Dynamics

Wednesday, January 13, 2021

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ENME232: Dynamics

CH 19: Planar Kinetics of a Rigid Body: Impulse and Momentum

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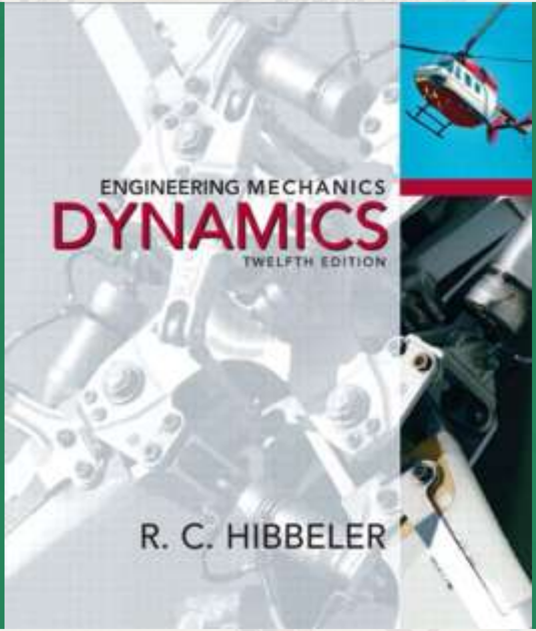
Text books

Engineering Mechanics: Dynamics

C. Hibbeler, 12th Edition, Prentice Hall, 2010

…and probably some more…



Recap of the Previous Classes Agenda

18 Planar Kinetics of a Rigid Body: Work and Energy 455

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Chapter's Agenda

19 Planar Kinetics of a Rigid Body: Impulse and Momentum 495

Part 1

Introduction

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Chapter objectives

- Develop formulations for the linear and angular momentum of a rigid body
- Apply the principles of linear and angular impulse and momentum to solve rigid body planar kinetic problems that involve force, velocity and time
- To discuss the application of the conservation of momentum

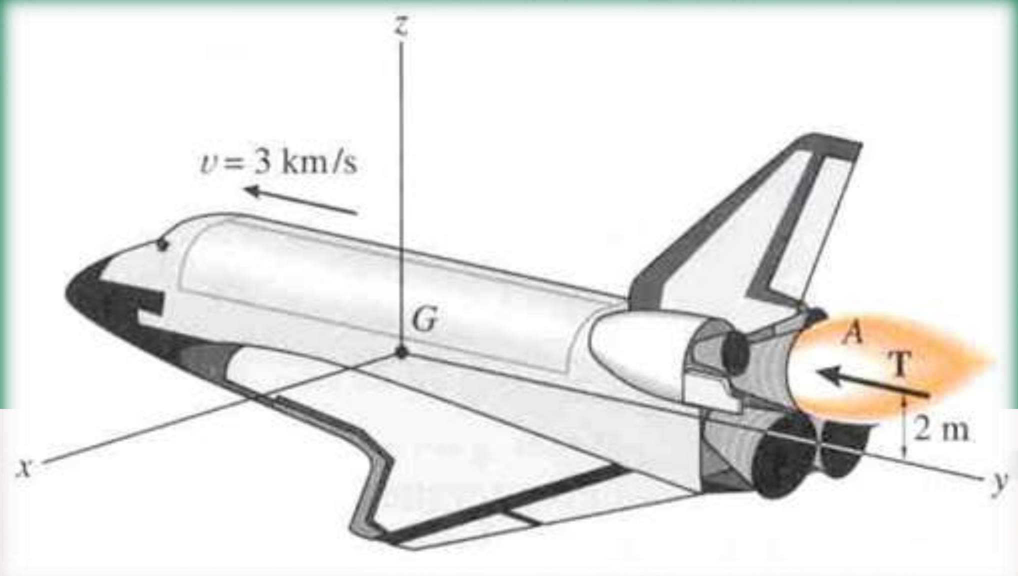
Today's Objectives

Students should be able to:

- Develop formulations for the linear and angular momentum of a body.
- Apply the principle of linear and angular impulse and momentum.
- Understand the conditions for conservation of linear and angular momentum.
- Use the condition of conservation of linear/ angular momentum.

Applications

The space shuttle has several engines that exert thrust on the shuttle when they are fired. By firing different engines, the pilot can control the motion and direction of the shuttle.



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Part 2

Linear and angular momentum (19.1)

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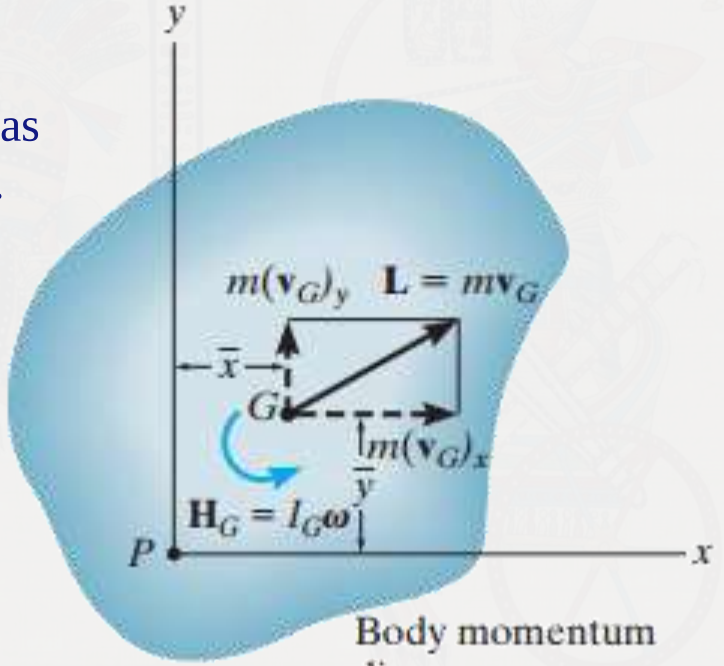
Linear and angular momentum (19.1)

The linear momentum of a rigid body is defined as

$$\boldsymbol{L} = m\,\boldsymbol{v}_G$$

This equation states that the linear momentum vector $\boldsymbol{L}$ has a magnitude equal to (mv_G) and a direction defined by $\boldsymbol{v}_G$.



Body momentum diagram

The angular momentum of a rigid body is defined as

$$\boldsymbol{H}_G = I_G\,\boldsymbol{\omega}$$

Remember that the direction of $\boldsymbol{H}_G$ is perpendicular to the plane of rotation.

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Linear and angular momentum (19.1)

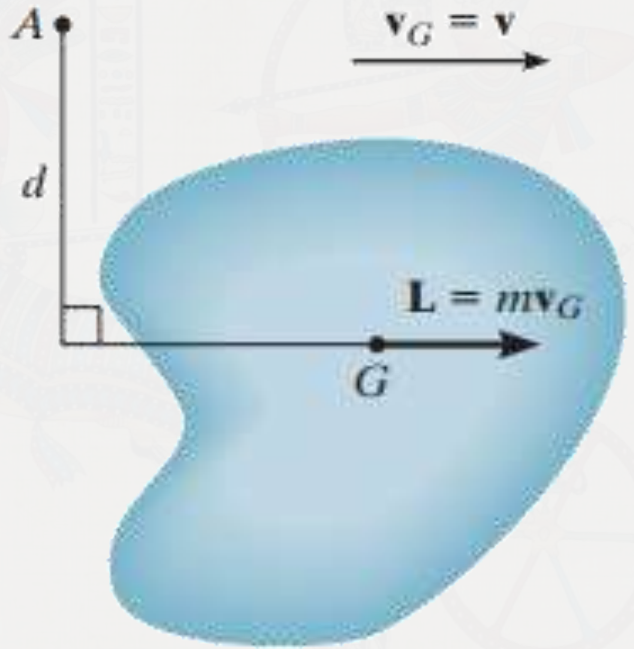
Translation.

When a rigid body undergoes rectilinear or curvilinear translation, its angular momentum is zero because $\omega = 0$.

Therefore:

$$L = m\, v_G$$

$$H_G = 0$$



Translation

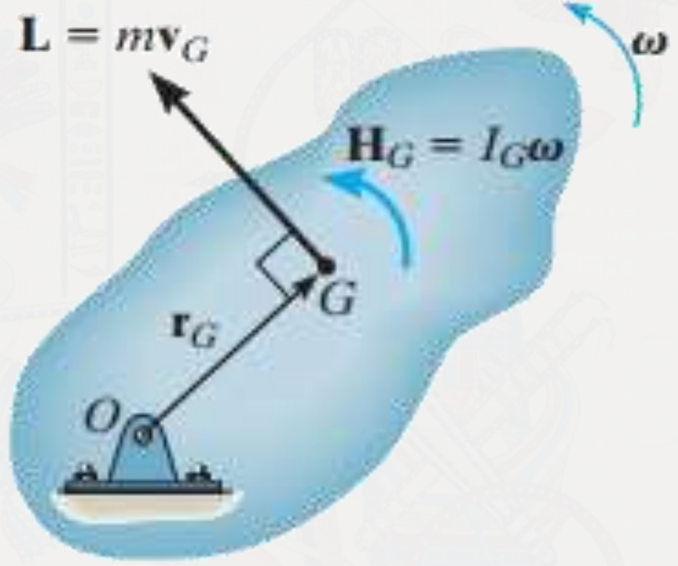
Linear and angular momentum (19.1)

Rotation about a fixed axis.

When a rigid body is rotating about a fixed axis passing through point O, the body's linear momentum and angular momentum about G are:

$$\boldsymbol{L} = m\,\boldsymbol{v}_G$$

$$H_G = I_G\,\boldsymbol{\omega}$$



Rotation about a fixed axis

It is sometimes convenient to compute the angular momentum of the body about the center of rotation O.

$$\boldsymbol{H}_O = (\,\boldsymbol{r}_G \times m\boldsymbol{v}_G) + I_G\boldsymbol{\omega} = I_O\,\boldsymbol{\omega}$$

Linear and angular momentum (19.1)

General plane motion.

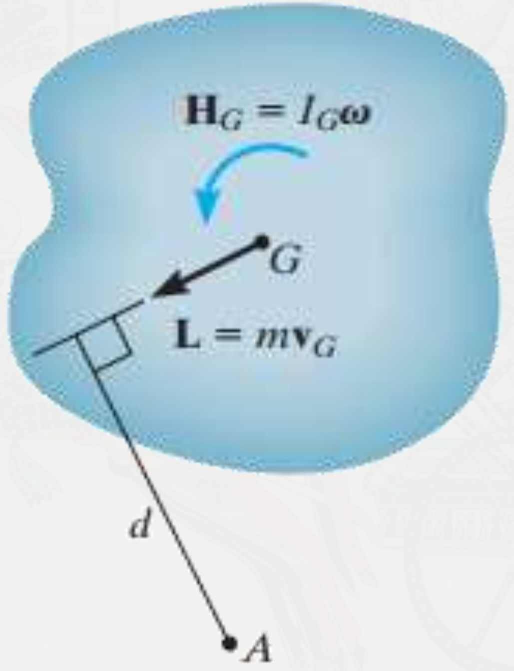
When a rigid body is subjected to general plane motion, both the linear momentum and the angular momentum computed about G are required.

$$\boldsymbol{L} = m\,\boldsymbol{v}_G$$

$$H_G = I_G\boldsymbol{\omega}$$

The angular momentum about point A is

$$H_A = I_G\omega + (d)mv_G$$



General plane motion

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Part 3

Principle of Impulse and Momentum (19.2)

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Principle of Impulse and Momentum (19.2)

As in the case of particle motion, the principle of impulse and momentum for a rigid body is developed by combining the equation of motion with kinematics. The resulting equations allow a direct solution to problems involving force, velocity, and time.

Linear impulse-linear momentum equation:

$$\boldsymbol{L}_1 + \sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt = \boldsymbol{L}_2 \quad \text{or} \quad (m\boldsymbol{v}_G)_1 + \sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt = (m\boldsymbol{v}_G)_2$$

Angular impulse-angular momentum equation:

$$(H_G)_1 + \sum \int_{t_1}^{t_2} M_G\,dt = (H_G)_2 \quad \text{or} \quad I_G\omega_1 + \sum \int_{t_1}^{t_2} M_G\,dt = I_G\omega_2$$

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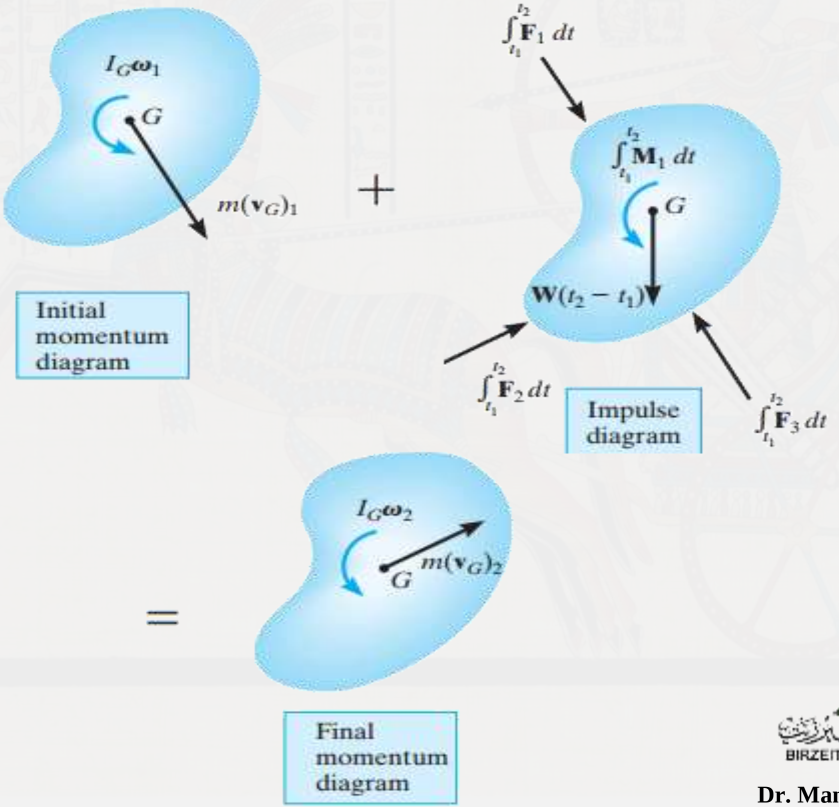
Principle of Impulse and Momentum (19.2)

The previous relations can be represented graphically by drawing the impulse-momentum diagram.

To summarize, if motion is occurring in the x-y plane, the linear impulse-linear momentum relation can be applied to the x and y directions and the angular momentum-angular impulse relation is applied about a z-axis passing through any point (i.e., G). Therefore, the principle yields three scalar equations (eqs 19-4) describing the planar motion of the body.



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Principle of Impulse and Momentum (19.2)

$$m(v_{Gx})_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_{Gx})_2$$

$$m(v_{Gy})_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_{Gy})_2$$

$$I_G \omega_1 + \Sigma \int_{t_1}^{t_2} M_G \, dt = I_G \omega_2$$

Procedure of analysis

- Establish the x, y, z inertial frame of reference.
- Draw the impulse-momentum diagrams for the body.
- Compute I_G, as necessary.
- Apply the equations of impulse and momentum (one vector and one scalar or the three scalar equations).
- If more than three unknowns are involved, kinematic equations relating the velocity of the mass center G and the angular velocity ω should be used in order to have additional equations.

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Part 4

Conservation of momentum (section 19.3)

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Applications

A skater spends a lot of time either spinning on the ice or rotating through the air. To spin fast, or for a long time, the skater must develop a large amount of angular momentum.

Conservation of momentum (section 19.3)

Recall that the linear impulse and momentum relationship is

$$\boldsymbol{L}_1 + \sum \int_{t_1}^{t_2} \cancelto{0}{\boldsymbol{F}}\, dt = \boldsymbol{L}_2 \quad \text{or} \quad (m\, \boldsymbol{v}_G)_1 + \sum \int_{t_1}^{t_2} \cancelto{0}{\boldsymbol{F}}\, dt = (m\, \boldsymbol{v}_G)_2$$

If the sum of all the linear impulses acting on the rigid body (or a system of rigid bodies) is zero, all the impulse terms are zero. Thus, the linear momentum for a rigid body is constant, or conserved. So $\boldsymbol{L}_1 = \boldsymbol{L}_2$.

$$\boldsymbol{L}_1 = \boldsymbol{L}_2 \text{ (for a system).}$$

This equation is referred to as the conservation of linear momentum. The conservation of linear momentum equation can be used if the linear impulses are small or non-impulsive.

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Conservation of angular momentum (section 19.3)

The angular impulse-angular momentum relationship is:

$$(\boldsymbol{H}_G)_1 + \sum\int_{t_1}^{t_2} \cancelto{0}{\boldsymbol{M}_G}\, dt = (\boldsymbol{H}_G)_2 \quad \text{or} \quad I_G\boldsymbol{\omega}_1 + \sum\int_{t_1}^{t_2} \cancelto{0}{\boldsymbol{M}_G}\, dt = I_G\boldsymbol{\omega}_2$$

Similarly, if the sum of all the angular impulses due to external forces acting on the rigid body (or a system of rigid bodies) is zero, all the impulse terms are zero. Thus, angular momentum is conserved. The resulting equation is referred to as the conservation of angular momentum or $(\boldsymbol{H}_G)_1 = (\boldsymbol{H}_G)_2$.

$$(\boldsymbol{H}_G)_1 = (\boldsymbol{H}_G)_2 \text{ (for a system).}$$

If the initial condition of the rigid body (or system) is known, conservation of momentum is often used to determine the final linear or angular velocity of a body just after an event occurs.

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Procedure of analysis

- Establish the x, y, z inertial frame of reference and draw FBDs.
- Write the conservation of linear momentum equation.
- Write the conservation of angular momentum equation about a fixed point or at the mass center G.
- Solve the conservation of linear or angular momentum equations in the appropriate directions.
- If the motion is complicated, kinematic equations relating the velocity of the mass center G and the angular velocity ω may be necessary.

Example 1

The 20-lb disk shown in Fig. 19–5*a* is acted upon by a constant couple moment of 4 lb·ft and a force of 10 lb which is applied to a cord wrapped around its periphery. Determine the angular velocity of the disk two seconds after starting from rest. Also, what are the force components of reaction at the pin?

Example 1

The 20-lb disk shown in Fig. 19–5*a* is acted upon by a constant couple moment of 4 lb · ft and a force of 10 lb which is applied to a cord wrapped around its periphery. Determine the angular velocity of the disk two seconds after starting from rest. Also, what are the force components of reaction at the pin?

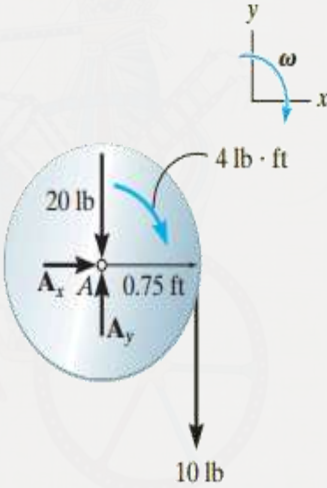
SOLUTION

Since angular velocity, force, and time are involved in the problems, we will apply the principles of impulse and momentum to the solution.

Free-Body Diagram. Fig. 19–5*b*. The disk's mass center does not move; however, the loading causes the disk to rotate clockwise.

The moment of inertia of the disk about its fixed axis of rotation is

$$I_A = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{20\text{ lb}}{32.2\text{ ft/s}^2}\right)(0.75\text{ ft})^2 = 0.1747\text{ slug}\cdot\text{ft}^2$$



$(\overset{+}{\rightarrow})$
$$m(v_{Ax})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Ax})_2$$
$$0 + A_x(2\text{ s}) = 0$$

$(+\uparrow)$
$$m(v_{Ay})_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_{Ay})_2$$
$$0 + A_y(2\text{ s}) - 20\text{ lb}(2\text{ s}) - 10\text{ lb}(2\text{ s}) = 0$$

$(\circlearrowright +)$
$$I_A\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\omega_2$$
$$0 + 4\text{ lb}\cdot\text{ft}(2\text{ s}) + [10\text{ lb}(2\text{ s})](0.75\text{ ft}) = 0.1747\omega_2$$

Solving these equations yields

$$A_x = 0 \qquad \textit{Ans.}$$
$$A_y = 30\text{ lb} \qquad \textit{Ans.}$$
$$\omega_2 = 132\text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$

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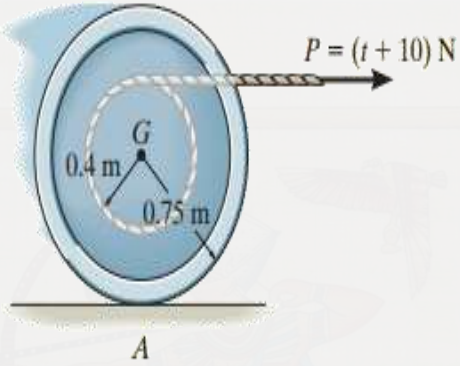
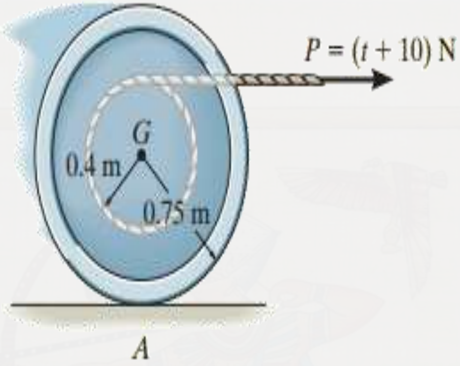
Example 2



The 100-kg spool shown in Fig. 19–6*a* has a radius of gyration $k_G = 0.35$ m. A cable is wrapped around the central hub of the spool, and a horizontal force having a variable magnitude of $P = (t + 10)$ N is applied, where t is in seconds. If the spool is initially at rest, determine its angular velocity in 5 s. Assume that the spool rolls without slipping at A.

Example 2

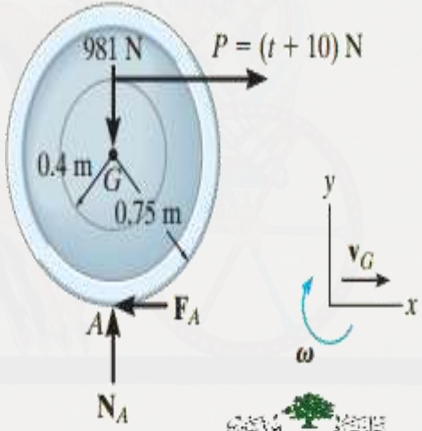
The 100-kg spool shown in Fig. 19–6*a* has a radius of gyration $k_G = 0.35$ m. A cable is wrapped around the central hub of the spool, and a horizontal force having a variable magnitude of $P = (t + 10)$ N is applied, where t is in seconds. If the spool is initially at rest, determine its angular velocity in 5 s. Assume that the spool rolls without slipping at A.



SOLUTION

Free-Body Diagram. From the free-body diagram, Fig. 19–6*b*, the *variable* force $\mathbf{P}$ will cause the friction force $\mathbf{F}_A$ to be variable, and thus the impulses created by both $\mathbf{P}$ and $\mathbf{F}_A$ must be determined by integration. Force $\mathbf{P}$ causes the mass center to have a velocity $\mathbf{v}_G$ to the right, and so the spool has a clockwise angular velocity $\boldsymbol{\omega}$.

Principle of Impulse and Momentum. A direct solution for $\boldsymbol{\omega}$ can be obtained by applying the principle of angular impulse and momentum about point A, the IC, in order to eliminate the unknown friction impulse.



$$(\circlearrowright +) \qquad I_A\omega_1 + \Sigma \int M_A\,dt = I_A\omega_2$$

$$0 + \left[\int_0^{5\,s} (t + 10)\text{ N } dt\right](0.75\text{ m} + 0.4\text{ m}) = [100\text{ kg }(0.35\text{ m})^2 + (100\text{ kg})(0.75\text{ m})^2]\omega_2$$

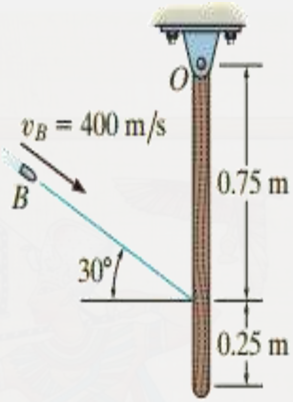
$$62.5(1.15) = 68.5\omega_2$$

$$\omega_2 = 1.05\text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$

NOTE: Try solving this problem by applying the principle of impulse and momentum about G and using the principle of linear impulse and momentum in the x direction.

Example 3

The 5-kg slender rod shown in Fig. 19–10*a* is pinned at *O* and is initially at rest. If a 4-g bullet is fired into the rod with a velocity of 400 m/s, as shown in the figure, determine the angular velocity of the rod just after the bullet becomes embedded in it.



Example 3

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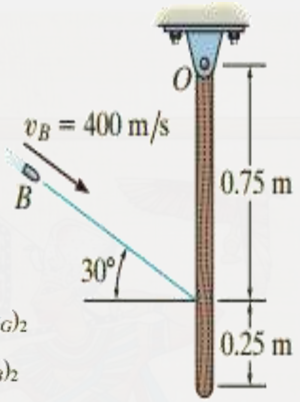
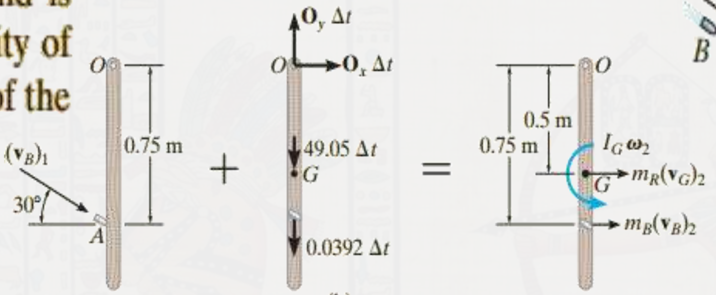


SOLUTION

Impulse and Momentum Diagrams. The impulse which the bullet exerts on the rod can be eliminated from the analysis, and the angular velocity of the rod just after impact can be determined by considering the bullet and rod as a single system. To clarify the principles involved, the impulse and momentum diagrams are shown in Fig. 19–10*b*. The momentum diagrams are drawn *just before and just after impact*. During impact, the bullet and rod exert equal but *opposite internal impulses* at *A*. As shown on the impulse diagram, the impulses that are external to the system are due to the reactions at *O* and the weights of the bullet and rod. Since the time of impact, Δt, is very short, the rod moves only a slight amount, and so the "moments" of the weight impulses about point *O* are essentially zero. Therefore angular momentum is conserved about this point.



(b)

Conservation of Angular Momentum. From Fig. 19–10*b*, we have

$$(\circlearrowleft +) \qquad \Sigma(H_O)_1 = \Sigma(H_O)_2$$

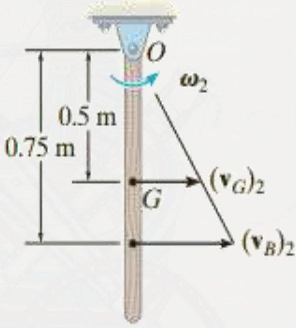
$$m_B(v_B)_1 \cos 30°(0.75\text{ m}) = m_B(v_B)_2(0.75\text{ m}) + m_R(v_G)_2(0.5\text{ m}) + I_G\omega_2$$

$$(0.004\text{ kg})(400 \cos 30°\text{ m/s})(0.75\text{ m}) =$$

$$(0.004\text{ kg})(v_B)_2(0.75\text{ m}) + (5\text{ kg})(v_G)_2(0.5\text{ m}) + \left[\tfrac{1}{12}(5\text{ kg})(1\text{ m})^2\right]\omega_2$$

or

$$1.039 = 0.003(v_B)_2 + 2.50(v_G)_2 + 0.4167\omega_2 \qquad (1)$$



(c)

Kinematics. Since the rod is pinned at *O*, from Fig. 19–10*c* we have

$$(v_G)_2 = (0.5\text{ m})\omega_2 \qquad (v_B)_2 = (0.75\text{ m})\omega_2$$

Substituting into Eq. 1 and solving yields

$$\omega_2 = 0.623\text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$

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Part 5

Planar kinetics of a rigid body: Eccentric Impact -19.4

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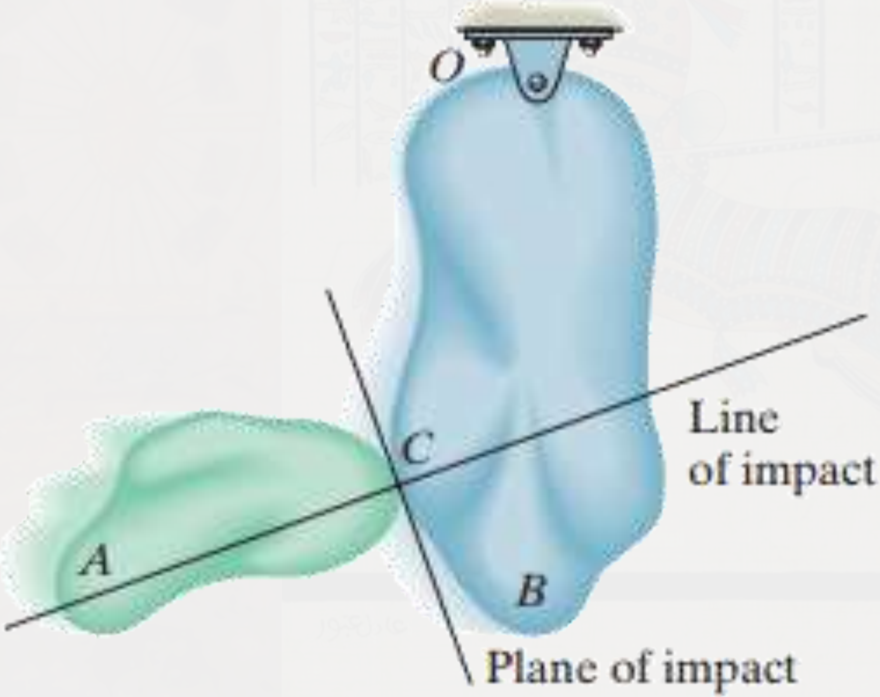
Applications

Here is an example of eccentric impact occurring between this bowling ball and pin.

Eccentric impact (19.4)

Eccentric impact occurs when the line connecting the *mass centers* of the two bodies *does not* coincide with the line of impact.* This type of impact often occurs when one or both of the bodies are constrained to rotate about a fixed axis.



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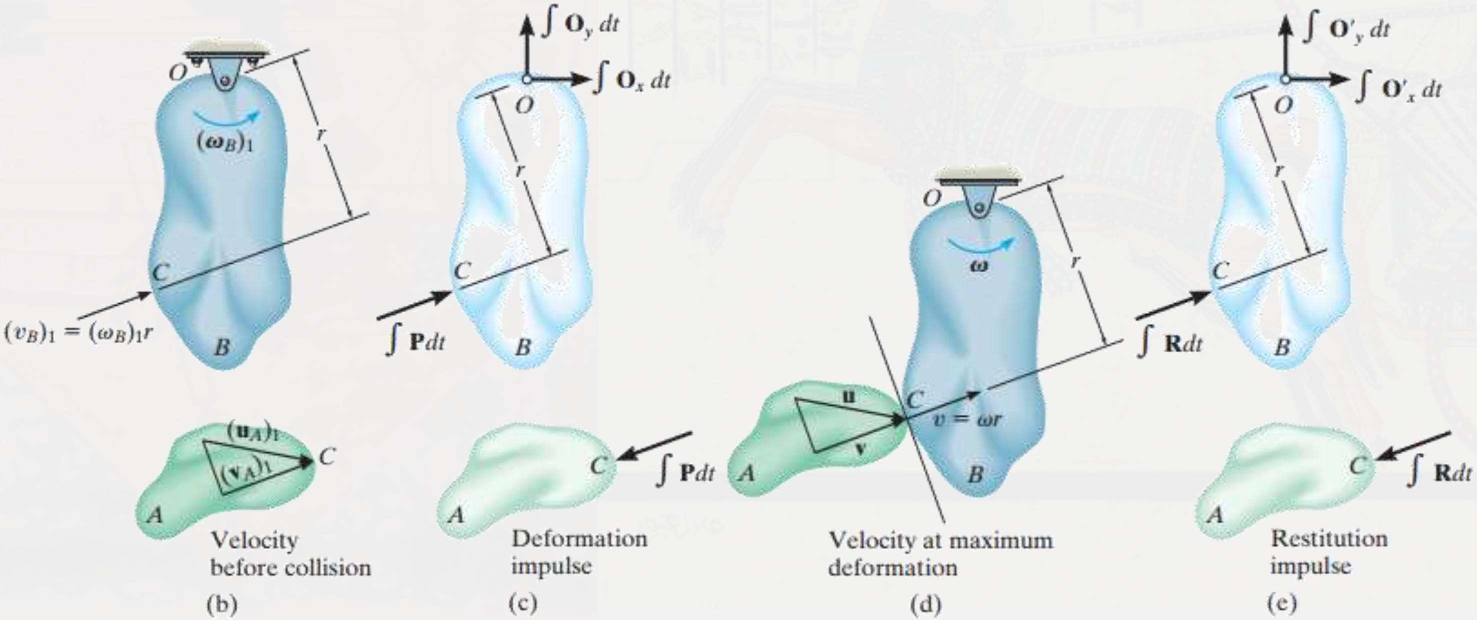
Eccentric impact (19.4)

- During the impact an equal but opposite impulsive force P is exerted between the bodies which deforms their shapes at the point of contact. The resulting impulse is shown on the impulse diagrams for both bodies, Fig. c.
- When the deformation at point C is a maximum, C on both the bodies moves with a common velocity v along the line of impact, Fig. d.



Velocity before collision (b) | Deformation impulse (c) | Velocity at maximum deformation (d) | Restitution impulse (e)

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Eccentric impact (19.4)

- A period of restitution then occurs in which the bodies tend to regain their original shapes. The restitution phase creates an equal but opposite impulsive force R acting between the bodies as shown on the impulse diagram, Fig. e.

- After restitution the bodies move apart such that point C on body B has a velocity $(V_B)_2$ and point C on body A has a velocity $(\mathbf{u}_A)_2$ Fig. f.

- In general, a problem involving the impact of two bodies requires determining the two unknowns $(V_B)_2$ and $(V_A)_2$. To solve such problems, two equations must be written.

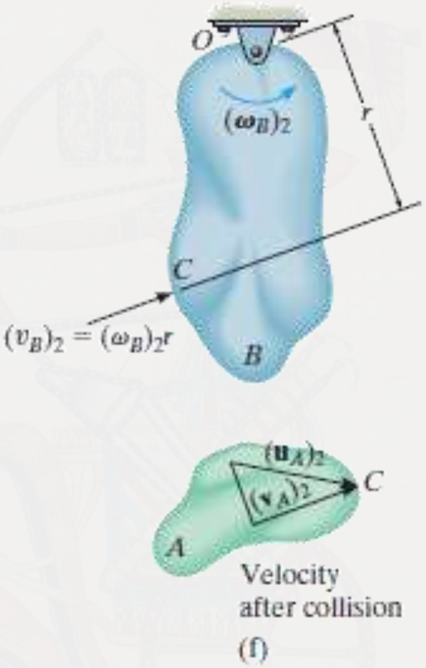

Fig. 19–11 (cont.)

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Eccentric impact (19.4)

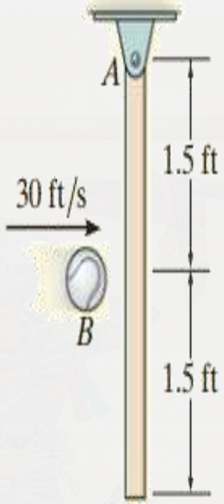
- The first equation generally involves application of the conservation of angular momentum to the two bodies. In the case of both bodies A and B. In the case of both bodies A and B, we can state that angular momentum is conserved about point O since the impulses at C are internal to the system and the impulses at O create zero moment (or zero angular impulse) about O.

- The second equation can be obtained using the definition of the coefficient of restitution, e, which is a ratio of the restitution impulse to the deformation impulse.

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

Example 4

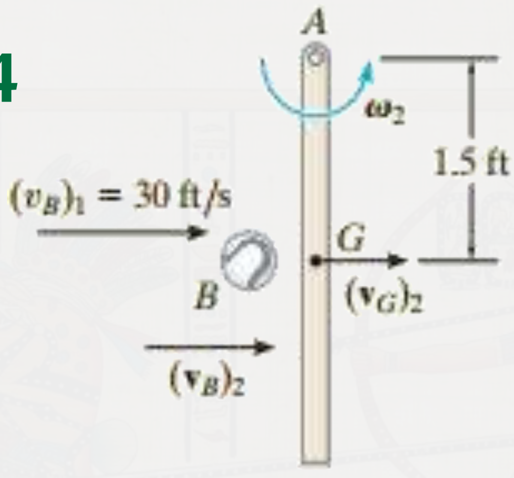
The 10-lb slender rod is suspended from the pin at A, Fig. 19–12a. If a 2-lb ball B is thrown at the rod and strikes its center with a velocity of 30 ft/s, determine the angular velocity of the rod just after impact. The coefficient of restitution is $e = 0.4$.



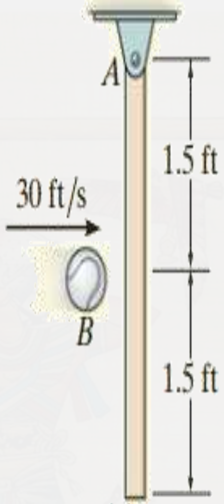
(a)

Example 4

The 10-lb slender rod is suspended from the pin at A, Fig. 19–12*a*. If a 2-lb ball B is thrown at the rod and strikes its center with a velocity of 30 ft/s, determine the angular velocity of the rod just after impact. The coefficient of restitution is $e = 0.4$.



(c)



(a)

SOLUTION

Conservation of Angular Momentum. Consider the ball and rod as a system, Fig. 19–12*b*. Angular momentum is conserved about point A since the impulsive force between the rod and ball is *internal*. Also, the *weights* of the ball and rod are *nonimpulsive*. Noting the directions of the velocities of the ball and rod just after impact as shown on the kinematic diagram, Fig. 19–12*c*, we require

$$(\circlearrowleft +) \qquad (H_A)_1 = (H_A)_2$$

$$m_B(v_B)_1(1.5 \text{ ft}) = m_B(v_B)_2(1.5 \text{ ft}) + m_R(v_G)_2(1.5 \text{ ft}) + I_G\omega_2$$

$$\left(\frac{2 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(30 \text{ ft/s})(1.5 \text{ ft}) = \left(\frac{2 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(v_B)_2(1.5 \text{ ft}) +$$

$$\left(\frac{10 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(v_G)_2(1.5 \text{ ft}) + \left[\frac{1}{12}\left(\frac{10 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(3 \text{ ft})^2\right]\omega_2$$

Since $(v_G)_2 = 1.5\omega_2$ then

$$2.795 = 0.09317(v_B)_2 + 0.9317\omega_2 \qquad (1)$$

Coefficient of Restitution. With reference to Fig. 19–12*c*, we have

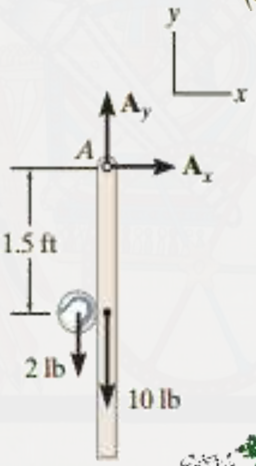
$$(\xrightarrow{+}) \qquad e = \frac{(v_G)_2 - (v_B)_2}{(v_B)_1 - (v_G)_1} \qquad 0.4 = \frac{(1.5 \text{ ft})\omega_2 - (v_B)_2}{30 \text{ ft/s} - 0}$$

$$12.0 = 1.5\omega_2 - (v_B)_2$$

Solving,

$$(v_B)_2 = -6.52 \text{ ft/s} = 6.52 \text{ ft/s} \leftarrow$$

$$\omega_2 = 3.65 \text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$



(b)

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Part 7

Help Session

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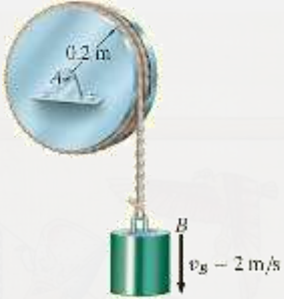
Help Session 1



The cylinder shown in Fig. 19–7*a* has a mass of 6 kg. It is attached to a cord which is wrapped around the periphery of a 20-kg disk that has a moment of inertia $I_A = 0.40\ \text{kg}\cdot\text{m}^2$. If the cylinder is initially moving downward with a speed of 2 m/s, determine its speed in 3 s. Neglect the mass of the cord in the calculation.

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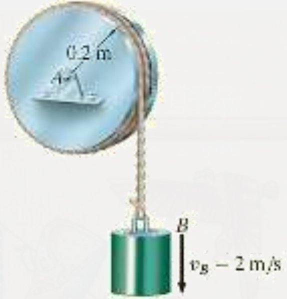

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SOLUTION I

Free-Body Diagram. The free-body diagrams of the cylinder and disk are shown in Fig. 19–7*b*. All the forces are *constant* since the weight of the cylinder causes the motion. The downward motion of the cylinder, $\mathbf{v}_B$, causes $\boldsymbol{\omega}$ of the disk to be clockwise.

Principle of Impulse and Momentum. We can eliminate $\mathbf{A}_x$ and $\mathbf{A}_y$ from the analysis by applying the principle of angular impulse and momentum about point A. Hence

Disk

$$(\circlearrowright +)\qquad I_A\omega_1 + \Sigma\int M_A\,dt = I_A\omega_2$$

$$0.40\ \text{kg}\cdot\text{m}^2(\omega_1) + T(3\ \text{s})(0.2\ \text{m}) = (0.40\ \text{kg}\cdot\text{m}^2)\omega_2$$

Cylinder

$$(+\uparrow)\qquad m_B(v_B)_1 + \Sigma\int F_y\,dt = m_B(v_B)_2$$

$$-6\ \text{kg}(2\ \text{m/s}) + T(3\ \text{s}) - 58.86\ \text{N}(3\ \text{s}) = -6\ \text{kg}(v_B)_2$$

Kinematics. Since $\omega = v_B/r$, then $\omega_1 = (2\ \text{m/s})/(0.2\ \text{m}) = 10$ rad/s and $\omega_2 = (v_B)_2/0.2\ \text{m} = 5(v_B)_2$. Substituting and solving the equations simultaneously for $(v_B)_2$ yields

$$(v_B)_2 = 13.0\ \text{m/s}\downarrow \qquad \textit{Ans.}$$

SOLUTION II

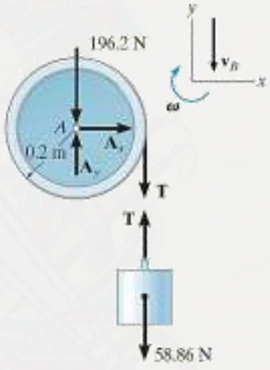
Impulse and Momentum Diagrams. We can obtain $(v_B)_2$ *directly* by considering the *system* consisting of the cylinder, the cord, and the disk. The impulse and momentum diagrams have been drawn to clarify application of the principle of angular impulse and momentum about point A, Fig. 19–7*c*.

Principle of Angular Impulse and Momentum. Realizing that $\omega_1 = 10$ rad/s and $\omega_2 = 5(v_B)_2$, we have

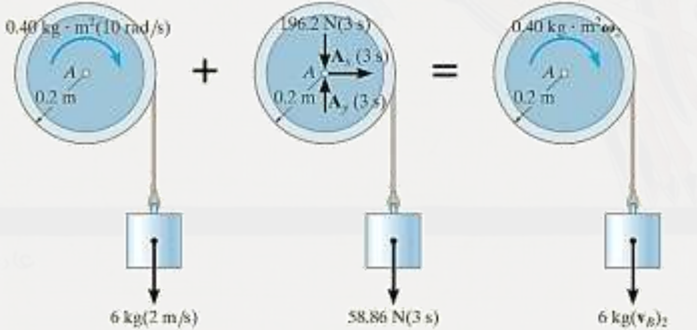
$$(\circlearrowright +)\left(\Sigma\begin{matrix}\text{syst. angular}\\ \text{momentum}\end{matrix}\right)_{A1} + \left(\Sigma\begin{matrix}\text{syst. angular}\\ \text{impulse}\end{matrix}\right)_{A(1-2)} = \left(\Sigma\begin{matrix}\text{syst. angular}\\ \text{momentum}\end{matrix}\right)_{A2}$$

$$(6\ \text{kg})(2\ \text{m/s})(0.2\ \text{m}) + (0.40\ \text{kg}\cdot\text{m}^2)(10\ \text{rad/s}) + (58.86\ \text{N})(3\ \text{s})(0.2\ \text{m})$$
$$= (6\ \text{kg})(v_B)_2(0.2\ \text{m}) + (0.40\ \text{kg}\cdot\text{m}^2)[5(v_B)_2(0.2\ \text{m})]$$

$$(v_B)_2 = 13.0\ \text{m/s}\downarrow \qquad \textit{Ans.}$$



(b)



(c)

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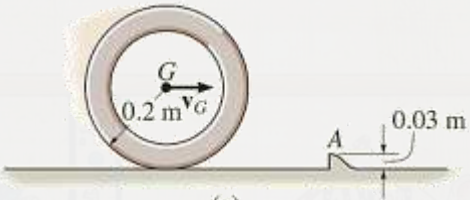
Help Session 2

The 10-kg wheel shown in Fig. 19–9*a* has a moment of inertia $I_G = 0.156\ \mathrm{kg \cdot m^2}$. Assuming that the wheel does not slip or rebound, determine the minimum velocity $\mathbf{v}_G$ it must have to just roll over the obstruction at A.



(a)

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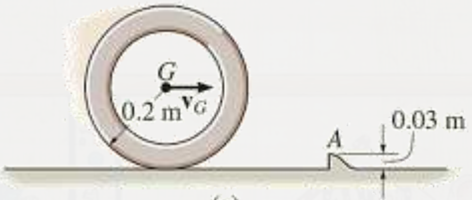


(a)

SOLUTION

Impulse and Momentum Diagrams. Since no slipping or rebounding occurs, the wheel essentially *pivots* about point A during contact. This condition is shown in Fig. 19–9*b*, which indicates, respectively, the momentum of the wheel *just before impact*, the impulses given to the wheel *during impact*, and the momentum of the wheel *just after impact*. Only two impulses (forces) act on the wheel. By comparison, the force at A is much greater than that of the weight, and since the time of impact is very short, the weight can be considered nonimpulsive. The impulsive force $\mathbf{F}$ at A has both an unknown magnitude and an unknown direction θ. To eliminate this force from the analysis, note that angular momentum about A is essentially *conserved* since $(98.1\Delta t)d \approx 0$.

Conservation of Angular Momentum. With reference to Fig. 19–9*b*,

$$(\circlearrowleft +) \qquad (H_A)_1 = (H_A)_2$$

$$r'm(v_G)_1 + I_G\omega_1 = rm(v_G)_2 + I_G\omega_2$$

$$(0.2\text{ m} - 0.03\text{ m})(10\text{ kg})(v_G)_1 + (0.156\text{ kg}\cdot\text{m}^2)(\omega_1) =$$
$$(0.2\text{ m})(10\text{ kg})(v_G)_2 + (0.156\text{ kg}\cdot\text{m}^2)(\omega_2)$$

Kinematics. Since no slipping occurs, in general $\omega = v_G/r = v_G/0.2\text{ m} = 5v_G$. Substituting this into the above equation and simplifying yields

$$(v_G)_2 = 0.8921(v_G)_1 \qquad (1)$$

Conservation of Energy.* In order to roll over the obstruction, the wheel must pass position 3 shown in Fig. 19–9*c*. Hence, if $(v_G)_2$ [or $(v_G)_1$] is to be a minimum, it is necessary that the kinetic energy of the wheel at position 2 be equal to the potential energy at position 3. Placing the datum through the center of gravity, as shown in the figure, and applying the conservation of energy equation, we have

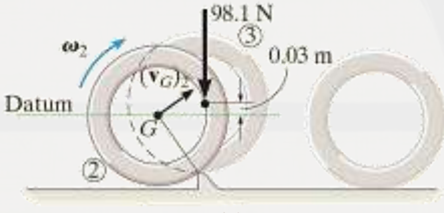
$$\{T_2\} + \{V_2\} = \{T_3\} + \{V_3\}$$

$$\left\{\tfrac{1}{2}(10\text{ kg})(v_G)_2^2 + \tfrac{1}{2}(0.156\text{ kg}\cdot\text{m}^2)\omega_2^2\right\} + \{0\} =$$
$$\{0\} + \{(98.1\text{ N})(0.03\text{ m})\}$$

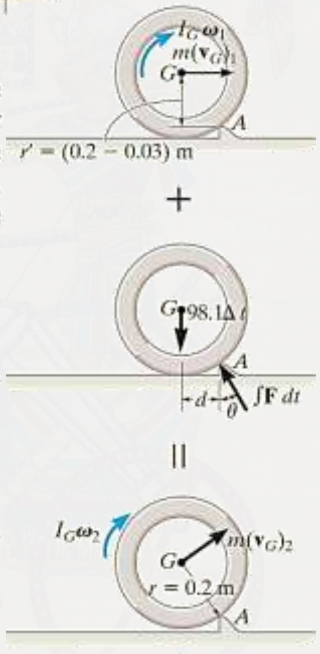
Substituting $\omega_2 = 5(v_G)_2$ and Eq. 1 into this equation, and solving,

$$(v_G)_1 = 0.729\text{ m/s} \rightarrow \qquad \textit{Ans.}$$

*This principle *does not apply during impact*, since energy is *lost* during the collision. However, just after impact, as in Fig. 19–9*c*, it can be used.



(c)



(b)

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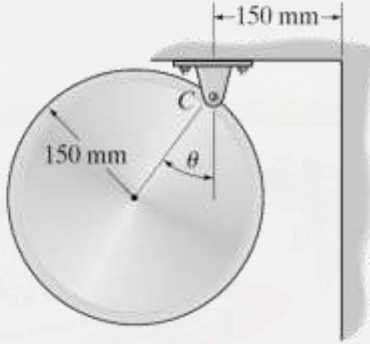
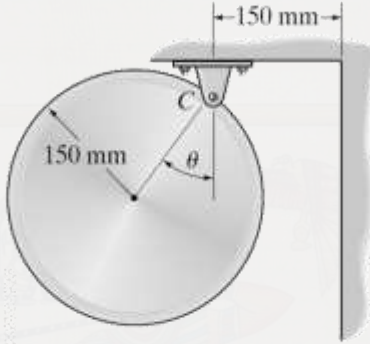
Help Session 3

19–51. The disk has a mass of 15 kg. If it is released from rest when $\theta = 30°$, determine the maximum angle θ of rebound after it collides with the wall. The coefficient of restitution between the disk and the wall is $e = 0.6$. When $\theta = 0°$, the disk hangs such that it just touches the wall. Neglect friction at the pin C.



Help Session 3



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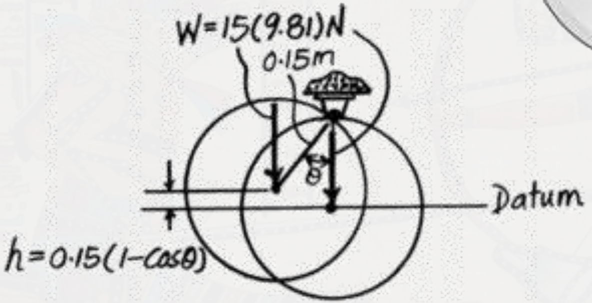
Datum at lower position of G.



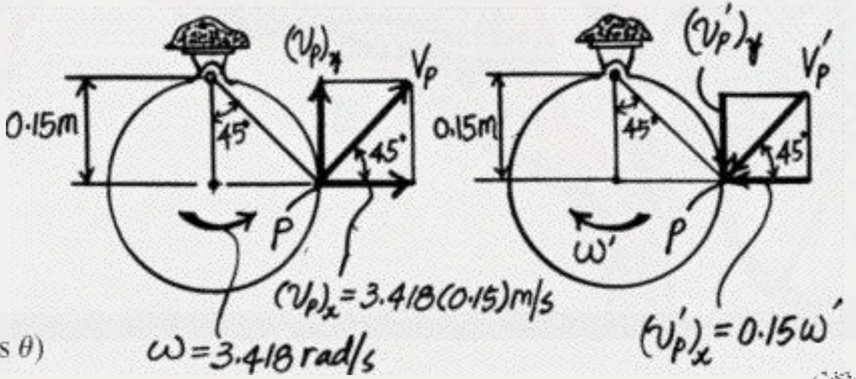
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (15)(9.81)(0.15)(1 - \cos 30°) = \frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right]\omega^2 + 0$$

$$\omega = 3.418 \text{ rad/s}$$

$$(\overset{+}{\rightarrow}) \qquad e = 0.6 = \frac{0 - (-0.15\omega')}{3.418(0.15) - 0}$$

$$\omega' = 2.0508 \text{ rad/s}$$



$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left[\frac{3}{2}(15)(0.15)^2\right](2.0508)^2 + 0 = 0 + 15(9.81)(0.15)(1 - \cos\theta)$$

$$\theta = 17.9°$$

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References

1. Engineering Mechanics: Dynamics, C. Hibbeler, 12th Edition, Prentice Hall, 2010.
2. Dr. Balasie PowerPoints, Dynamic Course, Birzeit University.

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