Dynamics

Wednesday, September 15, 2021

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ENME232: Dynamics

CH 12: Kinematics of a particle

Lecture 2: Sections 12.1-12.3

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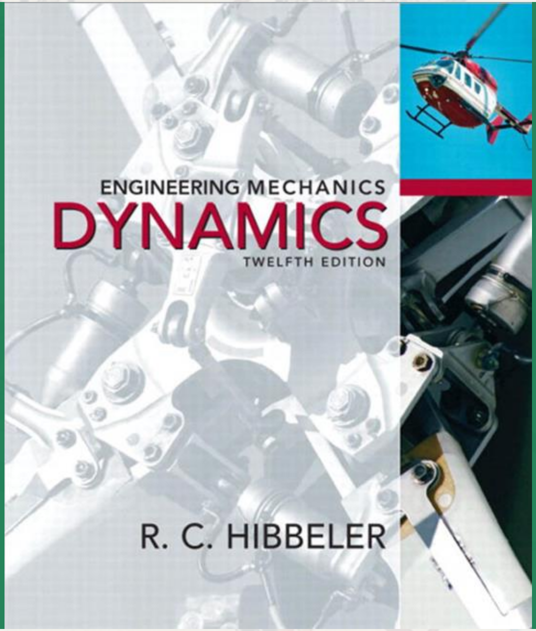
Text books

Engineering Mechanics: Dynamics

C. Hibbeler, 12th Edition, Prentice Hall, 2010

…and probably some more…



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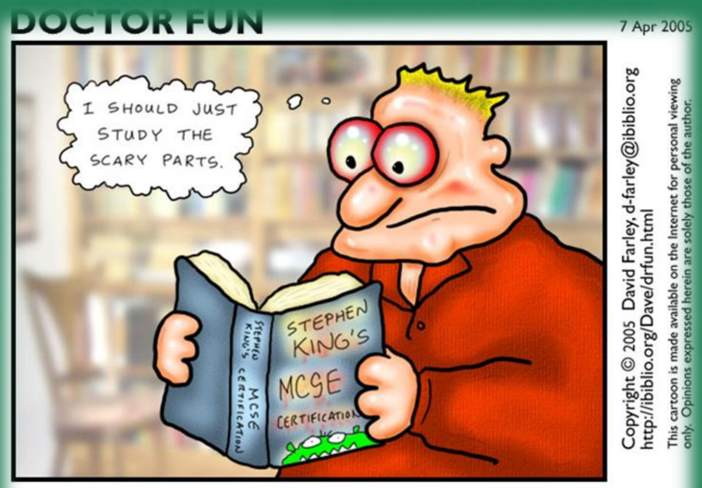
Text books

You revise some maths (i.e. trigonometric identities, derivatives and integrals)

... and some STATICS...



UNITS, Vector addition, FBD

(Hibbeler Statics: Ch. 1,2 and 5)

Today's Class Agenda

1. Objectives
2. Problem solving procedure
3. Introduction
4. Rectilinear kinematics: Continuous motion

   How to analyze problems,
5. Rectilinear kinematics: Erratic motion (Self Study)

Part 1

Objectives

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Objectives

- Concepts such as position, displacement, velocity and acceleration are introduced
- Study the motion of particles along a straight line. Graphical representation
- Investigation of a particle motion along a curved path. Use of different coordinate systems
- Analysis of dependent motion of two particles
- Principles of relative motion of two particles. Use of translating axis

Part 2

Problem solving procedure

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Problem solving procedure

1. Read the problem carefully (and read it again)
2. Physical situation and theory link
3. Draw diagrams and tabulate problem data
4. Coordinate system!!!
5. Solve equations and be careful with units
6. Be critical. A mass of an aeroplane can not be 50 g
7. Read the problem carefully

Part 3

Introduction

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An Overview of Mechanics

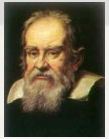
Mechanics: The study of how bodies react to the forces acting on them.

Statics: The study of bodies in equilibrium.
It is at rest/moves with constant velocity

Dynamics: ***Accelerated motion of a body***

1. **Kinematics** – concerned with the geometric aspects of motion
2. **Kinetics** - concerned with the forces causing the motion

Important contributors **Galileo Galilei, Newton, Euler**



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An Overview of Dynamics

Dynamics: Accelerated motion of a body

1. **Kinematics** – study of the geometry of motion.
   Relates displacement, velocity, acceleration, and time without reference to the cause of motion.

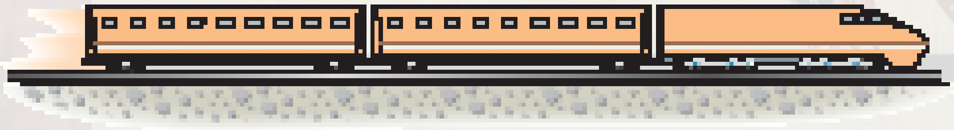
2. **Kinetics** - study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Sections' Objectives

Students should be able to:

1. Find the kinematic quantities (position, displacement, velocity, and acceleration) of a particle traveling along a straight path (12.2)

2. Determine position, velocity, and acceleration of a particle using graphs (12.3)



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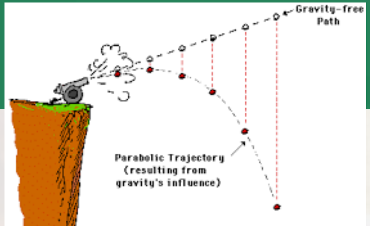
The Particle

The particle has a mass but negligible size and shape. Therefore we must limit application to those objects that have dimensions that are of no consequence in the analysis of the motion.

In most problems, we will be interested in bodies of finite size, such as rockets, projectiles, or vehicles. Each of these objects can be considered as a particle, as long as the motion is characterized by the motion of its mass center and any rotation of the body is neglected.



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The Motion

Rectilinear motion: position, velocity, and acceleration of a particle as it moves along a straight line.

Curvilinear motion: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

Part 4

Rectilinear kinematics: Continuous motion

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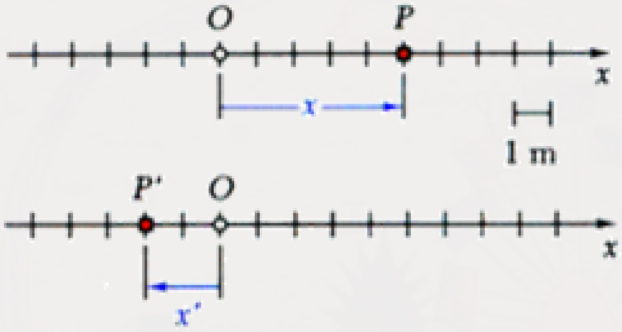
The Position

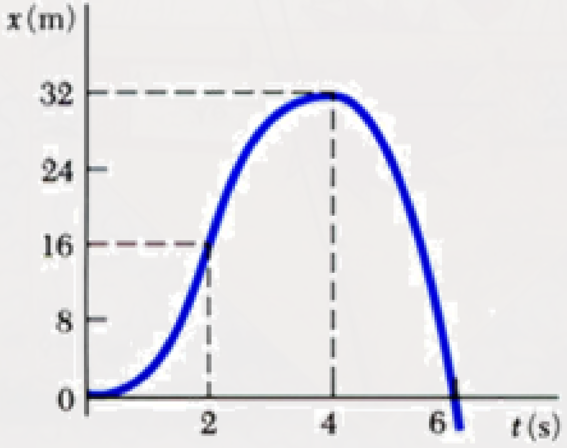


- ***Rectilinear motion:*** particle moving along a straight line
- ***Position coordinate:*** defined by positive or negative distance from a fixed origin on the line.

- The ***motion*** of a particle is known if the position coordinate for particle is known for every value of time t.

- May be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$



or in the form of a graph *x* vs. *t*.

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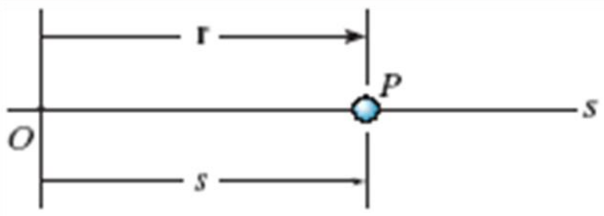
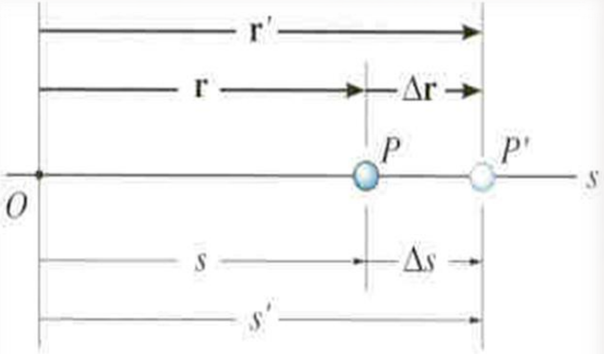
Rectilinear kinematics: Continuous motion



Position



Displacement

A particle travels along a straight-line path defined by **the coordinate axis s**

The **POSITION** of the particle at any instant, relative to the origin, O, is defined by the position vector ***r***, or the scalar s.
Scalar s can be positive or negative. Typical units for ***r*** and s are meters (m) or feet (ft).

The **Displacement** of the particle is defined as its change in position.

Scalar form: $\Delta s = s' - s$ Vector form: $\Delta \boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$

The total distance traveled by the particle, s_T, is a positive scalar that represents the total length of the path over which the particle travels.

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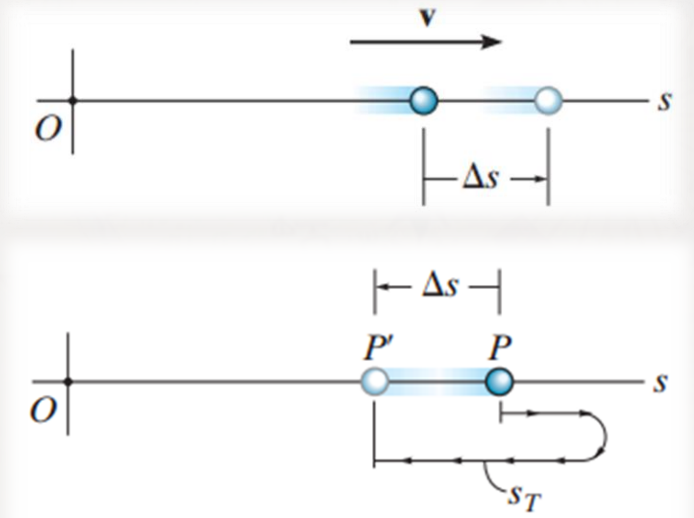
Velocity

Velocity is a measure of the rate of change in the position of a particle. It is a **vector** quantity (it has **both** magnitude and direction). The magnitude of the velocity is called speed, with units of m/s or ft/s.



Average velocity and
Average speed

The average velocity of a particle during a time interval Δt is

$$v_{avg} = \Delta \boldsymbol{r}/\Delta t$$

The **instantaneous velocity** is the time-derivative of position

$$\boldsymbol{v} = d\boldsymbol{r}/dt$$

Speed is the magnitude of velocity: $v = ds/dt$

Average speed is the total distance traveled divided by elapsed time: $(v_{sp})_{avg} = s_T/\Delta t$

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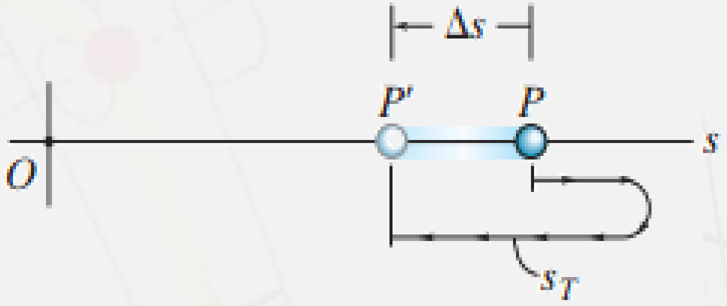
Average velocity vs. average speed

For example, the particle in Fig. 12–1*d* travels along the path of length s_T in time Δt, so its average speed is $(v_{sp})_{avg} = s_T/\Delta t$, but its average velocity is $v_{avg} = -\Delta s/\Delta t$.



Average velocity and
Average speed

(d)

Fig. 12–1 (cont.)

Acceleration

Acceleration. Provided the velocity of the particle is known at two points, the *average acceleration* of the particle during the time interval Δt is defined as

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

Here Δv represents the difference in the velocity during the time interval Δt, i.e., $\Delta v = v' - v$, Fig. 12–1*e*.

Acceleration

Fig. 12–1*e*.

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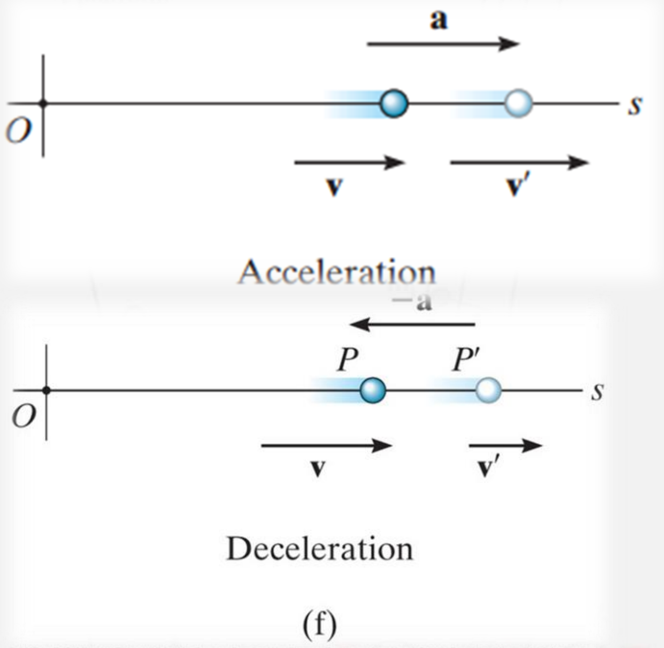
Acceleration

Acceleration is the rate of change in the velocity of a particle.
It is a vector quantity.
Typical units are m/s^2 or ft/s^2.



The instantaneous acceleration is the time derivative of velocity.

Scalar form: $a = dv/dt = d^2s/dt^2$

Vector form: $\boldsymbol{a} = d\mathbf{v}/dt$

Acceleration can be positive (speed increasing) or negative (speed decreasing).

As the book indicates, the derivative equations for velocity and acceleration can be manipulated to get $a\,ds = v\,dv$

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Acceleration (Constant Acc.)

The three kinematic equations can be integrated for the special case when **acceleration is constant ($a = a_c$)** to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81\ m/s^2 = 32.2\ ft/s^2$ downward. These equations are:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a_c\, dt \quad \text{yields} \quad v = v_o + a_c t$$

Velocity as a Function of Time

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2) a_c t^2$$

Position as a Function of Time

$$\int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a_c\, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

Velocity as a Function of Position

Important Points

- Dynamics: Accelerated motion of bodies
- Kinematics: Geometry of motion
- Average speed and average velocity
- Rectilinear kinematics or straight-line motion
- Acceleration is negative when particle is slowing down
- $a\,ds = v\,dv$; relation of acceleration, velocity, displacement

Analyzing problems in dynamics

Coordinate system

- ✓ Establish a position coordinate along the path and specify its fixed origin and positive direction
- ✓ Motion is along a straight line and therefore s, v and α can be represented as algebraic scalars
- ✓ Use an arrow alongside each kinematic equation in order to indicate positive sense of each scalar

Kinematic equations

- If any two of α, v, s and t are related, then a third variable can be obtained using one of the kinematic equations
- When performing integration, position and velocity must be known at a given instant (…so the constants or limits can be evaluated)
- Some equations must be used only when ***a is constant***

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Problems

EXAMPLE 12.1

The car in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.

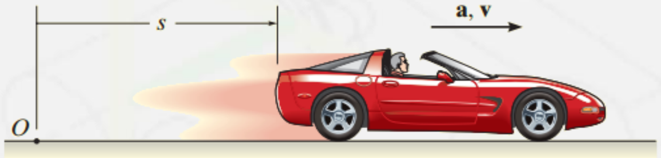

Fig. 12–2

SOLUTION

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

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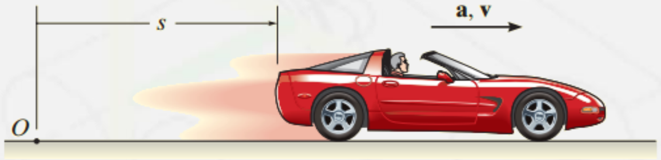


Fig. 12–2

SOLUTION

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft}$$

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a, v, and t.

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow$$

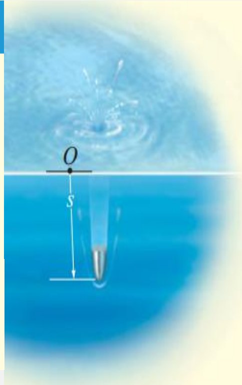
NOTE: The formulas for constant acceleration *cannot* be used to solve this problem, because the acceleration is a function of time.

*The *same result* can be obtained by evaluating a constant of integration C rather than using definite limits on the integral. For example, integrating $ds = (3t^2 + 2t)dt$ yields $s = t^3 + t^2 + C$. Using the condition that at $t = 0$, $s = 0$, then $C = 0$.

Problems

EXAMPLE 12.2

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3)\ \text{m/s}^2$, where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.



SOLUTION

Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at O, Fig. 12–3.

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Problems

EXAMPLE 12.2

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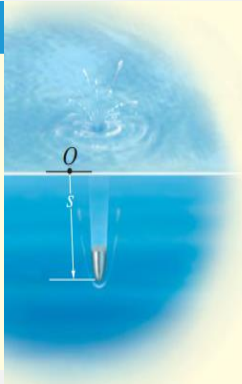

SOLUTION

Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at O, Fig. 12–3.

Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v, a, and t. (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60$ m/s when $t = 0$, yields

$$a = \frac{dv}{dt} = -0.4v^3$$

$$(+\downarrow) \quad \int_{60\text{ m/s}}^{v} \frac{dv}{-0.4v^3} = \int_0^t dt$$

$$\frac{1}{-0.4}\left(\frac{1}{-2}\right)\frac{1}{v^2}\bigg|_{60}^{v} = t - 0$$

$$\frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{(60)^2}\right] = t$$

$$v = \left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}\right\}\text{ m/s}$$

Here the positive root is taken, since the projectile will continue to move downward. When $t = 4$ s,

$$v = 0.559\text{ m/s}\downarrow \qquad \textit{Ans.}$$

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Problems

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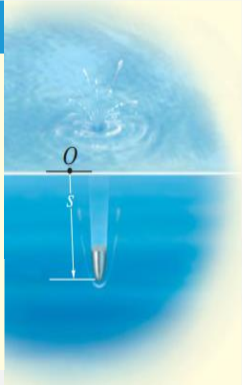

SOLUTION

Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at O, Fig. 12–3.

Position. Knowing $v = f(t)$, we can obtain the projectile's position from $v = ds/dt$, since this equation relates s, v, and t. Using the initial condition $s = 0$, when $t = 0$, we have

$$(+\downarrow)$$

$$v = \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2} dt$$

$$s = \frac{2}{0.8}\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2}\Bigg|_0^t$$

$$s = \frac{1}{0.4}\left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2} - \frac{1}{60}\right\}\ \mathrm{m}$$

When $t = 4$ s,

$$s = 4.43\ \mathrm{m}$$

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Problems

EXAMPLE 12.3

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

SOLUTION

Coordinate System. The origin O for the position coordinate s is taken at ground level with positive upward, Fig. 12–4.

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75$m/s when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81$ m/s^2 (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12–6, namely,

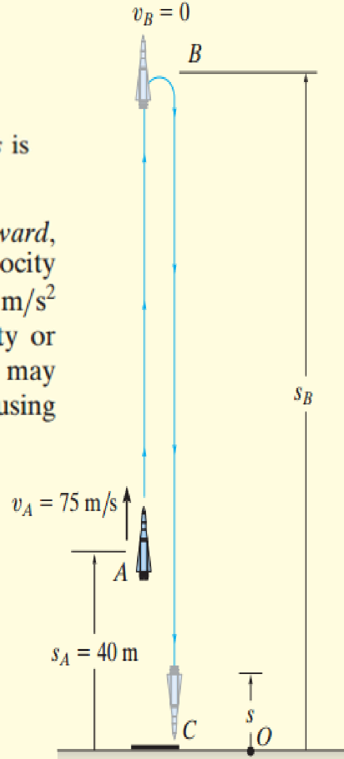

Fig. 12–4

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EXAMPLE 12.3

Problems

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SOLUTION

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$$(+\uparrow)$$

$$v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$0 = (75\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)(s_B - 40\text{ m})$$

$$s_B = 327\text{ m}$$

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12–6 between points B and C, Fig. 12–4.

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$

$$= 0 + 2(-9.81\text{ m/s}^2)(0 - 327\text{ m})$$

$$v_C = -80.1\text{ m/s} = 80.1\text{ m/s}\downarrow \qquad \textit{Ans.}$$

The negative root was chosen since the rocket is moving downward. Similarly, Eq. 12–6 may also be applied between points A and C, i.e.,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$

$$= (75\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)(0 - 40\text{ m})$$

$$v_C = -80.1\text{ m/s} = 80.1\text{ m/s}\downarrow \qquad \textit{Ans.}$$

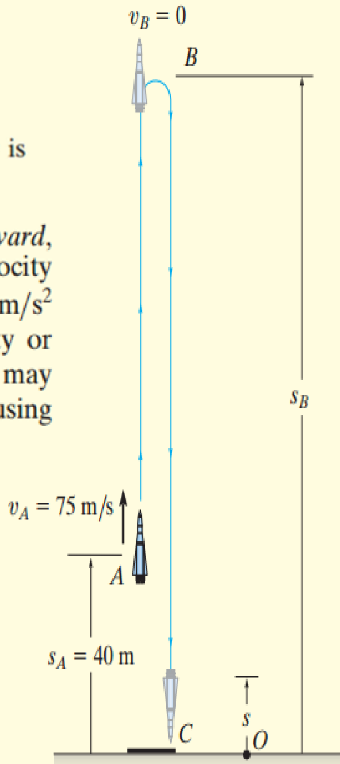
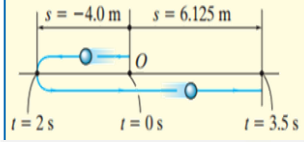


Fig. 12–4

NOTE: It should be realized that the rocket is subjected to a *deceleration* from A to B of 9.81 m/s^2, and then from B to C it is *accelerated* at this rate. Furthermore, even though the rocket momentarily comes to *rest* at B ($v_B = 0$) the acceleration at B is still 9.81 m/s^2 downward!

Problems

EXAMPLE 12.5



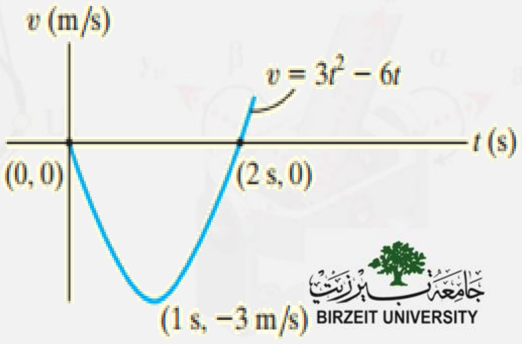
(a)

A particle moves along a horizontal path with a velocity of $v = (3t^2 - 6t)$ m/s, where t is the time in seconds. If it is initially located at the origin O, determine the distance traveled in 3.5 s, and the particle's average velocity and average speed during the time interval.

SOLUTION

Coordinate System. Here positive motion is to the right, measured from the origin O, Fig. 12–6*a*.

Distance Traveled. Since $v = f(t)$, the position as a function of time may be found by integrating $v = ds/dt$ with $t = 0$, $s = 0$.



(b)

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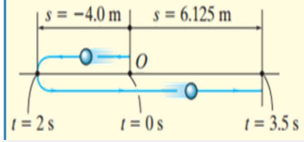
Problems

EXAMPLE 12.5



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SOLUTION

Coordinate System. Here positive motion is to the right, measured from the origin O, Fig. 12–6*a*.

Distance Traveled. Since $v = f(t)$, the position as a function of time may be found by integrating $v = ds/dt$ with $t = 0$, $s = 0$.

$$(\overset{+}{\rightarrow}) \quad ds = v\,dt$$

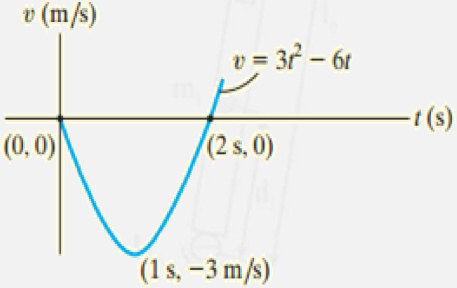
$$= (3t^2 - 6t)dt$$

$$\int_0^s ds = \int_0^t (3t^2 - 6t)\,dt$$

$$s = (t^3 - 3t^2)\text{m} \quad (1)$$

In order to determine the distance traveled in 3.5 s, it is necessary to investigate the path of motion. If we consider a graph of the velocity function, Fig. 12–6*b*, then it reveals that for $0 < t < 2$ s the velocity is *negative*, which means the particle is traveling to the *left*, and for $t > 2$ s the velocity is *positive*, and hence the particle is traveling to the *right*. Also, note that $v = 0$ at $t = 2$ s. The particle's position when $t = 0$, $t = 2$ s, and $t = 3.5$ s can now be determined from Eq. 1. This yields

$$s|_{t=0} = 0 \qquad s|_{t=2\,\text{s}} = -4.0\text{ m} \qquad s|_{t=3.5\,\text{s}} = 6.125\text{ m}$$



(b)

The path is shown in Fig. 12–6*a*. Hence, the distance traveled in 3.5 s is

$$s_T = 4.0 + 4.0 + 6.125 = 14.125\text{ m} = 14.1\text{ m} \qquad \textit{Ans.}$$

Velocity. The *displacement* from $t = 0$ to $t = 3.5$ s is

$$\Delta s = s|_{t=3.5\,\text{s}} - s|_{t=0} = 6.125\text{ m} - 0 = 6.125\text{ m}$$

and so the average velocity is

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{6.125\text{ m}}{3.5\text{ s} - 0} = 1.75\text{ m/s} \rightarrow$$

The average speed is defined in terms of the *distance traveled* s_T. positive scalar is

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t} = \frac{14.125\text{ m}}{3.5\text{ s} - 0} = 4.04\text{ m/s}$$

Note: In this problem, the acceleration is $a = dv/dt = (6t - 6)$ m/s², which is not constant.

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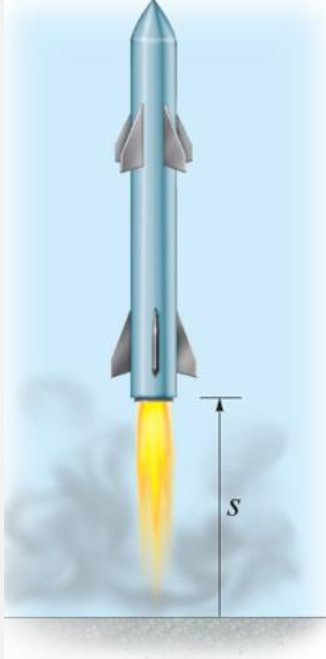
Problems (Solve it at your home)

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

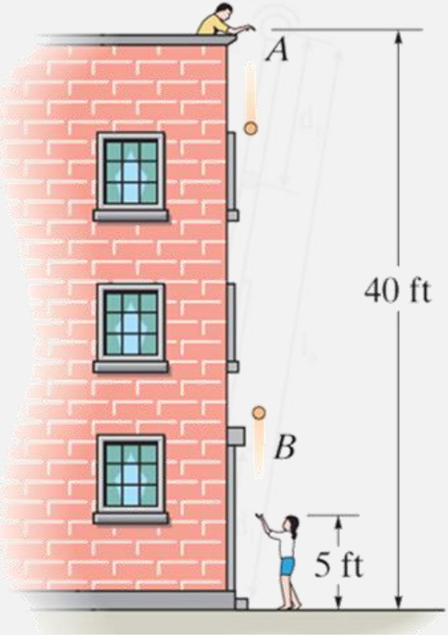
Problems (Solve it at your home)

12-22. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)$ m/s^2, where s is in meters. Determine the rocket's velocity when $s = 2$ km and the time needed to reach this altitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



Problems (Solve it at your home)

12-26. Ball *A* is released from rest at a height of 40 ft at the same time that a second ball *B* is thrown upward 5 ft from the ground. If the balls pass one another at a height of 20 ft, determine the speed at which ball *B* was thrown upward.



Part 5

Rectilinear kinematics: Erratic motion (Self Study)

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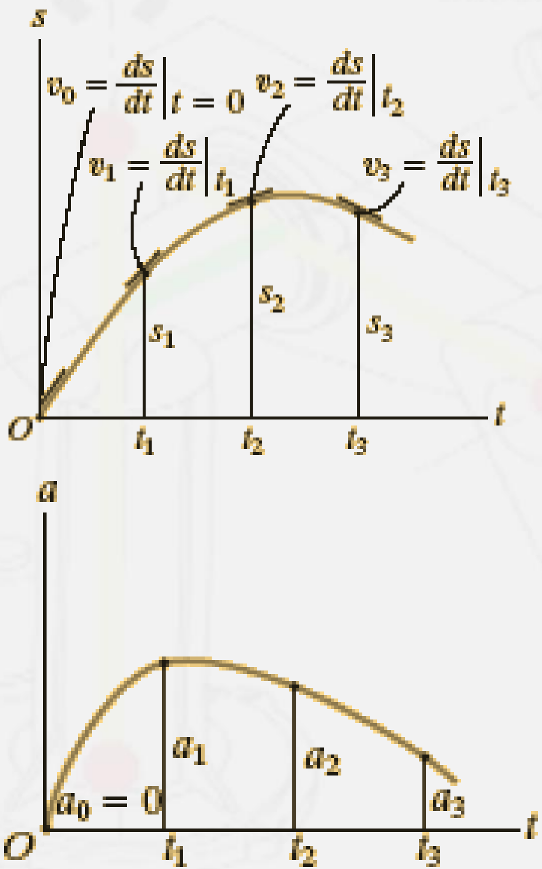
Erratic (discontinuous) motion

Graphing provides a good way to handle complex motions that would be difficult to describe with formulas. Graphs also provide a visual description of motion and reinforce the calculus concepts of differentiation and integration as used in dynamics

The approach builds on the facts that slope and differentiation are **linked** and that integration can be thought of as finding the area under a curve

s-t graph → construct v-t



Plots of position vs. time can be used to find velocity vs. time curves. Finding the slope of the line tangent to the motion curve at any point is the velocity at that point (or v = ds/dt)

Therefore, the v-t graph can be constructed by finding the slope at various points along the s-t graph

Also, the distance moved (displacement) of the particle is the area under the v-t graph during time Δt

s-t graph → construct v-t

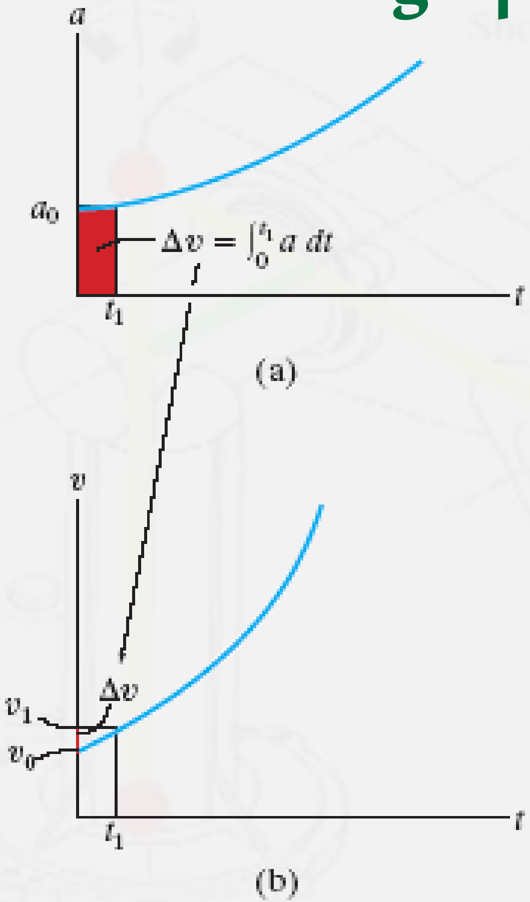


Fig. 12–10

Given the a-t curve, the change in velocity (Δv) during a time period is the area under the a-t curve.

So we can construct a v-t graph from an a-t graph if we know the initial velocity of the particle

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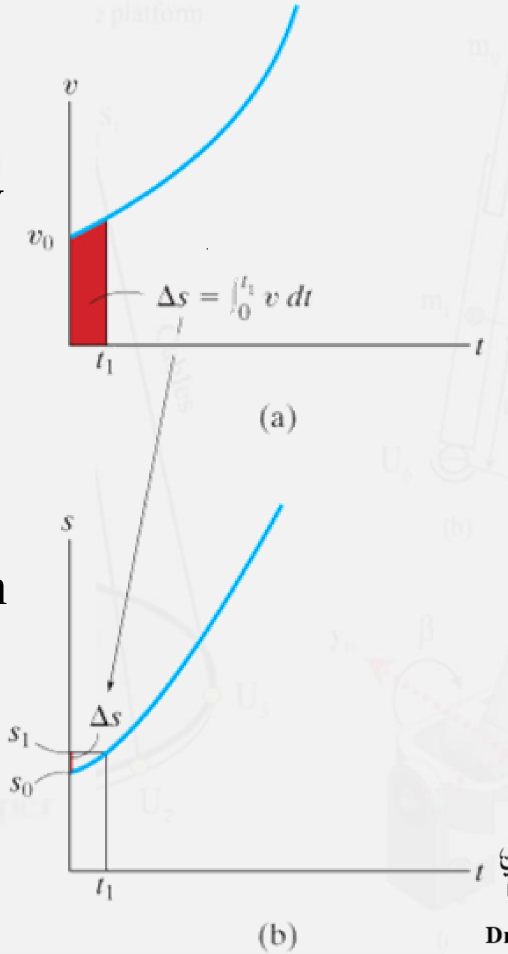
s-t graph ⇒ construct v-t

We begin with initial position S_0 and add algebraically increments Δs determined from the v-t graph

Equations described by v-t graphs may be integrated in order to yield equations that describe segments of the s-t graph



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Please remember the link!!!

Handle complex motions

↓

Graphing

↓

Visual description of motion

↓

Differentiation and integration

↓

Slope and area under curve

Explanation of Example 12.7

The test car in Fig. 12–12*a* starts from rest and travels along a straight track such that it accelerates at a constant rate for 10 s and then decelerates at a constant rate. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?

SOLUTION

v–t Graph. Since $dv = a\,dt$, the v–t graph is determined by integrating the straight-line segments of the a–t graph. Using the *initial condition* $v = 0$ when $t = 0$, we have

$$0 \le t < 10\text{ s}; \qquad a = 10; \qquad \int_0^v dv = \int_0^t 10\,dt, \qquad v = 10t$$

When $t = 10$ s, $v = 10(10) = 100$ m/s. Using this as the *initial condition* for the next time period, we have

$$10\text{ s} < t \le t'; \qquad a = -2; \qquad \int_{100}^v dv = \int_{10}^t -2\,dt, \qquad v = -2t + 120$$

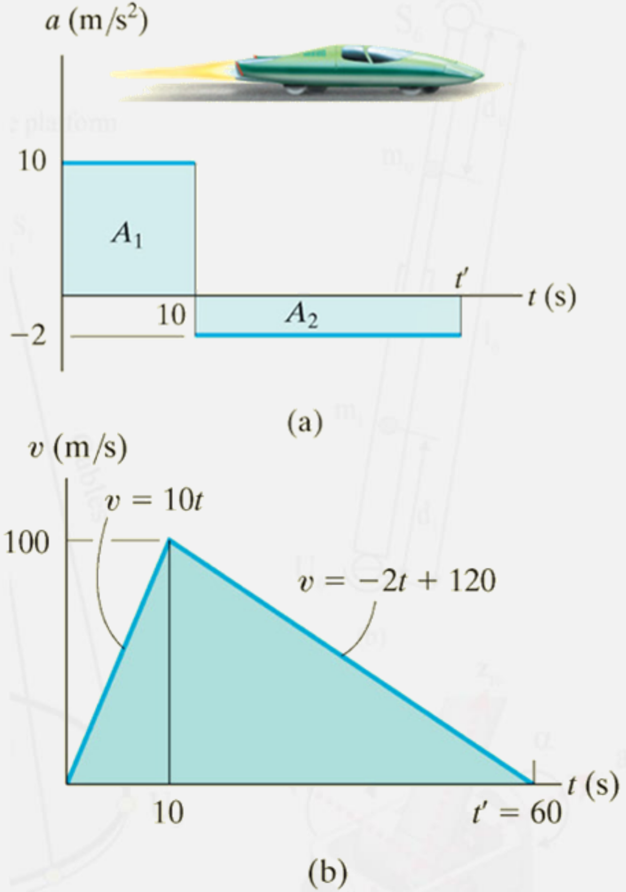
When $t = t'$ we require $v = 0$. This yields, Fig. 12–12*b*,

$$t' = 60\text{ s} \qquad \textit{Ans.}$$

A more direct solution for t' is possible by realizing that the area under the a–t graph is equal to the change in the car's velocity. We require $\Delta v = 0 = A_1 + A_2$, Fig. 12–12*a*. Thus

$$0 = 10\text{ m/s}^2(10\text{ s}) + (-2\text{ m/s}^2)(t' - 10\text{ s})$$

$$t' = 60\text{ s} \qquad \textit{Ans.}$$



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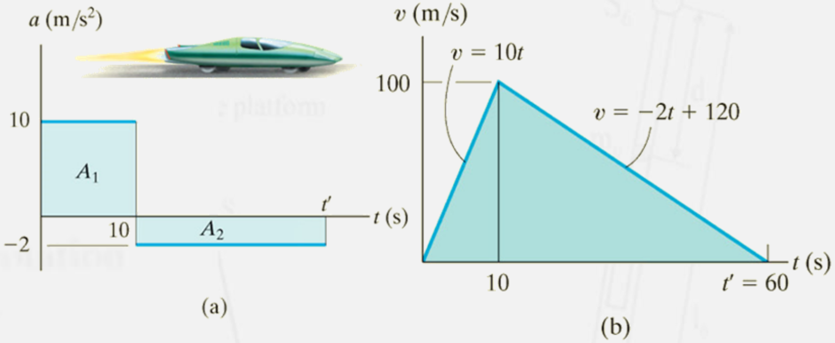
Explanation of Example 12.7

The test car in Fig. 12–12*a* starts from rest and travels along a straight track such that it accelerates at a constant rate for 10 s and then decelerates at a constant rate. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?



(a) (b)

s–t Graph. Since $ds = v\,dt$, integrating the equations of the v–t graph yields the corresponding equations of the s–t graph. Using the *initial condition* $s = 0$ when $t = 0$, we have

$$0 \le t \le 10 \text{ s}; \qquad v = 10t; \qquad \int_0^s ds = \int_0^t 10t\,dt, \qquad s = 5t^2$$

When $t = 10$ s, $s = 5(10)^2 = 500$ m. Using this *initial condition*,

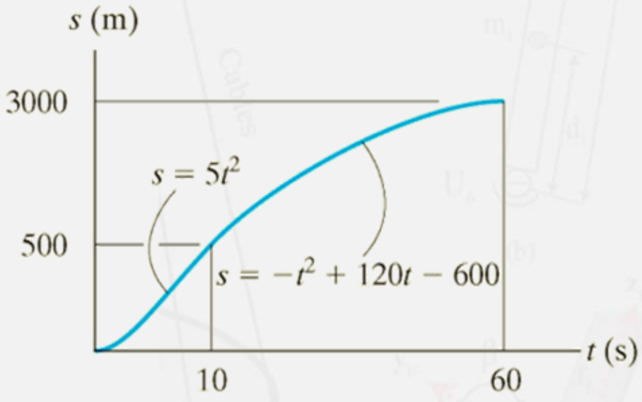
$$10 \text{ s} \le t \le 60 \text{ s}; \quad v = -2t + 120; \quad \int_{500}^s ds = \int_{10}^t (-2t + 120)\,dt$$

$$s - 500 = -t^2 + 120t - [-(10)^2 + 120(10)]$$

$$s = -t^2 + 120t - 600$$

When $t' = 60$ s, the position is

$$s = -(60)^2 + 120(60) - 600 = 3000 \text{ m} \qquad \textit{Ans.}$$



(c)

NOTE: A direct solution for s is possible when $t' = 60$ s, since the *triangular area* under the v–t graph would yield the displacement $\Delta s = s - 0$ from $t = 0$ to $t' = 60$ s. Hence,

$$\Delta s = \tfrac{1}{2}(60)(100) = 3000 \text{ m} \qquad \textit{Ans.}$$

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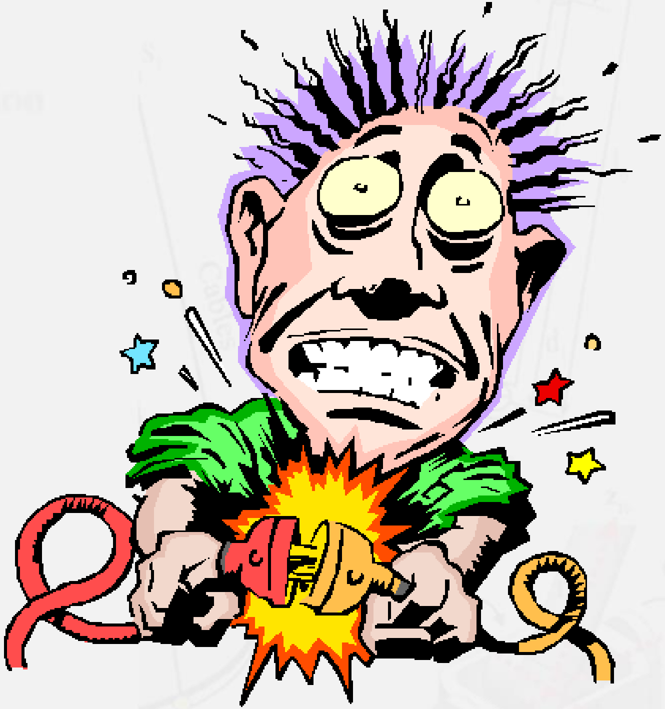
A couple of cases more…

A couple of cases that are a bit more …COMPLEX… and therefore need more attention!!!



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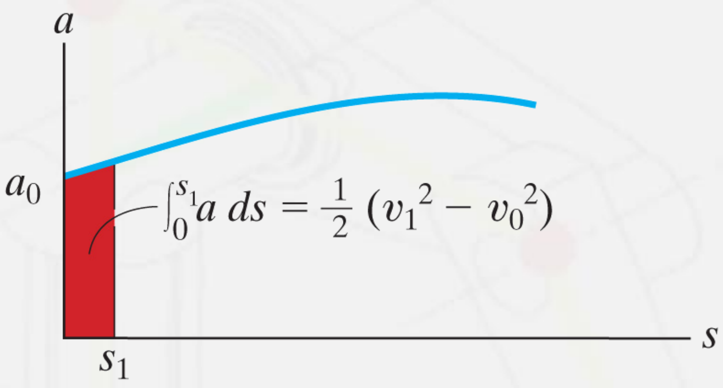
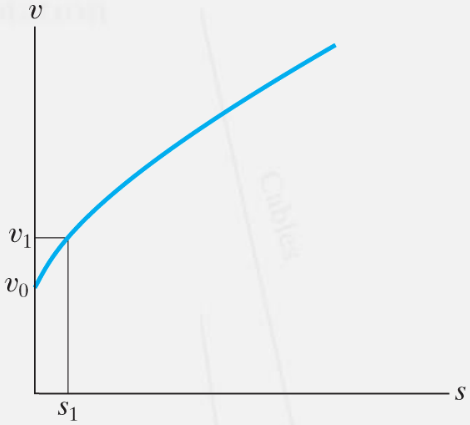
a-s graph ⟹ construct v-s

A more complex case is presented by the a-s graph. The area under the acceleration versus position curve represents the change in velocity. (recall ∫ a ds = ∫ v dv)



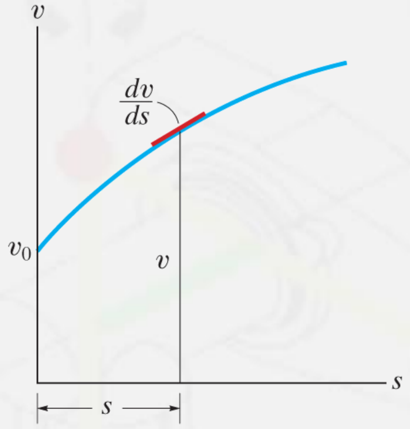
(a)



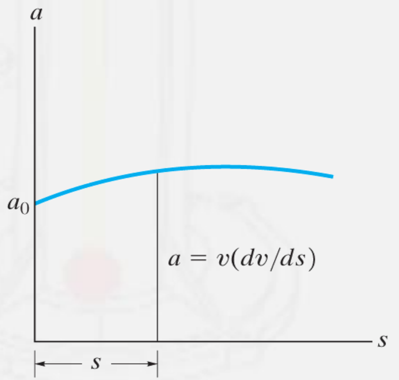
(b)

This equation can be solved for v_1, allowing you to solve for the velocity at a point. By doing this repeatedly, you can create a plot of velocity versus distance.

v-s graph ⇨ construct a-s



(a)



(b)

Another complex case is presented by the v-s graph. By reading the velocity v at a point on the curve and multiplying it by the slope of the curve (dv/ds) at this same point, we can obtain the acceleration at that point.

$$a = v\,(dv/ds)$$

Thus, we can obtain a plot of a vs. s from the v-s curve.

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Please think about it

If a particle in rectilinear motion has zero speed at some instant in time, is the acceleration necessarily zero at the same instant ?

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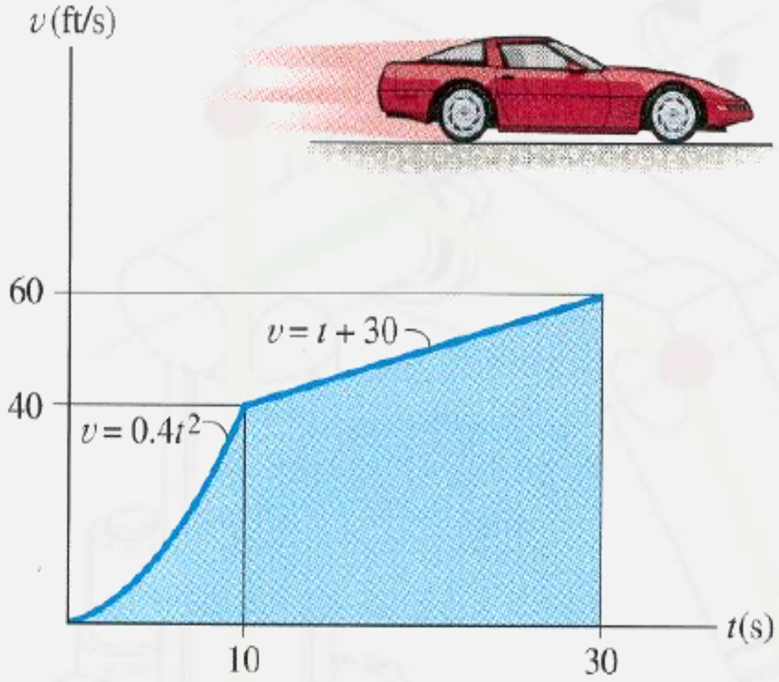
Groups think about this problem please



Given: The v-t graph shown

Find: The a-t graph, average speed, and distance traveled for the 30 s interval

Hints:

- ❖ Find slopes of the curves and draw the a-t graph.
- ❖ Find the area under the curve--that is the distance traveled.
- ❖ Finally, calculate average speed (using basic definitions!)

Groups think about this problem please

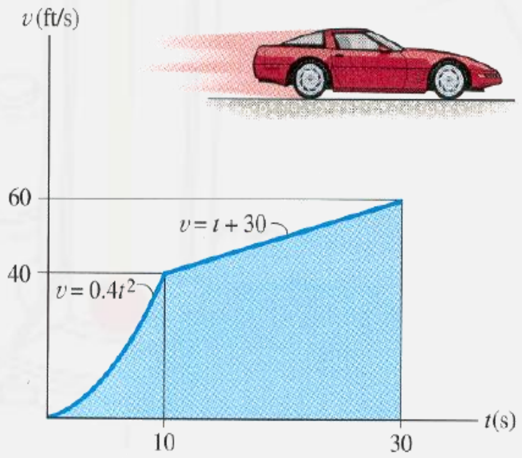
Given: The v-t graph shown

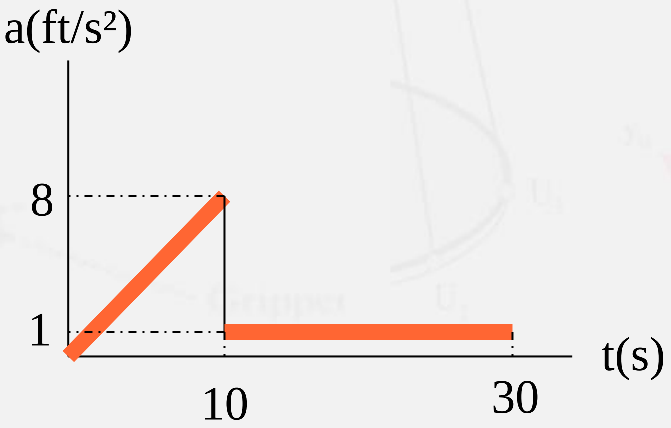
Find: The a-t graph, average speed, and distance traveled for the 30 s interval

Hints:
- ❖ Find slopes of the curves and draw the a-t graph.
- ❖ Find the area under the curve--that is the distance traveled.
- ❖ Finally, calculate average speed (using basic definitions!)

For $0 \leq t \leq 10$ $\quad a = dv/dt = 0.8\,t$ ft/s²

For $10 \leq t \leq 30$ $\quad a = dv/dt = 1$ ft/s²





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Solution to the problem

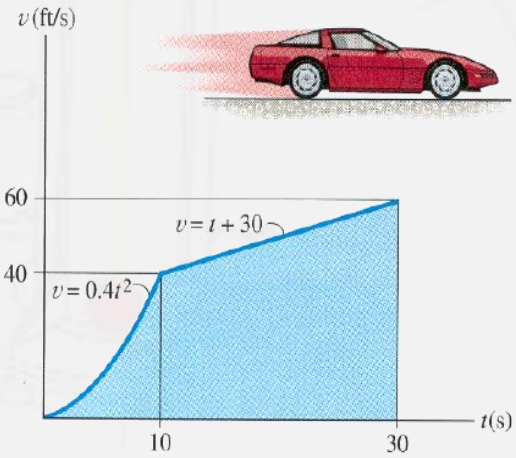
Given: The v-t graph shown

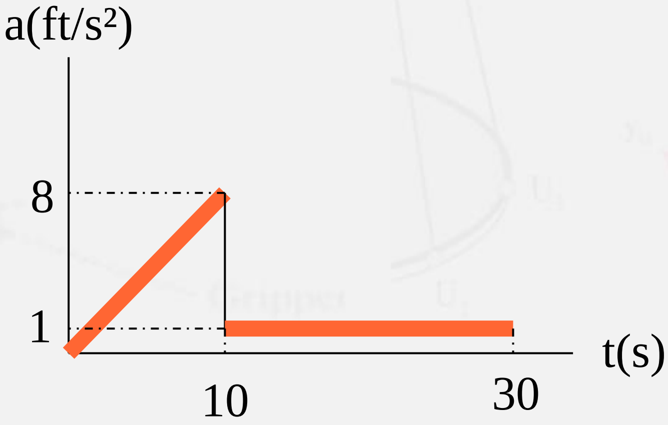
Find: The a-t graph, average speed, and distance traveled for the 30 s interval

Hints:
- ❖ Find slopes of the curves and draw the a-t graph.
- ❖ Find the area under the curve--that is the distance traveled.
- ❖ Finally, calculate average speed (using basic definitions!)

For $0 \leq t \leq 10$ $a = dv/dt = 0.8\,t$ ft/s²

For $10 \leq t \leq 30$ $a = dv/dt = 1$ ft/s²





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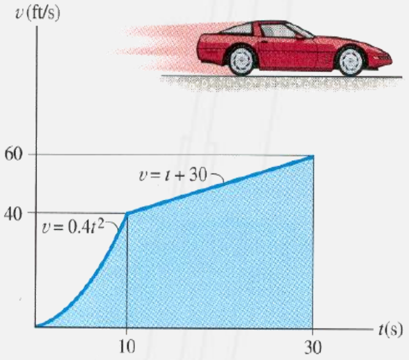
Solution to the problem (Contd.)

Given: The v-t graph shown

Find: The a-t graph, average speed, and distance traveled for the 30 s interval



Hints:
- ❖ Find slopes of the curves and draw the a-t graph.
- ❖ Find the area under the curve--that is the distance traveled.
- ❖ Finally, calculate average speed (using basic definitions!)

$$\Delta s_{0\text{-}10} = \int v\,dt = (1/3)\,(0.4)(10)^3 = 400/3 \text{ ft}$$

$$\Delta s_{10\text{-}30} = \int v\,dt = (0.5)(30)^2 + 30(30) - 0.5(10)^2 - 30(10)$$
$$= 1000 \text{ ft}$$

$$S_{T(0-30)} = 1000 + 400/3 = 1133.3$$

$$v_{avg(0\text{-}30)} = S_{T(0-30)} \,/\, \text{time}$$
$$= 1133.3/30$$
$$= 37.78 \text{ ft/s}$$

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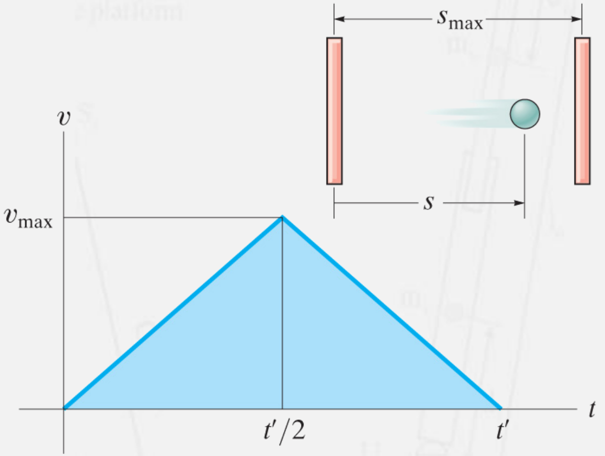
Try at home please (I)

12-42. The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



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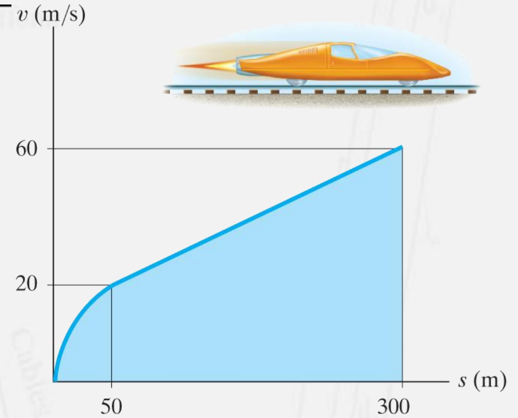
Try at home please (II)

12-53. Two cars start from rest side by side and travel along a straight road. Car *A* accelerates at 4 m/s^2 for 10 s and then maintains a constant speed. Car *B* accelerates at 5 m/s^2 until reaching a constant speed of 25 m/s and then maintains this speed. Construct the $a–t$, $v–t$, and $s–t$ graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

Try at home please (I)

12-65. The v–s graph was determined experimentally to describe the straight-line motion of a rocket sled. Determine the acceleration of the sled when $s = 100$ m, and when $s = 200$ m.



End of the Lecture

Let Learning Continue

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ENME232: Dynamics

CH 12: Kinematics of a particle

Lecture 3: Sections 12.4-12.6

Dr. Mamon M. Horoub
Assistant Professor,
Faculty of Engineering & Technology Department of
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Recap of the Previous Class Agenda

1. Objectives
2. Problem solving procedure
3. Introduction
4. Rectilinear kinematics: Continuous motion

   How to analyze problems,

   Problems
5. Rectilinear kinematics: Erratic motion (Self Study)

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Today's Class Agenda

1. General curvilinear motion
2. Curvilinear motion: Rectangular components
3. Motion of a projectile (Self Study)

Part 1

Objectives

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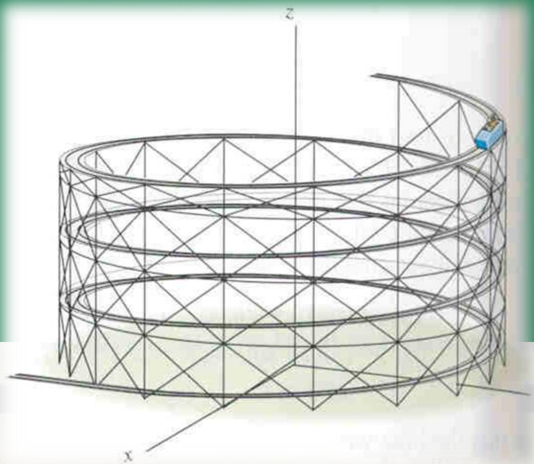
Sections' Objectives

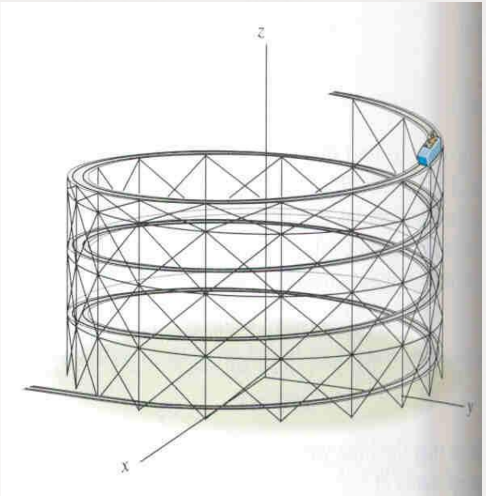
Students should be able to:

1. Describe the motion of a particle traveling along a curved path (12.4)
2. Relate kinematic quantities in terms of the rectangular components of the vectors (12.5)
3. Analyze the free-flight motion of a projectile (12.6, Self Study)

Curvilinear motion

Curvilinear motion occurs when a particle moves along a curved path. Since this path is often described in three dimensions, vector analysis will be used to formulate the particle's position, velocity, and acceleration.





Related applications

The path of motion of each plane in this formation can be tracked with radar and their x, y, and z coordinates (relative to a point on earth) recorded as a function of time

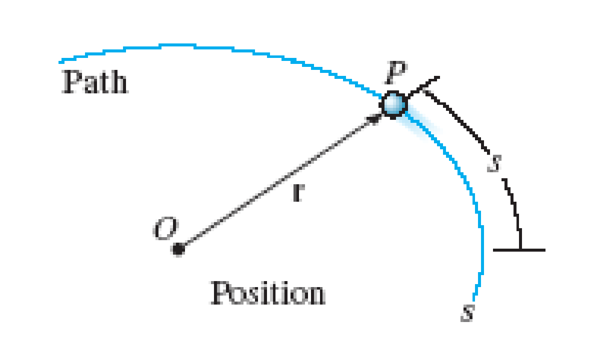
How can we determine the velocity or acceleration at any instant?

A roller coaster car travels down a fixed, helical path at a constant speed

If you are designing the track, why is it important to be able to predict the acceleration of the car?

General curvilinear motion

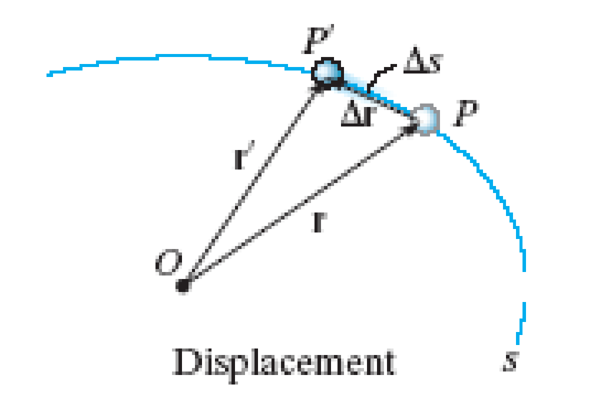
A particle moving along a curved path undergoes curvilinear motion. Since the motion is often three-dimensional, vectors are used to describe the motion



A particle moves along a curve defined by the path function, s

The position of the particle at any instant is designated by the vector

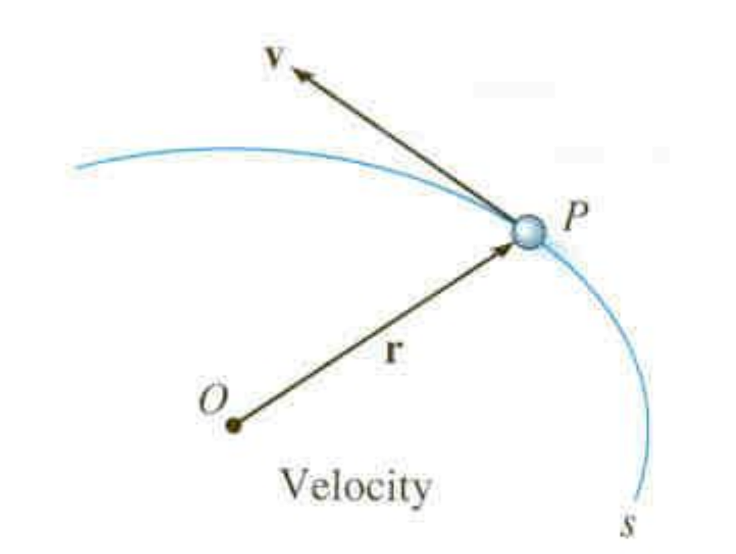
$\boldsymbol{r} = \boldsymbol{r}(t)$. Both the magnitude and direction of $\boldsymbol{r}$ may vary with time



If the particle moves a distance Δs along the curve during time interval Δt, the displacement is determined by vector subtraction: $\Delta \boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$

Velocity

Velocity represents the rate of change in the position of a particle



The average velocity of the particle during the time increment Δt is

$v_{avg} = \Delta \boldsymbol{r}/\Delta t$

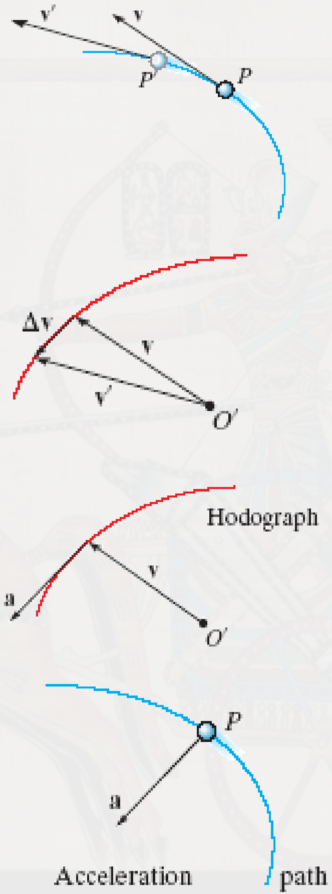
The instantaneous velocity is the time-derivative of position

$\boldsymbol{v} = d\boldsymbol{r}/dt$

The velocity vector, ***v***, is always tangent to the path of motion

The magnitude of ***v*** is called the speed. Since the arc length Δs approaches the magnitude of $\Delta \boldsymbol{r}$ as $t \rightarrow 0$, the speed can be obtained by differentiating the path function ($v = ds/dt$). Note that this is not a vector!

Acceleration



Acceleration represents the rate of change in the velocity of a particle

If a particle's velocity changes from $\boldsymbol{v}$ to $\boldsymbol{v'}$ over a time increment Δt, the average acceleration during that increment is:

$$\boldsymbol{a}_{avg} = \Delta \boldsymbol{v}/\Delta t = (\boldsymbol{v} - \boldsymbol{v'})/\Delta t$$

The instantaneous acceleration is the time-derivative of velocity:

$$\boldsymbol{a} = d\boldsymbol{v}/dt = d^2\boldsymbol{r}/dt^2$$

A plot of the locus of points defined by the arrowhead of the velocity vector is called a hodograph. The acceleration vector is tangent to the hodograph, but not, in general, tangent to the path function.

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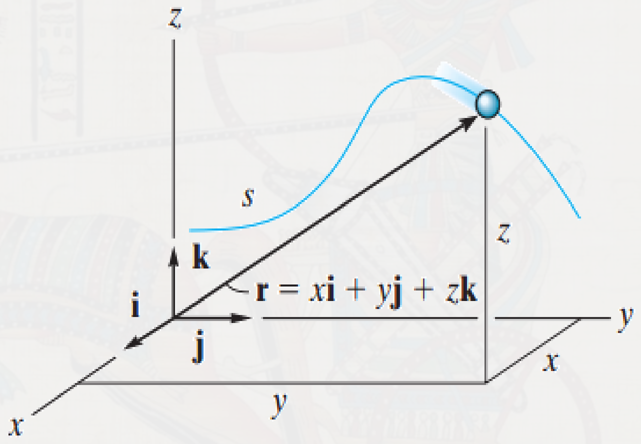
Curvilinear motion: Rectangular components

It is often convenient to describe the motion of a particle in terms of its x, y, z or rectangular components, relative to a fixed frame of reference

The position of the particle can be defined at any instant by the position vector

$$\boldsymbol{r} = \boldsymbol{x}\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$



Position

The x, y, z components may all be functions of time, i.e.,

x = x(t), y = y(t), and z = z(t)

The magnitude of the position vector is: $r = \sqrt{(x^2 + y^2 + z^2)}$

The direction of $\boldsymbol{r}$ is defined by the unit vector: $\boldsymbol{u_r} = (\boldsymbol{r}/r)$

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Rectangular components: Velocity

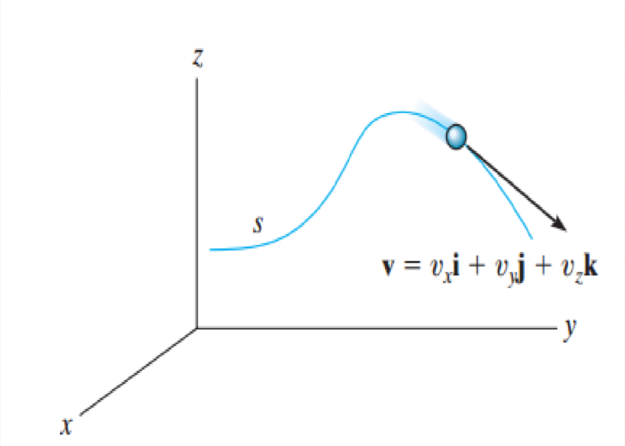
The velocity vector is the time derivative of the position vector:

$$\boldsymbol{v} = d\boldsymbol{r}/dt = d(x\boldsymbol{i})/dt + d(y\boldsymbol{j})/dt + d(z\boldsymbol{k})/dt$$

Since the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are constant in magnitude and direction, this equation reduces to

$$v = v_x\,\boldsymbol{i} + v_y\,\boldsymbol{j} + v_z\,\boldsymbol{k}$$

where; $v_x = dx/dt$, $v_y = dy/dt$, $v_z = dz/dt$



Velocity

The magnitude of the velocity vector is

$$v = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$$

The direction of $\boldsymbol{v}$ is tangent to the path of motion.

The direction of $\boldsymbol{v}$ is defined by the unit vector: $\boldsymbol{u}_v = (\boldsymbol{v}/v)$

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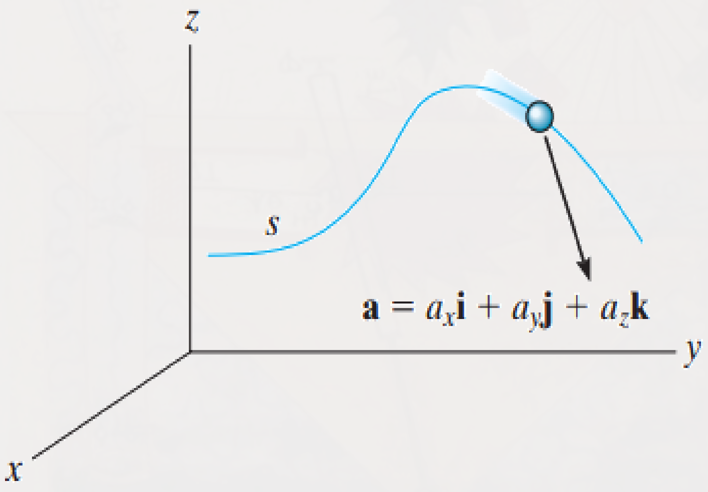
Rectangular components: Acceleration

The acceleration vector is the time derivative of the velocity vector (second derivative of the position vector):

$$\boldsymbol{a} = d\boldsymbol{v}/dt = d^2\boldsymbol{r}/dt^2 = a_x\ \boldsymbol{i} + a_y\ \boldsymbol{j} + a_z\ \boldsymbol{k}$$

where $a_x = dv_x/dt$, $a_y = dv_y/dt$, $a_z = dv_z/dt$



Acceleration

The magnitude of the acceleration vector is

$$a = v = \sqrt{(a_x)^2 + (a_y)^2 + (a_z)^2}$$

The direction of $\boldsymbol{a}$ is usually not tangent to the path of the particle

The direction of $\boldsymbol{a}$ is defined by the unit vector: $\boldsymbol{u_a} = (\boldsymbol{a}/a)$

Important points and analysis

1. Kinematic equations used because rectilinear motion occurs along each coordinate axis
2. Magnitudes of motion for x, y, z vector components can be found using Pythagorean theorem

Appendix C will help you with vectors

Use rectangular coordinate system to solve problems

Curvilinear motion can cause changes in both magnitude and direction of the position, velocity and acceleration vectors

By considering the component motions, the direction of motion of the particle is automatically taken into account

Velocity vector is always directed tangent to the path

In general the acceleration vector is not tangent to the path, but rather, to the hodograph

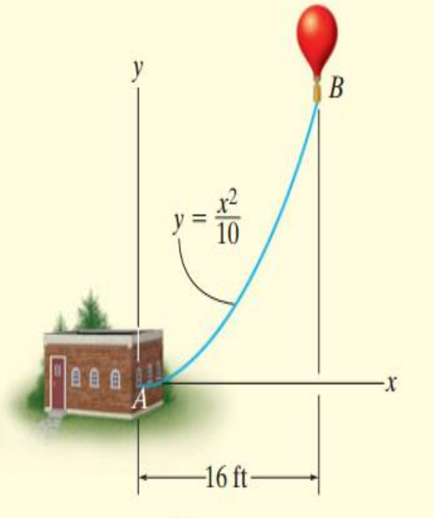
When using rectangular coordinates, the components along each of the axes do not change direction.

Only magnitude and algebraic sign will change

Problems

EXAMPLE 12.9

At any instant the horizontal position of the weather balloon in Fig. 12–18*a* is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

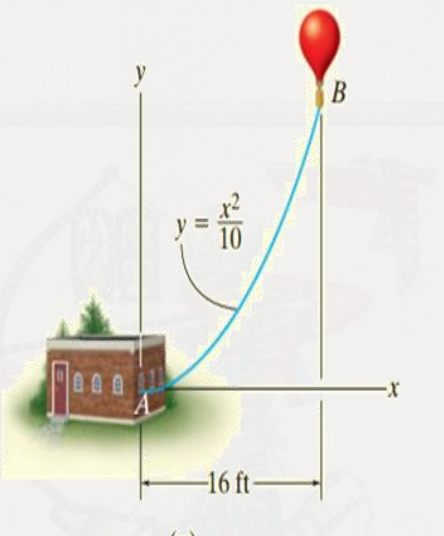


(a)

Problems

EXAMPLE 12.9

At any instant the horizontal position of the weather balloon in Fig. 12–18*a* is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.



(a)

SOLUTION

Velocity. The velocity component in the x direction is

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

To find the relationship between the velocity components we will use the chain rule of calculus. (See Appendix A for a full explanation.)

$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s} \qquad \textit{Ans.}$$

The direction is tangent to the path, Fig. 12–18*b*, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6^\circ \qquad \textit{Ans.}$$

Acceleration. The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

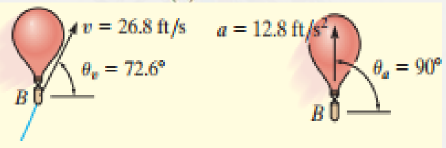
$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2 \qquad \textit{Ans.}$$

The direction of **a**, as shown in Fig. 12–18*c*, is

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90^\circ \qquad \textit{Ans.}$$



(b) (c)

NOTE: It is also possible to obtain v_y and a_y by first expressing $y = f(t) = (8t)^2/10 = 6.4t^2$ and then taking successive time derivatives.

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Problems

EXAMPLE 12.10

For a short time, the path of the plane in Fig. 12–19*a* is described by $y = (0.001x^2)$ m. If the plane is rising with a constant velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it is at $y = 100$ m.

Problems

EXAMPLE 12.10

For a short time, the path of the plane in Fig. 12–19*a* is described by $y = (0.001x^2)$ m. If the plane is rising with a constant velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it is at $y = 100$ m.

SOLUTION

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, since $v_y = 10$ m/s, then

$$y = v_y t; \qquad 100\text{ m} = (10\text{ m/s})\,t \qquad t = 10\text{ s}$$

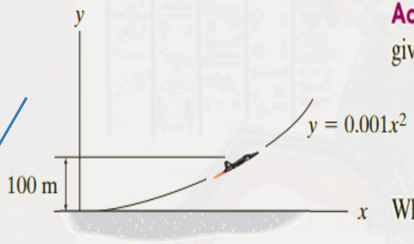
Velocity. Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

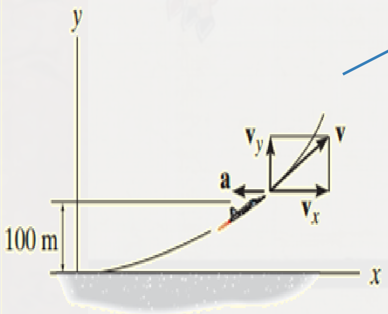
$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x \qquad (1)$$

$$10\text{ m/s} = 0.002(316.2\text{ m})(v_x)$$
$$v_x = 15.81\text{ m/s}$$

The magnitude of the velocity is therefore

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81\text{ m/s})^2 + (10\text{ m/s})^2} = 18.7\text{ m/s} \qquad \textit{Ans.}$$





Acceleration. Using the chain rule, the time derivative of Eq. (1) gives the relation between the acceleration components.

$$a_y = \dot{v}_y = 0.002\dot{x}v_x + 0.002x\dot{v}_x = 0.002(v_x^2 + xa_x)$$

When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

$$0 = 0.002((15.81\text{ m/s})^2 + 316.2\text{ m}(a_x))$$
$$a_x = -0.791\text{ m/s}^2$$

The magnitude of the plane's acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791\text{ m/s}^2)^2 + (0\text{ m/s}^2)^2}$$
$$= 0.791\text{ m/s}^2 \qquad \textit{Ans.}$$

These results are shown in Fig. 12–19*b*.

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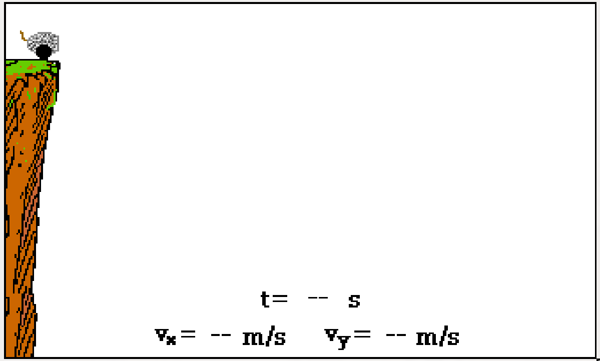
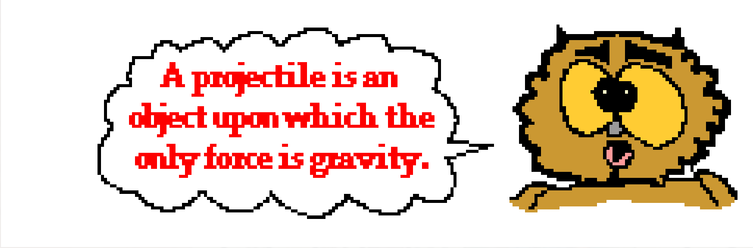
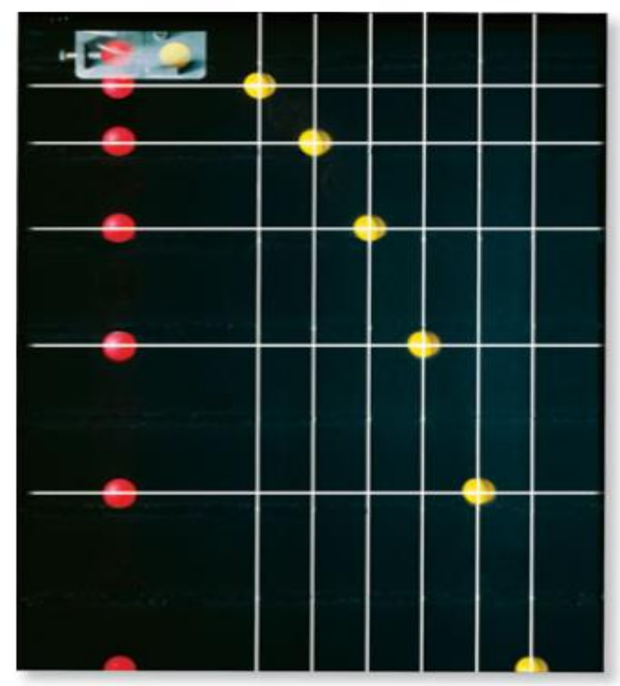
Projectile motion (Self Study)





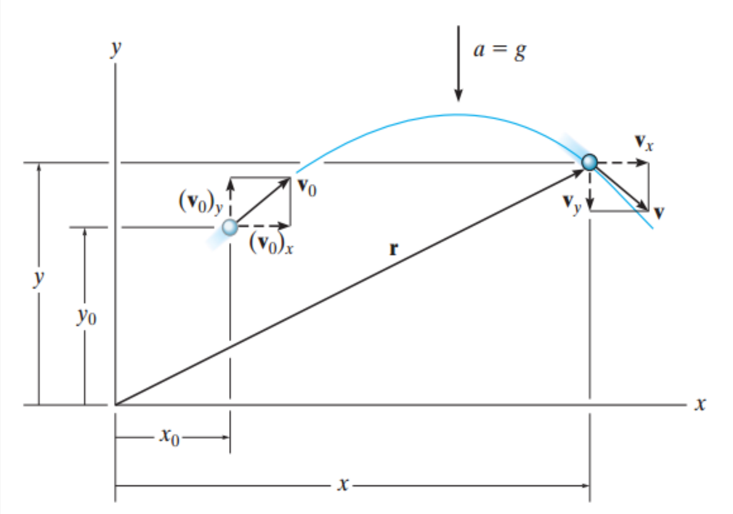
Motion of a projectile

Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration (i.e., gravity)



For illustration, consider the two balls on the left. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity. Each picture in this sequence is taken after the same time interval. Notice both balls are subjected to the same downward acceleration since they remain at the same elevation at any instant. Also, note that the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction is constant

Kinematic equations: Horizontal &Vertical motion



Since $a_x = 0$, the velocity in the horizontal direction remains constant ($v_x = v_{ox}$) and the position in the x direction can be determined by:

$$x = x_o + (v_{ox})(t)$$

Since the positive y-axis is directed upward, $a_y = -g$. Application of the constant acceleration equations yields:

$$v_y = v_{oy} - g(t)$$

$$y = y_o + (v_{oy})(t) - \frac{1}{2}g(t)^2$$

$$v_y^2 = v_{oy}^2 - 2g(y - y_o)$$

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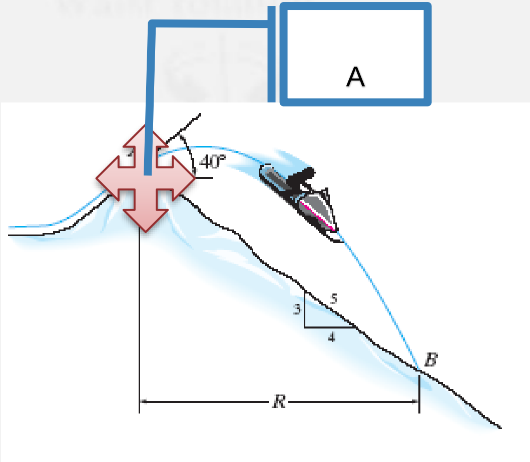
Example



Given: Snowmobile is going 15 m/s at point A.

Find: The horizontal distance it travels (R) and the time in the air.

Solution:

First, place the coordinate system at point A. Then write the equation for horizontal motion.

$\xrightarrow{+}$ $x_B = x_A + v_{Ax}t_{AB}$ and $v_{Ax} = 15 \cos 40°$ m/s

Now write a vertical motion equation. Use the distance equation.

$+\uparrow$ $y_B = y_A + v_{Ay}t_{AB} - 0.5g_c t_{AB}^2$ $v_{Ay} = 15 \sin 40°$ m/s

Note that $x_B = R$, $x_A = 0$, $y_B = -(3/4)R$, and $y_A = 0$.

Solving the two equations together (two unknowns) yields

$R = 19.0$ m $t_{AB} = 2.48$ s

Problems

EXAMPLE 12.12

The chipping machine is designed to eject wood chips at $v_O = 25$ ft/s as shown in Fig. 12–22. If the tube is oriented at 30° from the horizontal, determine how high, h, the chips strike the pile if at this instant they land on the pile 20 ft from the tube.

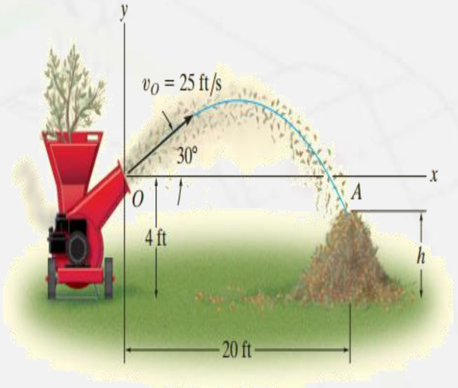


Fig. 12–22

SOLUTION

Coordinate System. When the motion is analyzed between points O and A, the three unknowns are the height h, time of flight t_{OA}, and vertical component of velocity $(v_A)_y$. [Note that $(v_A)_x = (v_O)_x$.] With the origin of coordinates at O, Fig. 12–22, the initial velocity of a chip has components of

$$(v_O)_x = (25 \cos 30°) \text{ ft/s} = 21.65 \text{ ft/s} \rightarrow$$

$$(v_O)_y = (25 \sin 30°) \text{ ft/s} = 12.5 \text{ ft/s} \uparrow$$

Also, $(v_A)_x = (v_O)_x = 21.65$ ft/s and $a_y = -32.2 \text{ ft/s}^2$. Since we do not need to determine $(v_A)_y$, we have

Horizontal Motion.

$$(\overset{+}{\rightarrow}) \qquad x_A = x_O + (v_O)_x t_{OA}$$

$$20 \text{ ft} = 0 + (21.65 \text{ ft/s}) t_{OA}$$

$$t_{OA} = 0.9238 \text{ s}$$

Vertical Motion. Relating t_{OA} to the initial and final elevations of a chip, we have

$$(+\uparrow) \qquad y_A = y_O + (v_O)_y t_{OA} + \tfrac{1}{2} a_c t_{OA}^2$$

$$(h - 4 \text{ ft}) = 0 + (12.5 \text{ ft/s})(0.9238 \text{ s}) + \tfrac{1}{2}(-32.2 \text{ ft/s}^2)(0.9238 \text{ s})^2$$

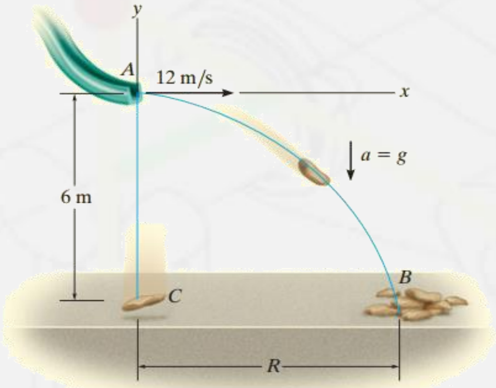
$$h = 1.81 \text{ ft} \qquad \textit{Ans.}$$

NOTE: We can determine $(v_A)_y$ by using $(v_A)_y = (v_O)_y + a_c t_{OA}$.

Problems

EXAMPLE 12.11

A sack slides off the ramp, shown in Fig. 12–21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up.



SOLUTION

Coordinate System. The origin of coordinates is established at the beginning of the path, point A, Fig. 12–21. The initial velocity of a sack has components $(v_A)_x = 12$ m/s and $(v_A)_y = 0$. Also, between points A and B the acceleration is $a_y = -9.81\ \text{m/s}^2$. Since $(v_B)_x = (v_A)_x = 12$ m/s, the three unknowns are $(v_B)_y$, R, and the time of flight t_{AB}. Here we do not need to determine $(v_B)_y$.

Vertical Motion. The vertical distance from A to B is known, and therefore we can obtain a direct solution for t_{AB} by using the equation

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$

$$-6\ \text{m} = 0 + 0 + \tfrac{1}{2}(-9.81\ \text{m/s}^2)t_{AB}^2$$

$$t_{AB} = 1.11\ \text{s} \qquad \textit{Ans.}$$

Horizontal Motion. Since t_{AB} has been calculated, R is determined as follows:

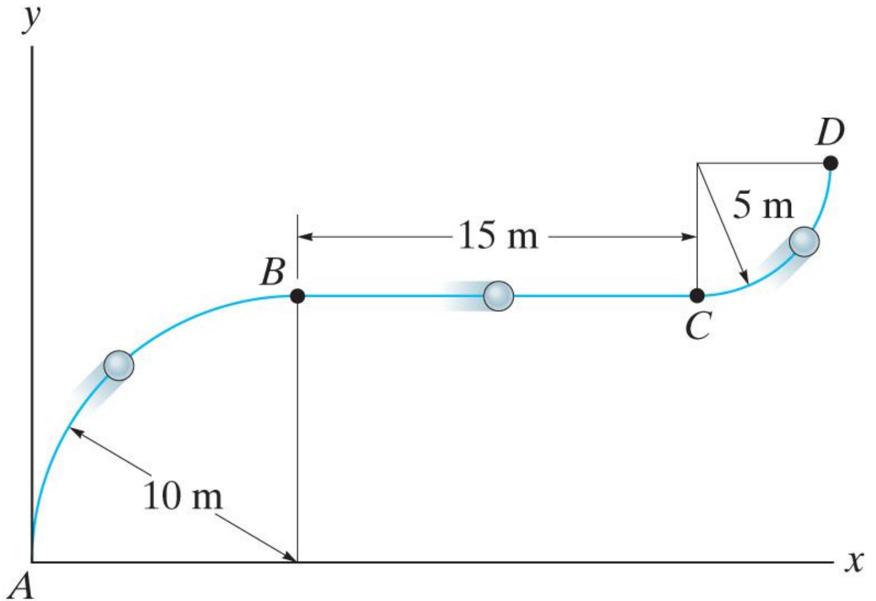
$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t_{AB}$$

$$R = 0 + 12\ \text{m/s}\ (1.11\ \text{s})$$

$$R = 13.3\ \text{m} \qquad \textit{Ans.}$$

Problems (Solve it at your home)

12-71. A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



End of the Lecture

Let Learning Continue

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ENME232: Dynamics

CH 12: Kinematics of a particle

Lecture 4: Sections 12.7-12.8

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Recap of the Previous Class Agenda

1. General curvilinear motion
2. Curvilinear motion: Rectangular components
3. Motion of a projectile (Self Study)

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Today's Class Agenda

1. Curvilinear Motion: Normal and Tangential Components
2. Curvilinear Motion: Cylindrical Components

Part 1

Objectives

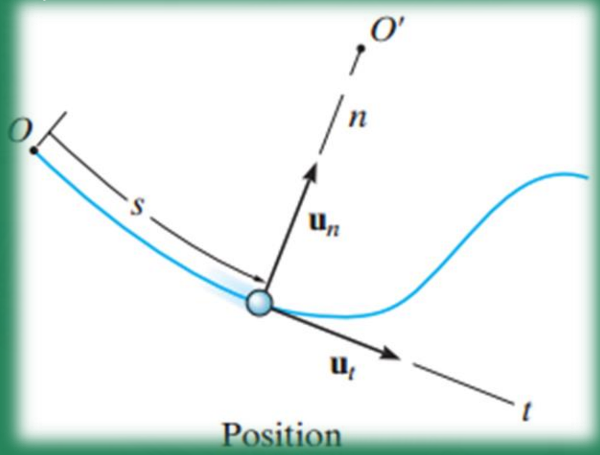
Sections' Objectives

Students should be able to:

1. Determine the normal and tangential components of velocity and acceleration of a particle traveling along a curved path. (Sec 12.7)
2. Determine velocity and acceleration components using cylindrical coordinates. (Sec 12.8)

Normal and tangential components I

When a particle moves along a curved path, it is sometimes convenient to describe its motion using coordinates other than Cartesian. When the path of motion is known, normal (n) and tangential (t) coordinates are often used



In the n-t coordinate system, the origin is located on the particle (the origin moves with the particle)

The t-axis is tangent to the path (curve) at the instant considered, positive in the direction of the particle's motion

The n-axis is perpendicular to the t-axis with the positive direction toward the center of curvature of the curve

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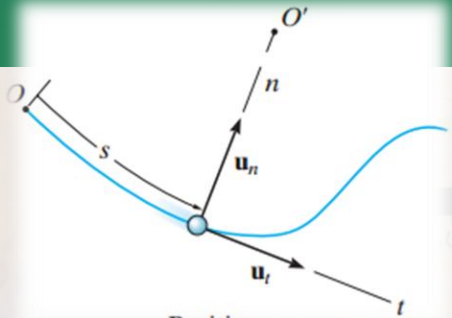
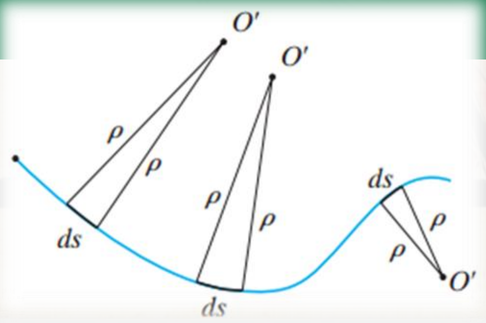
Normal and tangential components II

- The positive n and t directions are defined by the unit vectors $\boldsymbol{u}_n$ and $\boldsymbol{u}_t$, respectively.
- The center of curvature, O', always lies on the concave side of the curve.
- The radius of curvature, ρ, is defined as the perpendicular distance from the curve to the center of curvature at that point.
- The position of the particle at any instant is defined by the distance, s, along the curve from a fixed reference point.



Position



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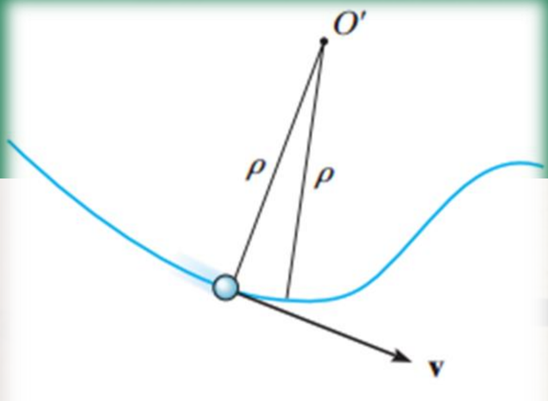
Velocity in the n-t coordinate system

The velocity vector is always tangent to the path of motion (t-direction)

The magnitude is determined by taking the time derivative of the path function, s(t)

$$\boldsymbol{v} = V\,\boldsymbol{u}_t \qquad \text{where } V = ds/dt = \dot{S}$$

Here V defines the magnitude of the velocity (speed) and $\boldsymbol{u}_t$ defines the direction of the velocity vector.



Velocity

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Acceleration in the n-t coordinate system I

Acceleration is the time rate of change of velocity:

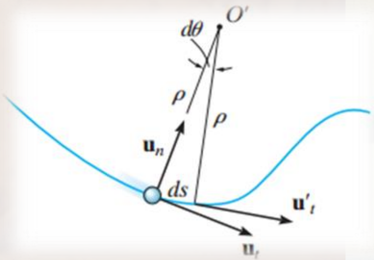
$$\boldsymbol{a} = d\boldsymbol{v}/dt = d(v\boldsymbol{u}_t)/dt = \dot{v}\boldsymbol{u}_t + v\,\dot{\boldsymbol{u}}_t$$

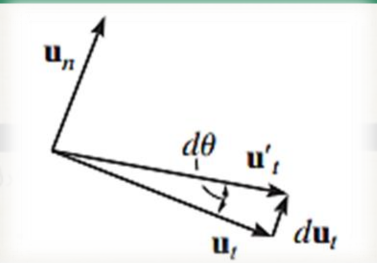
Here $\dot{v}$ represents the change in the magnitude of velocity and $\dot{\boldsymbol{u}}_t$ represents the rate of change in the direction of $\boldsymbol{u}_t$.

$\dot{u}_t$: note that as the particle moves along the arc ds in time dt

$$\dot{u}_t = du_t = 1(d\theta) = \dot{\theta}u_n = \frac{\dot{s}}{\rho} = \frac{v}{\rho}$$

After mathematical manipulation, the acceleration vector can be expressed as: $\boldsymbol{a} = \dot{v}\boldsymbol{u}_t + (v^2/\rho)\boldsymbol{u}_n = a_t\boldsymbol{u}_t + a_n\boldsymbol{u}_n$





Acceleration in the n-t coordinate system II

There are two components to the acceleration vector:

$$\boldsymbol{a} = a_t \boldsymbol{u}_t + a_n \boldsymbol{u}_n$$

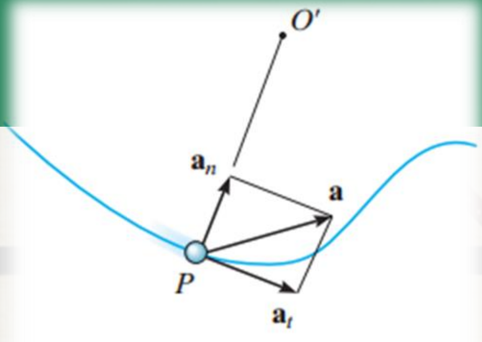
The tangential component is tangent to the curve and in the direction of increasing or decreasing velocity.

$$a_t = \dot{v} \quad \text{or} \quad a_t \, ds = v \, dv$$

The normal or centripetal component is always directed toward the center of curvature of the curve. $a_n = v^2/\rho$

The magnitude of the acceleration vector is

$$a = \sqrt{[(a_t)^2 + (a_n)^2]}$$



Acceleration

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Special cases of motion I

There are some special cases of motion to consider

1) The particle moves along a straight line.

$\rho = \infty \quad => \quad a_n = v^2/\rho = 0 \quad => \quad a = a_t = \dot{v}$

The tangential component represents the time rate of change in the magnitude of the velocity.

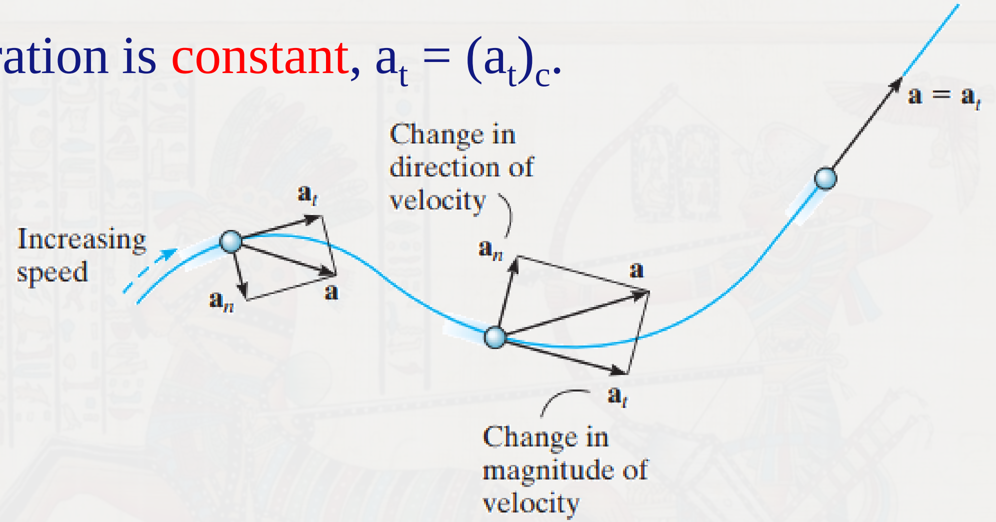
2) The particle moves along a curve at constant speed.

$a_t = \dot{v} = 0 \quad => \quad a = a_n = v^2/\rho$

The normal component represents the time rate of change in the direction of the velocity.

Special cases of motion II

3) The tangential component of acceleration is constant, $a_t = (a_t)_c$.



In this case,

$$s = s_o + v_o t + (1/2)(a_t)_c t^2$$

$$v = v_o + (a_t)_c t$$

$$v^2 = (v_o)^2 + 2(a_t)_c(s - s_o)$$

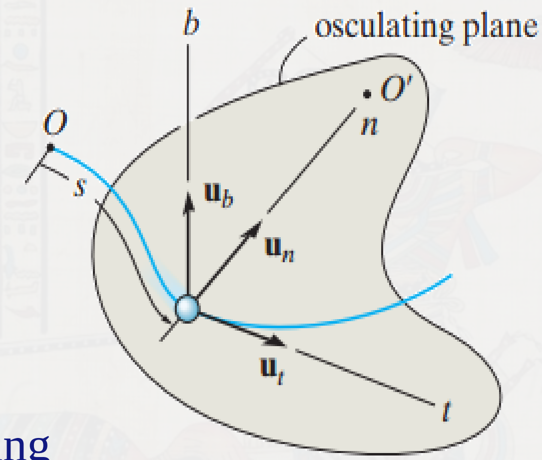
As before, s_o and v_o are the initial position and velocity of the particle at $t = 0$

4) The particle moves along a path expressed as $y = f(x)$.
The radius of curvature, ρ, at any point on the path can be calculated from

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{\left| d^2y/dx^2 \right|}$$

Three dimensional motion

If a particle moves along a space curve, the n and t axes are defined as before. At any point, the t-axis is tangent to the path and the n-axis points toward the center of curvature. The plane containing the n and t axes is called the osculating plane.



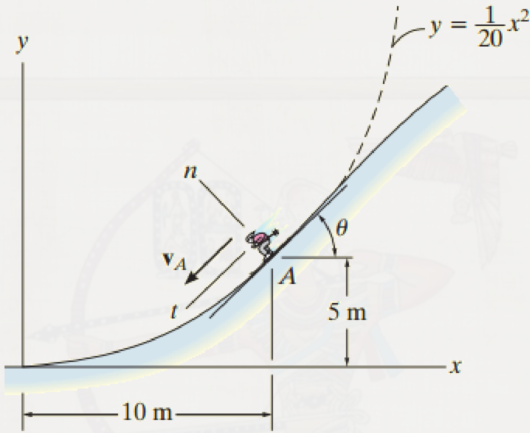
A third axis can be defined, called the binomial axis, b. The binomial unit vector, $\boldsymbol{u}_b$, is directed perpendicular to the osculating plane, and its sense is defined by the cross product $\boldsymbol{u}_b = \boldsymbol{u}_t \times \boldsymbol{u}_n$.

There is no motion, thus no velocity or acceleration, in the binomial direction.

Problem

EXAMPLE 12.14

When the skier reaches point A along the parabolic path in Fig. 12–27a, he has a speed of 6 m/s which is increasing at 2 m/s^2. Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.



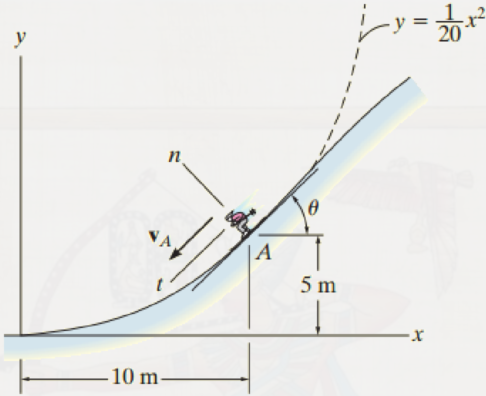
SOLUTION

Coordinate System. Although the path has been expressed in terms of its x and y coordinates, we can still establish the origin of the n, t axes at the fixed point A on the path and determine the components of $\mathbf{v}$ and $\mathbf{a}$ along these axes, Fig. 12–27a.

EXAMPLE 12.14

Problem

When the skier reaches point A along the parabolic path in Fig. 12–27a, he has a speed of 6 m/s which is increasing at 2 m/s^2. Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.



SOLUTION

Coordinate System. Although the path has been expressed in terms of its x and y coordinates, we can still establish the origin of the n, t axes at the fixed point A on the path and determine the components of $\mathbf{v}$ and $\mathbf{a}$ along these axes, Fig. 12–27a.

Velocity. By definition, the velocity is always directed tangent to the path. Since $y = \frac{1}{20}x^2$, $dy/dx = \frac{1}{10}x$, then at $x = 10$ m, $dy/dx = 1$. Hence, at A, $\mathbf{v}$ makes an angle of $\theta = \tan^{-1}1 = 45°$ with the x axis, Fig. 12–27a. Therefore,

$$v_A = 6\text{ m/s} \qquad 45° \measuredangle$$ *Ans.*

The acceleration is determined from $\mathbf{a} = \dot{v}\mathbf{u}_t + (v^2/\rho)\mathbf{u}_n$. However, it is first necessary to determine the radius of curvature of the path at A (10 m, 5 m). Since $d^2y/dx^2 = \frac{1}{10}$, then

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{\left|\frac{1}{10}\right|}\right|_{x=10\text{ m}} = 28.28\text{ m}$$

The acceleration becomes

$$\begin{aligned}\mathbf{a}_A &= \dot{v}\mathbf{u}_t + \frac{v^2}{\rho}\mathbf{u}_n \\ &= 2\mathbf{u}_t + \frac{(6\text{ m/s})^2}{28.28\text{ m}}\mathbf{u}_n \\ &= \{2\mathbf{u}_t + 1.273\mathbf{u}_n\}\text{m/s}^2\end{aligned}$$

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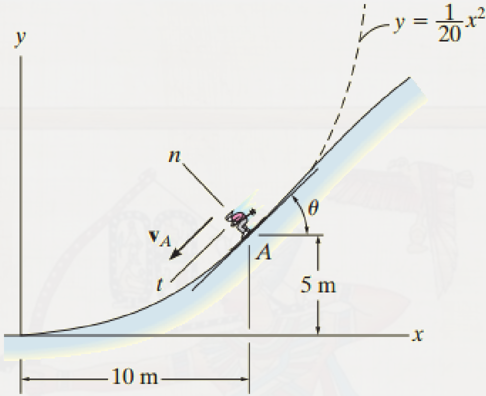
EXAMPLE 12.14

Problem

When the skier reaches point A along the parabolic path in Fig. 12–27*a*, he has a speed of 6 m/s which is increasing at 2 m/s². Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.



SOLUTION

Coordinate System. Although the path has been expressed in terms of its x and y coordinates, we can still establish the origin of the n, t axes at the fixed point A on the path and determine the components of $\mathbf{v}$ and $\mathbf{a}$ along these axes, Fig. 12–27*a*.

Velocity. By definition, the velocity is always directed tangent to the path. Since $y = \frac{1}{20}x^2$, $dy/dx = \frac{1}{10}x$, then at $x = 10$ m, $dy/dx = 1$. Hence, at A, $\mathbf{v}$ makes an angle of $\theta = \tan^{-1}1 = 45°$ with the x axis, Fig. 12–27*a*. Therefore,

$$v_A = 6 \text{ m/s} \quad 45° \qquad \textit{Ans.}$$

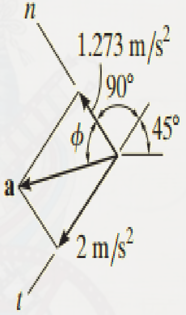
As shown in Fig. 12–27*b*,

$$a = \sqrt{(2 \text{ m/s}^2)^2 + (1.273 \text{ m/s}^2)^2} = 2.37 \text{ m/s}^2$$

$$\phi = \tan^{-1}\frac{2}{1.273} = 57.5°$$

Thus, $45° + 90° + 57.5° - 180° = 12.5°$ so that,

$$a = 2.37 \text{ m/s}^2 \quad 12.5° \qquad \textit{Ans.}$$



(b)

NOTE: By using n, t coordinates, we were able to readily solve this problem through the use of Eq. 12–18, since it accounts for the separate changes in the magnitude and direction of $\mathbf{v}$.

Fig. 12–27

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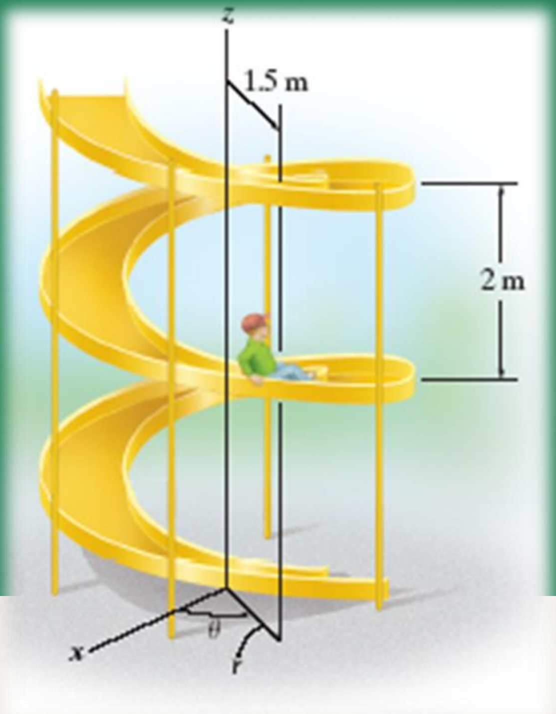
Curvilinear motion: Cylindrical components (12.8)

Applications

Sometimes the motion of the particle is constrained on a path that is best described using cylindrical coordinates. If motion is restricted to the plane, then polar coordinates are used.

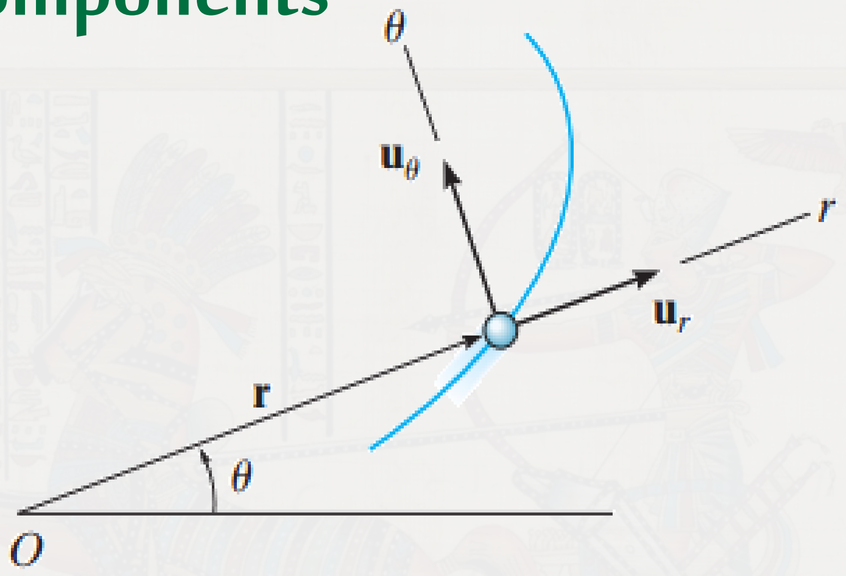
The cylindrical coordinate system is used in cases where the particle moves along a 3-D curve.



(spiral motion)

Cylindrical components

We can express the location of P in polar coordinates as using a radial coordinate r, which extends outward from the fixed origin O to the particle, and a transverse coordinate θ which is the counterclockwise angle between a fixed reference line and the r axis.

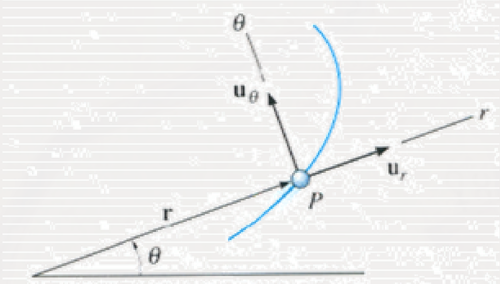


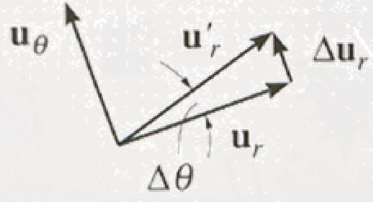
$r = \mathrm{r}\boldsymbol{u}_r$.

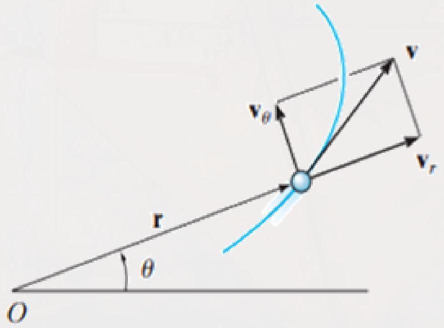
u_r is in the direction of increasing r when θ is held fixed, and is in a direction of increasing θ when r is held fixed. and u_θ is in a direction of increasing θ when r is held fixed.

Note that these directions are perpendicular to one another.

Velocity (Polar coordinates)







Velocity

The instantaneous velocity is defined as:

$$v = d\mathbf{r}/dt = d(r u_r)/dt$$

$$v = \dot{r} u_r + r \frac{du_r}{dt}$$

Using the chain rule:

$$du_r/dt = (du_r/d\theta)(d\theta/dt)$$

We can prove that $du_r/d\theta = u_\theta$ so $du_r/dt = \dot{\theta} u_\theta$

Therefore: $v = \dot{r} u_r + r\dot{\theta} u_\theta$

Thus, the velocity vector has two components: $\dot{r}$, called the radial component, and $r\dot{\theta}$, called the transverse component. The speed of the particle at any given instant is the sum of the squares of both components or

$$v = \sqrt{(r\dot{\theta})^2 + (\dot{r})^2}$$

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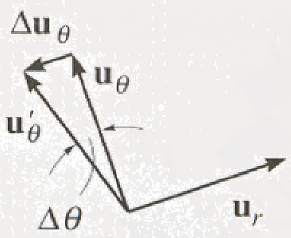
Acceleration (Polar coordinates)

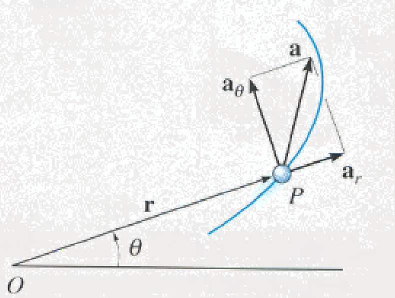
The instantaneous acceleration is defined as:



$$a = dv/dt = d(\dot{r}u_r + r\dot{\theta}u_\theta)/dt$$

After manipulation, the acceleration can be expressed as

$$a = (\ddot{r} - r\dot{\theta}^2)\, u_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})u_\theta$$



Acceleration

The term $(\ddot{r} - r\dot{\theta}^2)$ is the radial acceleration or a_r

The term $(r\ddot{\theta} + 2\dot{r}\dot{\theta})$ is the transverse acceleration or a_θ

The magnitude of acceleration is $a = \sqrt{((\ddot{r} - r\dot{\theta}^2))^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$

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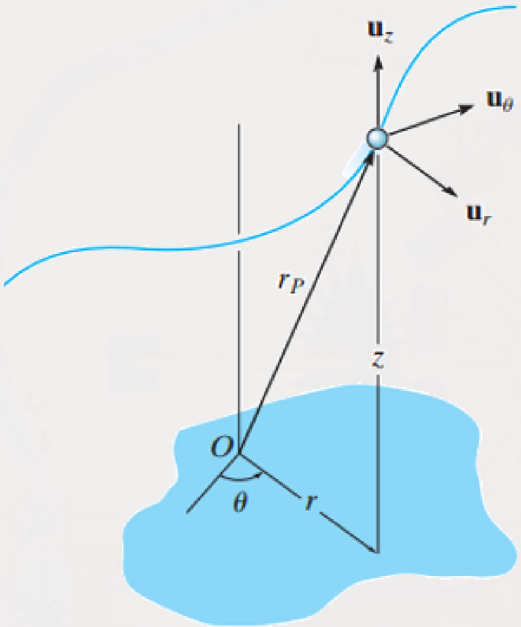
Cylindrical coordinates



If the particle P moves along a space curve, its position can be written as

$$\boldsymbol{r_P} = \mathrm{r}\boldsymbol{u_r} + \mathrm{z}\boldsymbol{u_z} + \theta u_\theta$$

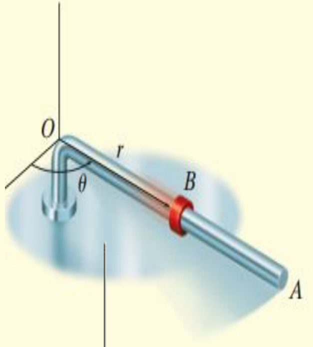
Taking time derivatives and using the chain rule:

Velocity: $\boldsymbol{v_P} = \dot{\mathrm{r}}\boldsymbol{u_r} + \mathrm{r}\dot{\theta}\boldsymbol{u_\theta} + \dot{\mathrm{z}}\boldsymbol{u_z}$

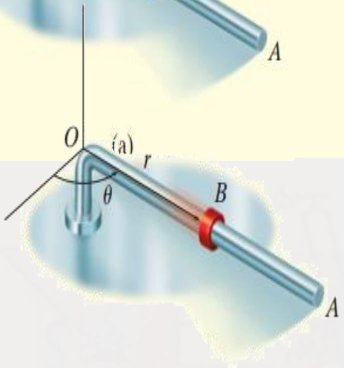
Acceleration: $\boldsymbol{a_P} = (\ddot{\mathrm{r}} - \mathrm{r}\dot{\theta}^2)\boldsymbol{u_r} + (\mathrm{r}\ddot{\theta} + 2\dot{\mathrm{r}}\dot{\theta})\boldsymbol{u_\theta} + \ddot{\mathrm{z}}\boldsymbol{u_z}$

Problem

EXAMPLE 12.18



(a)



(a)

The rod OA in Fig. 12–33a rotates in the horizontal plane such that $\theta = (t^3)$ rad. At the same time, the collar B is sliding outward along OA so that $r = (100t^2)$ mm. If in both cases t is in seconds, determine the velocity and acceleration of the collar when $t = 1$ s.

SOLUTION

Coordinate System. Since time-parametric equations of the path are given, it is not necessary to relate r to θ.

Velocity and Acceleration. Determining the time derivatives and evaluating them when $t = 1$ s, we have

$$r = 100t^2\Big|_{t=1\,\text{s}} = 100\,\text{mm} \quad \theta = t^3\Big|_{t=1\,\text{s}} = 1\text{ rad} = 57.3^\circ$$

$$\dot{r} = 200t\Big|_{t=1\,\text{s}} = 200\text{ mm/s} \quad \dot{\theta} = 3t^2\Big|_{t=1\,\text{s}} = 3\text{ rad/s}$$

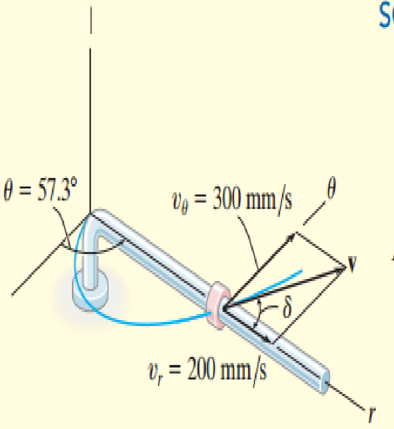
Problem

EXAMPLE 12.18

The rod *OA* in Fig. 12–33*a* rotates in the horizontal plane such that $\theta = (t^3)$ rad. At the same time, the collar *B* is sliding outward along *OA* so that $r = (100t^2)$ mm. If in both cases *t* is in seconds, determine the velocity and acceleration of the collar when $t = 1$ s.

SOLUTION

$$\ddot{r} = 200\Big|_{t=1\,\text{s}} = 200\ \text{mm/s}^2 \quad \ddot{\theta} = 6t\Big|_{t=1\,\text{s}} = 6\ \text{rad/s}^2.$$



(b)

As shown in Fig. 12–33*b*,

$$\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta$$

$$= 200\mathbf{u}_r + 100(3)\mathbf{u}_\theta = \{200\mathbf{u}_r + 300\mathbf{u}_\theta\}\ \text{mm/s}$$

The magnitude of **v** is

$$v = \sqrt{(200)^2 + (300)^2} = 361\ \text{mm/s} \qquad \textit{Ans.}$$

$$\delta = \tan^{-1}\left(\frac{300}{200}\right) = 56.3^\circ \qquad \delta + 57.3^\circ = 114^\circ \qquad \textit{Ans.}$$

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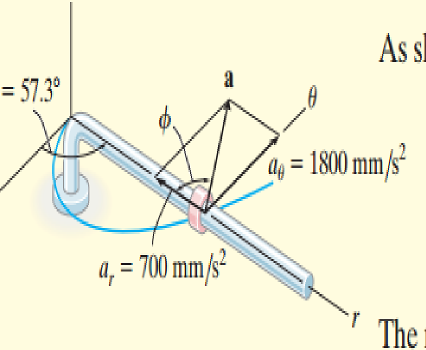
Problem

EXAMPLE 12.18

The rod OA in Fig. 12–33a rotates in the horizontal plane such that $\theta = (t^3)$ rad. At the same time, the collar B is sliding outward along OA so that $r = (100t^2)$ mm. If in both cases t is in seconds, determine the velocity and acceleration of the collar when $t = 1$ s.



(c)

Fig. 12–33

As shown in Fig. 12–33c,

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta$$

$$= [200 - 100(3)^2]\mathbf{u}_r + [100(6) + 2(200)3]\mathbf{u}_\theta$$

$$= \{-700\mathbf{u}_r + 1800\mathbf{u}_\theta\} \text{ mm/s}^2$$

The magnitude of **a** is

$$a = \sqrt{(700)^2 + (1800)^2} = 1930 \text{ mm/s}^2 \qquad \textit{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{1800}{700}\right) = 68.7^\circ \qquad (180^\circ - \phi) + 57.3^\circ = 169^\circ \qquad \textit{Ans.}$$

NOTE: The velocity is tangent to the path; however, the acceleration is directed within the curvature of the path, as expected.

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Problem

EXAMPLE 12.17

The amusement park ride shown in Fig. 12–32*a* consists of a chair that is rotating in a horizontal circular path of radius r such that the arm OB has an angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$. Determine the radial and transverse components of velocity and acceleration of the passenger. Neglect his size in the calculation.

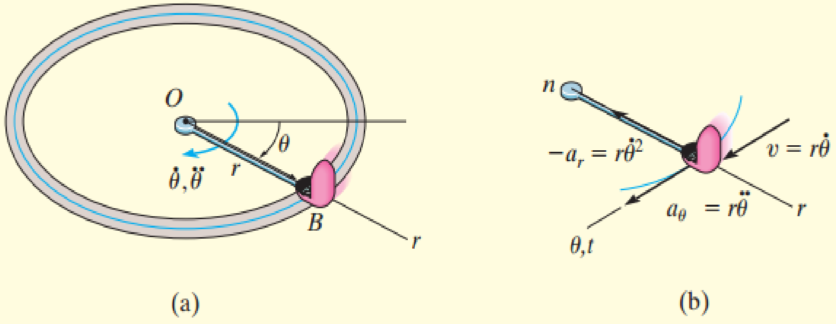
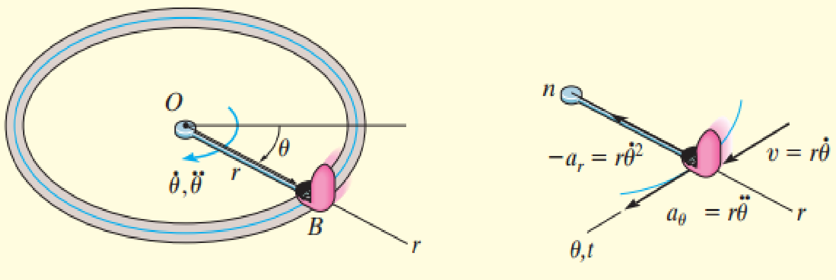


Fig. 12–32

Problem

EXAMPLE 12.17

The amusement park ride shown in Fig. 12–32*a* consists of a chair that is rotating in a horizontal circular path of radius r such that the arm OB has an angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$. Determine the radial and transverse components of velocity and acceleration of the passenger. Neglect his size in the calculation.



(a) (b)

Fig. 12–32

SOLUTION

Coordinate System. Since the angular motion of the arm is reported, polar coordinates are chosen for the solution, Fig. 12–32*a*. Here θ is not related to r, since the radius is constant for all θ.

Velocity and Acceleration. It is first necessary to specify the first and second time derivatives of r and θ. Since r is *constant*, we have

$$r = r \qquad \dot{r} = 0 \qquad \ddot{r} = 0$$

Thus,

$$v_r = \dot{r} = 0 \qquad \textit{Ans.}$$

$$v_\theta = r\dot{\theta} \qquad \textit{Ans.}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -r\dot{\theta}^2 \qquad \textit{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r\ddot{\theta} \qquad \textit{Ans.}$$

These results are shown in Fig. 12–32*b*.

These results are shown in Fig. 12–32*b*.

NOTE: The n, t axes are also shown in Fig. 12–32*b*, which in this special case of circular motion happen to be *collinear* with the r and θ axes, respectively. Since $v = v_\theta = v_t = r\dot{\theta}$, then by comparison,

$$-a_r = a_n = \frac{v^2}{\rho} = \frac{(r\dot{\theta})^2}{r} = r\dot{\theta}^2$$

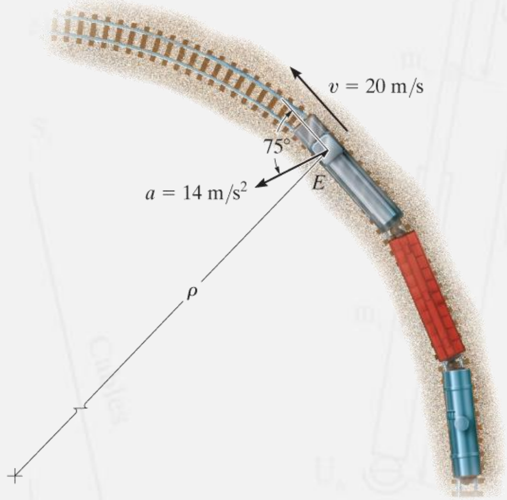
$$a_\theta = a_t = \frac{dv}{dt} = \frac{d}{dt}(r\dot{\theta}) = \frac{dr}{dt}\dot{\theta} + r\frac{d\dot{\theta}}{dt} = 0 + r\ddot{\theta}$$

Problems (Solve it at your home)

***12-100.** A car is traveling along a circular curve that has a radius of 50 m. If its speed is 16 m/s and is increasing uniformly at 8 m/s^2, determine the magnitude of its acceleration at this instant.

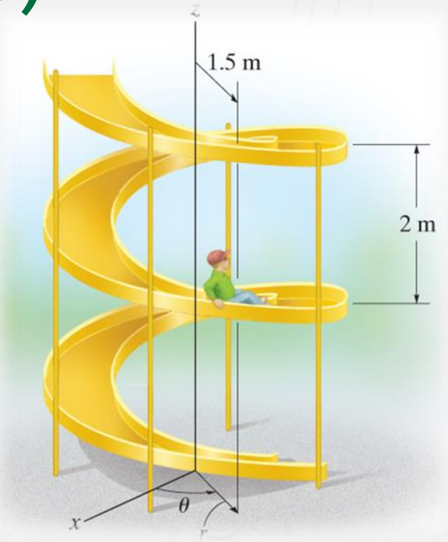
Problems (Solve it at your home)

12-111. At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



Problems (Solve it at your home)

12-153. The boy slides down the slide at a constant speed of 2 m/s. If the slide is in the form of a helix, defined by the equations $r = 1.5$ m and $z = -\theta/\pi$, determine the boy's angular velocity about the z axis, $\dot{\theta}$, and the magnitude of his acceleration.



ENME232: Dynamics

CH 12: Kinematics of a particle

Lecture 5: Sections 12.9-12.10

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Recap of the Previous Class Agenda

1. Curvilinear Motion: Normal and Tangential Components
2. Curvilinear Motion: Cylindrical Components

Today's Class Agenda

1. Absolute Dependent Motion Analysis of Two Particles
2. Relative-Motion of Two Particles Using Translating Axes

Part 1

Objectives

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Sections' Objectives

Students should be able to:

1. Relate the positions, velocities, and accelerations of particles undergoing dependent motion (Sec 12.9)
2. Understand translating frames of reference
3. Use translating frames of reference to analyze relative motion (Sec 12.10)

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Applications I

The cable and pulley system shown here can be used to modify the speed of block B relative to the speed of the motor. It is important to relate the various motions in order to determine the power requirements for the motor and the tension in the cable

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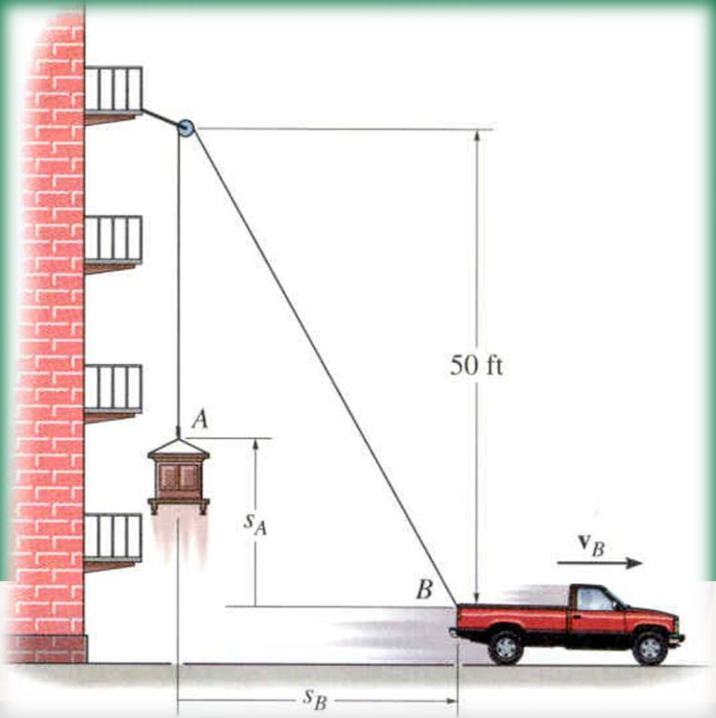
Applications II

Rope and pulley arrangements are often used to assist in lifting heavy objects. The total lifting force required from the truck depends on the acceleration of the cabinet.



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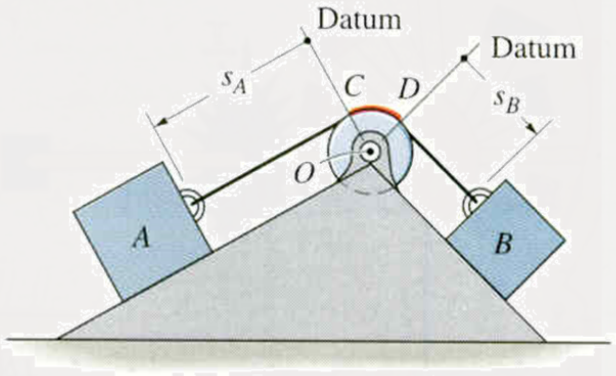
Dependent motion

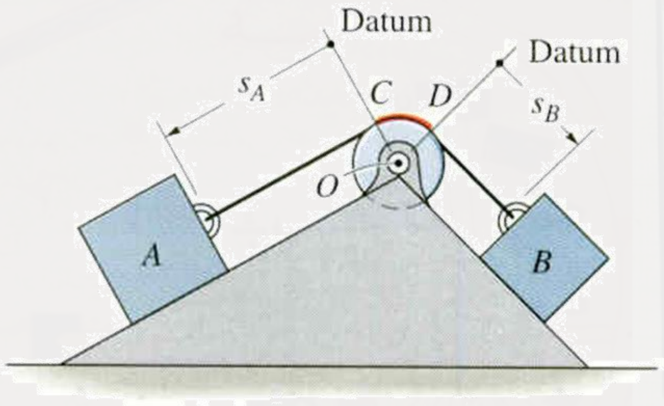
In many kinematics problems, the motion of one object will depend on the motion of another object



The blocks in this figure are connected by an inextensible cord wrapped around a pulley. If block A moves downward along the inclined plane, block B will move up the other incline

The motion of each block can be related mathematically by defining position coordinates, s_A and s_B. Each coordinate axis is defined from a fixed point or datum line, measured positive along each plane in the direction of motion of each block.

Dependent motion



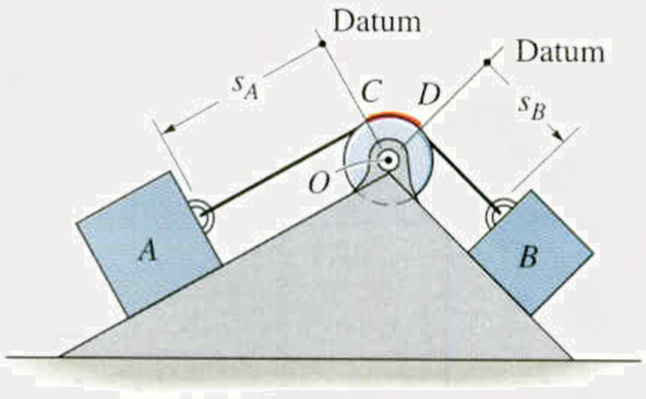
In this example, position coordinates s_A and s_B can be defined from fixed datum lines extending from the center of the pulley along each incline to blocks A and B

If the cord has a fixed length, the position coordinates s_A and s_B are related mathematically by the equation

$$s_A + l_{CD} + s_B = l_T$$

Here l_T is the total cord length and l_{CD} is the length of cord passing over arc CD on the pulley

Dependent motion



The velocities of blocks A and B can be related by differentiating the position equation. Note that l_{CD} and l_T remain constant,

so $dl_{CD}/dt = dl_T/dt = 0$

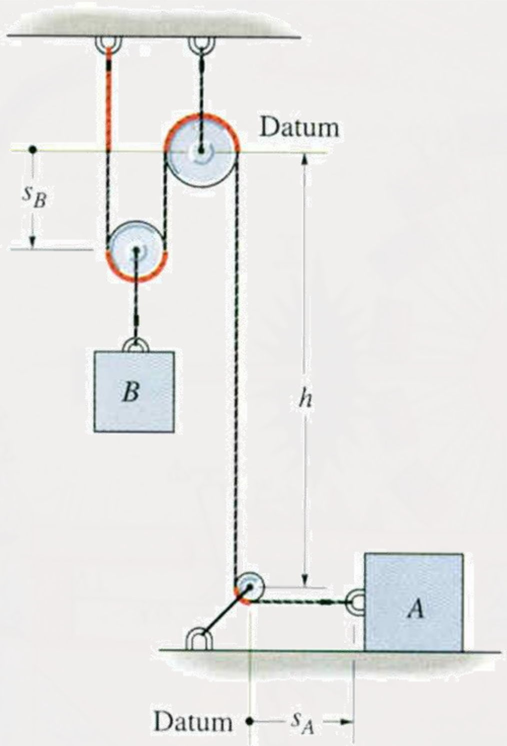
$$ds_A/dt + ds_B/dt = 0 \quad => \quad v_B = -v_A$$

-The negative sign indicates that as A moves down the incline (positive s_A direction), B moves up the incline (negative s_B direction)

-Accelerations can be found by differentiating the velocity expression

$$a_B = -a_A$$

Example

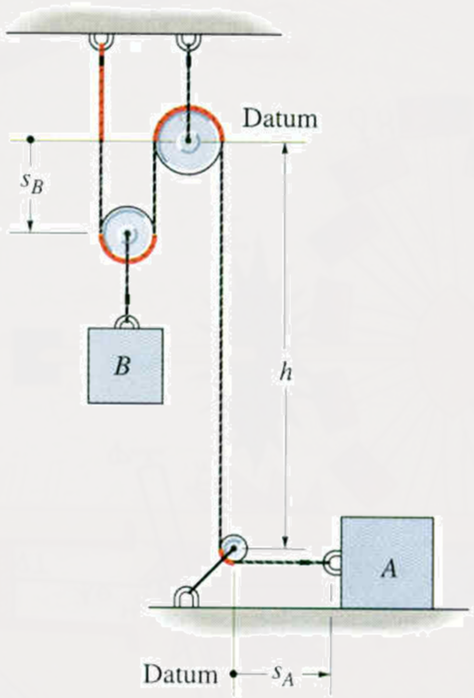


Consider a more complicated example. Position coordinates (s_A and s_B) are defined from fixed datum lines, measured along the direction of motion of each block

Note that s_B is only defined to the center of the pulley above block B, since this block moves with the pulley. Also, h is a constant

The red colored segments of the cord remain constant in length during motion of the blocks

Example



The position coordinates are related by the equation

$$2s_B + h + s_A = l$$

Where l is the total cord length minus the lengths of the red segments

Since l and h remain constant during the motion, the velocities and accelerations can be related by two successive time derivatives:

$2v_B = -v_A$ and $2a_B = -a_A$

When block B moves downward ($+s_B$), block A moves to the left ($-s_A$).
Remember to be consistent with the sign convention!

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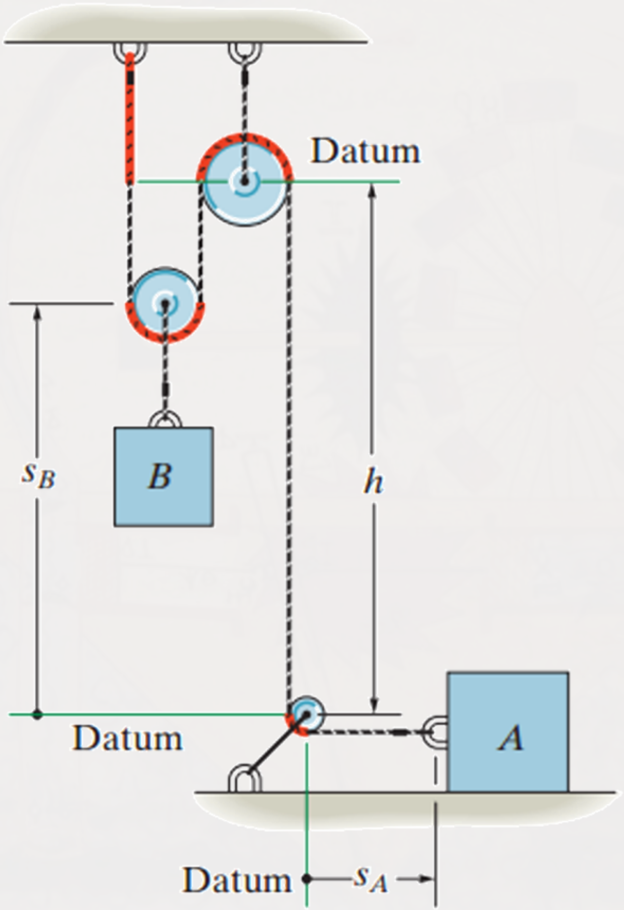
Example



This example can also be worked by defining the position coordinate for B (s_B) from the bottom pulley instead of the top pulley

The position, velocity, and acceleration relations then become

$$2(h - s_B) + h + s_A = l$$

and $2v_B = v_A$ $\quad$ $2a_B = a_A$

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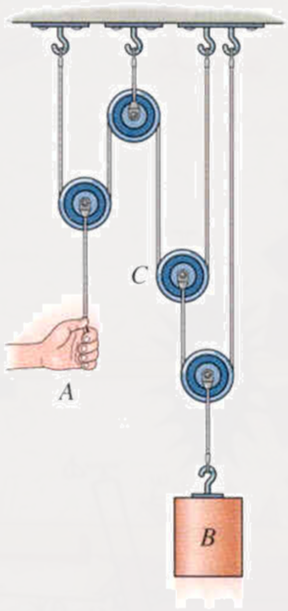
Dependent motion: Procedures for analysis

These procedures can be used to relate the dependent motion of particles moving along rectilinear paths (only the magnitudes of velocity and acceleration change, not their line of direction)

1) Define position coordinates from fixed datum lines, along the path of each particle. Different datum lines can be used for each particle
2) Relate the position coordinates to the cord length. Segments of cord that do not change in length during the motion may be left out
3) If a system contains more than one cord, relate the position of a point on one cord to a point on another cord. Separate equations are written for each cord
4) Differentiate the position coordinate equation(s) to relate velocities and accelerations. Keep track of signs!

Example



Given: In the figure on the left, the cord at A is pulled down with a speed of 8 ft/s

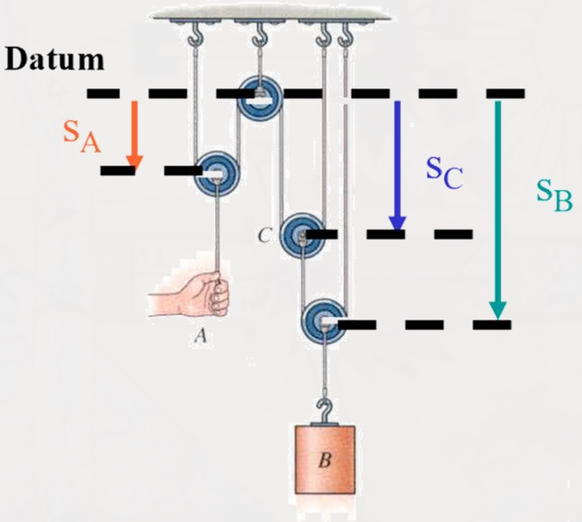
Find: The speed of block B

Plan: There are two cords involved in the motion in this example. The position of a point on one cord must be related to the position of a point on the other cord. There will be two position equations (one for each cord)

Example

Solution:

1) Define the position coordinates from a fixed datum line. Three coordinates must be defined: one for point A (s_A), one for block B (s_B), and one relating positions on the two cords. Note that pulley C relates the motion of the two cords



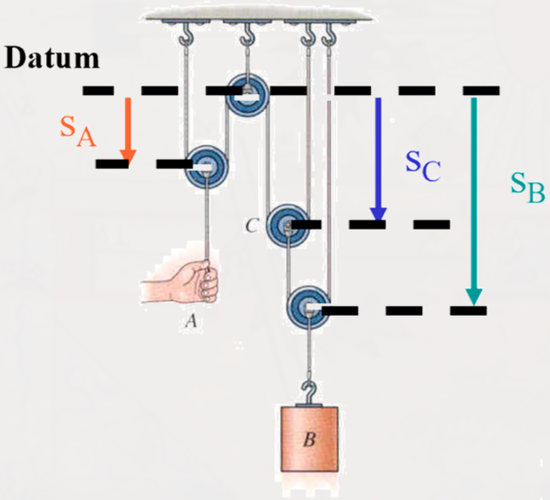
- Define the datum line through the top pulley (which has a fixed position).
- s_A can be defined to the center of the pulley above point A.
- s_B can be defined to the center of the pulley above B.
- s_C is defined to the center of pulley C.
- All coordinates are defined as positive down and along the direction of motion of each point/object.

Example

2) Write position/length equations for each cord. Define l_1 as the length of the first cord, minus any segments of constant length. Define l_2 in a similar manner for the second cord:

$$\text{Cord 1: } 2s_A + 2s_C = l_1$$

$$\text{Cord 2: } s_B + (s_B - s_C) = l_2$$



3) Eliminating s_C between the two equations, we get:

$$2s_A + 4s_B = l_1 + 2l_2$$

4) Relate velocities by differentiating this expression. Note that l_1 and l_2 are constant lengths.

$$2v_A + 4v_B = 0$$

$$\Rightarrow \quad v_B = -0.5v_A = -0.5(8) = -4 \text{ ft/s}$$

The velocity of block B is 4 ft/s up (negative s_B direction).

Example

Determine the speed of block B in Fig. 12–40 if the end of the cord at A is pulled down with a speed of 2 m/s.

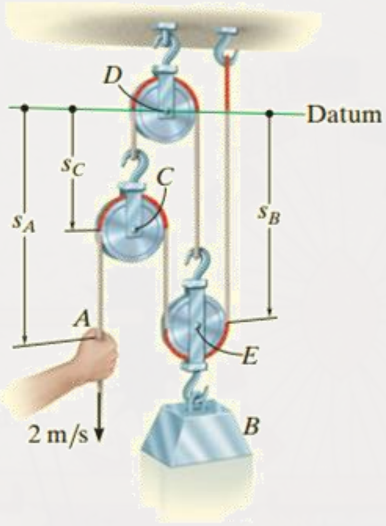


Fig. 12–40

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Example

Determine the speed of block B in Fig. 12–40 if the end of the cord at A is pulled down with a speed of 2 m/s.

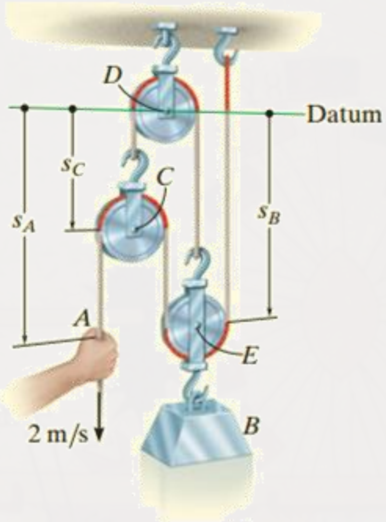

Fig. 12–40

Excluding the red colored segments of the cords in Fig. 12–40, the remaining constant cord lengths l_1 and l_2 (along with the hook and link dimensions) can be expressed as

$$s_C + s_B = l_1$$
$$(s_A - s_C) + (s_B - s_C) + s_B = l_2$$

Time Derivative. The time derivative of each equation gives

$$v_C + v_B = 0$$
$$v_A - 2v_C + 2v_B = 0$$

Eliminating v_C, we obtain

$$v_A + 4v_B = 0$$

so that when $v_A = 2$ m/s (downward),

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textit{Ans.}$$

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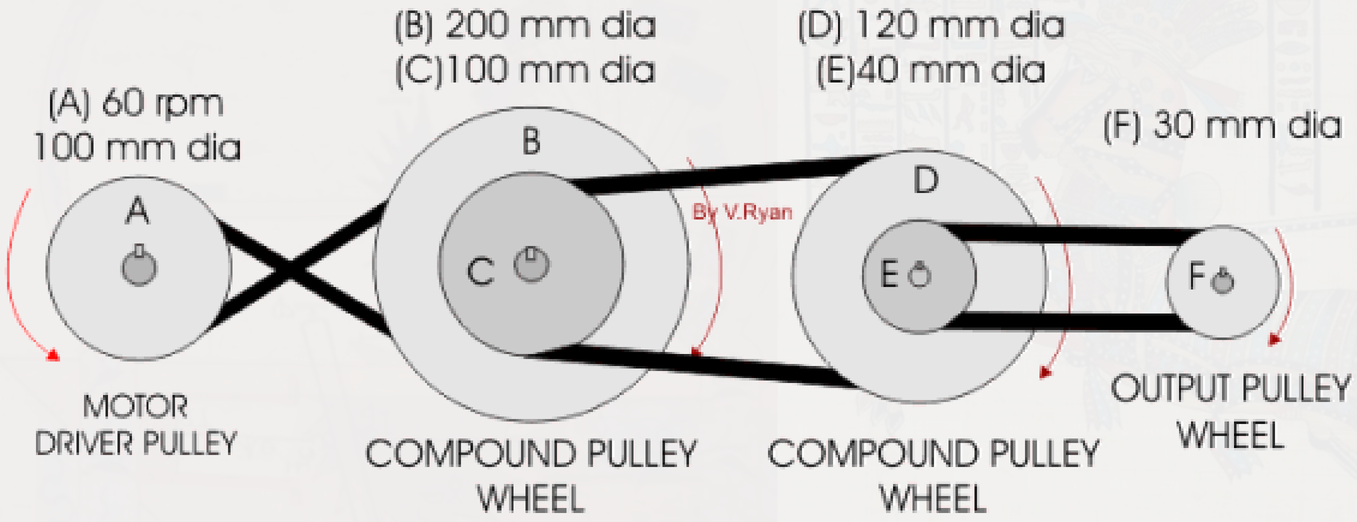
Think about it...



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Now it is time to move to 12.10...

Relative motion analysis of two particles using translating axis

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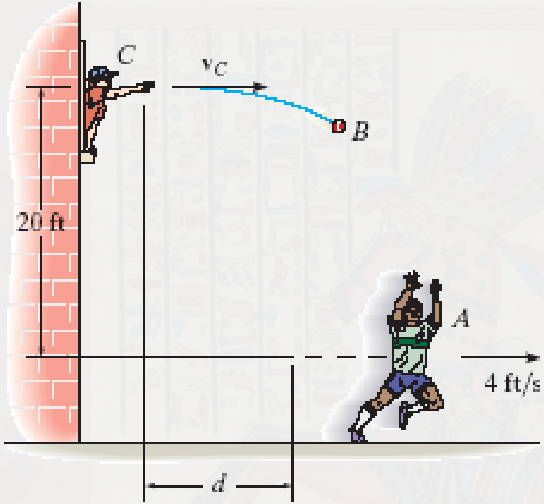
Applications I

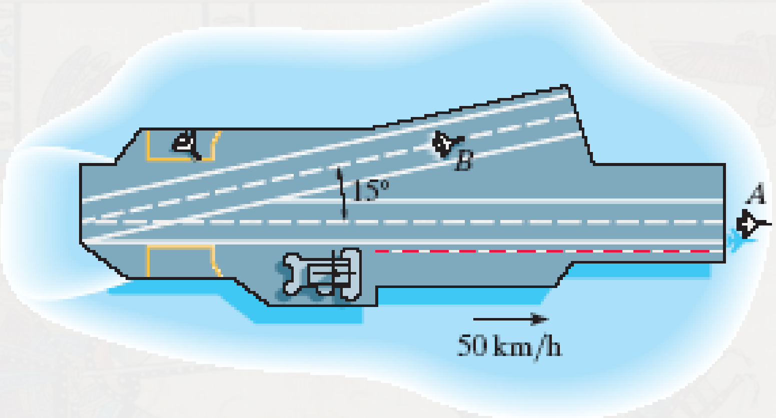


When you try to hit a moving object, the position, velocity, and acceleration of the object must be known. Here, the boy on the ground is at d = 10 ft when the girl in the window throws the ball to him

If the boy on the ground is running at a constant speed of 4 ft/s, how fast should the ball be thrown?

Applications II

When fighter jets take off or land on an aircraft carrier, the velocity of the carrier becomes an issue.



If the aircraft carrier travels at a forward velocity of 50 km/hr and plane A takes off at a horizontal air speed of 200 km/hr (measured by someone on the water), how do we find the velocity of the plane relative to the carrier?

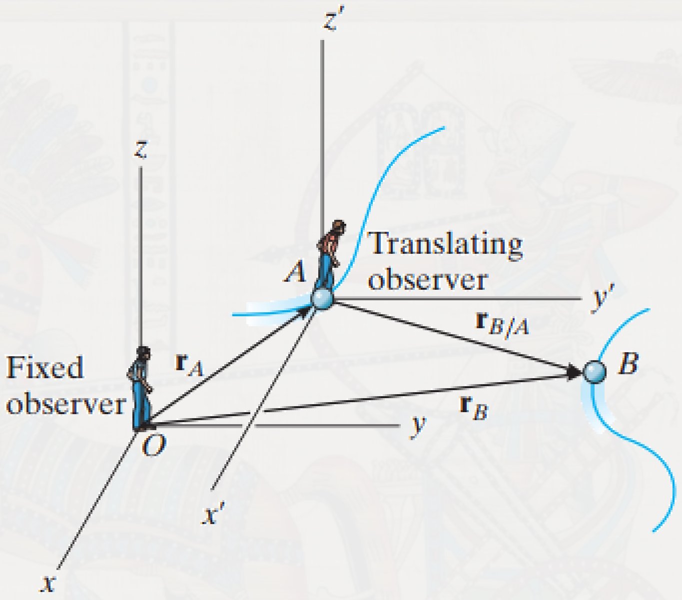
How would you find the same thing for airplane B?

How does the wind impact this sort of situation?

Relative position

The absolute position of two particles A and B with respect to the fixed x, y, z reference frame are given by $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$. The position of B relative to A is represented by

$$\boldsymbol{r}_{B/A} = \boldsymbol{r}_B - \boldsymbol{r}_A$$



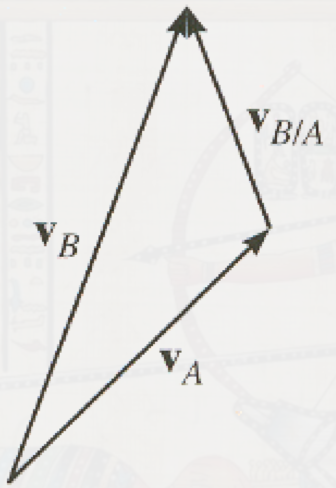
Therefore, if $\boldsymbol{r}_B = (10\,\boldsymbol{i} + 2\,\boldsymbol{j}\,)$ m

and $\boldsymbol{r}_A = (4\,\boldsymbol{i} + 5\,\boldsymbol{j}\,)$ m

then $\boldsymbol{r}_{B/A} = (6\,\boldsymbol{i} - 3\,\boldsymbol{j}\,)$ m

Relative velocity

To determine the relative velocity of B with respect to A, the time derivative of the relative position equation is taken.



$$\boldsymbol{v}_{B/A} = \boldsymbol{v}_B - \boldsymbol{v}_A$$

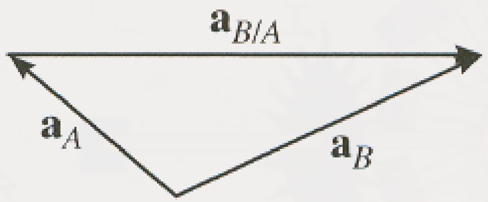
or

$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$$

In these equations, $\boldsymbol{v}_B$ and $\boldsymbol{v}_A$ are called absolute velocities and $\boldsymbol{v}_{B/A}$ is the relative velocity of B with respect to A.

Note that $\boldsymbol{v}_{B/A} = -\boldsymbol{v}_{A/B}$.

Relative acceleration



The time derivative of the relative velocity equation yields a similar vector relationship between the absolute and relative accelerations of particles A and B.

$$\boldsymbol{a}_{B/A} = \boldsymbol{a}_B - \boldsymbol{a}_A$$

or

$$\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{B/A}$$

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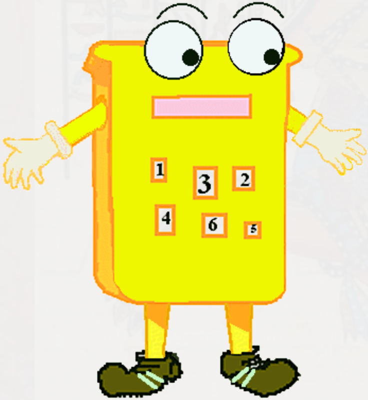
Solving problems

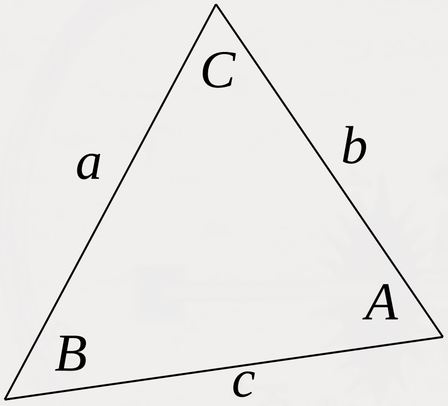
Since the relative motion equations are vector equations, problems involving them may be solved in one of two ways.

For instance, the velocity vectors in $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$ could be written as Cartesian vectors and the resulting scalar equations solved for up to two unknowns.



Alternatively, vector problems can be solved "graphically" by use of trigonometry. This approach usually makes use of the law of sines or the law of cosines.

Laws of sines and cosines



Since vector addition or subtraction forms a triangle, sine and cosine laws can be applied to solve for relative or absolute velocities and accelerations. For review, their formulations are provided below.

Law of Sines: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines:
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

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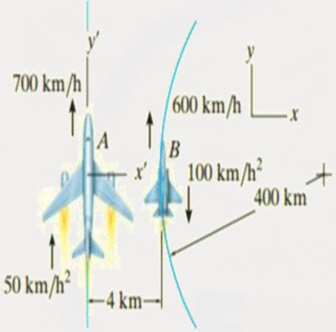
Problems

EXAMPLE 12.26



(a)

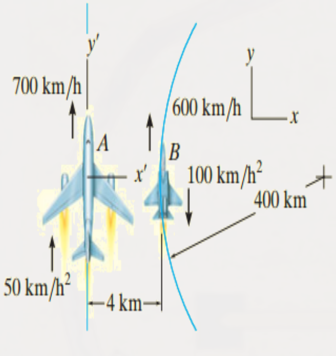
Plane A in Fig. 12–44*a* is flying along a straight-line path, whereas plane B is flying along a circular path having a radius of curvature of $\rho_B = 400$ km. Determine the velocity and acceleration of B as measured by the pilot of A.

SOLUTION

Velocity. The origin of the x and y axes are located at an arbitrary fixed point. Since the motion relative to plane A is to be determined, the *translating frame of reference* x', y' is attached to it, Fig. 12–44*a*. Applying the relative-velocity equation in scalar form since the velocity vectors of both planes are parallel at the instant shown, we have

Problems

EXAMPLE 12.26



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Plane A in Fig. 12–44a is flying along a straight-line path, whereas plane B is flying along a circular path having a radius of curvature of $\rho_B = 400$ km. Determine the velocity and acceleration of B as measured by the pilot of A.

SOLUTION

Velocity. The origin of the x and y axes are located at an arbitrary fixed point. Since the motion relative to plane A is to be determined, the *translating frame of reference* x', y' is attached to it, Fig. 12–44a. Applying the relative-velocity equation in scalar form since the velocity vectors of both planes are parallel at the instant shown, we have

$(+\uparrow)$
$$v_B = v_A + v_{B/A}$$
$$600\text{ km/h} = 700\text{ km/h} + v_{B/A}$$
$$v_{B/A} = -100\text{ km/h} = 100\text{ km/h}\downarrow \qquad \textit{Ans.}$$

Acceleration. Plane B has both tangential and normal components of acceleration since it is flying along a *curved path*. From Eq. 12–20, the magnitude of the normal component is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(600\text{ km/h})^2}{400\text{ km}} = 900\text{ km/h}^2$$

Applying the relative-acceleration equation gives

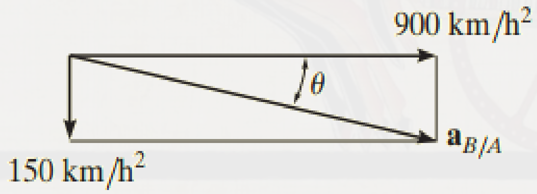
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$
$$900\mathbf{i} - 100\mathbf{j} = 50\mathbf{j} + \mathbf{a}_{B/A}$$

Thus,

$$\mathbf{a}_{B/A} = \{900\mathbf{i} - 150\mathbf{j}\}\text{ km/h}^2$$

From Fig. 12–44c, the magnitude and direction of $\mathbf{a}_{B/A}$ are therefore

$$a_{B/A} = 912\text{ km/h}^2 \qquad \theta = \tan^{-1}\frac{150}{900} = 9.46° \;\measuredangle \qquad \textit{Ans.}$$



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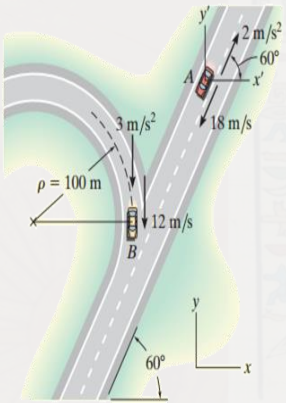
Problems

EXAMPLE 12.27

At the instant shown in Fig. 12–45*a*, cars *A* and *B* are traveling with speeds of 18 m/s and 12 m/s, respectively. Also at this instant, *A* has a decrease in speed of 2 m/s^2, and *B* has an increase in speed of 3 m/s^2. Determine the velocity and acceleration of *B* with respect to *A*.



SOLUTION

Velocity. The fixed *x*, *y* axes are established at an arbitrary point on the ground and the translating x', y' axes are attached to car *A*, Fig. 12–45*a*. Why? The relative velocity is determined from $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$. What are the two unknowns? Using a Cartesian vector analysis, we have

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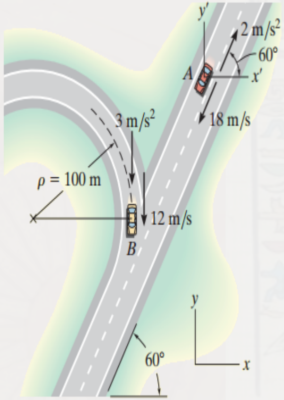
Problems

EXAMPLE 12.27

At the instant shown in Fig. 12–45*a*, cars *A* and *B* are traveling with speeds of 18 m/s and 12 m/s, respectively. Also at this instant, *A* has a decrease in speed of 2 m/s^2, and *B* has an increase in speed of 3 m/s^2. Determine the velocity and acceleration of *B* with respect to *A*.



(a)

SOLUTION

Velocity. The fixed *x*, *y* axes are established at an arbitrary point on the ground and the translating *x'*, *y'* axes are attached to car *A*, Fig. 12–45*a*. Why? The relative velocity is determined from $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$. What are the two unknowns? Using a Cartesian vector analysis, we have

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-12\mathbf{j} = (-18\cos 60°\mathbf{i} - 18\sin 60°\mathbf{j}) + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{9\mathbf{i} + 3.588\mathbf{j}\}\ \text{m/s}$$

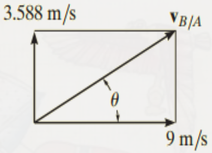
Thus,

$$v_{B/A} = \sqrt{(9)^2 + (3.588)^2} = 9.69\ \text{m/s} \qquad \textit{Ans.}$$

Noting that $\mathbf{v}_{B/A}$ has $+\mathbf{i}$ and $+\mathbf{j}$ components, Fig. 12–45*b*, its direction is

$$\tan\theta = \frac{(v_{B/A})_y}{(v_{B/A})_x} = \frac{3.588}{9}$$

$$\theta = 21.7° \qquad \textit{Ans.}$$



(b)

Acceleration. Car *B* has both tangential and normal components of acceleration. Why? The magnitude of the normal component is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(12\ \text{m/s})^2}{100\ \text{m}} = 1.440\ \text{m/s}^2$$

Applying the equation for relative acceleration yields

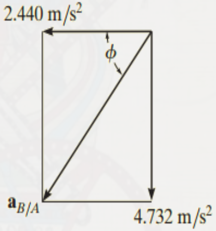
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(-1.440\mathbf{i} - 3\mathbf{j}) = (2\cos 60°\mathbf{i} + 2\sin 60°\mathbf{j}) + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{-2.440\mathbf{i} - 4.732\mathbf{j}\}\ \text{m/s}^2$$

Here $\mathbf{a}_{B/A}$ has $-\mathbf{i}$ and $-\mathbf{j}$ components. Thus, from Fig. 12–45*c*,

$$a_{B/A} = \sqrt{(2.440)^2 + (4.732)^2} = 5.32\ \text{m/s}^2 \qquad \textit{Ans.}$$



(c)

Here $\mathbf{a}_{B/A}$ has $-\mathbf{i}$ and $-\mathbf{j}$ components. Thus, from Fig. 12–45*c*,

$$a_{B/A} = \sqrt{(2.440)^2 + (4.732)^2} = 5.32\ \text{m/s}^2 \qquad \textit{Ans.}$$

$$\tan\phi = \frac{(a_{B/A})_y}{(a_{B/A})_x} = \frac{4.732}{2.440}$$

$$\phi = 62.7° \qquad \textit{Ans.}$$

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Next lecture;

Help Session

and may be…

Starting

Chapter 16…

ENME232: Dynamics

CH 12: Kinematics of a particle

Lecture 7: Help Session

Dr. Mamon M. Horoub
Assistant Professor,
Faculty of Engineering & Technology Department of
Mechanical and Mechatronics Engineering

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Recap of the Previous Class Agenda

1. Absolute Dependent Motion Analysis of Two Particles

2. Relative-Motion of Two Particles Using Translating Axes

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Today's Class Agenda

1. Solve different problems on CH12

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Problems

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

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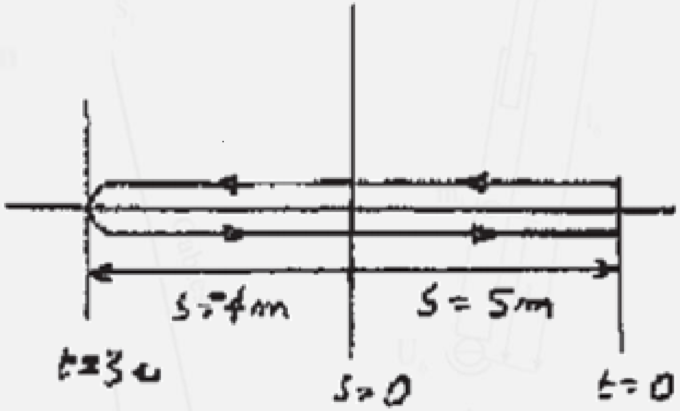
Problems

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.



SOLUTION

$$s = t^2 - 6t + 5$$

$$v = \frac{ds}{dt} = 2t - 6$$

$$a = \frac{dv}{dt} = 2$$

$$v = 0 \text{ when } t = 3$$

$$s|_{t=0} = 5$$

$$s|_{t=3} = -4$$

$$s|_{t=6} = 5$$

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{0}{6} = 0$$ **Ans.**

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t} = \frac{9 + 9}{6} = 3 \text{ m/s}$$ **Ans.**

$$a|_{t=6} = 2 \text{ m/s}^2$$ **Ans.**

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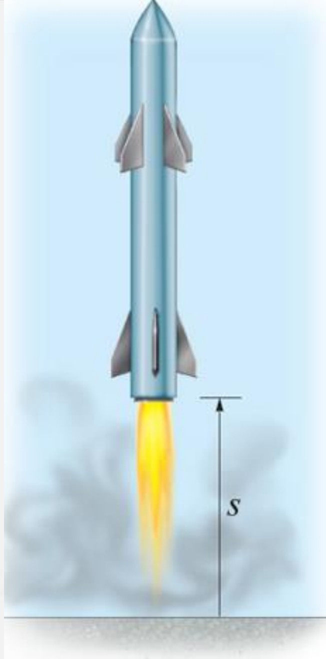
Problems

12-22. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the rocket's velocity when $s = 2$ km and the time needed to reach this altitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



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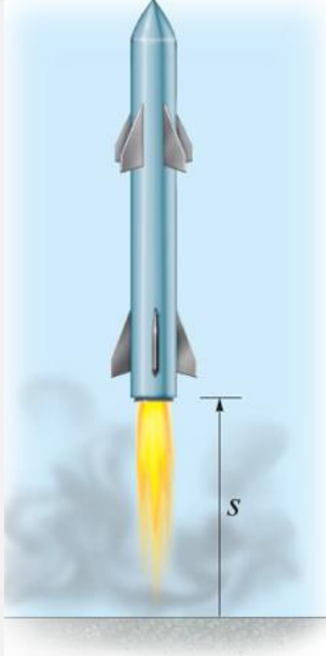
12-22. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the rocket's velocity when $s = 2$ km and the time needed to reach this altitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



Solution:

Show me the final answer↓

As the acceleration is not constant, we will need to integrate our acceleration to figure out the velocity. To do so, we will need to use the following equation:

$$ads = vdv$$

Take the integral of both sides:

$$\int_{s_0}^{s} ads = \int_{v_0}^{v} vdv$$

$$\int_0^s (6 + 0.02s) = \int_0^v vdv$$

(For the acceleration integral, the lower limit is 0 because the rocket starts at a height of 0 m. For the velocity integral, remember that the rocket starts from rest, meaning the lower limit is 0 m/s.)

$$(6s + \frac{0.02s^2}{2})\Big|_0^s = \frac{v^2}{2}\Big|_0^v$$

$$6s + \frac{0.02s^2}{2} = \frac{v^2}{2}$$

$$v = \sqrt{12s + 0.02s^2}$$

When the height is 2000 m, the velocity is:

$$v = \sqrt{12(2000) + 0.02(2000)^2}$$

$$v = 322.5 \text{ m/s}$$

To find the time, remember that:

$$v = \frac{ds}{dt}$$

$$dt = \frac{ds}{v}$$

Again, take the integral of both sides:

$$\int_0^t dt = \int_0^s \frac{ds}{v}$$

(substitute the velocity equation we found)

$$\int_0^t dt = \int_0^{2000} \frac{ds}{\sqrt{12s + 0.02s^2}}$$

if it's hard to visualize the right side of this integral, remember that you can write it like so:

$$\int_0^t dt = \int_0^{2000} \frac{1}{\sqrt{12s + 0.02s^2}} ds$$

(This is a complicated integral, however, you can see the integral solved here: https://goo.gl/iWgq1f)

t=19.27 s

https://www.questionsolutions.com/the-acceleration-of-a-rocket-traveling-upward

Final Answers:

$v = 322.5$ m/s

$t = 19.27$ s

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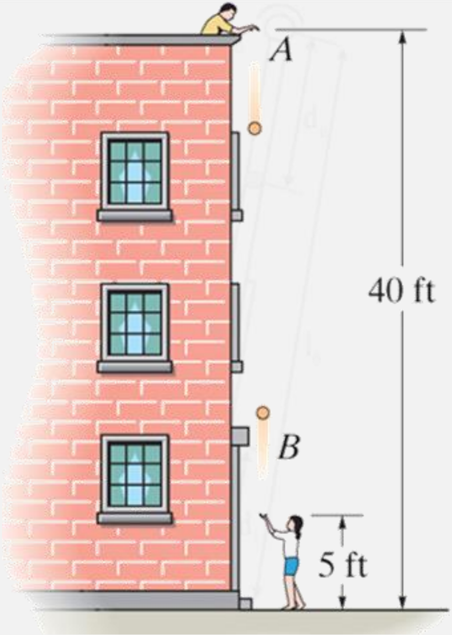
Problems

12-26. Ball *A* is released from rest at a height of 40 ft at the same time that a second ball *B* is thrown upward 5 ft from the ground. If the balls pass one another at a height of 20 ft, determine the speed at which ball *B* was thrown upward.



Plan: Both balls experience a constant downward acceleration of 32.2 ft/s² due to gravity. Apply the formulas for constant acceleration, with $a_c = -32.2\ ft/s^2$.

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Problems

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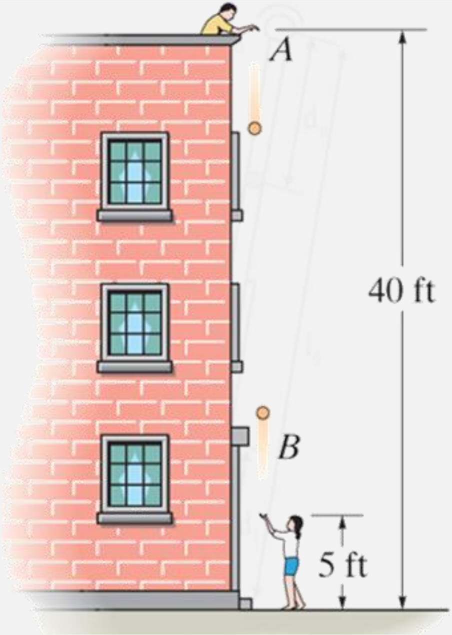

Plan: Both balls experience a constant downward acceleration of 32.2 ft/s² due to gravity. Apply the formulas for constant acceleration, with $a_c = -32.2$ ft/s².

$$s = s_0 + v_0 t + \frac{1}{2} a t^2$$

Ball A:

$$20 = 40 + 0 + \frac{1}{2}(-32.2)t^2$$

$$t = 1.11\ s$$

Ball B:

$$20 = 5 + v_0(1.11) + \frac{1}{2}(-32.2)(1.11^2)$$

$$v_0 = 31.4 \quad ft/s$$

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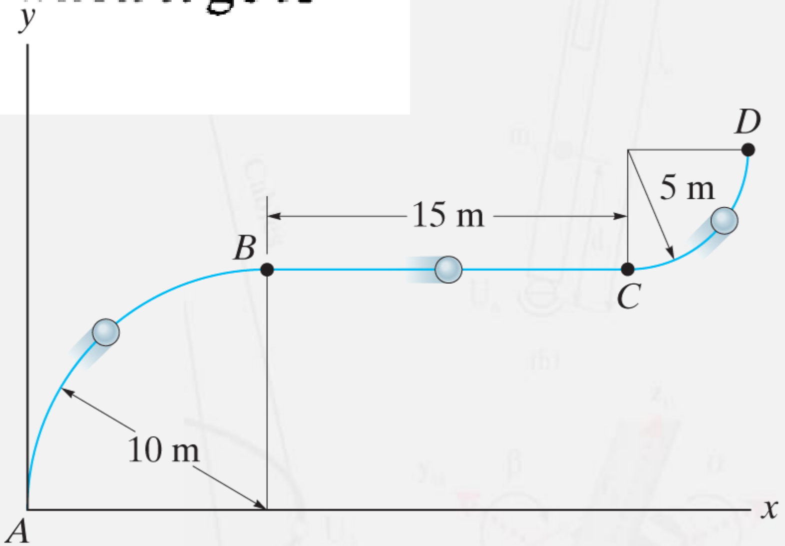
Problems (Solve it at your home)

12-71. A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



Problems (Solve it at your home)

12-71. A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.

Solution:

We must first figure out the total distance traveled by the particle. To do so, we must realize that each curved section is in fact $\frac{1}{4}$*th* of the circumference of a circle. The length of each curved part is:

$$l_1 = (\frac{1}{4})(2)(\pi)(10) = 15.71 \text{ m}$$

$$l_2 = (\frac{1}{4})(2)(\pi)(5) = 7.85 \text{ m}$$

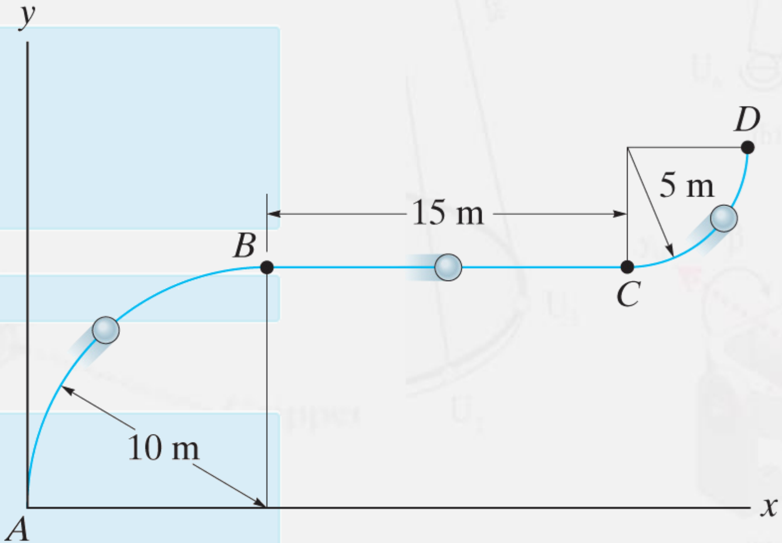
(Remember, the circumference of a circle is $c = (2)(\pi)(r)$, where r is the radius)

The total distance the particle traveled = 15.71 + 15 + 7.85 = 38.56 m

Thus, the speed is:

$$\text{speed} = \frac{distance}{time}$$

$$\text{speed} = \frac{38.56}{2+4+3} = 4.28 \text{ m/s}$$



Problems (Solve it at your home)

***12-100.** A car is traveling along a circular curve that has a radius of 50 m. If its speed is 16 m/s and is increasing uniformly at 8 m/s^2, determine the magnitude of its acceleration at this instant.

Problems (Solve it at your home)

***12-100.** A car is traveling along a circular curve that has a radius of 50 m. If its speed is 16 m/s and is increasing uniformly at 8 m/s^2, determine the magnitude of its acceleration at this instant.

Solution:

The tangential acceleration, a_t is equal to 8 m/s^2. We now need to find the normal acceleration since the car is travelling along a circular curve. We can use the following formula to do so:

$$a_n = \frac{v^2}{\rho}$$

(Where a_n is normal acceleration, v is velocity, and ρ is the radius of the circle)

$$a_n = \frac{16^2}{50} = 5.12 \text{ m}/s^2$$

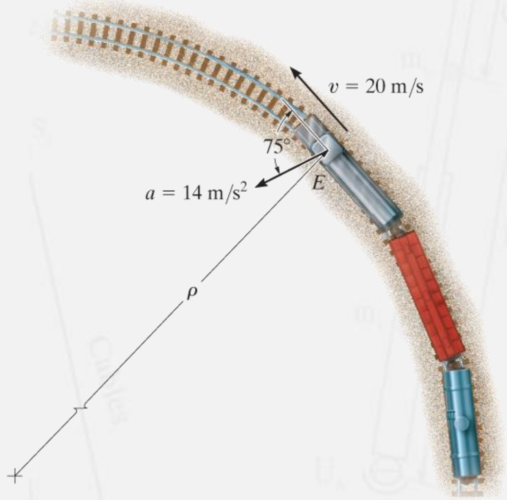
The magnitude of acceleration is:

$$a = \sqrt{(a_t)^2 + (a_n)^2}$$

$$a = \sqrt{8^2 + 5.12^2} = 9.5 \text{ m}/s^2$$

Problems (Solve it at your home)

12-111. At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



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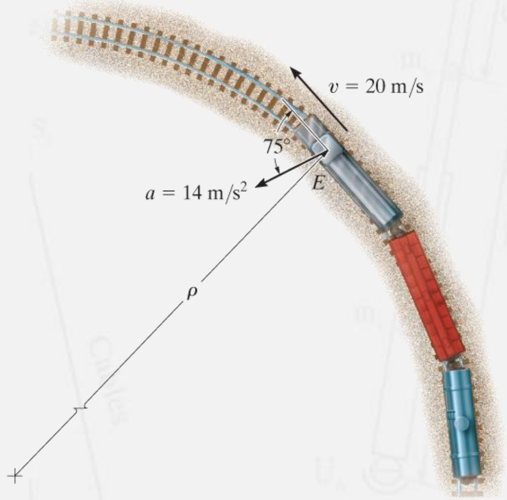

SOLUTION

$$a_t = 14 \cos 75° = 3.62 \text{ m/s}^2$$

$$a_n = 14 \sin 75°$$

$$a_n = \frac{(20)^2}{\rho}$$

$$\rho = 29.6 \text{ m}$$

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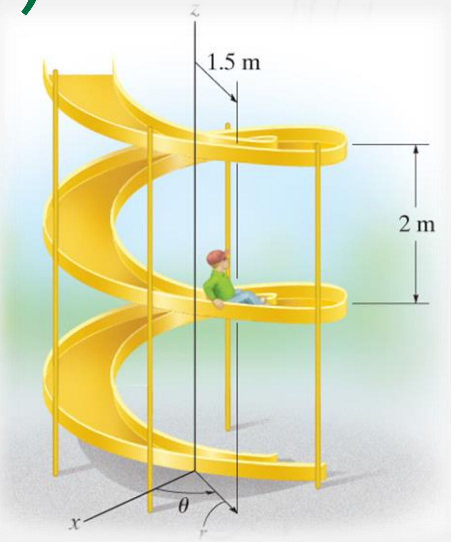
Problems (Solve it at your home)

12-153. The boy slides down the slide at a constant speed of 2 m/s. If the slide is in the form of a helix, defined by the equations $r = 1.5$ m and $z = -\theta/\pi$, determine the boy's angular velocity about the z axis, $\dot{\theta}$, and the magnitude of his acceleration.



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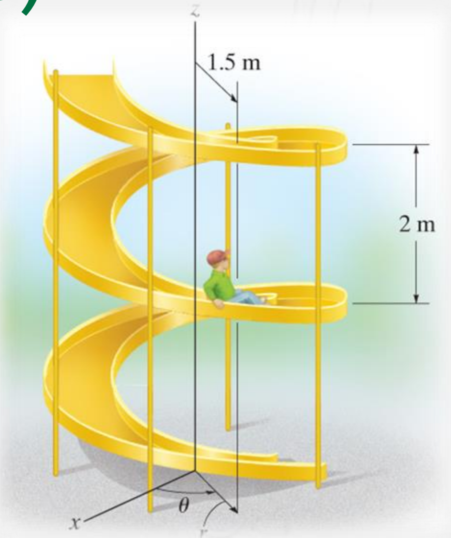

Solution

$\boldsymbol{v}_P = \dot{r}\boldsymbol{u}_r + r\dot{\theta}\boldsymbol{u}_\theta + \dot{z}\boldsymbol{u}_z$

$= 0\,u_r + 1.5\dot{\theta}\,u_\theta + \left(\frac{-\dot{\theta}}{\pi}\right)u_z$

$2 = \sqrt{(1.5\dot{\theta})^2 + \left(\frac{-\dot{\theta}}{\pi}\right)^2}$

$\Rightarrow \dot{\theta} = \dfrac{2}{\sqrt{(1.5)^2 + \left(\frac{1}{\pi}\right)^2}}$

$= 1.304$ rad/s ANS

$\boldsymbol{a}_P = (\ddot{r} - r\dot{\theta}^2)\boldsymbol{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\boldsymbol{u}_\theta + \ddot{z}\boldsymbol{u}_z$

$= (0 - r\dot{\theta}^2)u_r + (0 + 0)u_\theta + 0\,u_z$

$\Rightarrow a = \sqrt{(-r\dot{\theta}^2)^2}$

$= r\dot{\theta}^2$

$= 1.5(1.304)^2$

$= 2.55$ m/s² ANS

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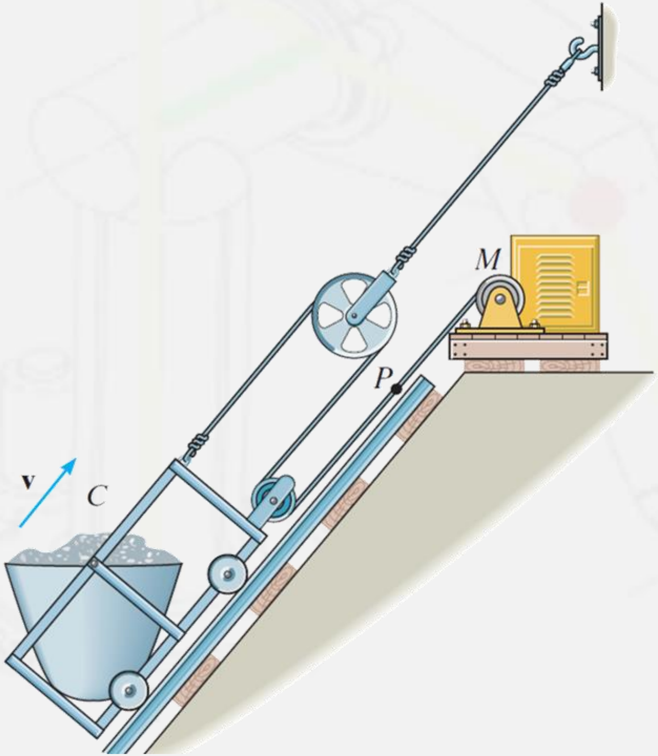
Problems (Help Session)

12–195. The mine car C is being pulled up the incline using the motor M and the rope-and-pulley arrangement shown. Determine the speed v_P at which a point P on the cable must be traveling toward the motor to move the car up the plane with a constant speed of $v = 2$ m/s.



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Problems (Help Session)

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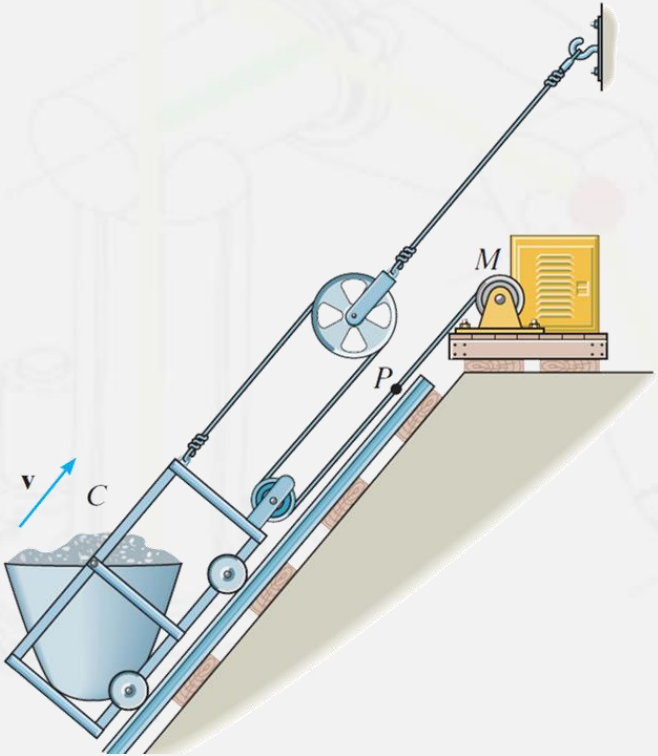

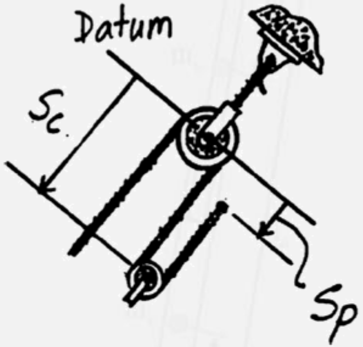

$$2s_C + (s_C - s_P) = l$$

Thus,

$$3v_C - v_P = 0$$

Hence,

$$v_P = 3(-2) = -6 \text{ m/s} = 6 \text{ m/s} \quad \nearrow$$

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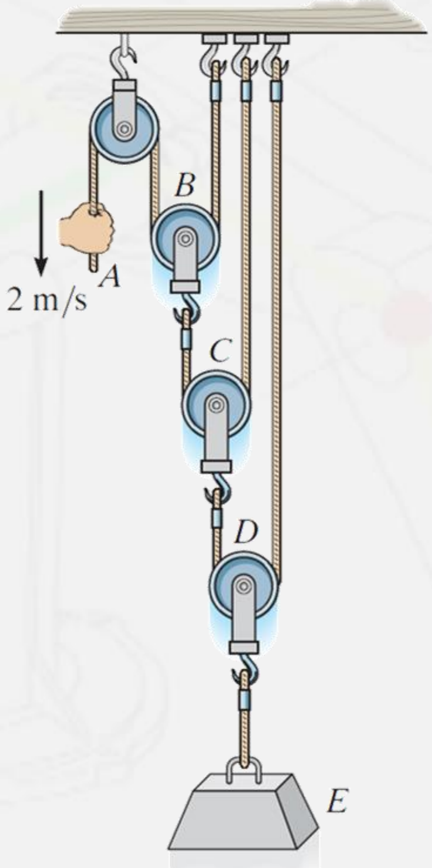
Problems (Help Session)

***12–208.** If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block E rises.



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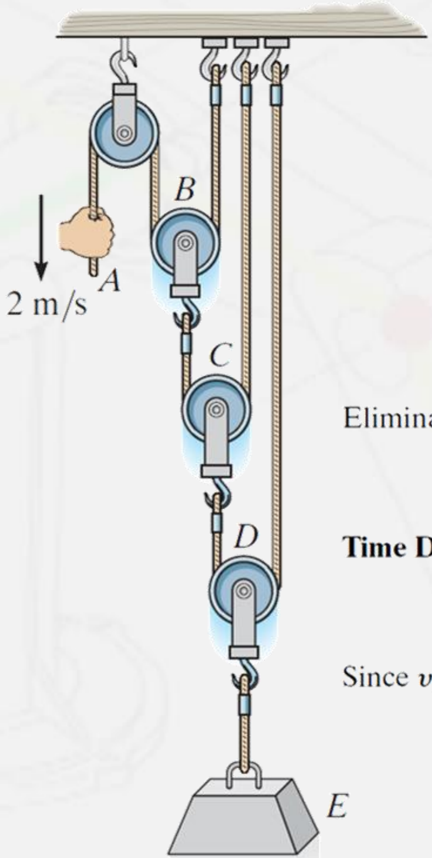
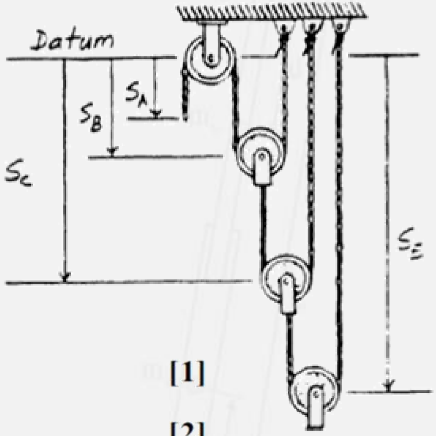
Problems (Help Session)

***12–208.** If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block E rises.





$$2s_B + s_A = l_1 \quad [1]$$

$$s_C + (s_C - s_B) = l_2 \quad [2]$$

$$s_E + (s_E - s_C) = l_3 \quad [3]$$

Eliminating s_C and s_B from Eqs. [1], [2] and [3], we have

$$s_A + 8s_E = l_1 + 2l_2 + 4l_3$$

Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 8v_E = 0 \quad [4]$$

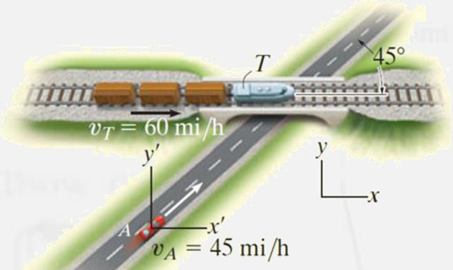
Since $v_A = 2$ m/s, from Eq. [3]

$$(+\downarrow) \qquad 2 + 8v_E = 0$$

$$v_E = -0.250 \text{ m/s} = 0.250 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Problems (Help Session)

A train travels at a constant speed of 60 mi/h, crosses over a road as shown in Fig. 12–43*a*. If the automobile *A* is traveling at 45 mi/h along the road, determine the magnitude and direction of the velocity of the train relative to the automobile.



(a)

SOLUTION I

Vector Analysis. The relative velocity $\mathbf{v}_{T/A}$ is measured from the translating x', y' axes attached to the automobile, Fig. 12–43*a*. It is determined from $\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$. Since $\mathbf{v}_T$ and $\mathbf{v}_A$ are known in *both* magnitude and direction, the unknowns become the *x* and *y* components of $\mathbf{v}_{T/A}$. Using the *x*, *y* axes in Fig. 12–43*a*, we have

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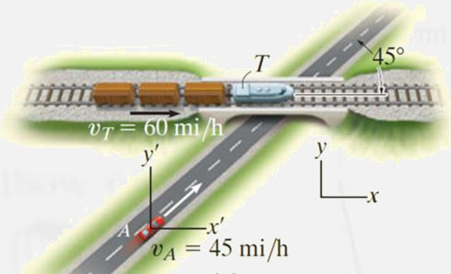

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SOLUTION I

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$$\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$$

$$60\mathbf{i} = (45\cos 45^\circ\mathbf{i} + 45\sin 45^\circ\mathbf{j}) + \mathbf{v}_{T/A}$$

$$\mathbf{v}_{T/A} = \{28.2\mathbf{i} - 31.8\mathbf{j}\}\ \text{mi/h} \qquad \textit{Ans.}$$

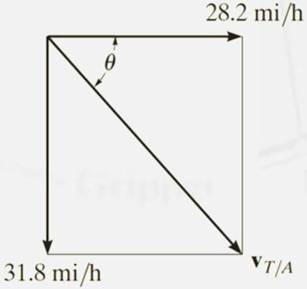
The magnitude of $\mathbf{v}_{T/A}$ is thus

$$v_{T/A} = \sqrt{(28.2)^2 + (-31.8)^2} = 42.5\ \text{mi/h} \qquad \textit{Ans.}$$

From the direction of each component, Fig. 12–43*b*, the direction of $\mathbf{v}_{T/A}$ is

$$\tan\theta = \frac{(v_{T/A})_y}{(v_{T/A})_x} = \frac{31.8}{28.2}$$

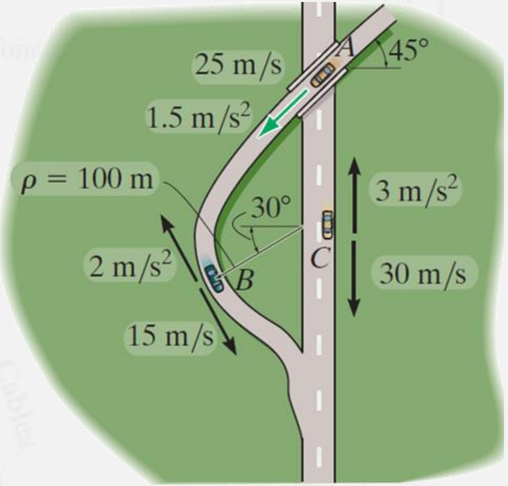
$$\theta = 48.5^\circ \qquad \textit{Ans.}$$



Note that the vector addition shown in Fig. 12–43*b* indicates the correct sense for $\mathbf{v}_{T/A}$. This figure anticipates the answer and can be used to check it.

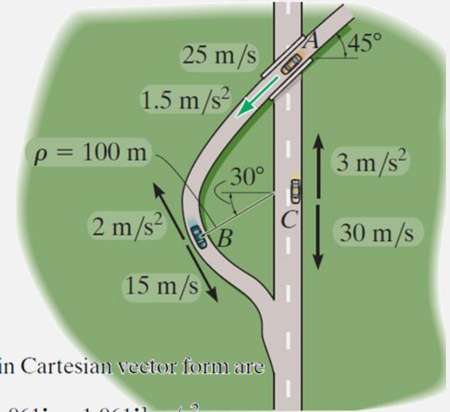
Problems (Help Session)

***12–216.** Car *A* travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2. At this same instant car *C* is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car *A* relative to car *C*.



Problems (Help Session)

*12–216. Car A travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car A relative to car C.



Velocity: The velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [-25\cos 45°\mathbf{i} - 25\sin 45°\mathbf{j}]\text{ m/s} = [-17.68\mathbf{i} - 17.68\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-17.68\mathbf{i} - 17.68\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{A/C}$$

$$\mathbf{v}_{A/C} = [-17.68\mathbf{i} + 12.32\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{A/C}$ is given by

$$v_{A/C} = \sqrt{(-17.68)^2 + 12.32^2} = 21.5\text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{A/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{12.32}{17.68}\right) = 34.9° \qquad \textbf{Ans.}$$

Acceleration: The acceleration of cars A and B expressed in Cartesian vector form are

$$\mathbf{a}_A = [-1.5\cos 45°\mathbf{i} - 1.5\sin 45°\mathbf{j}]\text{ m/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$-1.061\mathbf{i} - 1.061\mathbf{j} = 3\mathbf{j} + \mathbf{a}_{A/C}$$

$$\mathbf{a}_{A/C} = [-1.061\mathbf{i} - 4.061\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{A/C}$ is given by

$$a_{A/C} = \sqrt{(-1.061)^2 + (-4.061)^2} = 4.20\text{ m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{A/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{4.061}{1.061}\right) = 75.4° \qquad \textbf{Ans.}$$

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Prepare your self for;

a QUIZ

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2. Dr. Balasie PowerPoints, Dynamic Course, Birzeit University.

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