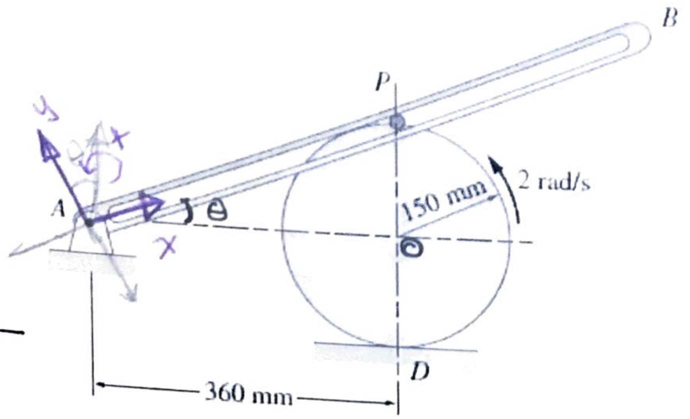
1) The disk is rolling without slipping on the horizontal surface with constant counterclockwise angular velocity of 2 rad/s. The pin P, attached to the disk, slides inside the slot in the rotating arm AB. Calculate the angular velocity and angular acceleration of AB when the system is in the position shown.

Clearly label your coordinate system(s).

~~30~~ marks 35



$|\vec{v}_o| = 0.15 \times 2 = 0.3 \ m/s \leftarrow$

$\vec{v}_o = -0.3\cos 22.6\,\hat{i} + 0.3\sin 22.6\,\hat{j}$ $\quad \theta = \tan^{-1}\frac{150}{360} = 22.6^\circ$ 2

$\vec{v}_o = -0.277\,\hat{i} + 0.115\,\hat{j} \ m/s$ 5

$|\vec{a}_o| = 0$ 3

P, O $\rightarrow$ same rigid body

$\vec{v}_P = \vec{v}_o + \vec{\omega} \times \vec{r}_{P/O}$

5 $= -0.277\,\hat{i} + 0.115\,\hat{j} + 2\hat{k} \times (0.15\sin 22.6\,\hat{i} + 0.15\cos 22.6\,\hat{j})$

$= -0.277\,\hat{i} + 0.115\,\hat{j} + 0.115\,\hat{j} - 0.277\,\hat{i}$

$\boxed{\vec{v}_P = -0.554\,\hat{i} + 0.23\,\hat{j} \ m/s}$

$\vec{a}_P = \vec{a}_o + \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \vec{r}_{P/O}$

5 $= 0 + 0 - (2)^2 (0.15\sin 22.6\,\hat{i} + 0.15\cos 22.6\,\hat{j})$

$\boxed{\vec{a}_P = -0.23\,\hat{i} - 0.554\,\hat{j} \ m/s^2}$

P / A → not the same rigid body.

$|r_{P/A}| = 0.39$ m

$$\vec{v}_P = \vec{v}_A + \vec{\Omega} \times \vec{r}_{P/A} + \vec{v}_{P/A}$$

7

$$-0.554\hat{i} + 0.23\hat{j} = 0 + \Omega\hat{k} \times 0.39\hat{i} + v_{P/A}\hat{i}$$

$$-0.554\hat{i} + 0.23\hat{j} = 0.39\Omega\hat{j} + v_{P/A}\hat{i}$$

$$\boxed{v_{P/A} = -0.554 \text{ m/s}}$$

$$\boxed{\Omega = 0.59 \text{ rad/s}}$$ #

$$\vec{a}_P = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{P/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{P/A}) + 2\vec{\Omega} \times \vec{v}_{P/A} + \vec{a}_{P/A}$$

8

$$-0.23\hat{i} - 0.544\hat{j} = 0 + \dot{\Omega}\hat{k} \times 0.39\hat{i} + 0.59\hat{k} \times (0.59\hat{k} \times 0.39\hat{i}) + 2(0.59\hat{k} \times -0.554\hat{i}) + a_{P/A}\hat{i}$$

$$-0.23\hat{i} - 0.544\hat{j} = 0.39\dot{\Omega}\hat{j} - 0.136\hat{i} - 0.65\hat{j} + a_{P/A}\hat{i}$$

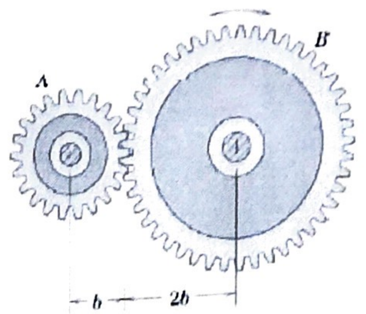
$$-0.23 = -0.136 + a_{P/A}$$

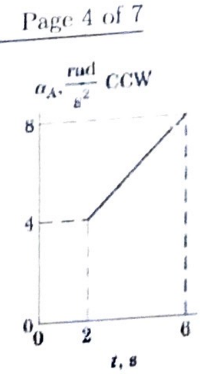
$$\boxed{a_{P/A} = -0.094 \text{ m/s}^2}$$

$$-0.544 = 0.39\dot{\Omega} - 0.65$$

$$\boxed{\dot{\Omega} = 0.27 \text{ rad/s}^2}$$ ##

2) Gear B is rotating clockwise with an angular velocity of 300 rev/min. At $t = 2$ s, gear A is given a counterclockwise angular acceleration α_A which varies with time for a duration of 4 s as shown in the graph. Determine ω_B when $t = 6$ s.





35 marks

Before Applying The acceleration

$$300 \frac{rev}{min} = \frac{300 \times 2\pi}{1 \times 60} \frac{rad}{s}$$

$$\omega_{B_1} = 31.42 \text{ rad/s}$$

5

$$\omega_{A_1} r_A = \omega_{B_1} r_B$$

5

$$\omega_{A_1} \times \cancel{b} = 31.42 (2\cancel{b})$$

$$\boxed{\omega_{A_1} = 62.8 \text{ rad/s}}$$

To find the function of The acceleration $\alpha_A(t)$

$$\text{slope} = \frac{8-4}{6-2} = \frac{4}{4} = 1$$

$$\alpha_A(t) = t + b$$

$$4 = 2 + b \rightarrow b = 2$$

$$\boxed{\alpha_A(t) = t + 2}$$

5

$$\int_2^6 \alpha_A \, dt = \int_{62.8}^{\omega_{A_2}} d\omega_A$$

5

2

$$\frac{t^2}{2} + 2t \Big|_2^6 = \omega_{A_2} - 62.8$$

$$\frac{36}{2} + 12 - \frac{4}{2} - 4 = \omega_{A_2} - 62.8$$

$$\omega_{A_2} = 24 + 62.8$$

$$\boxed{\omega_{A_2} = 86.8 \text{ rad/s}}$$

$$\omega_{A_2} r_A = \omega_{B_2} r_B$$

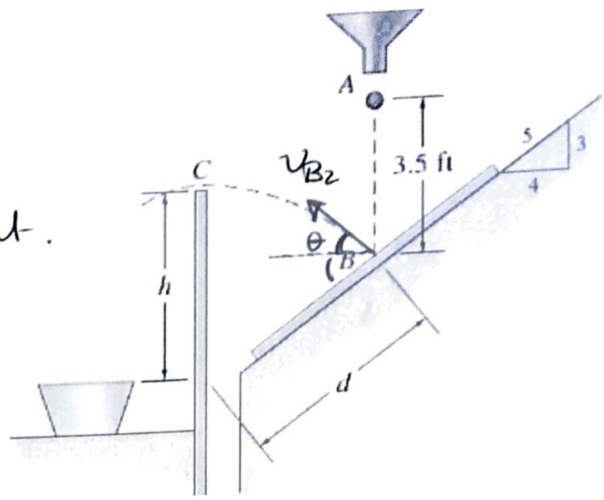
$$86.8 \times b = \omega_{B_2} \times 2b$$

$$\boxed{\omega_{B_2} = 43.4 \text{ rad/s}}$$

5

Question 2 continues on the next page...

3) Before a cranberry can make it to your dinner plate, it must pass a bouncing test which rates its quality. If cranberries having an $e = 0.8$ are to be accepted, determine the dimensions d and h for the barrier so that when a cranberry falls from rest at A it strikes the plate at B and bounces over the barrier at C. C is the maximum height. Clearly label your coordinate system(s).



A – B Conservation of Energy.

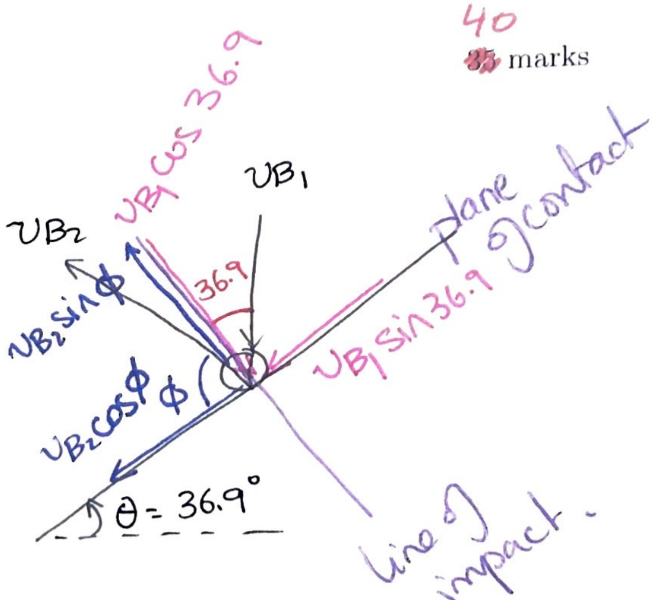
$$T_A + V_A = T_B + V_B$$

5 $$0 + mg \times 3.5 = \frac{1}{2} m v_B^2 + 0$$

$$\boxed{v_B = 15.01 \text{ ft/s}}$$

40 ~~35~~ marks

at B → oblique impact



Along the plane of Contact

$$v_{B1}\sin 36.9 = v_{B2}\cos\phi$$ 5

$$\boxed{v_{B2}\cos\phi = 9.01} \quad (1)$$

Along the line of impact

$$e = \frac{v_{B2}\sin\phi - 0}{0 + v_{B1}\cos 36.9} = 0.8$$ 5

$$\boxed{v_{B2}\sin\phi = 9.6} \quad (2)$$

Solving (1) & (2)

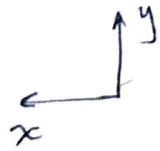
$$\phi = 46.8°$$

$$\boxed{v_{B2} = 13.17 \text{ ft/s}}$$

5

Question 3 continues on the next page...

B – C projectile.



$\theta = 46.8 - 36.9 = 9.9^\circ$ 2

Horizontally.

$x = v_{B_2} \cos 9.9\, t$ 5

$$\boxed{d \cos 36.9 = 13.17 \cos 9.9\, t_{BC}} \text{ —— (1)}$$

Vertically.

$h - d \sin 36.9 = v_{B_2} \sin 9.9\, t - \frac{1}{2}(32.2)\, t^2$ 5

$$\boxed{h - d \sin 36.9 = 13.17 \sin 9.9\, t - \frac{1}{2}(32.2)\, t^2} \text{ —— (2)}$$

Vertically.

$v_{Cy} = v_{By} - g\, t_{B/C}$

5

$0 = 13.17 \sin 9.9 - 32.2\, t_{B-C}$

$$\boxed{t_{B-C} = 0.07\ s}$$

$\boxed{d = \text{~~[illegible]~~ ft}}$ # $\boxed{h = \text{~~[illegible]~~ ft}}$ #

1.13 3 0.76