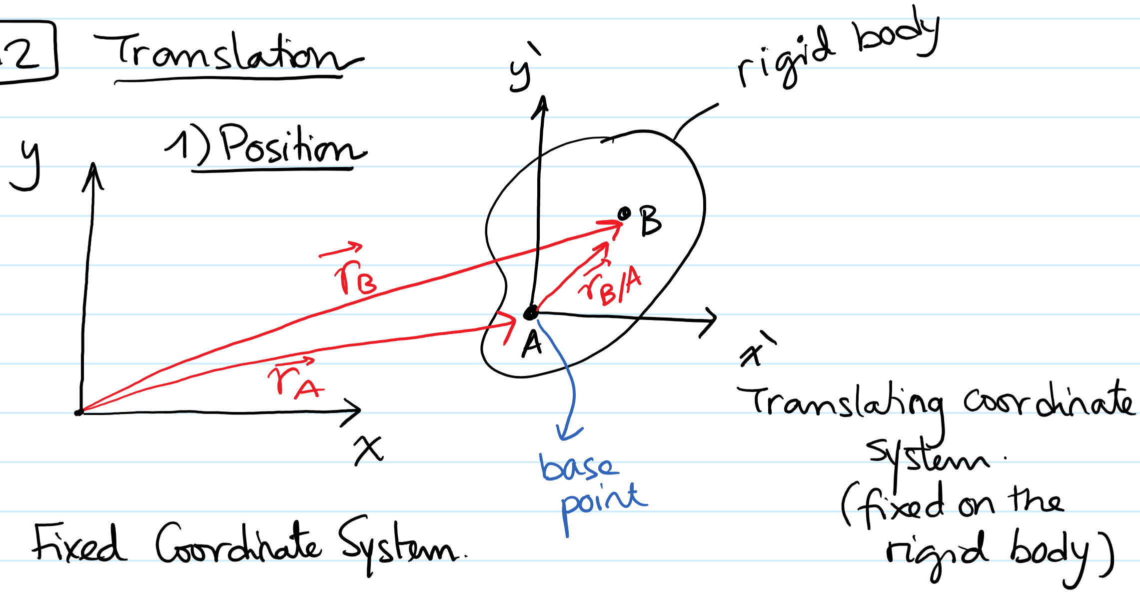
3) General plane motion: Combination of translation & Rotation → translation within the reference plane, rotation about an axis perpendicular

' to the reference plane.



16.2 Translation

1) Position



Fixed Coordinate System.

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

relative position (r_B with respect to A)

2) Velocity

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_{B/A}}{dt}$$

→ zero, A, B are fixed on the body

$$\vec{v}_B = \vec{v}_A$$

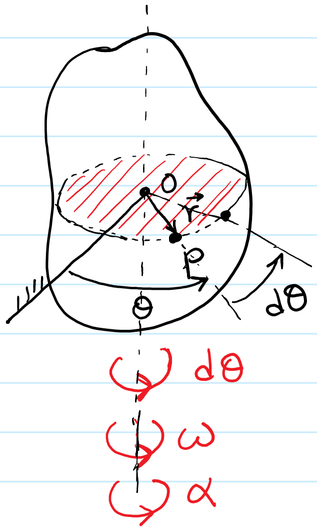
3) Acceleration

$$\vec{a}_B = \vec{a}_A$$

* When no rotation occurs, all points on the

* When no rotation occurs, all points on the rigid body move with the same velocity & Acceleration.

$\boxed{16.3}$ Rotation about a fixed axis



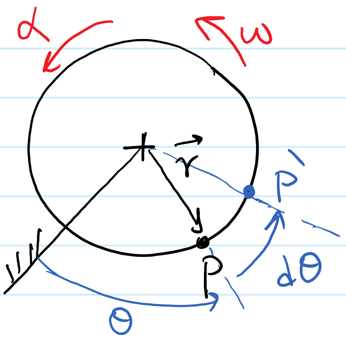
1) Angular Position (θ)
measured from a reference line to $\vec{r}$, $\vec{r}$ points from origin to P.

2) Angular displacement ($d\vec{\theta}$)

$|d\vec{\theta}| = d\theta$ [degrees, rad, rev]

1 revolution = 2π rad
1 rad = $57.3°$

Direction $\Rightarrow$ use right hand rule.



Top View

CCW along axis of rotation

3) Angular velocity ($\vec{\omega}$)

$+$ $\omega = \dfrac{d\theta}{dt}$ (same direction as $d\vec{\theta}$)

[rad/s]

3) Angular Acceleration ($\vec{\alpha}$)

$+$ $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$ (same direction as $d\theta, \omega$) (motion)

[rad/s²]

sense of direction depends on

$[rad/s^2]$ sense of direction depends on whether ω is ↑ or ↓

Eliminating $dt \Rightarrow \alpha\, d\theta = \omega\, d\omega$

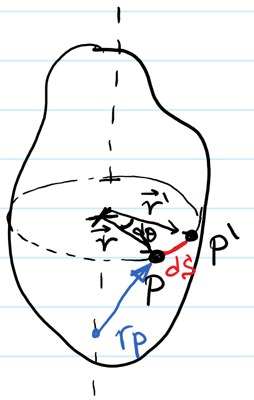
* Constant Angular Acceleration: $\alpha = \alpha_c$

$+\circlearrowleft \quad \omega = \omega_0 + \alpha_c t$

$+\circlearrowleft \quad \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$

$+\circlearrowleft \quad \omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

* Motion of Point "P", relation between linear & Angular parameters:



1) Position & Displacement

$$\boxed{ds = r\, d\theta}$$

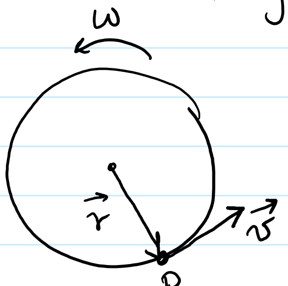
$$\boxed{ds = dr}$$

2) Velocity

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\dot{r} = v = r\omega \quad \text{(magnitude)}$$

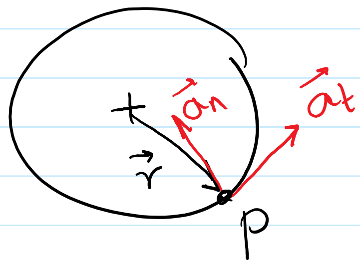
in general $\Rightarrow \vec{v} = \vec{\omega} \times \vec{r}_p$



$\vec{r}_p$ any vector pointing from a point on the axis of rotation to point (P)

$\vec{r}$ is a special case of $\vec{r}_p$ (easiest to use)

(easiest to use)



$$a_t = \frac{dv}{dt} = \frac{d(r\omega)}{dt} = r\frac{d\omega}{dt}$$

$$\boxed{a_t = r\alpha} \quad \text{(magnitude)}$$

$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

$$a_n = \frac{v^2}{\rho} = \frac{(r\omega)^2}{r}$$

$$\boxed{a_n = r\omega^2} \quad \text{(magnitude)}$$

Vector Form:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

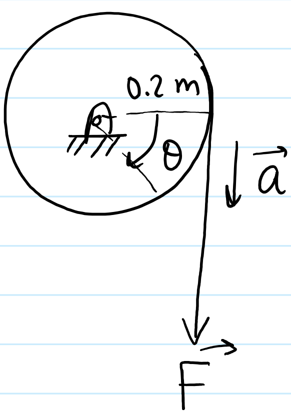
$$= \frac{d}{dt}\vec{\omega} \times \vec{r} + \vec{\omega} \times \frac{d}{dt}\vec{r}$$

$$\vec{\alpha} \times \vec{r} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})}_{-\omega^2\vec{r}}$$

In vector form: $\vec{a} = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r}$

$$|\vec{a}| = \sqrt{a_t^2 + a_n^2}$$

example 16.1



at rest @ $\theta = 0°$

$F \rightarrow a = 4t \ m/s^2$

Find $\omega(t)$
$\theta(t)$

$$(a_p)_t = 4t = \alpha r$$

$$4t = \alpha\, 0.2$$

$$\boxed{\alpha(t) = 20t}$$

$$\alpha = \frac{d\omega}{dt} \Rightarrow d\omega = \alpha\, dt$$

$$\int_0^{\omega} d\omega = \int_0^{t} 20t\, dt$$

$$\omega = \frac{20t^2}{2}\Big|_0^t$$

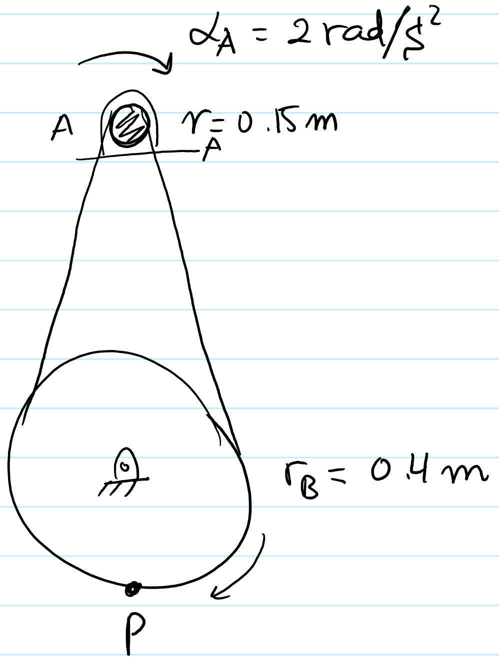
$$\boxed{\omega = 10t^2}$$

$$\omega = \frac{d\theta}{dt} \Rightarrow \int_0^\theta d\theta = \int_0^t \omega\, dt$$

$$\theta = \int_0^t 10t^2\, dt$$

$$\boxed{\theta = \frac{10}{3}t^3\Big|_0^t = \frac{10t^3}{3}}$$

example 16.2



Find v_p, a_p after motor turns 2 revolutions

$\theta_A = 2(2\pi)$ rad.

$$\boxed{\theta_A = 12.57 \text{ rad}}$$

$\alpha_A \rightarrow$ constant

$$\omega_A^2 = \omega_{0A}^2 + 2\alpha_A(\theta - \theta_0)$$

$$\omega_A^2 = 0 + 2(2)(12.57 - 0)$$

$$\omega_A = 7.09 \text{ rad/s}$$

* Belt has same speed & a_t for pulley & wheel

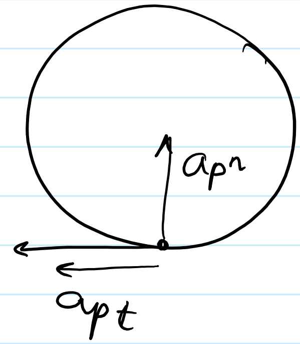
$$v = \omega_A r_A = \omega_B r_B$$

$$\boxed{\omega_B = 2.659 \text{ rad/s}}$$

$$\alpha_A r_A = \alpha_B r_B$$

$\alpha_A r_A = \alpha_B r_B$

$$\boxed{\alpha_B = 0.75 \text{ rad}/s^2}$$



thus $v_p = \omega_B r_B = 1.06 \, m/s$

$(a_p)_t = \alpha_B r_B = 0.3 \, m/s^2$

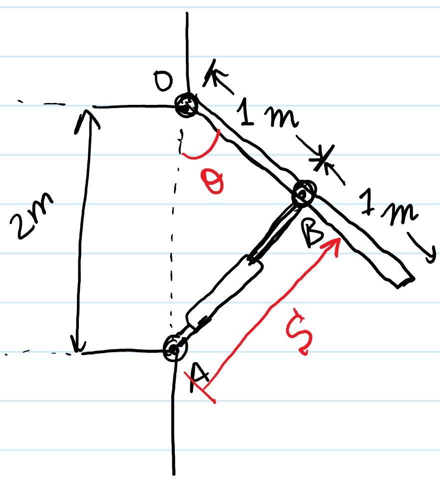
$(a_p)_n = \omega_B^2 r_B = 2.827 \, m/s^2$

$a_p = \sqrt{(a_p)_t^2 + (a_p)_n^2} = 2.84 \, m/s^2$

$\boxed{16.4}$ Absolute Motion Analysis:

1) Specify a translational coordinate s
2) " " rotational coordinate θ
3) Relate s, θ through geometry.

example 16.5



hydraulic cylinder AB

Cylinder extends at a constant rate $0.5 \, m/s$

find ω, α at $\theta = 30°$

Law of cosines: (OAB)

$$(AB)^2 = (OA)^2 + (OB)^2 - 2(OA)(OB)\cos\theta$$

$$s^2 = (2)^2 + (1)^2 - 2(2)(1)\cos\theta$$

$$s^2 = (2)^2 + (1)^2 - 2(2)(1)\cos\theta$$

$$\boxed{s^2 = 5 - 4\cos\theta}$$

@ $\theta = 30^\circ \Rightarrow \boxed{s = 1.239\ m}$

$$2s\frac{ds}{dt} = 0 - 4(-\sin\theta)\frac{d\theta}{dt} \Rightarrow s\frac{ds}{dt} = 2\sin\theta\frac{d\theta}{dt}$$

$$2s\,v_s = +4\sin\theta\,\omega$$

$$\omega = 0.62\ rad/s$$

Second time derivative

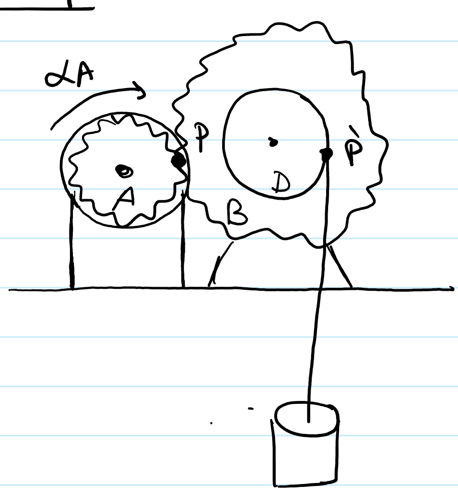
$$s\frac{d^2s}{dt^2} + \frac{ds}{dt}\frac{ds}{dt} = 2\left(\sin\theta\frac{d\omega}{dt} + \omega\cos\theta\frac{d\theta}{dt}\right)$$

constant rate

$$v_s^2 = 2\alpha\sin\theta + \omega^2\cos\theta$$

$$\boxed{\alpha = -0.415\ rad/s^2}$$ Angular Deceleration

example



$r_A = 75$ mm
$r_B = 225$ mm
$r_D = 125$ mm.

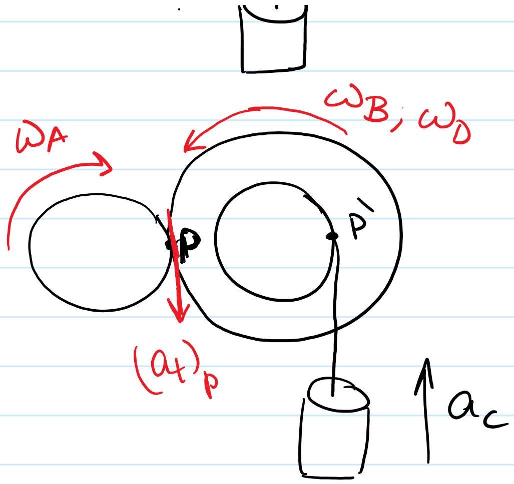
$\alpha_A = 4.5\ rad/s^2$

starting from rest.

D is rigidly attached to B

D is rigidly attached to B

Find v_C after 3 s



$$(a_t)_P = \alpha_A r_A = 4.5 \times 0.075 = 0.3375 \ m/s^2$$

$$(a_t)_P = \alpha_B r_B \Rightarrow \alpha_B = \frac{0.3375}{0.225} = 1.5 \ rad/s$$

$$\alpha_D = \alpha_B = 1.5 \ rad/s^2$$

$$(\vec{a}_t)_{P'} = \alpha_D r_D = 1.5 \times 0.125 = 0.1875 \ m/s^2$$

$\alpha_A, \alpha_B, \alpha_D \longrightarrow$ constants.

$$\omega_D = \omega_{D_0} + \alpha_D t$$

$$= 0 + 1.5(3) = 4.5 \ rad/s$$

$$\boxed{v_C = r_D \omega_D = 0.125 \times 4.5 = 0.5625 \ m/s}$$

$$\theta_D = \theta_{D_0} + \omega_{D_0} t + \frac{1}{2}\alpha_D t^2$$

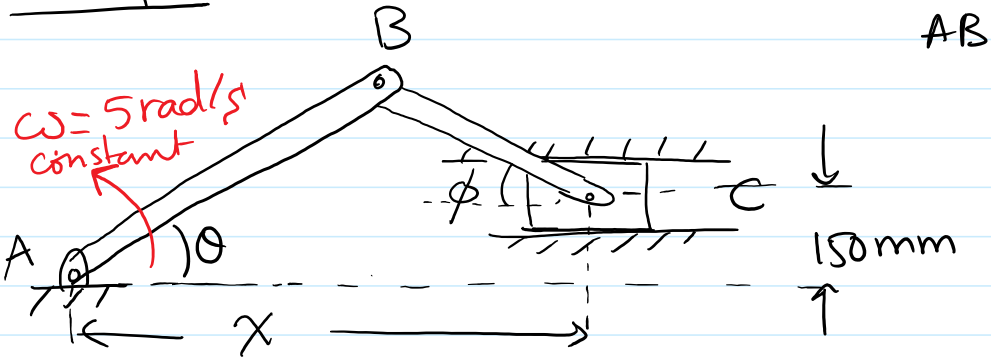
$$= 0 + 0 + \frac{1}{2}(1.5)(3)^2$$

$$\theta_D = 6.75 \ rad$$

$$S_C = \theta_D r_D = 6.75 \times 0.125 = 0.844 \ m$$

$$\boxed{S_c = 844 \text{ mm}}$$

Example



BC = 300 mm
AB = 600 mm

Find ω_{BC}

v_c

@ $\theta = 30°$

$$x = 0.6 \cos\theta + 0.3 \cos\phi \Rightarrow @\ \theta = 30° \Rightarrow x = 0.78 \text{ m}$$

$$0.6 \sin\theta = 0.3 \sin\phi + 0.150 \Rightarrow @\ \theta = 30 \Rightarrow \phi = 30°$$

$$\frac{dx}{dt} = -0.6 \sin\theta \frac{d\theta}{dt} - 0.3 \sin\phi \frac{d\phi}{dt}$$

$$0.6 \cos\theta \frac{d\theta}{dt} = 0.3 \cos\phi \frac{d\phi}{dt}$$

$$v_c = -0.6 \sin 30\, \omega_{AB} - 0.3 \sin 30\, \omega_{BC}$$

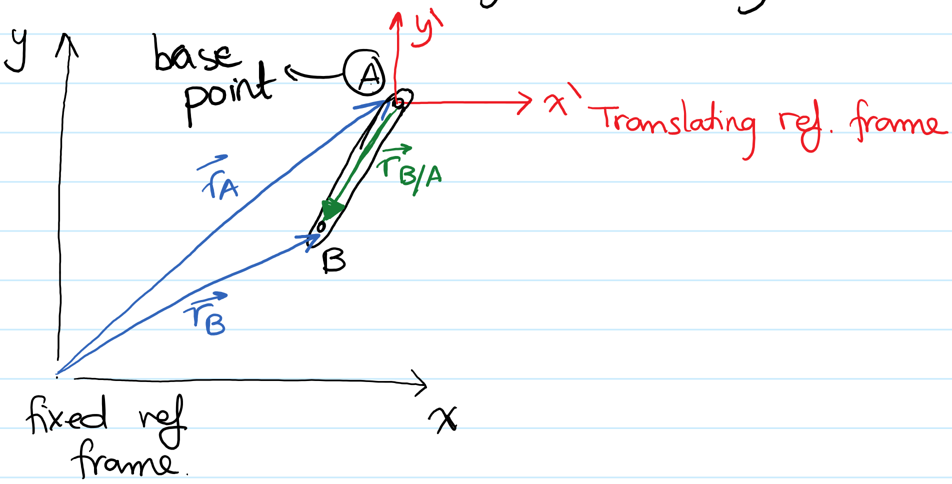
$$0.6 \cancel{\cos 30}\, \omega_{AB} = 0.3 \cancel{\cos 30}\, \omega_{BC}$$

$$\boxed{\omega_{BC} = 2\, \omega_{AB} = 10 \text{ rad/s}}$$

$$\boxed{v_c = -3 \text{ m/s}}$$

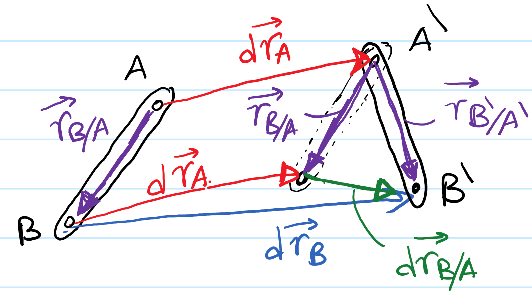
16.5 Relative Motion Analysis & Velocity

16.5 Relative Motion Analysis : Velocity



1) Position : $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

2) Displacement :



$$\underbrace{d\vec{r}_B}_{\text{Absolute Motion}} = \underbrace{d\vec{r}_A}_{\text{Translation of (A)}} + \underbrace{d\vec{r}_{B/A}}_{\text{Rotation about (A)}}$$

3) Velocity

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_{B/A}}{dt}$$

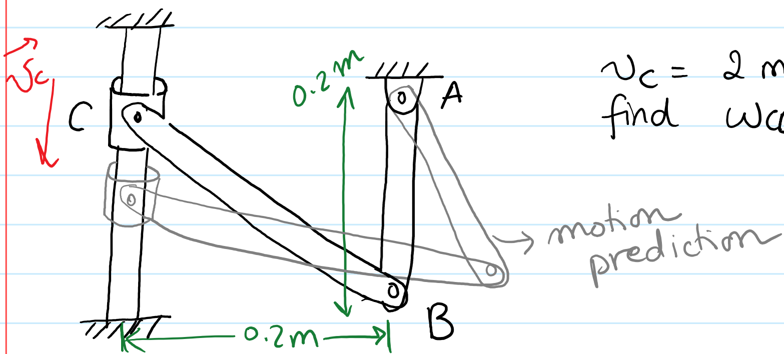
$$\vec{v}_B = \vec{v}_A + \underbrace{\vec{v}_{B/A}}_{\text{due to rotation}}$$ (in case of translation and No rotation $\vec{v}_{B/A} = 0$)

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{BA} \times \vec{r}_{B/A}$$

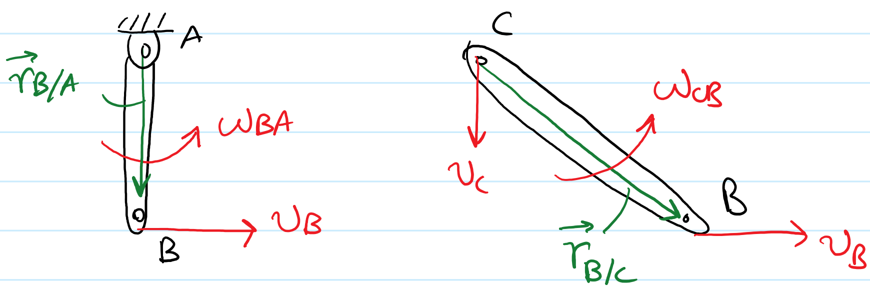
* Important Note

1) B is the point of interest
2) Choose A such that its motion is completely known.

example 16.8



$v_c = 2$ m/s ↓
find ω_{CB}



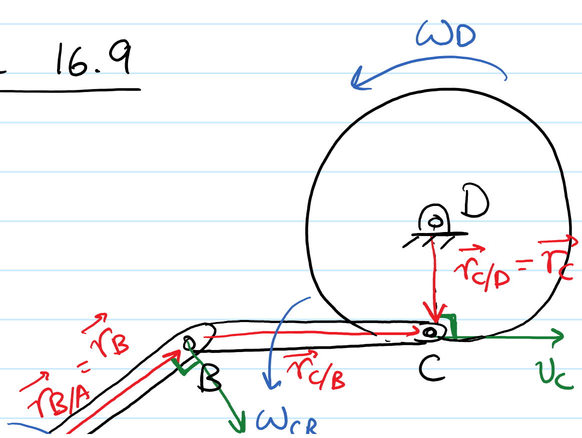
$$\vec{v}_B = \vec{v}_c + \vec{\omega}_{CB} \times \vec{r}_{B/C}$$

$$v_B \hat{i} = -2\hat{j} + \left(\omega_{CB}\hat{k} \times (0.2\hat{i} - 0.2\hat{j})\right)$$

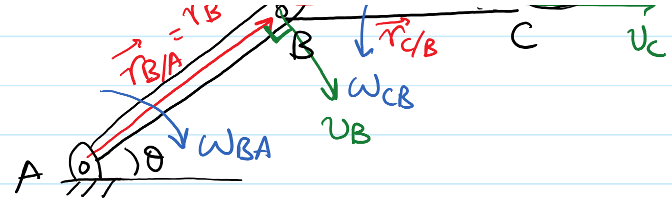
$$v_B \hat{i} = -2\hat{j} + 0.2\omega_{CB}\hat{j} + 0.2\omega_{CB}\hat{i}$$

$$\boxed{\omega_{CB} = 10 \text{ rad/s} \circlearrowleft}$$

example 16.9



AB = 0.2 m
BC = 0.2 m
$r_D = 0.1$ m
$\theta = 60°$
$\omega_{BA} = 30$ rad/s



$\omega_{BA} = 30$ rad/s

Find ω_{CB}
ω_D

* AB $\rightarrow$ rotation about fixed axis

$$\vec{v}_B = \vec{\omega}_{BA} \times \vec{r}_B$$

$$= -30\hat{k} \times (0.2\cos 60\,\hat{i} + 0.2\sin 60\,\hat{j})$$

$$\vec{v}_B = \{5.2\hat{i} - 3\hat{j}\}\ \text{m/s}$$

* BC $\rightarrow$ general planar motion

$$\vec{v}_C = \vec{v}_B + \omega_{BC} \times \vec{r}_{C/B}$$

$$v_C\hat{i} = 5.2\hat{i} - 3\hat{j} + (\omega_{BC}\hat{k} \times 0.2\hat{i})$$

$$\boxed{\omega_{BC} = 15\ \text{rad/s} \circlearrowleft}$$

$$\boxed{\vec{v}_C = 5.2\hat{i}\ \text{m/s}}$$

CD $\rightarrow$ rotation about a fixed axis

$$\vec{v}_C = \vec{\omega}_D \times \vec{r}_C$$

$$5.2\hat{i} = \omega_D\hat{k} \times -0.1\hat{j}$$

$$\boxed{\omega_D = 52\ \text{rad/s} \circlearrowleft}$$

16.6 Instantaneous Center of zero Velocity

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

Choose A such that $\vec{v}_A = 0$

$$\vec{v}_B = \vec{\omega} \times \vec{r}_{B/A}$$

* If A does not exist as a fixed point in the system
→ find A such that $\vec{v}_A = 0$ at a certain instant

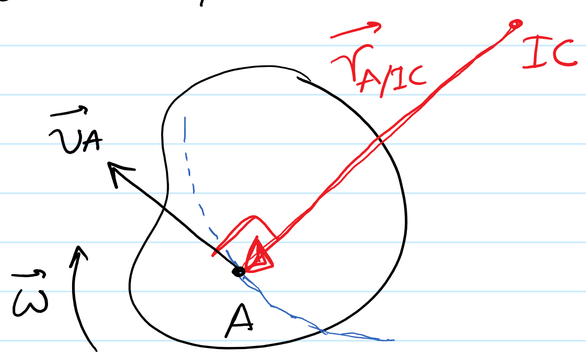
~~A~~ → IC : Instant center of zero velocity.
It lies on the instant axis of zero velocity which is ⊥ plane.

$$\vec{V}_B = \vec{v}_{IC} + \vec{\omega} \times \vec{r}_{B/IC}$$

$$\boxed{\vec{v}_B = \vec{\omega} \times \vec{r}_{B/IC}}$$ (Note $\vec{r}_{B/IC} \perp \vec{v}_B$ always)

Location of IC :

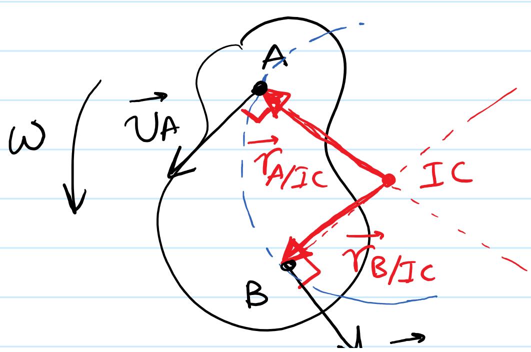
1) Given $\vec{v}_A$, $\vec{\omega}$



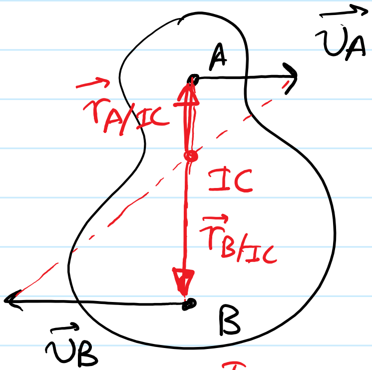
$$\vec{r}_{A/IC} \perp \vec{v}_A$$

$$|r_{A/IC}| = \frac{v_A}{\omega}$$

2) Given $\vec{v}_A$, $\vec{v}_B$ (non-parallel)



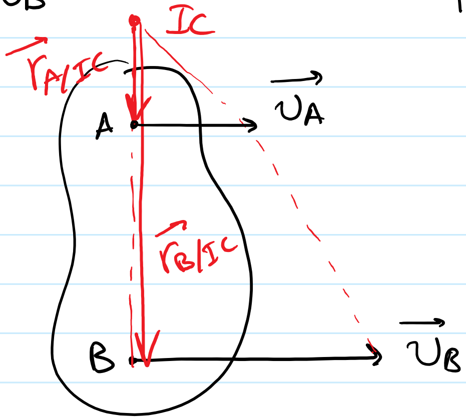
3) Given $\vec{v}_A$, $\vec{v}_B$ (parallel)



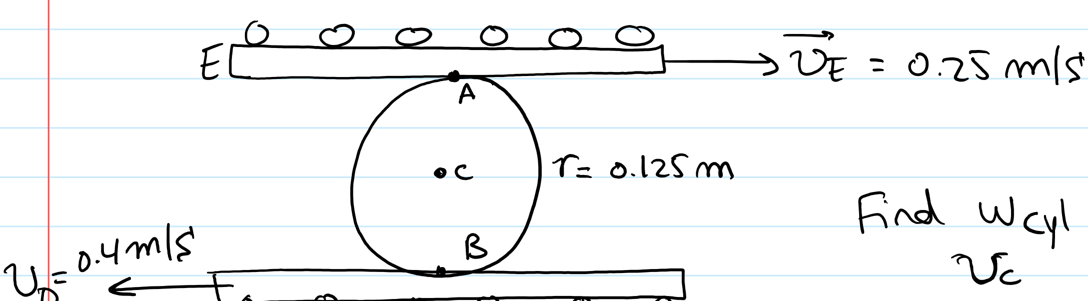
(proportional triangles)

$$|\vec{r}_{A/IC}| = \frac{v_A}{\omega}$$

$$|\vec{r}_{B/IC}| = \frac{v_B}{\omega}.$$

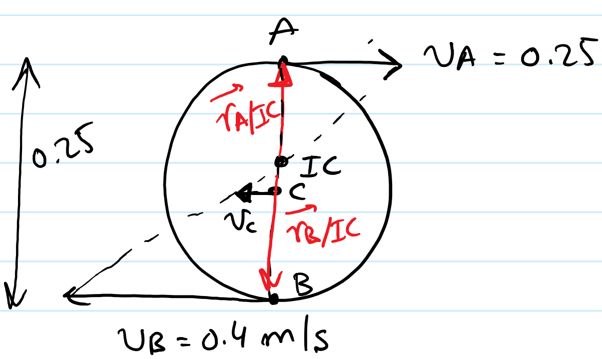


example 16.12



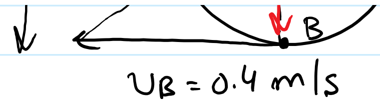
Find ω_{cyl}

v_c



$$\frac{v_A}{r_{A/IC}} = \frac{v_B}{r_{B/IC}}$$

$$r_{B/IC} = 0.25 - r_{A/IC}$$



$$r_{B/IC} = 0.25 - r_{A/IC}$$

$$r_{B/IC} = 0.1538 \text{ m}$$

$$v_B = \omega \, r_{B/IC}$$

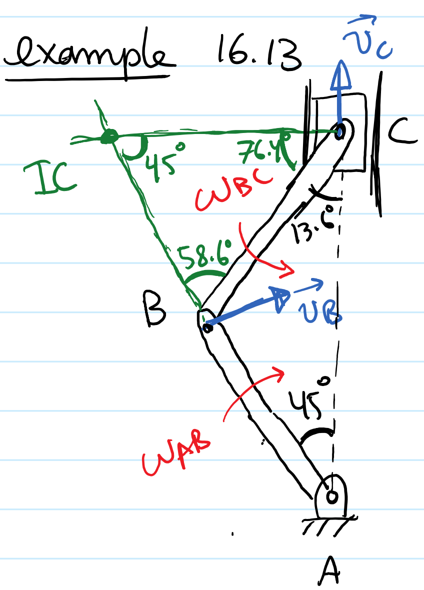
$$0.4 = \omega \, (0.1538) \Rightarrow \boxed{\omega = 2.6 \text{ rad/s} \circlearrowright}$$

To find $v_C = \omega \, r_{C/IC}$

$$= 2.6 \, (0.1538 - 0.125)$$

$$\boxed{v_C = 0.075 \text{ m/s} \leftarrow}$$

example 16.13



$AB = 0.25$ ft
$BC = 0.75$ ft

$\omega_{AB} = 10$ rad/s

Find v_C at the instant shown.

v_B, v_C non parallel.

$$\vec{v}_A = 0 \Rightarrow v_B = \omega_{AB} \, r_B$$

$$v_B = 10 \times 0.25 = 2.5 \text{ ft/s}$$

following sine rule

$$\frac{BC}{\sin 45} = \frac{r_{B/IC}}{\sin 76.4} \Rightarrow r_{B/IC} = 1.031 \text{ ft}$$

$$BC = r_{C/IC} \Rightarrow \vec{r}_{C/IC} = 0.9056 \text{ ft}$$

$$\frac{BC}{\sin 45} = \frac{r_{C/IC}}{\sin 58.6} \Rightarrow \vec{r}_{C/IC} = 0.9056 \text{ ft}$$

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = 2.425 \text{ rad/s} \circlearrowleft$$

$$v_C = \omega_{BC}\, r_{C/IC}$$

$$\boxed{v_C = 2.20 \text{ ft/s}} \uparrow$$

$\boxed{16.7}$ Relative - Motion Analysis : Acceleration

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

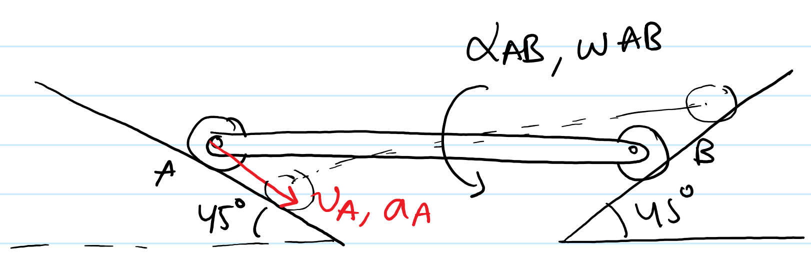
$$\underbrace{\vec{a}_B}_{\text{acceleration of point B}} = \vec{a}_A + \underbrace{(\vec{a}_{B/A})_t}_{\vec{\alpha} \times \vec{r}_{B/A}} + \underbrace{(\vec{a}_{B/A})_n}_{-\omega^2 \vec{r}_{B/A}}$$

$\vec{a}_A$: acceleration of point A

$(\vec{a}_{B/A})_t$: tangential acceleration
mag: $\alpha r_{B/A}$
dir: $\perp r_{B/A}$

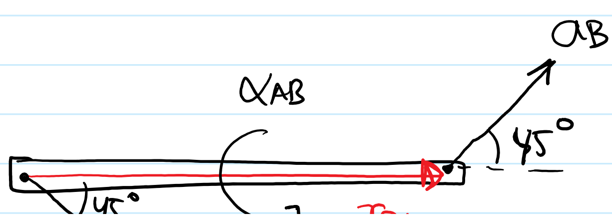
$(\vec{a}_{B/A})_n$: Normal acceleration
mag: $\omega^2 r_{B/A}$
dir: from B to A

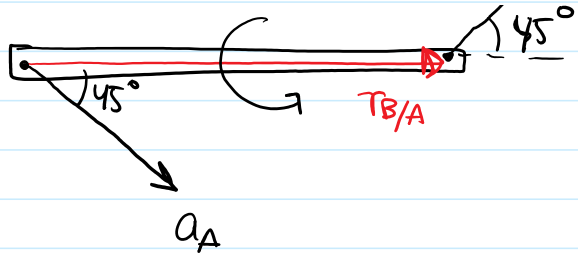
example 16.14



length = 10 m
$v_A = 2$ m/s
$a_A = 3$ m/s^2

find α_{AB} when the rod is horizontal





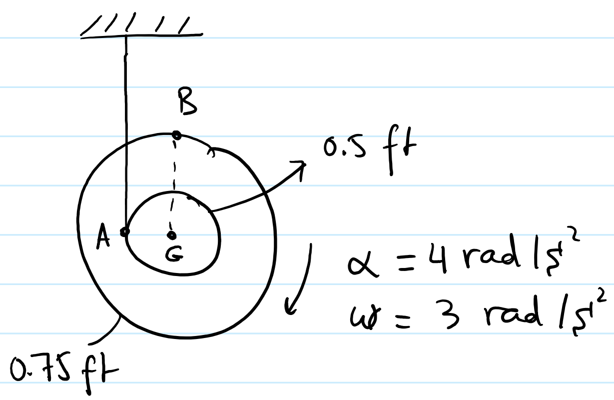
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

$$a_B \cos 45\, \hat{i} + a_B \sin 45\, \hat{j} = 3\cos 45\, \hat{i} - 3 \sin 45\, \hat{j} + (\alpha \hat{k} \times 10\hat{i}) - (0.283)^2 (10\,\hat{i})$$

$$a_B = 1.87 \text{ m/s}^2$$

$$\boxed{\alpha_{AB} = 0.344 \text{ rad} \circlearrowleft}$$

example 16.16



Find $\vec{a}_B$

$$\vec{a}_G = \alpha r = 4\,(0.5) = 2 \text{ ft/s}^2$$ (see example 16.15)

$$\vec{a}_B = \vec{a}_G + \vec{\alpha} \times \vec{r}_{B/G} - \omega^2 \vec{r}_{B/G}$$

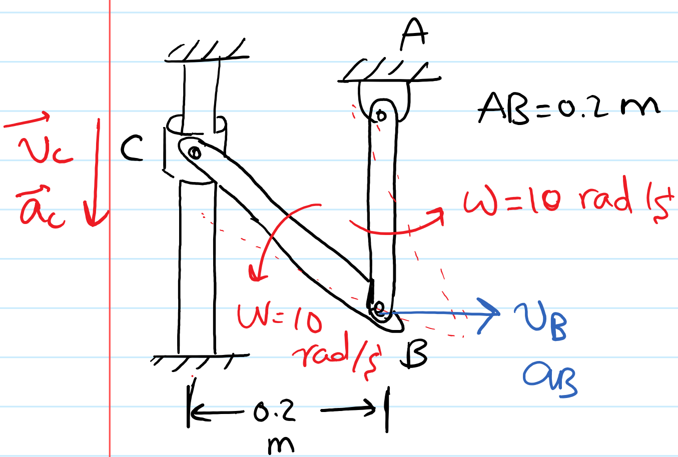
$$= -2\hat{j} + -4\hat{k} \times 0.75\hat{j} - (3)^2 (0.75\hat{j})$$

$$\vec{a}_B = 3\hat{i} - 8.75\hat{j} \text{ ft/s}^2$$

$$|\vec{a}_B| = \sqrt{3^2 + 8.75^2} = 9.25 \text{ ft/s}^2$$

$$\theta = \tan^{-1}\frac{8.75}{3} = 71.1^\circ$$

<u>example 16.17</u>



$v_c = 2$ m/s
$a_c = 1$ m/s²

Find α_{AB}, α_{BC}

$$\vec{a}_B = \cancel{\vec{a}_A} + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$

$$= \alpha_{AB}\hat{k} \times -0.2\hat{j} - (10)^2(-0.2\hat{j})$$

$$\boxed{\vec{a}_B = 0.2\,\alpha_{AB}\,\hat{i} + 20\hat{j}}$$

$$\vec{a}_B = \vec{a}_C + \vec{\alpha}_{CB} \times \vec{r}_{B/C} - \omega_{CB}^2 \vec{r}_{B/C}$$

$$0.2\alpha_{AB}\hat{i} + 20\hat{j} = -\hat{j} + \alpha_{CB} k \times (0.2\hat{i} - 0.2\hat{j}) - 10^2(0.2\hat{i} - 0.2\hat{j})$$

$$0.2\,\alpha_{AB} = 0.2\,\alpha_{BC} - 20$$

$$20 = -1 + 0.2\,\alpha_{CB} + 20$$

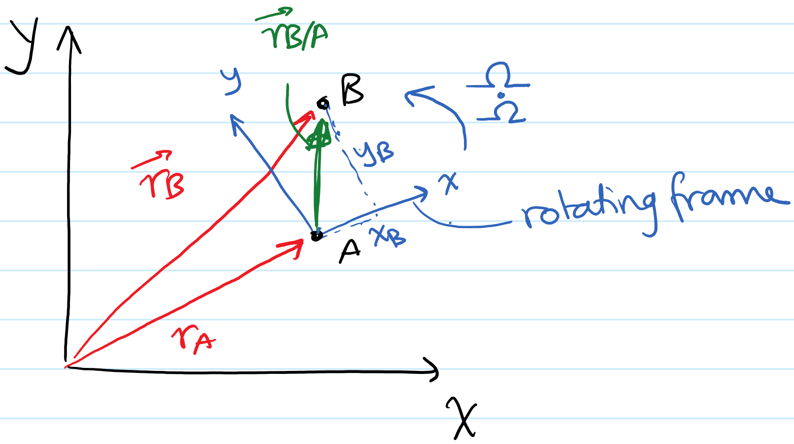
$$\boxed{\alpha_{CB} = 5 \text{ rad/s}^2 \circlearrowleft}$$

$$\boxed{\alpha_{AB} = -95 \text{ rad/s}^2 = 95 \text{ rad/s}^2 \circlearrowright}$$

$$\boxed{\alpha_{AB} = -95 \text{ rad}/s^2 = 95 \text{ rad}/s^2 \circlearrowright}$$

$\boxed{16.8}$ Relative Motion Analysis Using Rotating axes.

→ for analyzing motion of points that are <u>not</u> on the same body and when bodies are not pin-connected.



$$\vec{r}_{B/A} = x_B\hat{i} + y_B\hat{j}$$

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

* <u>Velocity</u>

$$\vec{v}_B = \vec{v}_A + \frac{d\,\vec{r}_{B/A}}{dt}$$

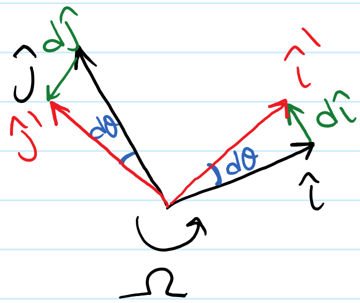
$$\frac{d\vec{r}_{B/A}}{dt} = \frac{d}{dt}\left(x_B\hat{i} + y_B\hat{j}\right)$$

$$= \frac{d}{dt}x_B\,\hat{i} + x_B\frac{d\hat{i}}{dt} + \frac{dy_B}{dt}\hat{j} + y_B\frac{d\hat{j}}{dt}$$

$$= \left(\frac{d}{dt}x_B\,\hat{i} + \frac{dy_B}{dt}\hat{j}\right) + \left(x_B\frac{d\hat{i}}{dt} + y_B\frac{d\hat{j}}{dt}\right)$$

$$= \left(\vec{v}_{B/A}\right)_{xyz} + \left(x_B\left(\vec{\Omega}\times\hat{i}\right) + y_B\left(\vec{\Omega}\times\hat{j}\right)\right)$$

$$= (v_{B/A})_{xyz} + \left(x_B (\vec{\Omega} \times \hat{i}) + y_B (\vec{\Omega} \times \hat{j}) \right)$$



$|d\hat{i}| = 1\, d\theta$ (mag)

$d\hat{i} = \hat{j}$ (dir)

$d\hat{i} = d\theta\, \hat{j}$ (vector equ)

$$\frac{d\hat{i}}{dt} = \frac{d\theta}{dt}\hat{j} = \Omega \hat{j}$$

$$\frac{d\hat{j}}{dt} = -\frac{d\theta}{dt}\hat{i}$$

In general in 3D $\quad \dfrac{d\hat{i}}{dt} = \vec{\Omega} \times \hat{i}$

$$\frac{d\hat{j}}{dt} = \vec{\Omega} \times \hat{j}.$$

$$\frac{d\vec{r}_{B/A}}{dt} = (\vec{v}_{B/A})_{xyz} + (\vec{\Omega} \times x_B \hat{i}) + (\vec{\Omega} \times y_B \hat{j})$$

$$= (\vec{v}_{B/A})_{xyz} + \vec{\Omega} \times (x_B \hat{i} + y_B \hat{j})$$

$$\boxed{\frac{d\vec{r}_{B/A}}{dt} = (\vec{v}_{B/A})_{xyz} + \vec{\Omega} \times \vec{r}_{B/A}}$$

Thus : $\quad \vec{v}_B = \vec{v}_A + (\vec{v}_{B/A})_{xyz} + \vec{\Omega} \times \vec{r}_{B/A}$

$\vec{v}_B$ wrt XYZ $\quad$ $\vec{v}_A$ wrt XYZ $\quad$ $\vec{v}_{B/A}$ wrt xyz $\quad$ angular vel of xyz $\quad$ wrt xyz

* Acceleration

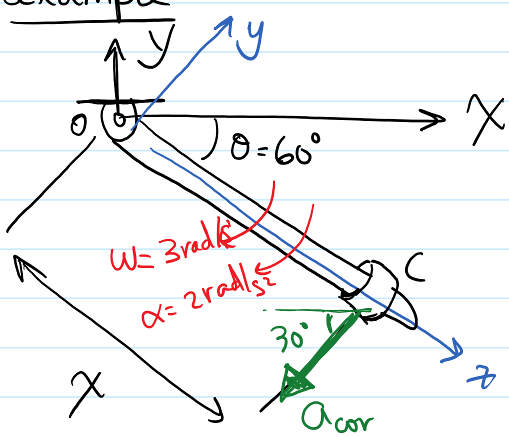
$$\frac{d\vec{v}_B}{dt} = \frac{d\vec{v}_A}{dt} + \frac{d(\vec{v}_{B/A})_{xyz}}{dt} + \frac{d}{dt}(\vec{\Omega} \times \vec{r}_{B/A})$$

$$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{xyz} + \vec{\Omega} \times (\vec{v}_{B/A})_{xyz}$$
$$+ \vec{\Omega} \times \left((\vec{v}_{B/A})_{xyz} + \vec{\Omega} \times \vec{r}_{B/A}\right) + \dot{\vec{\Omega}} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A}) + \underbrace{2\,\vec{\Omega} \times (\vec{v}_{B/A})_{xyz}}$$
$$+ \underbrace{(\vec{a}_{B/A})_{xyz}}$$

Coriolis Acceleration always $\perp$ to $\vec{\Omega}$ and $(\vec{v}_{B/A})_{xyz}$

example



when $x = 0.2$ m
$v_c = 2$ m/s
$a_c = 3$ m/s^2
relative to the rod

find v_c, a_c

a_{cor}

$$\vec{v}_C = \vec{v}_O + \vec{\Omega} \times \vec{r}_{C/O} + (\vec{v}_{C/O})_{xyz}$$

$$\vec{v}_C = 0 + -3\hat{k} \times 0.2\hat{i} + 2\hat{i}$$

$$\boxed{\vec{v}_C = \{2\hat{i} - 0.6\hat{j}\} \text{ m/s}}$$

$$a_{cor} = 2\,\vec{\Omega} \times (\vec{v}_{C/O})_{xyz}$$

$$\boxed{a_{cor} = 2(-3\hat{k}) \times (2\hat{i}) = -12\hat{j} \text{ m/s}^2}$$

$$\vec{a}_C = \vec{a}_O + \dot{\vec{\Omega}} \times \vec{r}_{C/O} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{C/O}) + \vec{a}_{cor}$$

$+ (\vec{a}_{C/O})_{xyz}$

$= 0 + (-2\hat{k}) \times (0.2\hat{i}) + \left(-3\hat{k} \times \left(-3\hat{k} \times 0.2\hat{i}\right)\right) - 12\hat{j}$

$+ 3\hat{i}$

$$\boxed{\vec{a}_C = \{1.2\hat{i} - 12.4\hat{j}\}\ m/s^2}$$