

Mechanism is a set of rigid links connected by joints

Rigid Bodies / links interconnected by joints designed for a given job.

Joint is a connection between two or more bodies

Degrees of Freedom DOF

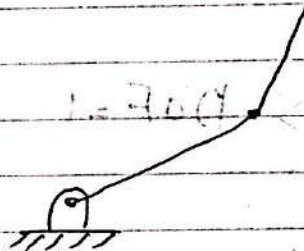
Number of independent parameters / inputs needed to control the motion of the joints

* Two types of spaces =
Motion in 2D-plane (x, y, θ)
Motion in 3D-space (x, y, z)
3 DOF
6 DOF

Types of Joints:

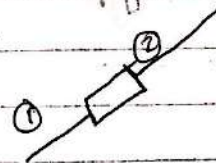
① Revolute Joints

DOF = 1



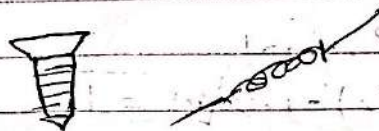
② Prismatic Joints

DOF = 1



③ Screw Joints

DOF = 1



④ Cylindrical Joints

DOF = 2

⑤ Spherical Joints: Can rotate in 3 angles

DOF = 3

* Mobility: (m)

Number of independent parameters/inputs needed to completely control the motion of mechanism

$M = 1$ DOF

in 2D: $m = 3(n-1) - 2 \times J_1 - J_2$ Kutzbach

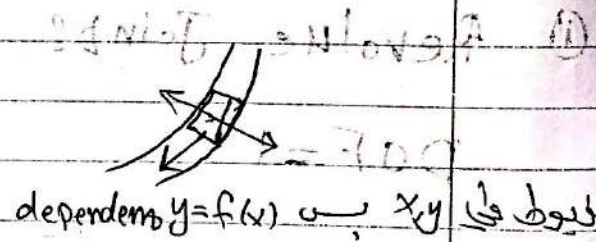
n: number of bodies including the ground only one to

J_1 : number of joints that have one DOF.

J_2 : " " " " " two DOF

* Contact joints

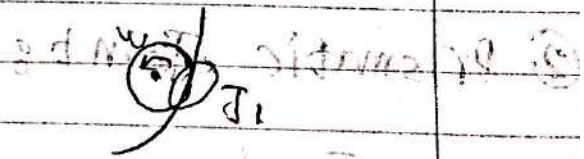
① Sliding only: \Rightarrow DOF = 1



② Rolling without sliding:

DOF = 1

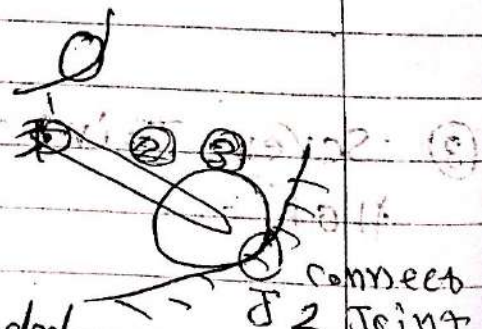
No slipping



③ Rolling with sliding \Rightarrow DOF = 2

Ex $n = 3, J_1 = 2, J_2 = 1$

$$m = 3(3-1) - 2(2) - 1 \Rightarrow m = 1$$



$m > 0 \Rightarrow$ Independent parameter are needed

$m = 0 \Rightarrow$ Determinate structure (No motion)

$m < 0 \Rightarrow$ Indeterminate

Mechanics

Kutzbach Criteria

$$m = 3(n-1) - 2J_1 - J_2$$

$$m = 0$$

$$m > 0$$

$$m < 0$$

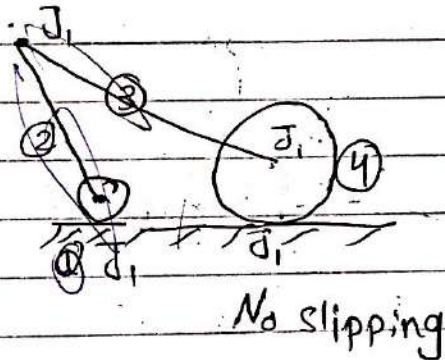
Example:

$$n = 4$$

$$J_1 = 4$$

$$J_2 = 0$$

$$\Rightarrow m = 1$$



$$n = 3$$

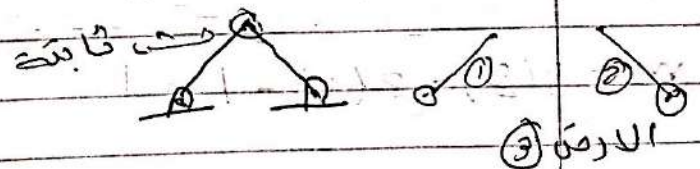
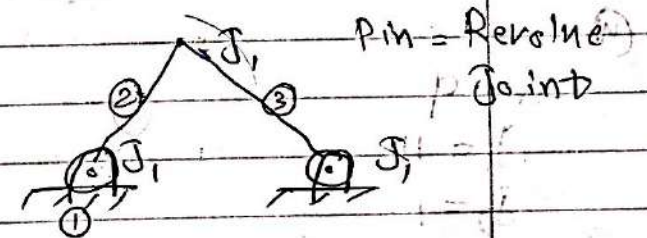
$$J_1 = 3$$

$$J_2 = 0$$

$$\Rightarrow m = 3(3-1) - 2(3) = 0$$

\Rightarrow Fixed structure

No motion



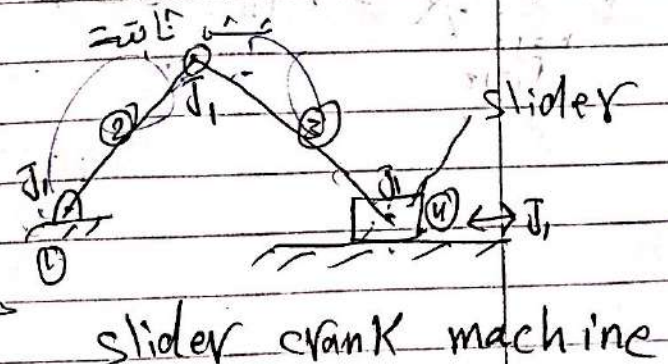
$$\textcircled{3} \cdot n = 4$$

$$J_1 = 4$$

$$J_2 = 0$$

$$\Rightarrow m = 1$$

في حركة
بما أنو الي يتحرك في الحركة؟
سبب التطبيق



سین ۳ و ۴
۵ و ۶

③ بیشتر داخل و بعضی علیه ساز تعلقاتی بین 4 و 5
dependencies

$$\left. \begin{array}{l} n=5 \\ j_1=1 \\ j_2=0 \end{array} \right\} \Rightarrow m = 3(5-1) - 2(1) = 0$$

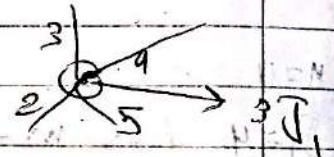
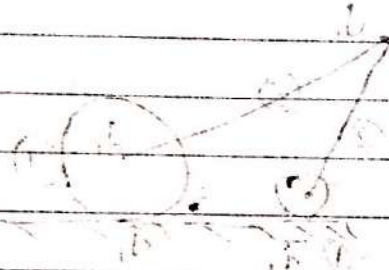
OR: $n=4$: ground, triangle, A, B

$$J_1 = 4;$$

$$m = 0$$

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5,28, 4,28, 3,2

Pin داخل slot داخل J_2

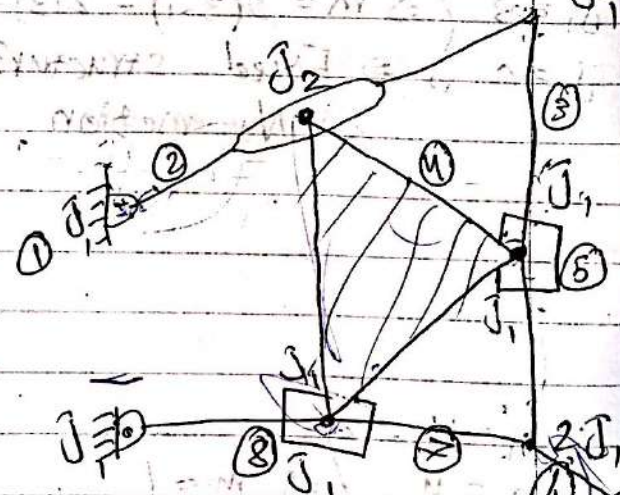
④

$n = 9$
 $J_1 = 11$
 $J_2 = 1$

$$m = 3(8) - 2(11) - 1 = 1$$

ولو كان بدل sbt  بدل $m=1$

$$\begin{aligned} n &= 10 \\ J_1 &= 13 \\ J_2 &= 0 \end{aligned}$$



بين الاسود والاحمر واحمر
بين الاسود والازرق واحمر sliding

No-slip

Exception to Kutzbach Criteria:

$$n=5$$

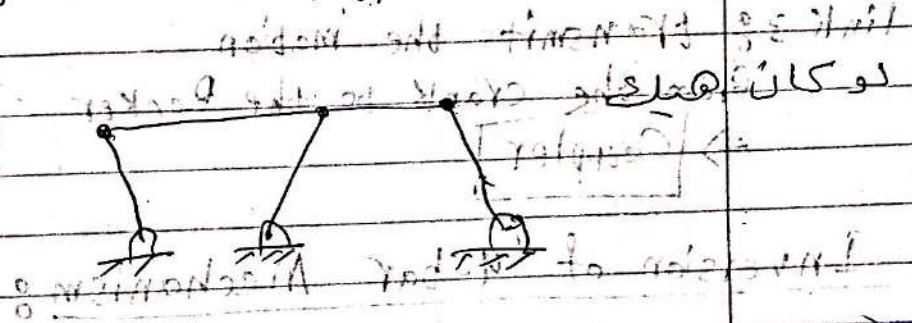
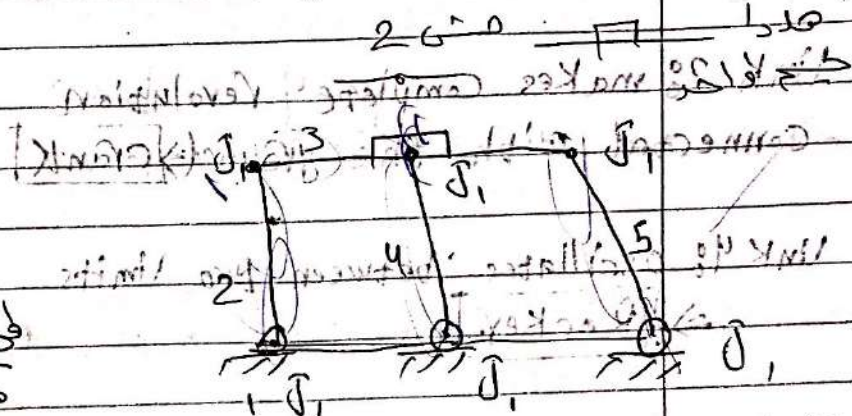
$$J_1=6$$

$$J_2=0$$

$$m=3(4)-2(6)=0$$

لها حركة لانه احوال متساوية ومتوازية

تتحرك لانه يهبط تعارض



Expo:

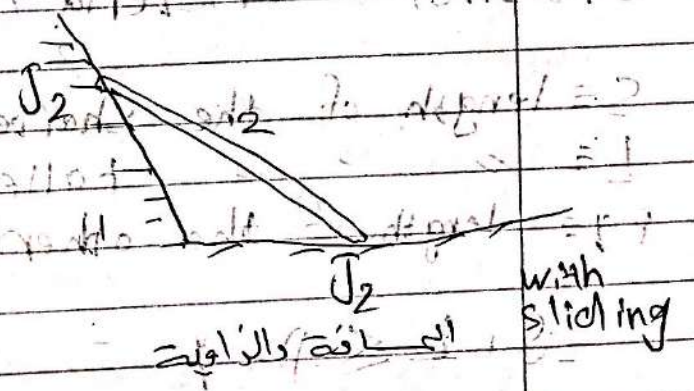
$$n=2$$

$$J_1=0$$

$$J_2=2$$

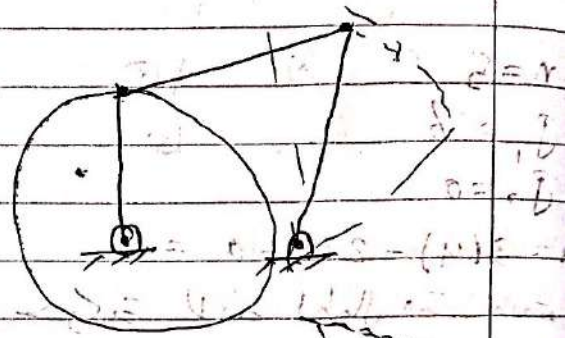
$$m=1$$

$$m=3(1)-2(2)=1$$



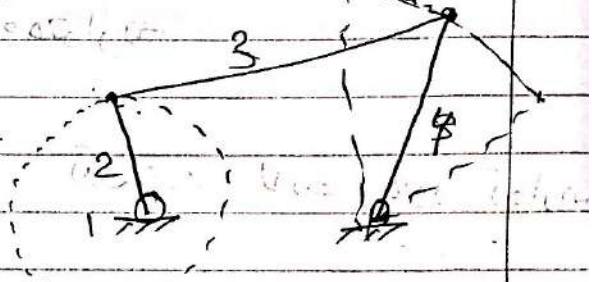
* Four Bar Mechanism

link 2: makes complete revolution
connected with the ground Crank



link 4: oscillates between two limits
Rocker

link 3: transmits the motion
from the crank to the Rocker
Coupler



Inversion of 4-bar Mechanism:

Grashof's criteria / mechanism

4 bar, 4 rev

S = length of the shortest link

L = length of the longest link

p, q = length of the other two links

$$S + L \leq p + q$$

\Rightarrow at least one link can make complete revolution

If shortest link is one of sides \Rightarrow crank

" " " " the ground \Rightarrow double cranks

\Rightarrow double Rocker No crank
only 4 rev

* Types of Four bar Mechanism

Grashof's Mechanism $S + L \leq P + Q$
 one of the side links is the shortest one. Crank-Rocker

1) $S \equiv$ fixed link
 All links make complete revolution. Double Crank

2) Double Rocker when shortest link \equiv connecting Rod

3) $S + L = P + Q$ change point Mechanism

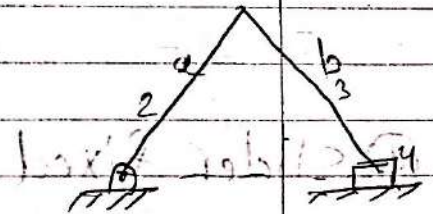
5) Non Grashof Mechanism Tripple Rocker
 $S + L > P + Q$ No link makes complete revolution

Slider Crank Mechanisms

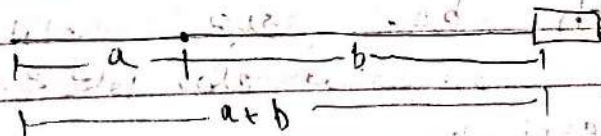
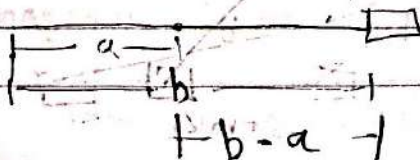
link 2: makes complete revolution \equiv crank

link 4: slider

link 3: connecting Rod

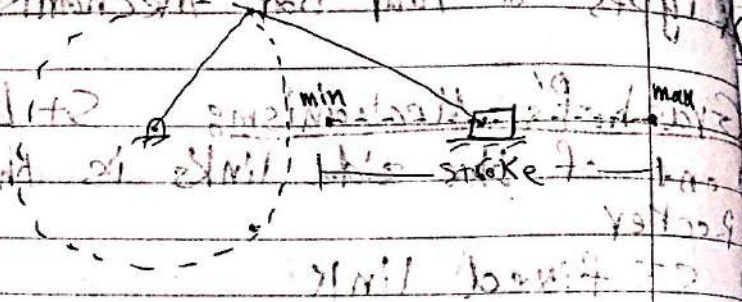


Stroke \equiv the total distance by the slider
 distance between the two extreme left & right positions of the slider



the stroke = $(a+b) - (b-a)$ minimum stroke to point to point
 Stroke = $2a$

$$p + q \geq l + 2$$



Types of Slider Crank Mechanisms

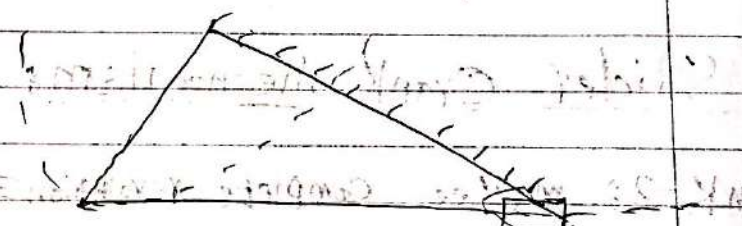
① Slider crank



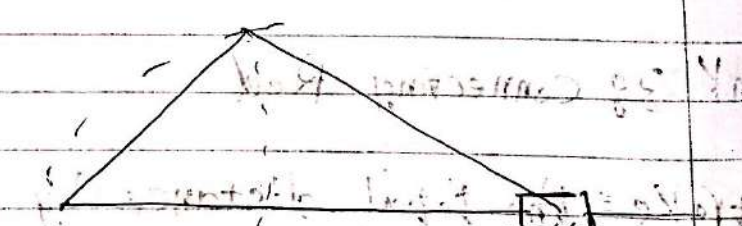
② Crank fixed



③ Connecting Rod Fixed



④ Slider fixed



⑤ Offset slider crank Mechanism

$$b > a + h$$

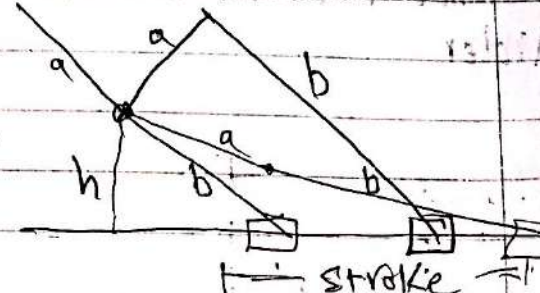
إذا كان $b > a + h$ فإن الحركة تكون دورية

$$r_1 = \sqrt{(a+b)^2 - h^2}$$

$$r_2 = \sqrt{(b-a)^2 - h^2}$$

$$\text{Stroke} = r_1 - r_2$$

$$\text{Stroke} = \sqrt{(a+b)^2 - h^2} - \sqrt{(b-a)^2 - h^2}$$



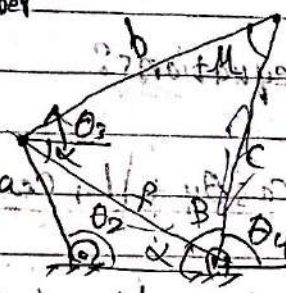
Position Analysis

- 1 Graphical Analysis
- 2 Analytical Analysis using complex Number

Four bar Mechanism

Graphical Analysis

Given θ_2 find θ_4



$$f^2 = a^2 + b^2 - 2ab \cos(\theta_2)$$

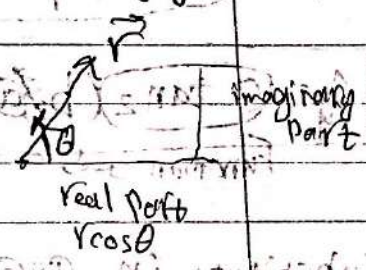
$$\frac{f}{\sin \theta_2} = \frac{a}{\sin \alpha} \Rightarrow \text{I have } \alpha$$

$$\vec{r} = r e^{j\theta} = r(\cos \theta + j \sin \theta)$$

$$= r \cos \theta + j r \sin \theta$$

$$b^2 = f^2 + c^2 - 2fc \cos(B) \Rightarrow \text{I have } B$$

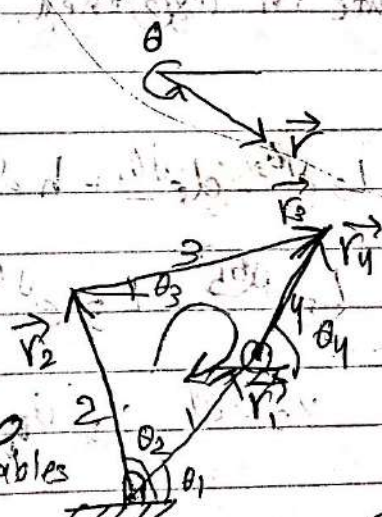
$$\Rightarrow \theta_4 = 180 - (B + \alpha)$$



Analytical Method: Position loop equation

$$\vec{r} = r(\cos \theta + j \sin \theta)$$

$$= r e^{j\theta}$$



$$\vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_1 = 0$$

$$L_2 e^{j\theta_2} + L_3 e^{j\theta_3} - L_4 e^{j\theta_4} - L_1 e^{j\theta_1} = 0$$

$$L_2(\cos \theta_2 + j \sin \theta_2) + L_3(\cos \theta_3 + j \sin \theta_3) - L_4(\cos \theta_4 + j \sin \theta_4) - L_1(\cos \theta_1 + j \sin \theta_1) = 0$$

variables

$\theta_2 \equiv$ input & (θ_3 & θ_4) are outputs, θ_1 is constant (ground)

Real Part: $L_2 \cos \theta_2 + L_3 \cos \theta_3 - L_4 \cos \theta_4 - L_1 \cos \theta_1 = 0 \quad \text{--- (1)}$

Imaginary Part: $L_2 \sin \theta_2 + L_3 \sin \theta_3 - L_4 \sin \theta_4 - L_1 \sin \theta_1 = 0 \quad \text{--- (2)}$

Position loop equations are non-linear relative to the variables.

To solve the equations:

$$L_3 \cos \theta_3 = L_4 \cos \theta_4 + \boxed{L_1 \cos \theta_1 - L_2 \cos \theta_2} \quad \text{--- (1)}$$

$$L_3 \sin \theta_3 = L_4 \sin \theta_4 + \boxed{L_1 \sin \theta_1 - L_2 \sin \theta_2} \quad \text{--- (2)}$$

$$\textcircled{1}^2 + \textcircled{2}^2 = L_3^2 = (L_4 \cos \theta_4 + A)^2 + (L_4 \sin \theta_4 + B)^2$$

$$L_3^2 = \underbrace{L_4^2 \cos^2 \theta_4}_{a} + A^2 + 2AL_4 \cos \theta_4 + \underbrace{L_4^2 \sin^2 \theta_4}_{b} + B^2 + 2BL_4 \sin \theta_4$$

$$\textcircled{3} = \underbrace{L_3^2 - L_4^2 - A^2 - B^2}_{c} = \underbrace{2L_4 A}_{a} \cos \theta_4 + \underbrace{2L_4 B}_{b} \sin \theta_4$$

$$\theta_4 = \boxed{a \tan^{-1} \left(\frac{b}{a} \right) \pm a \tan^{-1} \left(\frac{\sqrt{a^2 + b^2 - c^2}}{c} \right)} \quad \text{2 sol. for } \theta_4$$

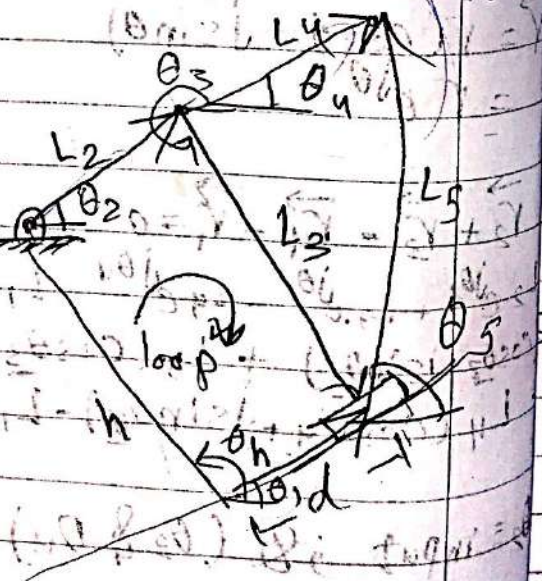
Function \tan^{-1}

Substitute in (1) & (2) then take $\tan \theta_3$ (find $\cos \theta_3$ & $\sin \theta_3$ then $\tan \theta_3$)

$$L_2 e^{j\theta_2} + L_3 e^{j\theta_3} - d e^{j\theta_1} + h e^{j\theta_4} = 0 \quad \begin{matrix} F_{11} \\ F_{12} \end{matrix}$$

$$L_4 e^{j\theta_4} - L_5 e^{j\theta_5} - L_3 e^{j\theta_3} = 0 \quad \begin{matrix} F_{31} \\ F_{41} \end{matrix}$$

$$x_1 = \theta_3 \quad x_2 = d \quad x_3 = \theta_4 \quad x_4 = \theta_5$$



Slider Crank Mechanism Position Analysis Using Graphical Method

UPLOADED BY AHMAD JUNDI

given θ_2 find X ?

$$X = a \cos \theta_2 + b \cos B$$

$$a \sin \theta_2 = b \sin B$$

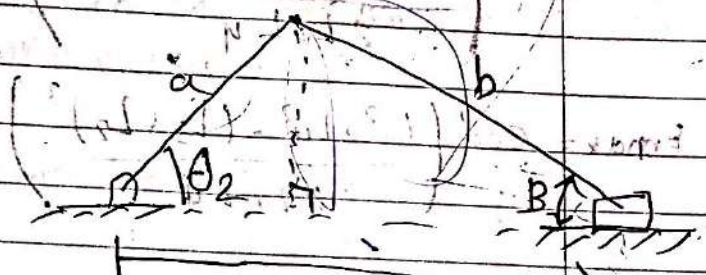
$$\sin B = \frac{a \sin \theta_2}{b}$$

$$(\sin B)^2 = \left(\frac{a \sin \theta_2}{b} \right)^2 \quad \text{OR}$$

$$1 - (\cos B)^2 = \left(\frac{a \sin \theta_2}{b} \right)^2$$

$$\cos B = \sqrt{1 - \left(\frac{a \sin \theta_2}{b} \right)^2}$$

$$X = a \cos \theta_2 + b \sqrt{1 - \left(\frac{a \sin \theta_2}{b} \right)^2}$$



$$B = \sin^{-1} \left(\frac{a \sin \theta_2}{b} \right)$$

لأنه \sin^{-1} معرفة بالزاوية

فقد رتبنا القانون الجيب لأنه في
ممكنة مع الزاوية المستقيمة

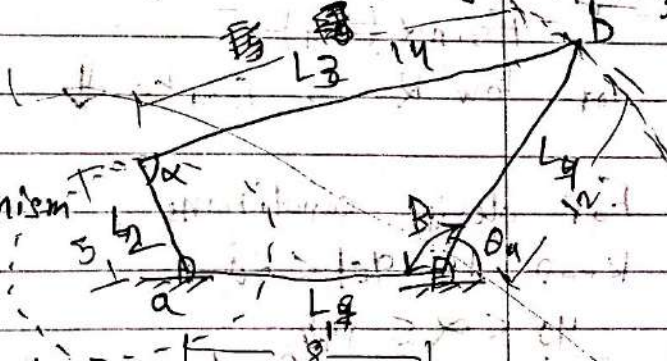
Four Bar Mechanism: Dead Point Configurations (Range of Motion)
given $L_1, L_2, L_3, L_4 \rightarrow$ find
range of motion of Link 4

Sol: Type of the four bar Mechanism

$$DoS + L < P + q ?$$

$$5 + 18 < 8 + 12$$

$$19 < 20 \quad \checkmark \Rightarrow \text{Grashof's mechanism}$$



Shortest link is one of the side links \rightarrow crank Rocker

θ_4 is minimum when B is maximum, B is maximum when the distance between a & b is maximum, \Rightarrow when $\alpha = 180^\circ$

the same with θ_4 minimum $\Rightarrow \alpha = 0$

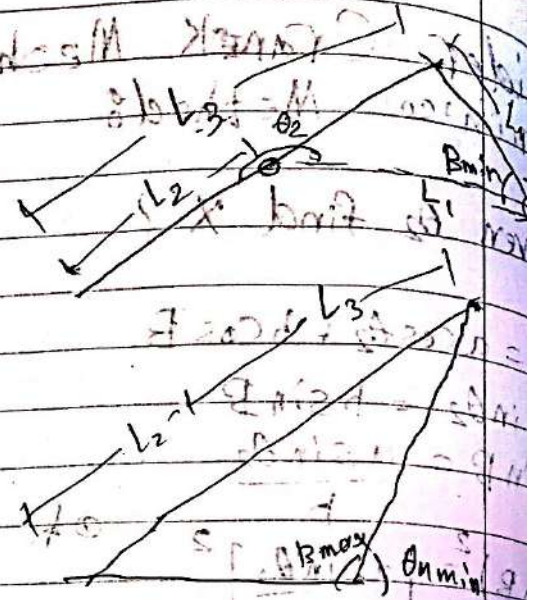
$$\theta_{4 \max} = 180 - B_{\min}$$

$$(L_3 - L_2)^2 = L_1^2 + L_4^2 - 2L_1L_4 \cos B_{\min}$$

$$B_{\min} = \cos^{-1} \left(\frac{L_1^2 + L_4^2 - (L_3 - L_2)^2}{2L_1L_4} \right)$$

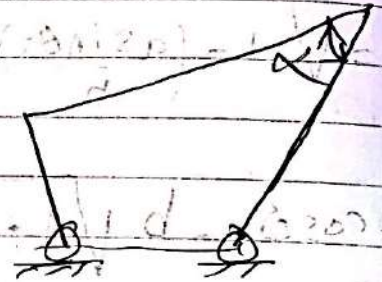
$$B_{\max} = \cos^{-1} \left(\frac{L_1^2 + L_4^2 - (L_3 + L_2)^2}{2L_1L_4} \right)$$

Range: $\theta_{4 \min} < \theta_4 \leq \theta_{4 \max}$
by sin law find θ_2



Transmission Angle:
→ Only for crank rocker
between the coupler & the rocker.

Motion is transmitted to link 4 (output)
through the coupler. (force)
Coupler → two force member



$$T = F \sin \alpha (L_4)$$

for low $\alpha \rightarrow$ Torque ↓

For design conditions

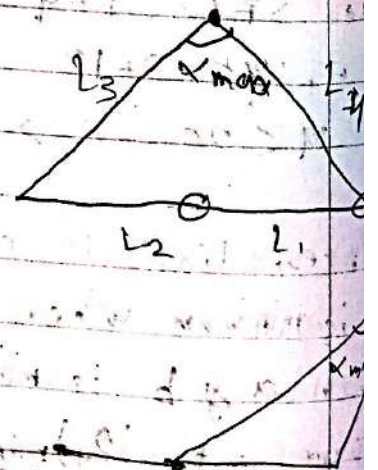
$$\text{Keep } |\alpha - 90| \leq 50^\circ$$

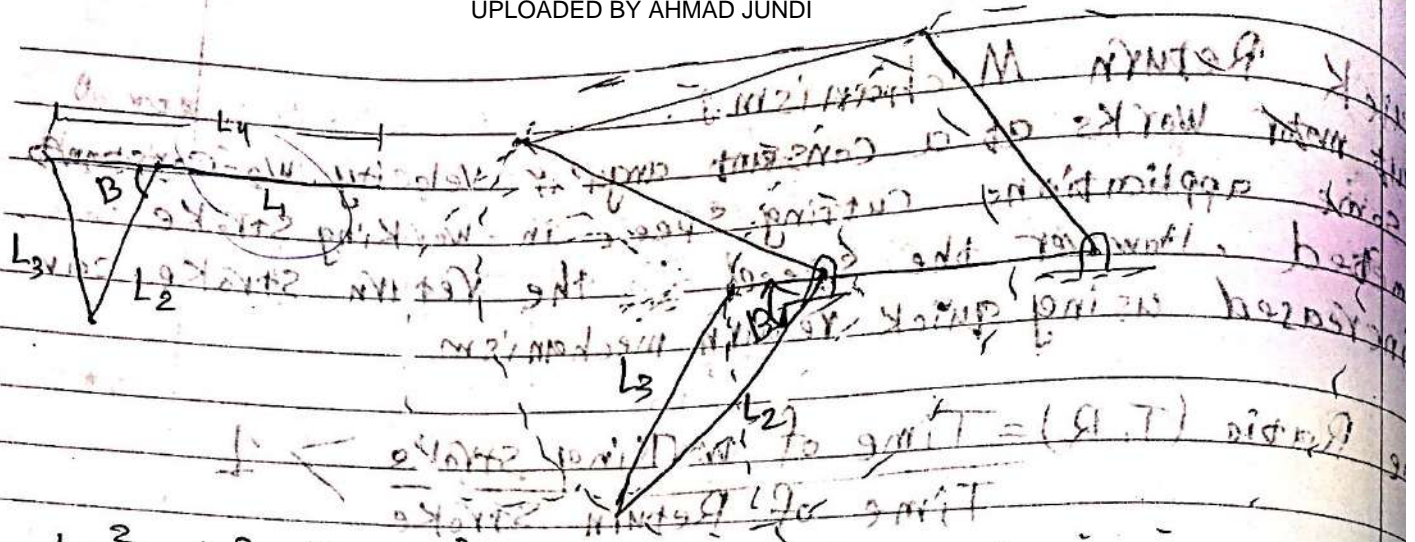
$$40^\circ \leq \alpha \leq 140^\circ$$

⇒ It is needed to find α_{\min} & α_{\max}

$$\alpha_{\min} = \cos^{-1} \left(\frac{L_4^2 + L_3^2 - (L_1 + L_2)^2}{2L_3L_4} \right)$$

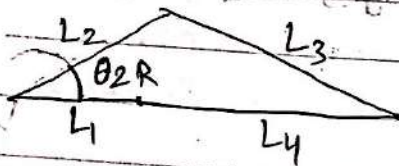
$$\alpha_{\max} = \cos^{-1} \left(\frac{L_4^2 + L_3^2 - (L_1 - L_2)^2}{2L_3L_4} \right)$$





$$L_3^2 = L_1^2 + (L_4 - L_1)^2 - 2L_1(L_4 - L_1)\cos B$$

$$B = 180^\circ \Rightarrow \theta_{2L} = 180^\circ + B$$



$$\frac{\Delta \theta_1}{\Delta t} = \frac{\Delta \theta_2}{\Delta t} = \frac{\Delta \theta_3}{\Delta t} = \frac{\Delta \theta_4}{\Delta t}$$

$$L_3^2 = L_2^2 + (L_1 + L_4)^2 - 2L_2(L_1 + L_4)\cos \theta_{2R} \Rightarrow \theta_{2R} = 180^\circ$$

$$\Delta \theta_W = \theta_{2L} - \theta_{2R}$$

$$\Delta \theta_R = 360^\circ - \Delta \theta_W$$

$$T.R = \frac{\Delta \theta_W}{\Delta \theta_R}$$

$$\text{Stroke} = 2a$$

② Crank Shaper Mechanism

$$B = \cos^{-1}\left(\frac{L_2}{L_1}\right)$$

$$\theta_{2R} = 270^\circ + B$$

$$\theta_{2L} = 270^\circ - B$$

$$\Delta\theta_R = 2B$$

$$\Delta\theta_W = 360^\circ - 2B$$

$$T.R. = \frac{\Delta\theta_W}{\Delta\theta_A}$$

$$\text{Stroke} = 2L_1 \cos B$$

موتوري اضع

③ Whitworth mechanism

$$L_2 > L_1$$

$$B = \cos^{-1}\left(\frac{L_1}{L_2}\right)$$

$$T.R. = \frac{360^\circ - 2B}{2B}$$

$$\text{Stroke} = 2a$$

go to animation

④ offset slider Mechanism

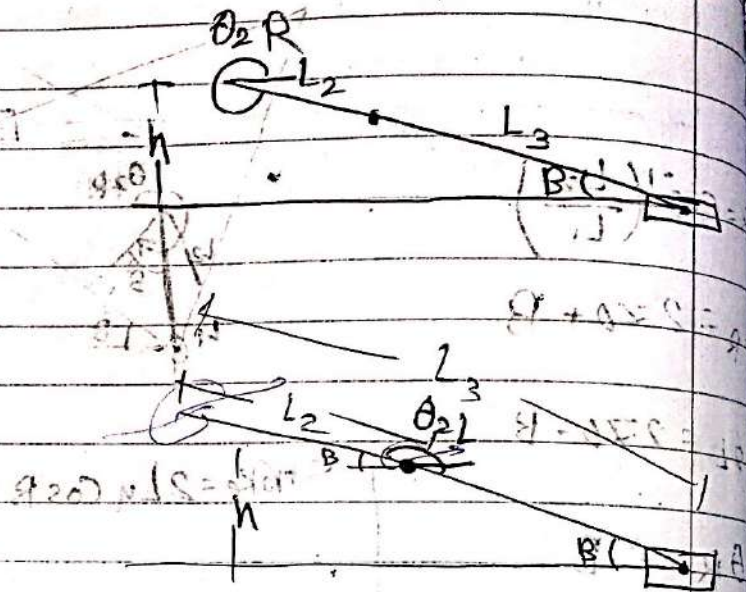
$$B = \sin^{-1} \left(\frac{h}{L_3 + L_2} \right)$$

$$\theta_{2R} = 360 - B$$

$$B_2 = \sin^{-1} \left(\frac{h}{L_3 - L_2} \right)$$

$$\theta_{2L} = 180 - B_2$$

$$\Delta\theta = \theta_{2R} - \theta_{2L}$$



Other Mechanisms

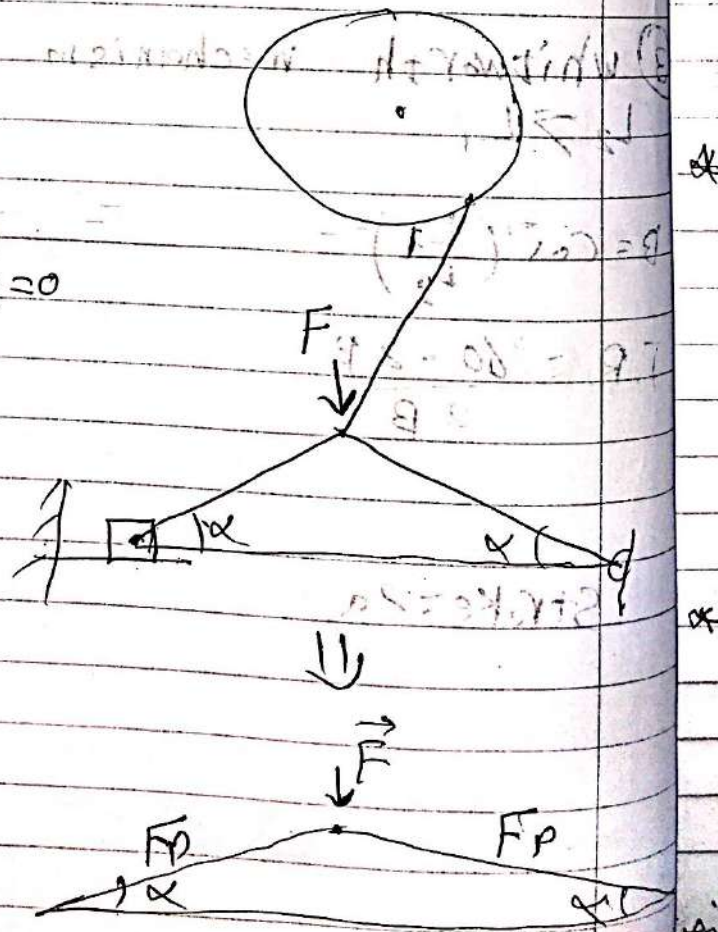
- Toggle Mechanism

$$\Sigma F_y = 0$$

$$F = 2F_p \sin \alpha$$

$$F_p = \frac{F}{2 \sin \alpha} \quad \text{as } \alpha \rightarrow 0 \Rightarrow \sin \alpha \rightarrow 0$$

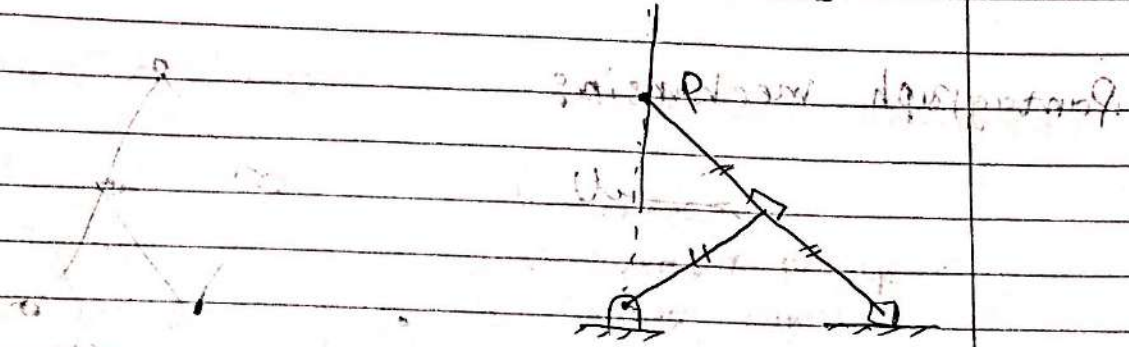
$$F \rightarrow \infty$$



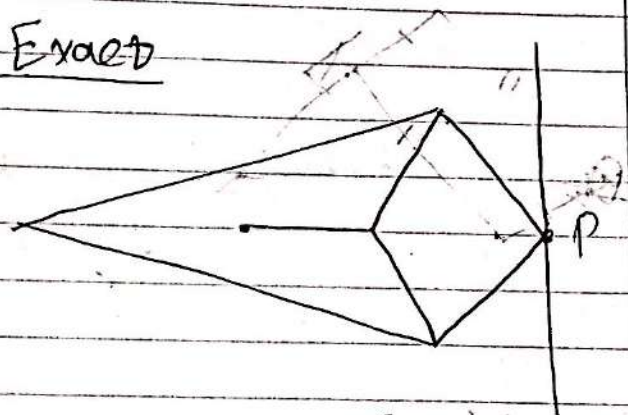
* Straight line Mechanisms

A mechanism that has a point that moves in a straight line (exact/approximate)

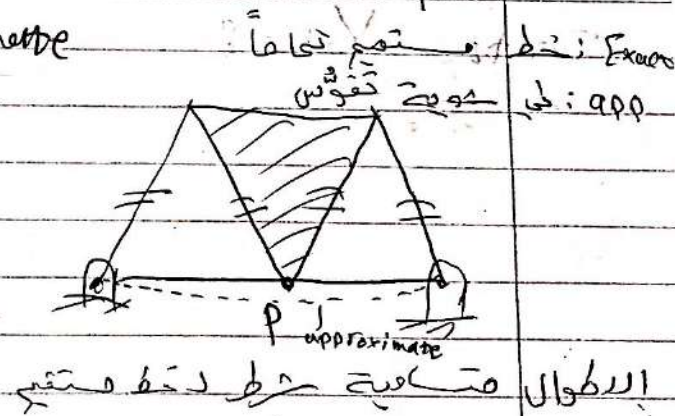
* Scott Russel : Exact Straight line mechanism



* Peaucellier's Mechanism Exact

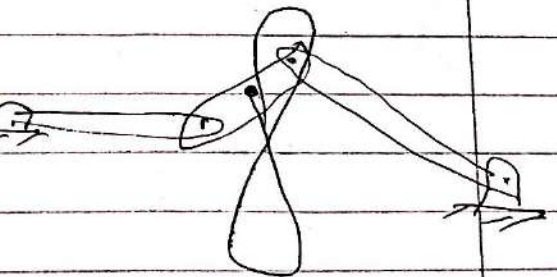


* Robert's mechanism Approximate



* WATT : Approximate

$$\frac{A}{B} = \frac{a}{b}$$

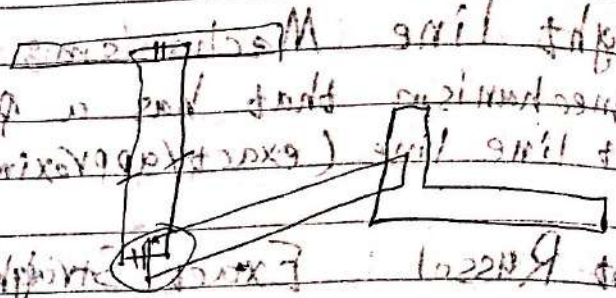


نسبة أطوال السليتين المتساويتين / إذا متساويتين تكونا متساويتين
نسبة أطوال السليتين المتساويتين / إذا متساويتين تكونا متساويتين

Parallel Mechanism

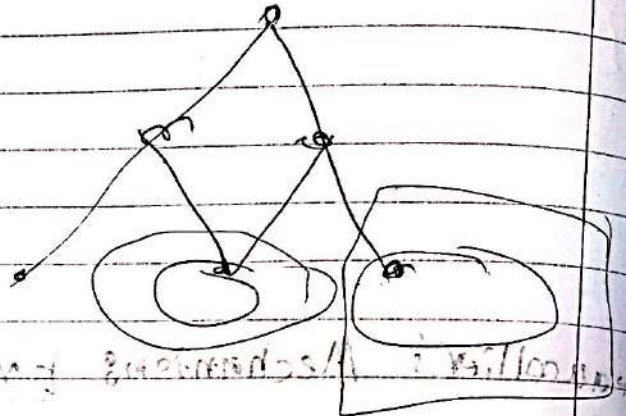
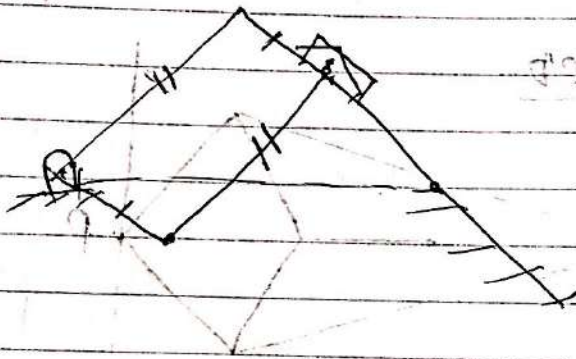
as a base, orientation, line, and

(6 degrees of freedom)

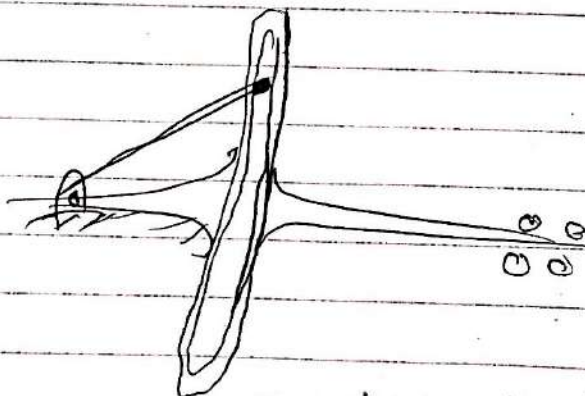


Pantograph mechanism

will



Scott Yoke



geneva mechanism

8. Velocity and acceleration analysis (continued)

Chapter 2: Velocity and Acceleration Analysis

* Velocity Analysis:

Acceleration

① Velocity polygons

① acceleration polygons

② Instant centers

② complex number

③ Complex numbers

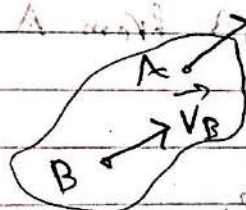
① Velocity Polygons:

* Types of motion of Rigid body in plane:

① Translation

$$\vec{V}_B = \vec{V}_A$$

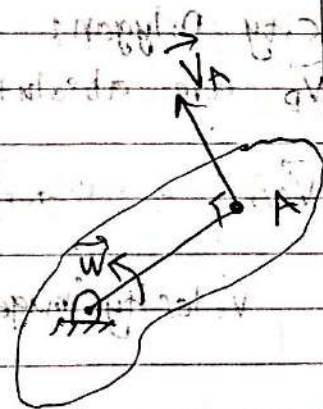
$$\vec{\omega} = 0$$



② Rotation about a fixed axis:

$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/O}$$

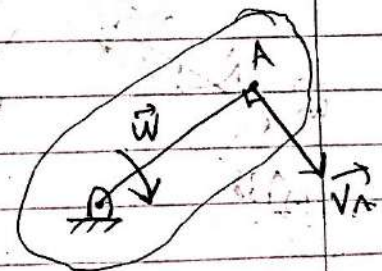
$\vec{r}_{A/O}$ = Position vector of point A relation to O "from O to A"



$$\vec{V}_A \perp \vec{\omega}$$

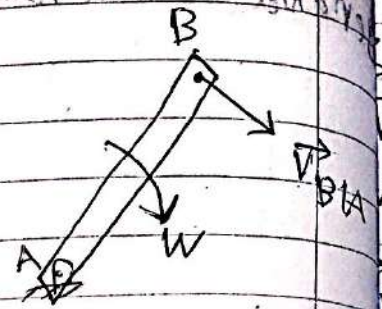
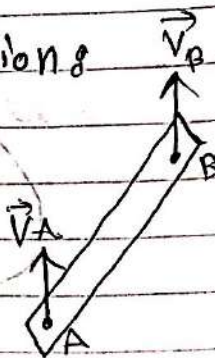
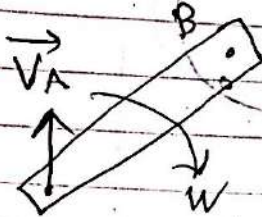
$$\vec{V}_A \perp \vec{r}_{A/O}$$

\vec{V}_A "In the direction of $\vec{\omega}$ (rotation)"



$$V_A = \omega r_{A/O} \quad \text{magnitude}$$

③ General Plane Motion



General plane motion

translation as 'A'

Rotation about A (Relative)

$$\vec{V}_{B/A} = \vec{V}_B - \vec{V}_A$$

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

$$\vec{V}_{B/A} = \vec{W} \times \vec{r}_{B/A}$$

$$\vec{V}_{B/A} \perp \vec{r}_{B/A} \text{ \& } \vec{W}$$

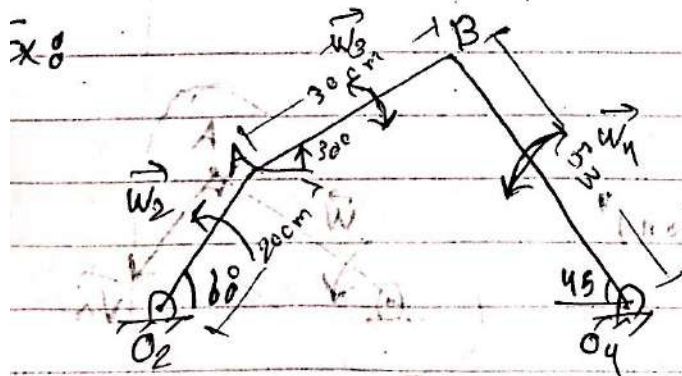
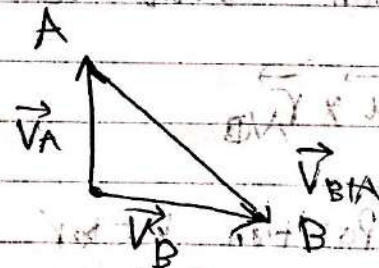
$\vec{V}_{B/A}$ "in the direction of \vec{W} " "rotation sense"
 $\vec{r}_{B/A} \equiv$ Position vector of B w.r.t A from A to B

* Velocity Polygons

\vec{V}_A & \vec{V}_B are absolute velocities.

$\vec{V}_A = \vec{V}_{A/O_2} \Rightarrow$ Vector from 'O' to 'A'

Velocity image of link AB



given $W_2 = 2 \text{ rad/s}$ in the direction shown find W_3 & V_B

Start with known links:

$$\vec{V}_A = \vec{\omega}_2 \times \vec{r}_{A/O_2} + \vec{V}_{O_2}$$

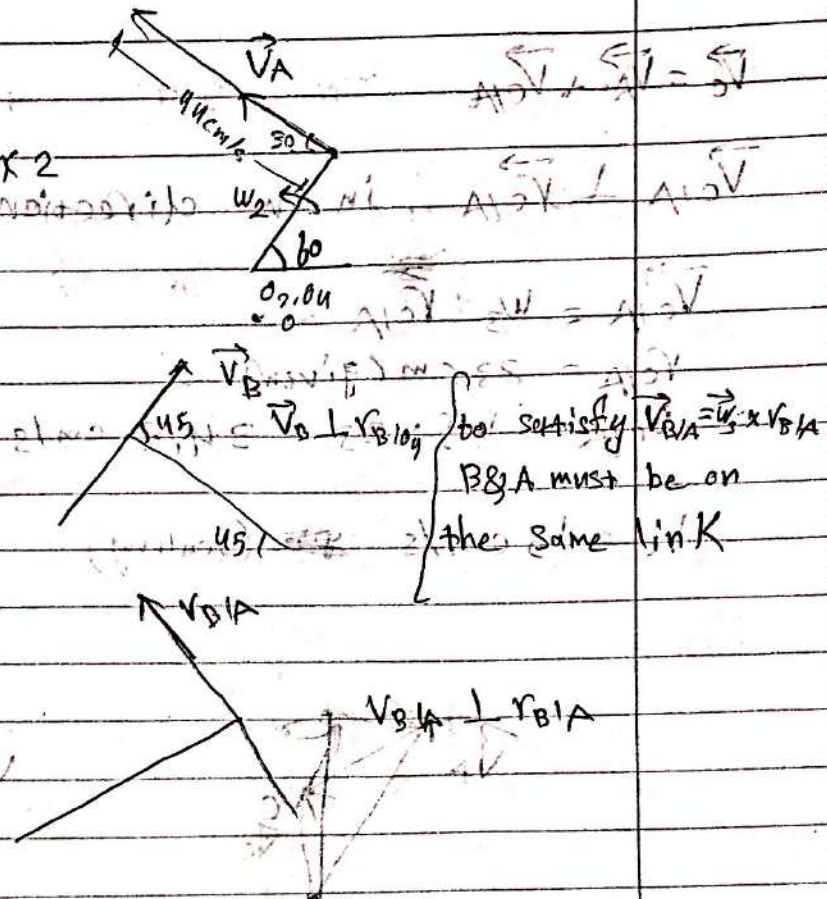
$$V_A = (2)(22) = 44 \text{ cm/s}$$

$$\vec{V}_B = \vec{\omega}_4 \times \vec{r}_{B/O_4}$$

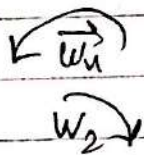
$$\vec{V}_B \perp \vec{r}_{B/O_4}$$

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$$

$$\vec{V}_{B/A} = \vec{\omega}_3 \times \vec{r}_{B/A}$$

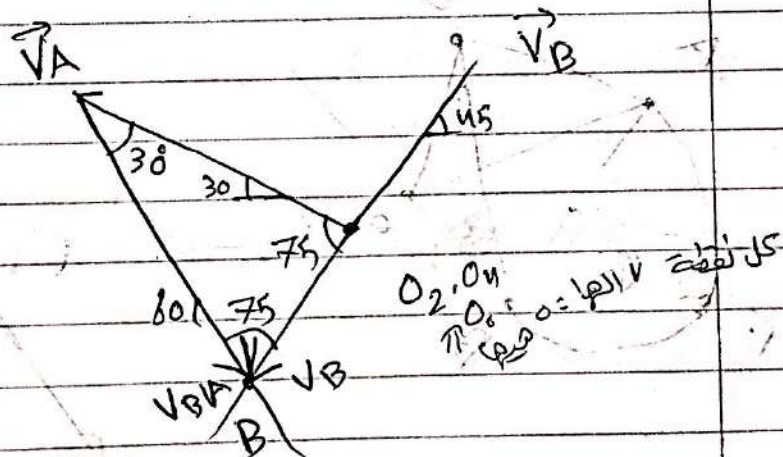


Velocity image:



$$\vec{V}_{B/A} = 46 \text{ cm/s}$$

$$V_B = 20 \text{ cm/s}$$



$$\omega_4 = \frac{V_B}{r_{B/O_4}} = \frac{20}{53} \text{ rad/s counter-clockwise}$$

$$\omega_3 = \frac{V_{B/A}}{r_{B/A}} = \frac{46}{30} \text{ rad/s clockwise}$$

$$\vec{V}_C = \vec{V}_A + \vec{V}_{C/A}$$

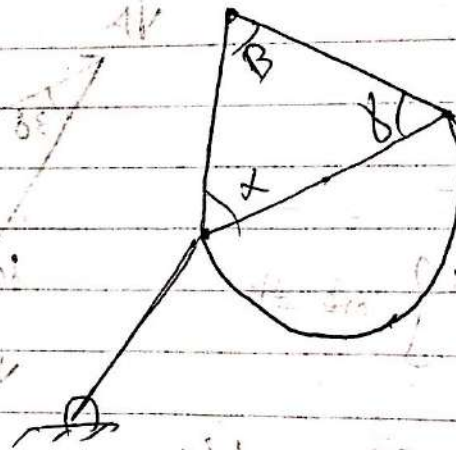
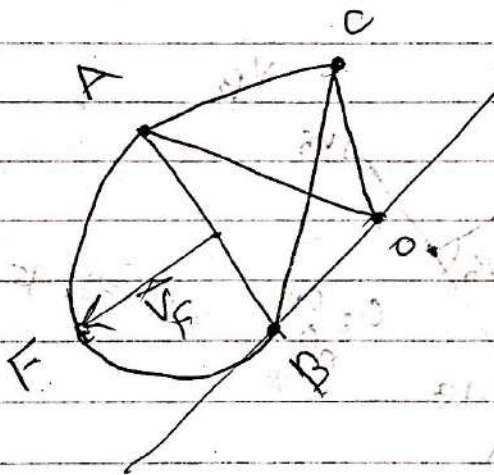
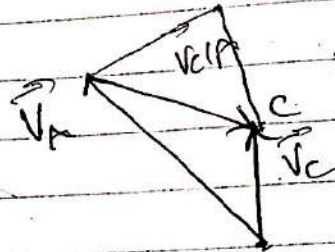
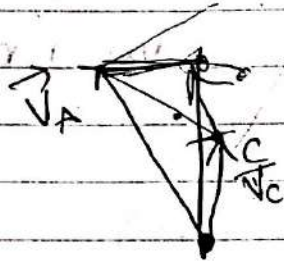
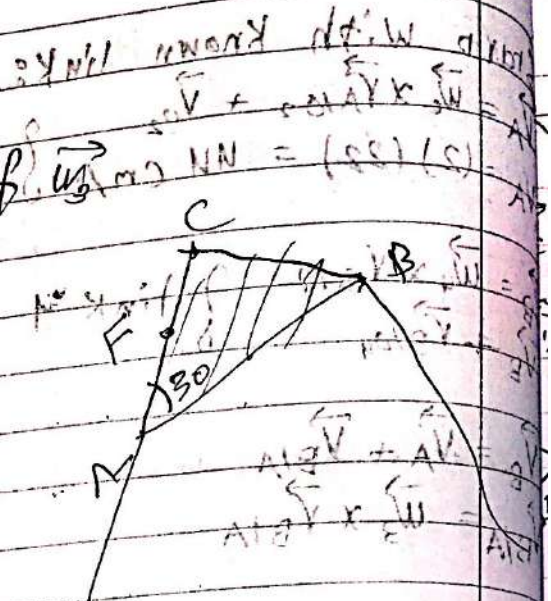
$\vec{V}_{C/A} \perp \vec{V}_A$ in the direction of \vec{V}_A

$$\vec{V}_{C/A} = \omega_3 \cdot \vec{r}_{C/A}$$

$$r_{C/A} = 23 \text{ cm (given)}$$

$$V_{C/A} = 1.5 \times 23 = 34.5 \text{ cm/s}$$

$$V_C = 15 \text{ cm/s (scaling)}$$



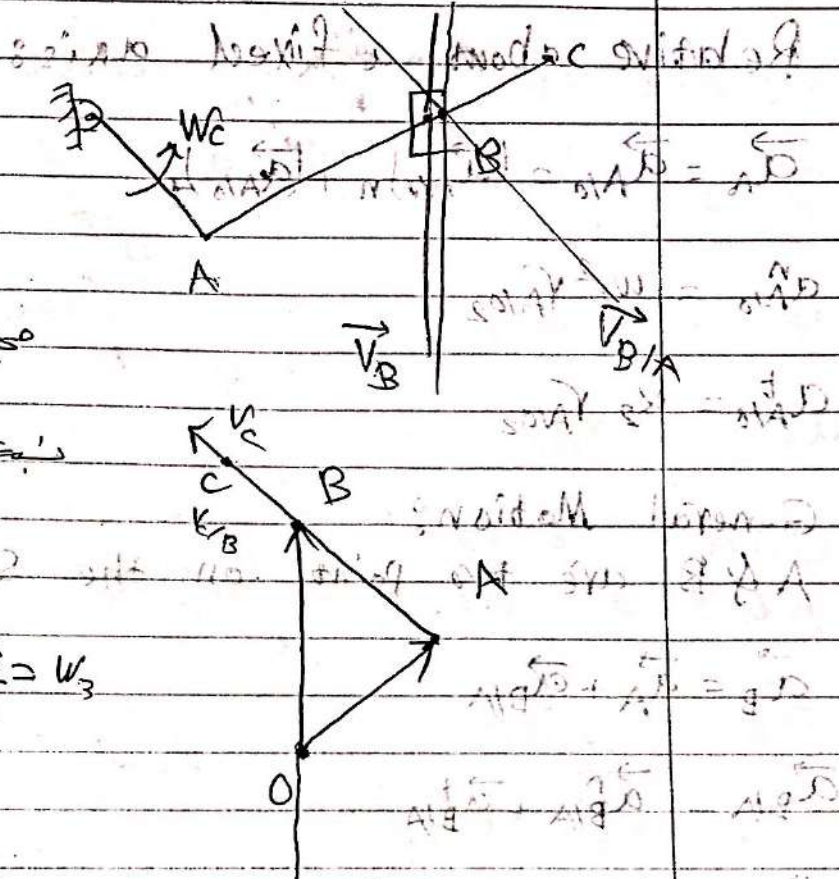
Relative Motion on the same link: $\omega_{AB} = \omega_{BA}$ $\omega_{AB} = \omega_{BA}$

$\vec{V}_C = \vec{V}_A + \vec{V}_{CA}$
 مجهول قتي و اتجاه \vec{V}_C مجهول قتي و اتجاه \vec{V}_{CA}
 معطى \vec{V}_A و \vec{V}_{CA}

$\Rightarrow \vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$
 معيّنات فقط قيمة v_B , $v_{B/A}$ بقدر الطول

$$\frac{V_{CIB}}{V_{CIA}} = \frac{V_{CIB}}{V_{CIA}} \quad \text{بما أن النسبة متساوية}$$

$$\frac{V_{C/B}}{V_{A/B}} = \frac{V_{C/B}}{V_{A/B}}, \quad \frac{V_{C/B}}{V_{C/B}} = \frac{V_{A/B}}{V_{A/B}} = W_3$$



Relative Motion on the same link

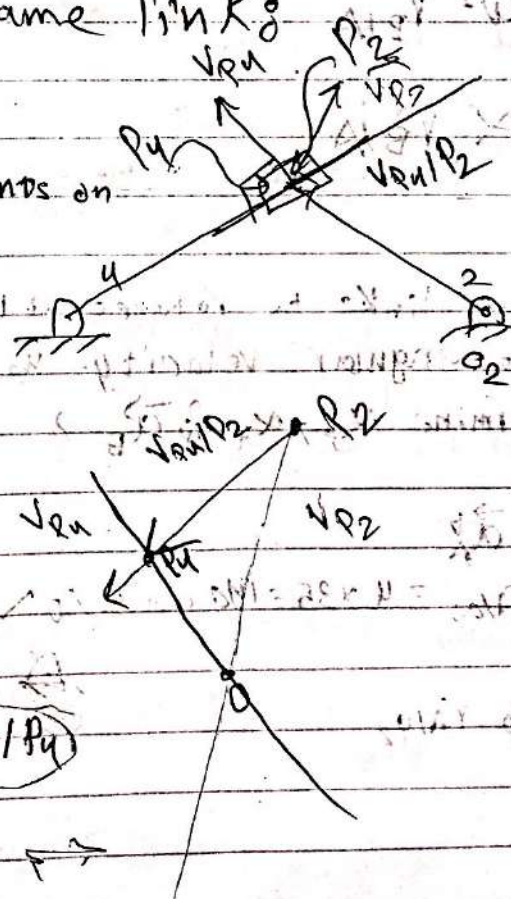
$$\begin{aligned} \vec{V}_{P_2/P_4} &= \vec{V}_{P_2} - \vec{V}_{P_4} \\ \vec{V}_{P_4/P_2} &= \vec{V}_{P_4} - \vec{V}_{P_2} = -\vec{V}_{P_2/P_4} \end{aligned} \quad \left. \begin{array}{l} \text{assume two} \\ \text{coincident points on} \\ \text{both links} \end{array} \right\}$$

$$\vec{V}_{PQ} = \vec{V}_{P2} + \vec{V}_{Q/P2}$$

$$V_{P2} = \omega_2 \times r_{P2/O2} = 1 \times 50 = 50 \text{ mm/s}$$

حركة P_2 الى P_1 نقطة
على straight line link 4

$$\vec{a}_{p_2} = \vec{a}_{p_1} + (\vec{a}_{p_2/p_1})_{rel} + 2\vec{\omega}_y \times V_{p_2/p_1}$$



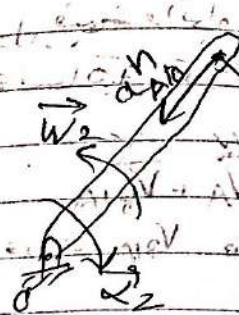
Relative Motion: Analysis of Acceleration using Polygon

Relative about a fixed axis:

$$\vec{a}_A = \vec{a}_{A/O} = (\vec{a}_{A/O})_n + (\vec{a}_{A/O})_t$$

$$a_{A/O}^n = \omega_2^2 r_{A/O_2}$$

$$a_{A/O}^t = \alpha_2 r_{A/O_2}$$



General Motion:

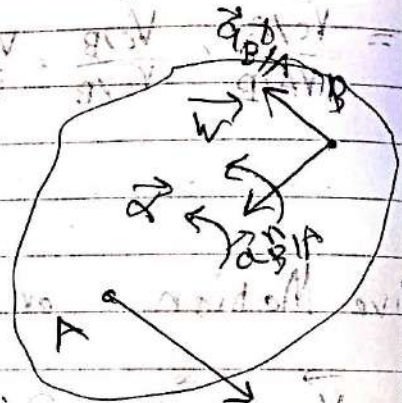
A & B are two points on the same rigid link

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_{B/A} = \vec{a}_{B/A}^n + \vec{a}_{B/A}^t$$

$$a_{B/A}^n = \omega^2 r_{B/A}$$

$$a_{B/A}^t = \alpha r_{B/A}$$



Ex8 given links to rotates at a

constant angular velocity $\omega_2 = 2 \text{ rad/s}$

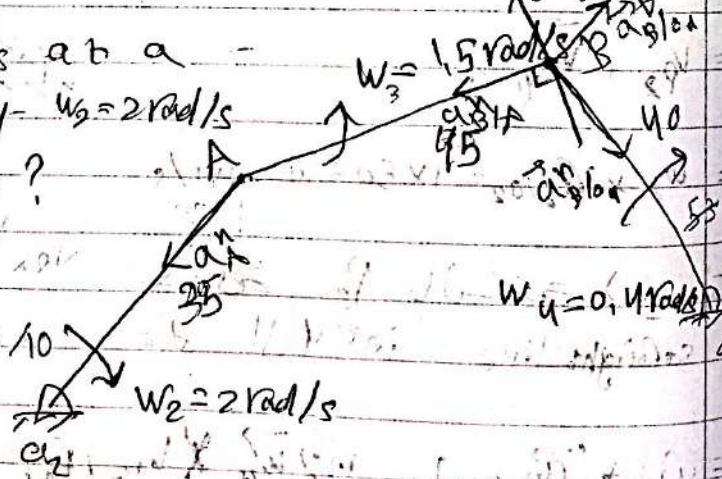
Shown, determine α_3 , α_4 & \vec{a}_B ?

$$\alpha_2 = 0$$

$$\vec{a}_A = \vec{a}_A^n + \vec{a}_A^t$$

$$a_A^n = \omega_2^2 r_{A/O_2} = 4 \times 35 = 140 \text{ cm/s}^2$$

$$a_A^t = 0 = \alpha_2 r_{A/O_2}$$



لما يكون عيني جسم يدور حولي تتحرك بكوني عيني

UPLOADED BY AHMAD JUNDI

لما ادور واسموت جسم تتحرك

of the axis

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}^n + \vec{a}_{B/A}^t$$

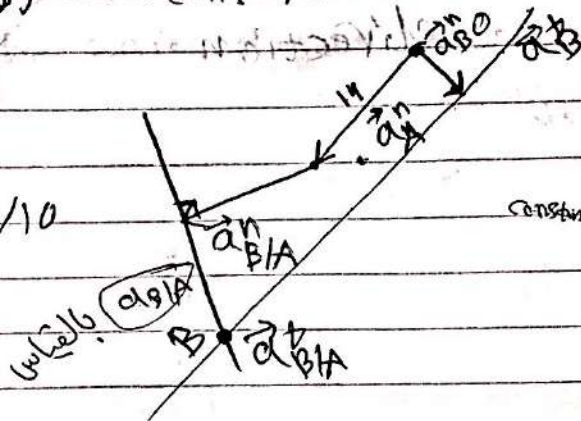
$$a_{B/A}^n = \omega_3^2 r_{B/A} = (1.5)^2 (4.5) = 10.1 \text{ cm/s}^2 / 10$$

$$\vec{a}_B = \vec{a}_B^n + \vec{a}_B^t$$

$$\vec{a}_B^n = \omega_4^2 r_{P/O_4}$$

$$= (0.4)^2 \times 10 = 6.4 / 10$$

$$\alpha_3 = \frac{a_{B/A}}{r_{B/A}}$$



Ex:

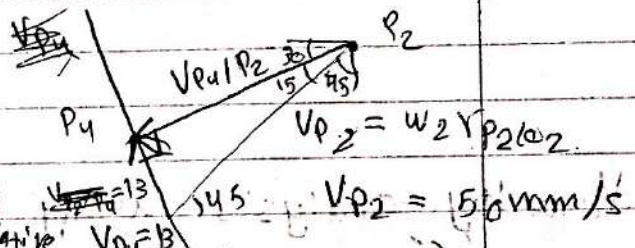
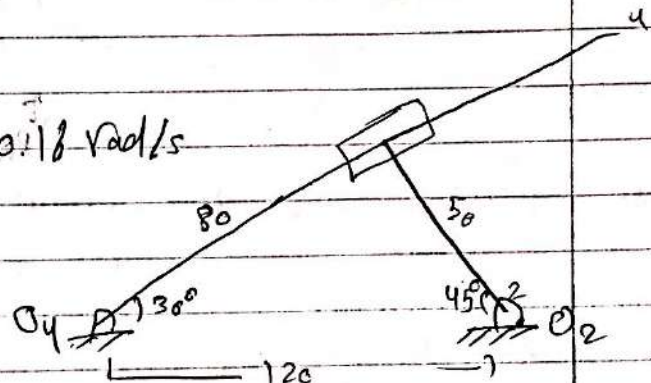
$$\omega_4 = v_{P_4} / r_{P_4/O_4} = \frac{13}{80} = 0.16 \text{ rad/s}$$

$$a_{P_4}^n = \omega_4^2 r_{P_4/O_4}$$

$$a_{P_4}^n = 2.1 \text{ mm/s}^2$$

$$a_{P_2}^n = \omega_2^2 r_{P_2/O_2} = 12^2 \times 50 = 50 \text{ mm/s}^2$$

$$a_{P_2}^t = \alpha_2 \cdot r_{P_2/O_2} = 0 \cdot r_{P_2/O_2} = 0$$



$$a_{P_2}^n + a_{P_2}^t - 2\omega_4 \times v_{P_4/P_2} = a_{P_4}^n + a_{P_4}^t + (a_{P_2/P_4})_{\text{relative}}$$

$$a_{P_2}^n + a_{P_2}^t + 2\omega_4 \times v_{P_4/P_2} = a_{P_4}^n + a_{P_4}^t + (a_{P_2/P_4})_{\text{rel}}$$

في هذا المثال، نأخذ v_{P_2/P_4} و a_{P_2/P_4} لأن P_2 تتحرك بنسبة P_4 تتحرك بنسبة P_2 الأسفل. a_{P_2/P_4} بنسبة P_4 تتحرك بنسبة P_2 الأسفل. a_{P_2/P_4} بنسبة P_4 تتحرك بنسبة P_2 الأسفل. a_{P_2/P_4} بنسبة P_4 تتحرك بنسبة P_2 الأسفل.

$$(2\omega_4 \times v_{P_4/P_2}) = 2 \times 0.16 \times 48.3 = 15.5 \text{ magnitude}$$

$v = \omega \times r$ Perpendicular to r with direction to ω Example

$2\omega \times v_{\text{relative}}$: direction \perp to the track

a_{relative} : = with = =

direction

60°

38°

$$2\vec{\omega} \times \vec{r}_{P_1/P_2}$$

\vec{r}_{P_1/P_2}

\vec{r}_{P_2/P_1}

\vec{r}_{P_2/P_1}

Velocity analysis using instantaneous center

نقطة سرعة الجسم عند هذه النقطة هي انه النقطة التي يدور حولها

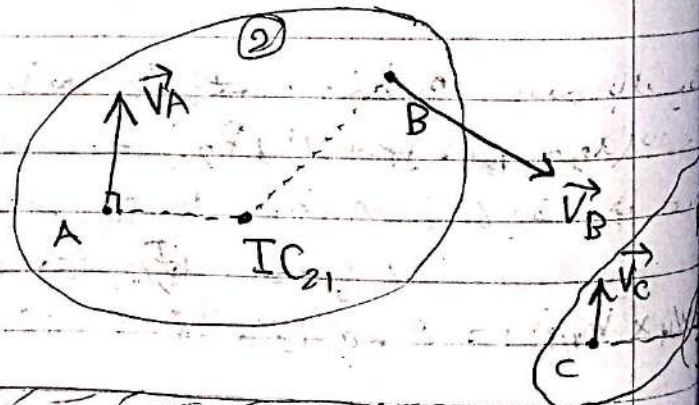
$$\vec{V}_{IC_{12}} = 0$$

$$\vec{V}_B = \vec{V}_{IC_{12}} + \vec{V}_{B/IC_{12}}$$

$$\vec{V}_B = 0 + \vec{V}_{B/IC_{12}}$$

$$\vec{V}_{B/IC_{12}} = \vec{\omega}_2 \times \vec{r}_{B/IC_{12}}$$

$$V_{B/IC_{12}} = \omega_2 r_{B/IC_{12}}$$



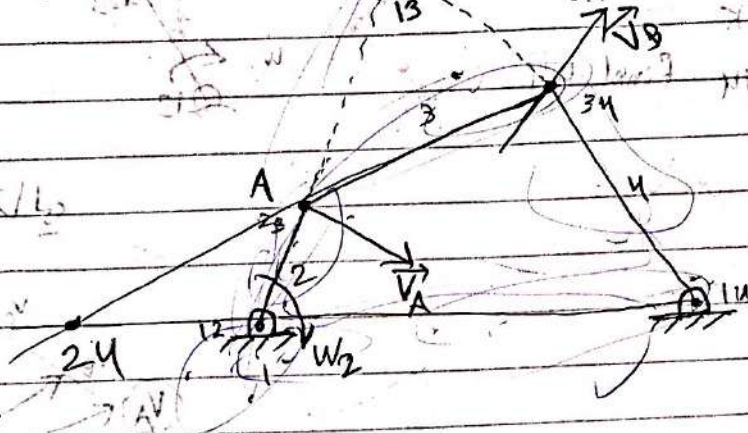
IC \equiv A point in the plane at which the velocity is equal on both bodies.

A point about which the bodies appear to be rotating relative to each other.

$$\# \text{ of ICs} = \frac{n(n-1)}{2}$$

$$\# \text{ ICs} = \frac{4 \times 3}{2} = 6$$

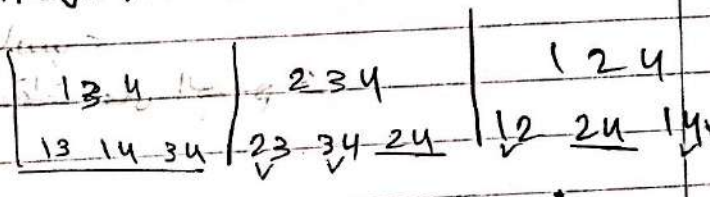
12 13 14
23 24 34
24 \rightarrow how?
 \downarrow



Kennedy's Theorem

Any three bodies moving in plane have three relative instantaneous centers that lie on a straight line.

11 21 31
12 13 23 \rightarrow on the straight line



$V_{24} = r\omega$ Known Rotation about fixed axis

try Joss \leftarrow link 4 \rightarrow 24

Primary ICs:

- ① At Pins
- ② At Contact points, without slipping

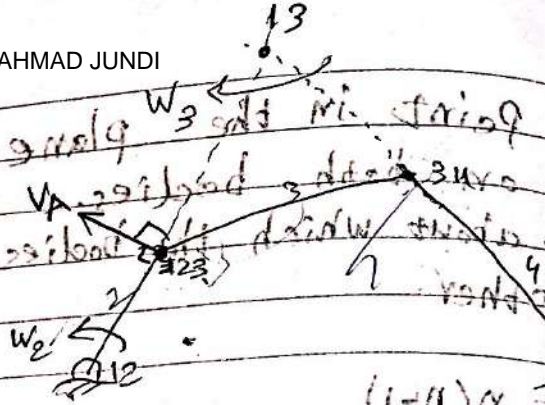


$$\vec{V}_A = \omega_2 \vec{r}_{A/12} \text{ find } \omega_2$$

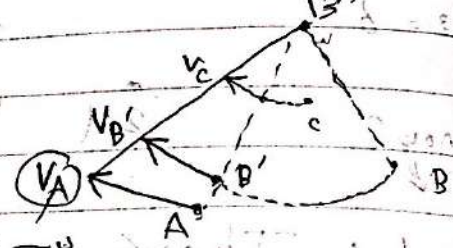
$$\vec{V}_A = \omega_3 \vec{r}_{A/13} \text{ find } \omega_3$$

$$\vec{V}_B = \omega_4 \vec{r}_{B/14} \text{ find } \omega_4$$

$$\vec{V}_3 = -\omega_3 \hat{k}$$



أول ω_3 كالمالي

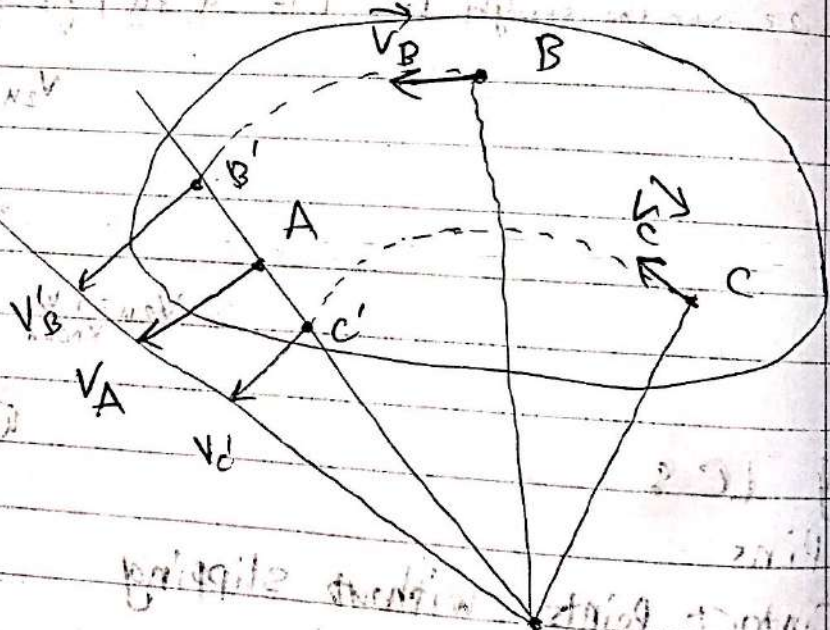


برسم طولها حسب $V_A = \frac{1}{2} \times \dots$

$$\frac{V_B}{V_A} = \frac{r_{B/14}}{r_{A/12}}$$

بـ تحرك الآلة الحاسب

الانحراف للإيجار للسرعة أي نقطة على الجسم

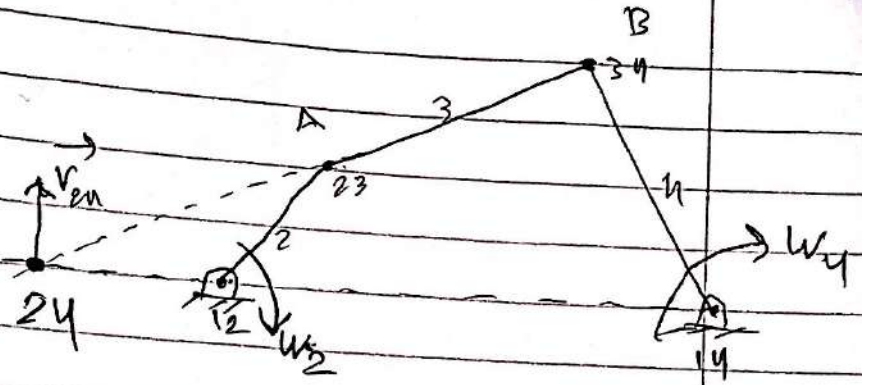


13 ω_3 بطول حسيه V_B والدائري منه يعرفه

$$\omega_2 = 2 \text{ rad/s}$$

$$V_{24} = \omega_2 \cdot r_{24/12}$$

$$V_{24} = 2 \times 16 = 32 \text{ cm/s}$$

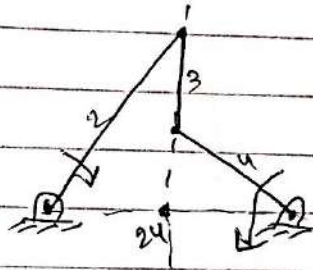


$$V_{24} = \omega_4 \cdot r_{24/14}$$

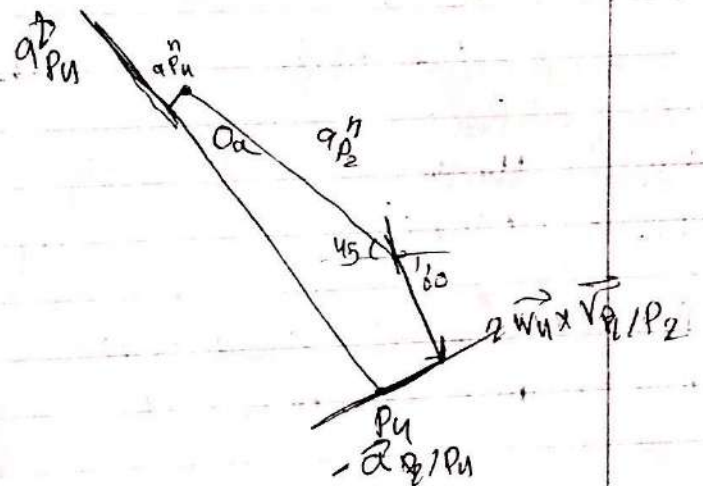
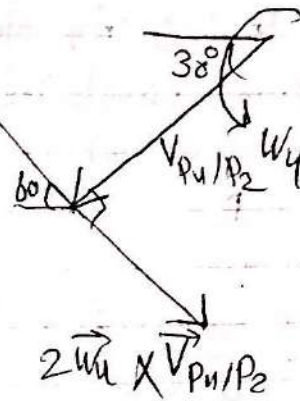
$$\omega_4 = \frac{V_{24}}{r_{24/14}} = \frac{32}{80} = 0.375$$

$$V_{14/24} = (\omega_4 - \omega_2) \cdot r_{14/24}$$

$$V_{24/14} = 0 \text{ or } \omega_2$$



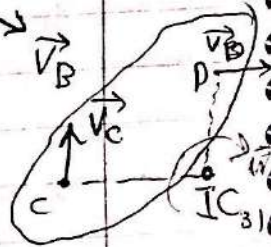
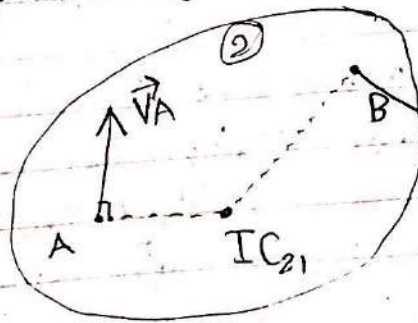
direction



Velocity analysis using Instantaneous Center
 IC نقطة سرعتها صفر عند لمسة السطح به انه النقطة التالية بتدور حوله.

$$\begin{aligned}\vec{V}_{IC_{12}} &= 0 \\ \vec{V}_B &= \vec{V}_{IC_{12}} + \vec{V}_{B/IC_{12}} \\ \vec{V}_B &= 0 + \vec{V}_{B/IC_{12}} \\ \vec{V}_{B/IC_{12}} &= \vec{\omega}_2 \times \vec{r}_{B/IC_{12}}\end{aligned}$$

$$V_{B/IC_{12}} = \omega_2 r_{B/IC_{12}}$$

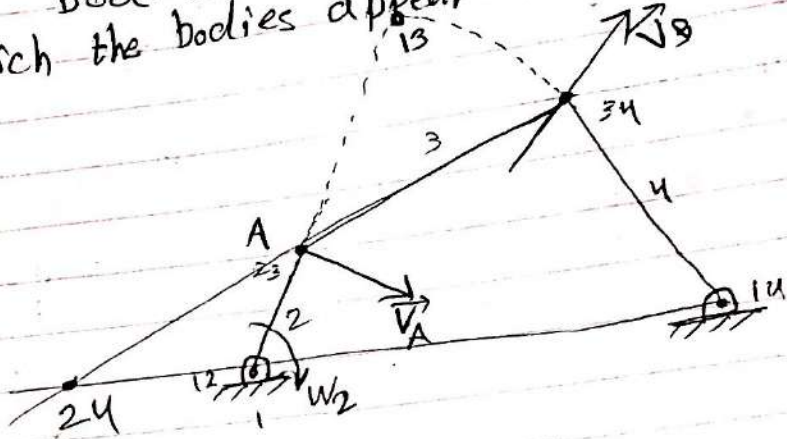


IC \equiv A point in the plane at which the velocity is equal on both bodies.
 ② A point about which the bodies appear to be rotating relative to each other.

$$\# \text{ of ICs} = \frac{n(n-1)}{2}$$

$$\# \text{ ICs} = \frac{4 \times 3}{2} = 6$$

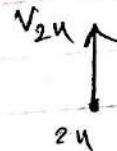
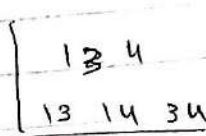
12 13 14
23 24 34
how?
↓



Kennedy's Theorem
 Any three bodies moving in plane have three relative instantaneous centers that lie on a straight line.

1, 2, 3

12 13 23 \rightarrow on the straight line



$V_{24} = r\omega$ Known Rotation about fixed axis \leftarrow link 2 or 4

try $\omega_{34} \leftarrow$ link 3 or 4

Primary ICs

① At Pins

② At Contact points, without slipping

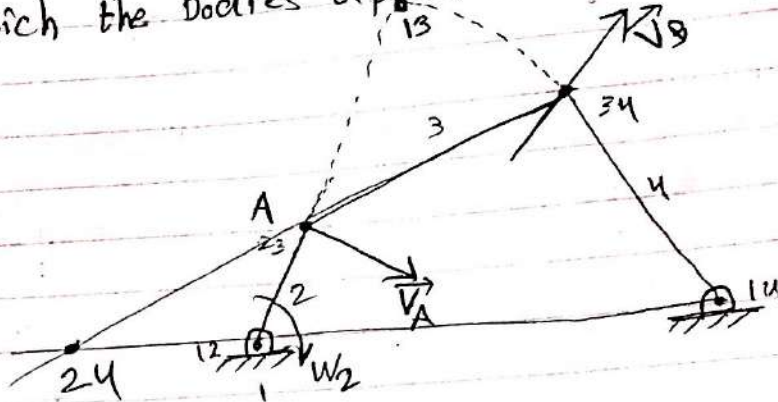


IC \equiv A point in the plane at which the velocity is equal on both bodies.
 ② A point about which the bodies appear to be rotating relative to each other.

$$\# \text{ of } IC_s = \frac{n(n-1)}{2}$$

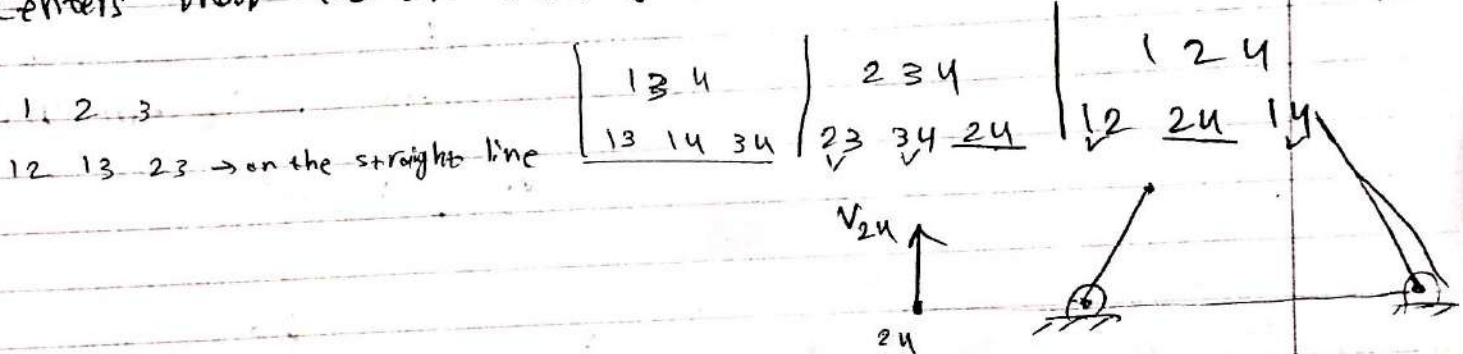
$$\# IC_s = \frac{4 \times 3}{2} = 6$$

12 13 14
23 24 34
24 34 14
how?
↓



Kennedy's Theorem

Any three bodies moving in plane have three relative instantaneous centers that lie on a straight line.



$v_{24} = r \omega$ Known Rotation about fixed axis link 2 & 4

link 4 \leftarrow link 4 \rightarrow 24

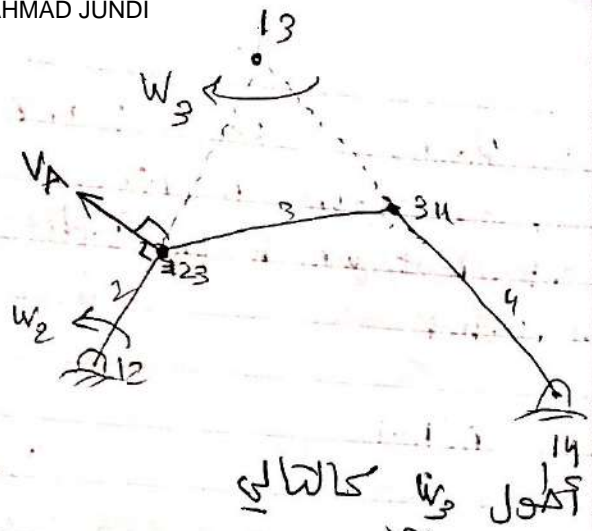
Primary IC:

① At Pins

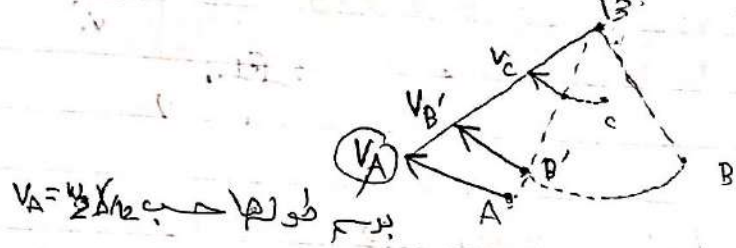
② At Contact points, without slipping



$$\begin{aligned}\vec{V}_A &= \omega_2 \vec{r}_{A/12} \\ \vec{V}_A &= \omega_3 \vec{r}_{A/13} \quad \text{find } \omega_3 \\ \vec{V}_B &= \omega_3 \vec{r}_{B/13} \\ \omega_3 &= -\omega_2 \hat{k} \\ \vec{V}_B &= \omega_4 \vec{r}_{B/14} \quad \text{find } \omega_4\end{aligned}$$



يمكن الحصول على النتائج كالتالي

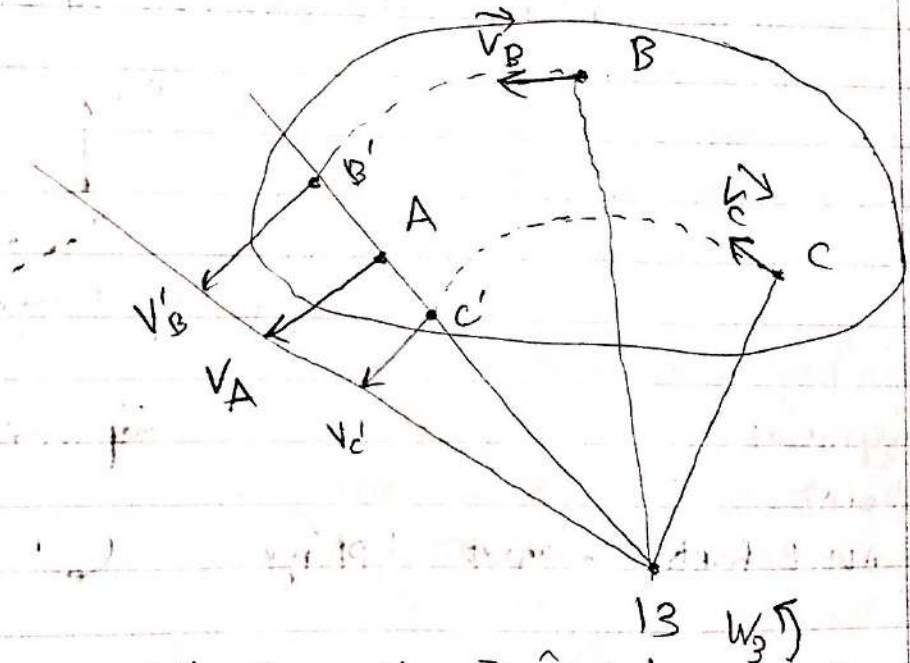


برسم طولها حسب $V_A = \omega_2 r_{A/12}$

$$\frac{V_{B'}}{r_{B'/13}} = \frac{V_A}{r_{A/13}}$$

تساوي مثلثات:

حيث يتحرك الآلة الجاسية
كذلك الامر للاجبار لتسوية أي نقطة على الجسم 3.



تساوي مثلثات بطول قسمة V_C و $V_{B'}$ والدائري حتى يعرف ω_3

$$\omega_2 = 2 \text{ rad/s}$$

$$V_{24} = \omega_2 r_{24/12}$$

$$V_{24} = 2 \times 16 = 32 \text{ cm/s}$$

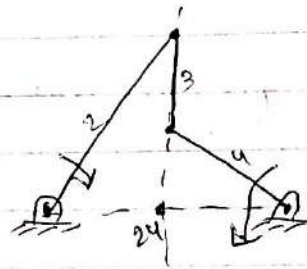


$$V_{24} = \omega_4 r_{24/14}$$

$$\omega_4 = \frac{V_{24}}{r_{24/14}} = \frac{32}{80} = 0.375$$

$$V_{41/24} = (\omega_4 - \omega_2) r_{41/24}$$

$$V_{24}/r_{24/14} = 0 \text{ or } \sin 2$$

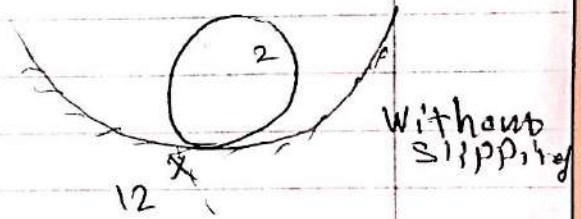


③ Contacts

Ⓐ without slipping (Rolling)

Ⓑ Sliding without Rolling

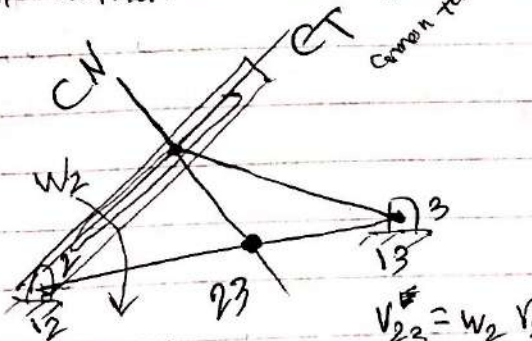
at the center of curvature of the relative path between the two bodies



Ⓒ Rolling with Sliding

(Pin inside a slot)

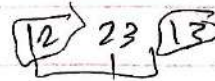
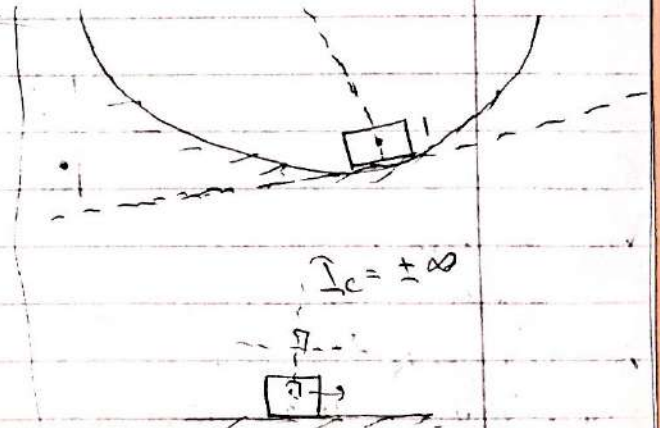
a point somewhere on the common normal direction.



$$V_{23} = \omega_2 r_{23/12}$$

$$V_{23} = \omega_3 r_{23/13}$$

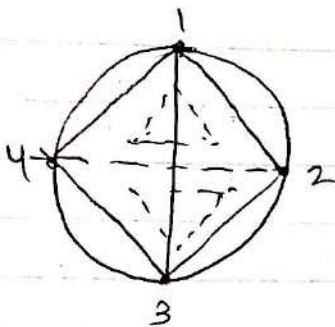
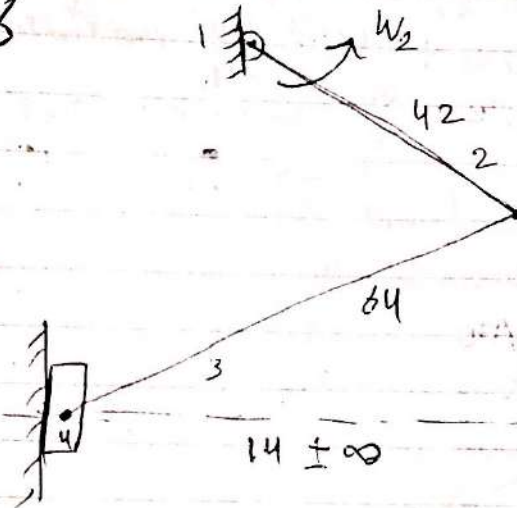
$$\frac{\omega_2}{\omega_3} = \frac{r_{23/12}}{r_{23/13}}$$



X:

$$\# \text{ of } I_c = \frac{n(n-1)}{2} = \frac{4(3)}{2} = 6$$

Given $\omega_2 = 2 \text{ rad/s}$
find \vec{V}_s slider

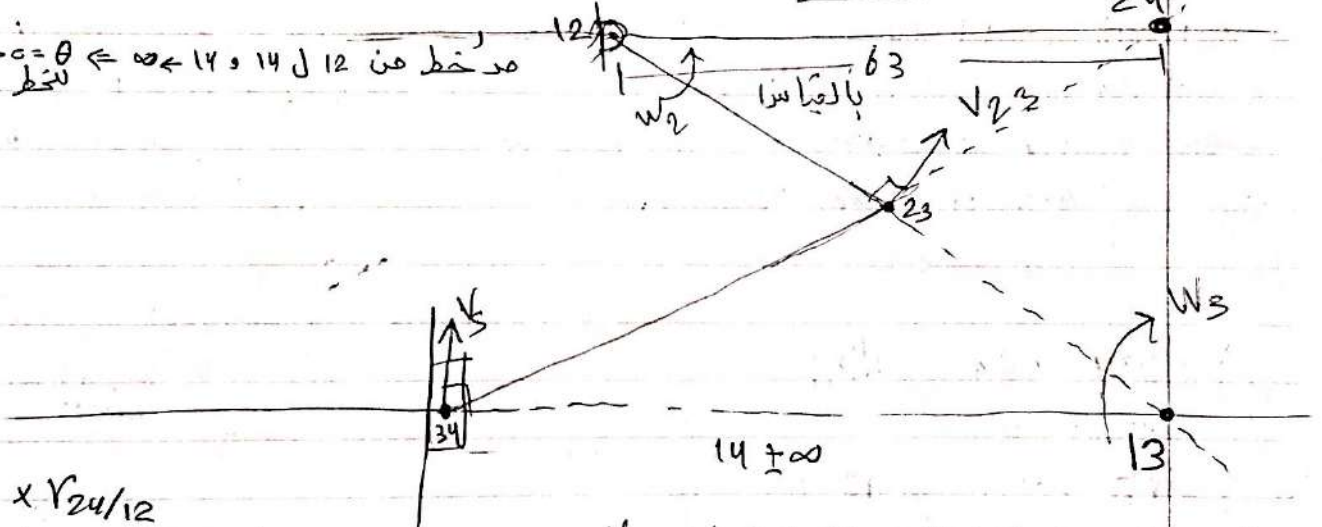


عدان ادجو 24، حانا الفصل مشترك مع مثلثين: 432، 412

$$\leftarrow \text{تقاطع امتدادهم يحدد نقطة 24} \leftarrow \begin{bmatrix} 12 & 14 \\ 23 & 34 \end{bmatrix} \begin{matrix} 24 \\ 24 \end{matrix}$$

$$13 \leftarrow \begin{bmatrix} 12 & 23 \\ 14 & 34 \end{bmatrix} \begin{matrix} 13 \\ 13 \end{matrix}$$

مخطط من 12 و 14 و 14 و 14 $\leftarrow \infty \leftarrow \theta = 0$ نقطة



$$V_{24} = \omega_2 \times r_{24/12} = 2 \times 63 = 126 \text{ cm/s}$$

$$s = 126 \text{ cm/s} \uparrow$$

$$\omega_4 = \frac{V_{24}}{r_{24/14}} = \frac{126}{\infty} = 0$$

$$\vec{V}_{24} = \vec{V}_{24} + \omega_4 \times r_{34/24}$$

$$\vec{V}_{34} = \vec{V}_{24}$$

اتجاه V_{24} مرصون بـ ω_2 ، ω_3 مرصون بـ V_{23}
بـ ω_4 مرصون بـ V_{34}

$$V_{23} = \omega_2 V_{23/12} = 2 \times 42 = 84 \text{ cm/s}$$

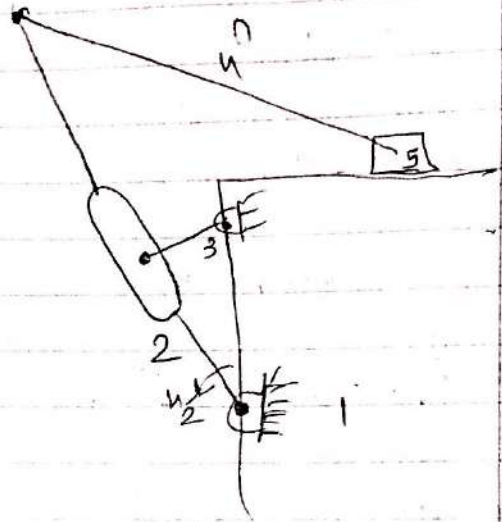
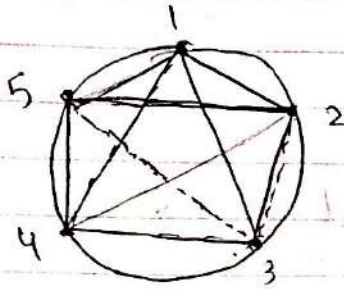
$$V_{23} = \omega_3 V_{23/13} \Rightarrow \omega_3 = \frac{84}{24} = 3.5 \text{ rad/s}$$

$$V_{34} = \omega_3 V_{34/13} = \frac{84}{24} \times 12 = 42 \text{ cm/s}$$

Ex: given ω_2 find \vec{V}_5

$$\omega_2 = 2 \text{ rad/s CCW}$$

$$I_C = \frac{5(4)}{2} = 10$$



$\boxed{12 \quad 13} \quad 23$ with CN pin slot

$$\boxed{12 \quad 15} \quad 25 \Rightarrow 25 \checkmark$$

$$\boxed{24 \quad 45} \quad 25$$

$$\boxed{15 \quad 13} \quad 35 \Rightarrow \text{كله بطله} \Rightarrow \text{تفويض} \Rightarrow \text{طريقه تازه}$$

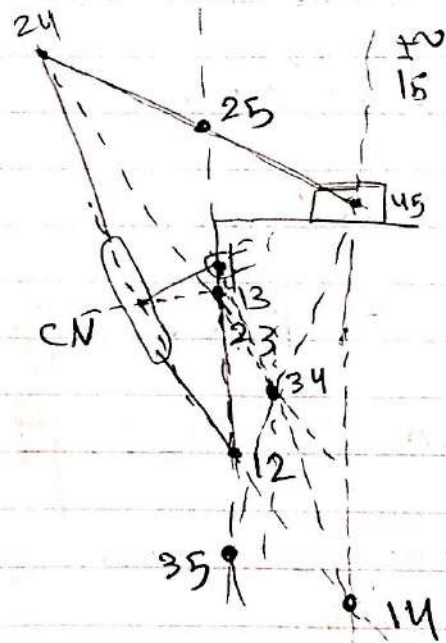
$$\boxed{25 \quad 23} \quad 35$$

$$\boxed{15 \quad 45} \quad 14 \checkmark$$

$$\boxed{12 \quad 24} \quad 14 \checkmark$$

$$\boxed{31 \quad 14} \quad 34 \text{ وقف}$$

$$\boxed{32 \quad 24} \quad 34 \text{ بنظره} \Rightarrow \text{كله} \Rightarrow V_{\text{slider}}$$



from ω_2 find V_{23} then ω_3 then V_{34} then ω_4 then V_{45}

$$V_{23} = \omega_2 V_{23/12}$$

$$\omega_2 = \frac{V_{23}}{V_{23/12}}, \quad V_{24} = \omega_4 V_{24/12}, \quad \omega_4 = \frac{V_{24}}{V_{24/14}}, \quad V_{45} = \omega_4 V_{45/15} \checkmark$$

$$O/R \Rightarrow V_{35} = \omega_3 V_{35/13}, \quad V_{\text{slider}} = V_{35}$$

Velocity and acceleration analysis using complex numbers

Position loop:

Equation:

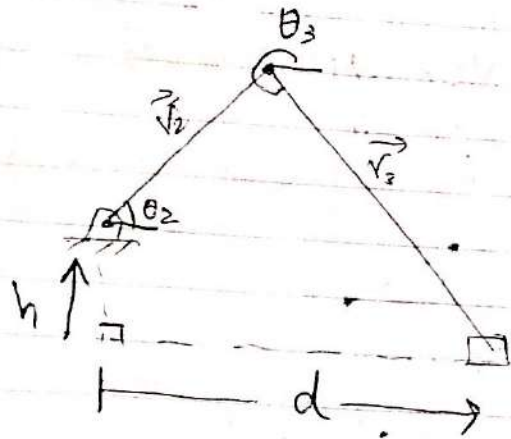
$$\vec{r}_2 + \vec{r}_3 - \vec{d} + \vec{h} = 0$$

$$2e^{j\theta_2} + L_3e^{j\theta_3} - d e^{j\theta} + h e^{j\theta_0} = 0$$

$$2e^{j\theta_2} + L_3e^{j\theta_3} - d + jh = 0$$

Velocity equation: $\vec{v} = \frac{d\vec{r}}{dt}$

$$L_2 j e^{j\theta_2} \dot{\theta}_2 + L_3 j e^{j\theta_3} \dot{\theta}_3 - \dot{d} + 0 = 0$$



Acceleration: $\vec{a} = \frac{d\vec{v}}{dt}$

$$-L_2 j (\ddot{\theta}_2 e^{j\theta_2} + \dot{\theta}_2^2 e^{j\theta_2}) + L_3 j (\ddot{\theta}_3 e^{j\theta_3} + \dot{\theta}_3^2 e^{j\theta_3}) - \ddot{d} = 0$$

$$\frac{d e^{jx}}{dt} = \frac{d e^{jx}}{dx} \frac{dx}{dt}$$

$$= j e^{jx} \frac{dx}{dt}$$

$$\text{Velocity: } L_2 j e^{j\theta_2} \dot{\theta}_2 + L_3 j e^{j\theta_3} \dot{\theta}_3 - \dot{d} = 0$$

$$\Rightarrow L_2 j \dot{\theta}_2 (\cos\theta_2 + j \sin\theta_2) + L_3 j \dot{\theta}_3 (\cos\theta_3 + j \sin\theta_3) - \dot{d} = 0$$

$$\text{Real Part: } -L_2 \dot{\theta}_2 \sin\theta_2 - L_3 \dot{\theta}_3 \sin\theta_3 - \dot{d} = 0$$

$$\text{Imaginary Part: } L_2 \dot{\theta}_2 \cos\theta_2 + L_3 \dot{\theta}_3 \cos\theta_3 = 0$$

given input $\dot{\theta}_2 = \text{known}$

Unknown: $\dot{\theta}_3, \dot{d}$

$$\begin{bmatrix} L_3 \sin\theta_3 & 1 \\ L_3 \cos\theta_3 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_3 \\ \dot{d} \end{bmatrix} = \begin{bmatrix} -L_2 \sin\theta_2 \\ -L_2 \cos\theta_2 \end{bmatrix} \dot{\theta}_2$$

$$A\vec{x} = \vec{b}$$

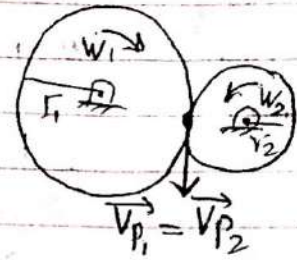
$$\vec{x} = A^{-1} \vec{b}$$

Gear Trains

Types of gear trains:

① Simple gear train
one gear on each shaft

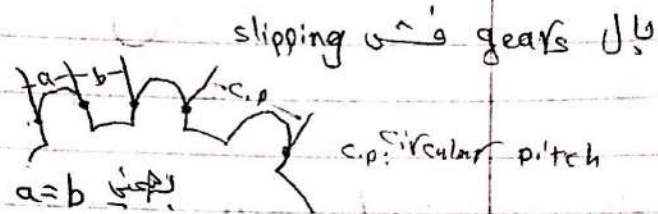
$$\vec{V}_P = \vec{V}_{P_2} = \omega_1 r_1 = \omega_2 r_2$$



$$V_R = \frac{\omega_1}{\omega_2} = -\frac{r_2}{r_1} \equiv \text{Velocity Ratio}$$

$$\frac{\omega_1}{\omega_2} = -\frac{r_2}{r_1}$$

$$\frac{\omega_2}{\omega_3} = -\frac{r_3}{r_2} ; \quad \frac{\omega_3}{\omega_4} = -\frac{r_4}{r_3}$$



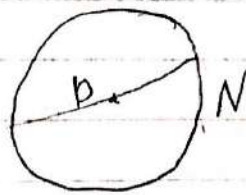
$$V_R = \frac{\omega_{in}}{\omega_{out}} = \frac{\omega_1}{\omega_4} = \frac{\omega_1}{\omega_2} \cdot \frac{\omega_2}{\omega_3} \cdot \frac{\omega_3}{\omega_4} = -\frac{r_4}{r_1} \quad \text{Gears 2 \& 3 are idle gears}$$

$$\frac{\omega_1}{\omega_4} = \frac{\omega_1}{\omega_2} \cdot \frac{\omega_2}{\omega_3} \cdot \frac{\omega_3}{\omega_4} = \left(-\frac{r_2}{r_1}\right) \left(-\frac{r_3}{r_2}\right) \left(-\frac{r_4}{r_3}\right) \Rightarrow \text{to increase the distance between the input \& output gears.}$$

(-) to invert velocity direction

* To enable to mesh, two gears, both must have the same circular pitch

$$\text{Circular Pitch} = \frac{\pi D}{N} \quad \text{for pitch circle}$$



$$\frac{\pi D_1}{N_1} = \frac{\pi D_2}{N_2} = m_1 = m_2$$

$$m = \frac{\pi D}{N}$$

$$\text{Module} = m = \frac{D}{N}$$

all calculation (Velocity Ratio) are made basing on the pitch circle

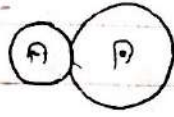
$$\text{Module} = m = \frac{D}{N} \quad \text{SI metric}$$

$$\text{Diametral Pitch } P = \frac{N}{D} \quad \text{English units}$$

$$\frac{\omega_1}{\omega_2} = -\frac{r_2}{r_1} = -\frac{D_2}{D_1} = -\frac{m_2 N_2}{m_1 N_1} = -\frac{N_2}{N_1}$$

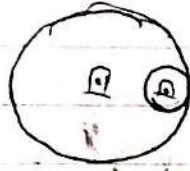
* Connection between gears:

① External :



$$\frac{\omega_1}{\omega_2} = -\frac{N_2}{N_1}$$

② Internal



$$\omega_R = \frac{N_2}{N_1} = \frac{\omega_1}{\omega_2}$$

فصل الب، انجاز البعدين تقص

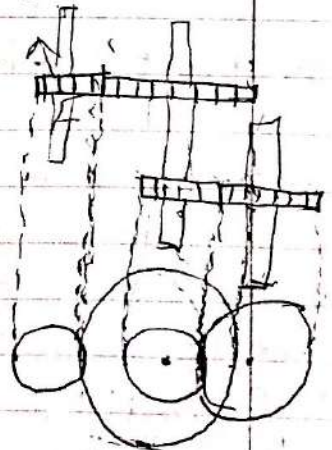
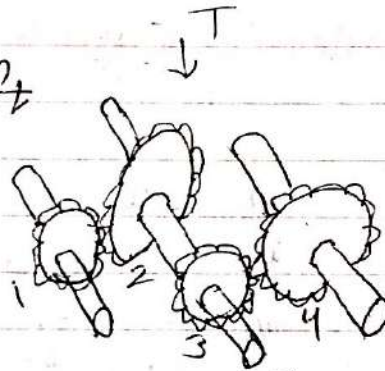
② Compound Gear Train:

More than one gear on one shaft

$$\frac{\omega_1}{\omega_2} = -\frac{N_2}{N_1}$$

$$\omega_3 = \omega_2$$

$$\frac{\omega_3}{\omega_4} = -\frac{N_4}{N_3}$$

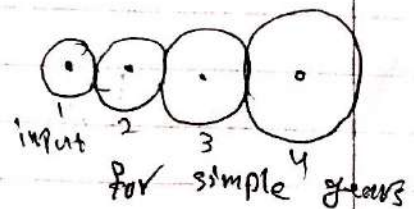


$$\omega_R = \frac{\omega_{in}}{\omega_{out}} = \frac{\omega_1}{\omega_4} = \frac{\omega_1}{\omega_2} \frac{\omega_2}{\omega_3} \frac{\omega_3}{\omega_4}$$

$$\omega_R = -\frac{N_2}{N_1} (1) \left(-\frac{N_4}{N_3}\right) = \frac{N_2 N_4}{N_1 N_3} = \frac{\omega_1}{\omega_4}$$

$$\omega_R = \frac{\omega_{in}}{\omega_{out}} = \frac{\text{Product of Driven teeth}}{\text{Product of Driver teeth}} = \frac{-N_2 - N_4}{N_1 N_3}$$

$$\omega_R = \frac{\omega_{in}}{\omega_{out}} = \frac{\text{Driven}}{\text{Driver}} = -\frac{N_2}{N_1} -\frac{N_3}{N_2} -\frac{N_4}{N_3} = -\frac{N_4}{N_1}$$



Power:

$T_{in} W_{in} = T_{out} W_{out}$

$$\frac{W_{in}}{W_{out}} = \frac{T_{out}}{T_{in}}$$

AHMAD JUNDI
 $y = \frac{w_{in}}{w_{out}}$ input بلف بسوطة y اصناف output
 T_{out} اكبر من T_{in} y اصناف
 T_{out} أعلى ← قوة أعلى

$$V_R = \frac{W_{in}}{W_{out}} = \frac{\pi \text{ driven}}{\pi \text{ driver}}$$

velocity Ratio

$$\prod_{i=1}^n i = 1 \times 2 \times 3$$

$$\text{Train Value (e)} = \frac{w_{out}}{w_{in}} = \frac{\pi_{Driver}}{\pi_{Driven}}$$

③ Planetary Gears :

a. Arm is the fixed ground

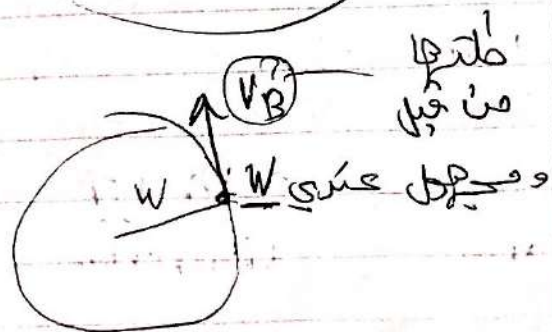
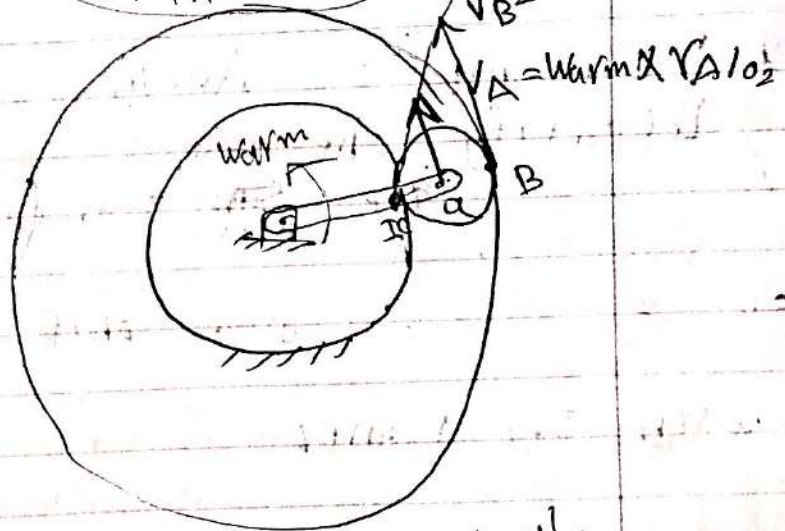
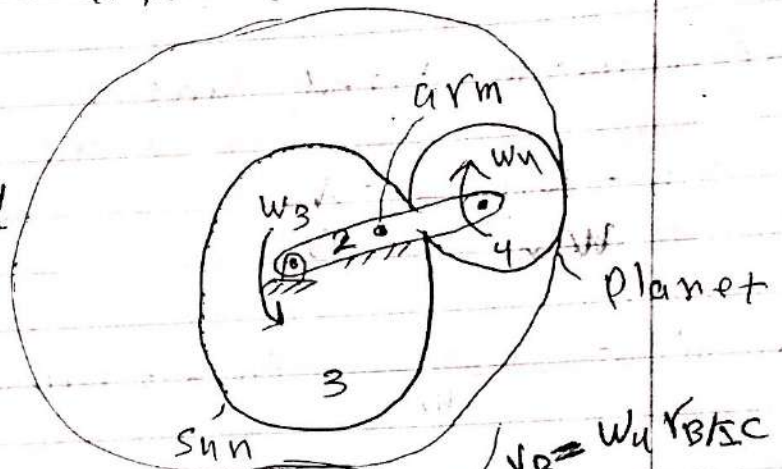
$$\frac{w_3}{w_4} = -\frac{N_4}{N_3}, \frac{w_3}{w_5} = -\frac{N_4}{N_3} \cdot \frac{N_5}{N_4}$$

b. Sun is fixed.

$$W_3$$

3
 0 نقطة التقاء الجزيئين بسرعة
 لأنه $\omega = \text{sum } W$

٧ انما هو صغير لأنه في Warm .



Relations Relative to the arm:

$$\frac{W_{out}/arm}{W_{in}/arm} = \frac{T_{Driver}}{T_{Driven}}$$

نسبة السرعات الى سرعة الدعامه
ثابتة في كل حال، gears ال

$$\Rightarrow \frac{W_{out} - W_{arm}}{W_{in} - W_{arm}} = \frac{-N_3 \times N_4}{N_4 \times N_5}$$

$$\boxed{\frac{W_5 - W_{arm}}{W_3 - W_{arm}} = -\frac{N_3}{N_5}}$$

According to fixed suns

$$\frac{W_5 - W_{arm}}{0 - W_{arm}} = -\frac{N_3}{N_5}$$

$$\text{Ex: } \frac{W_5 - 2}{-2} = \frac{-20}{40} \Rightarrow W_5 = 3 \text{ rad/s (ccw)}$$

* Procedure of Analysis:

- ① Identify the arm.
- ② = the ~~sun~~ planet gear (its axis rotating and is supported by the arm)
- ③ = gears that are (directly connected) in mesh

with the planet

الدعامه الى كل gear

Call then the input & output

$$\frac{W_{out}/arm}{W_{in}/arm} = \frac{T_{Driver}}{T_{Driven}}$$

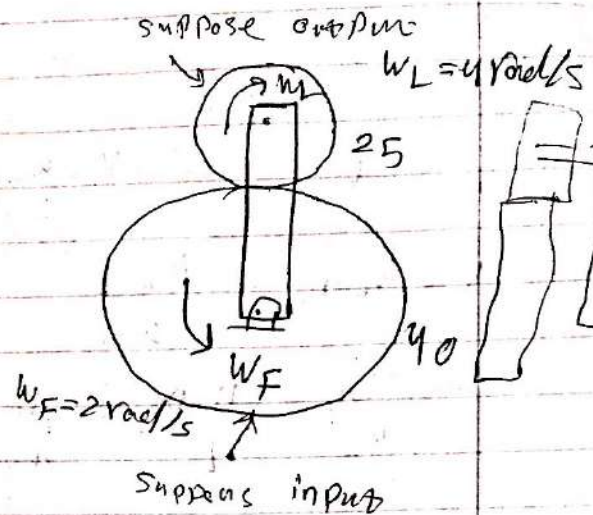
Ex: Find ω_{arm} ?

Planet = Gear (L)

$$\frac{\omega_{out} / \omega_{arm}}{\omega_{in} / \omega_{arm}} = \frac{T_{driver}}{T_{driven}} = -\frac{N_F}{N_L}$$

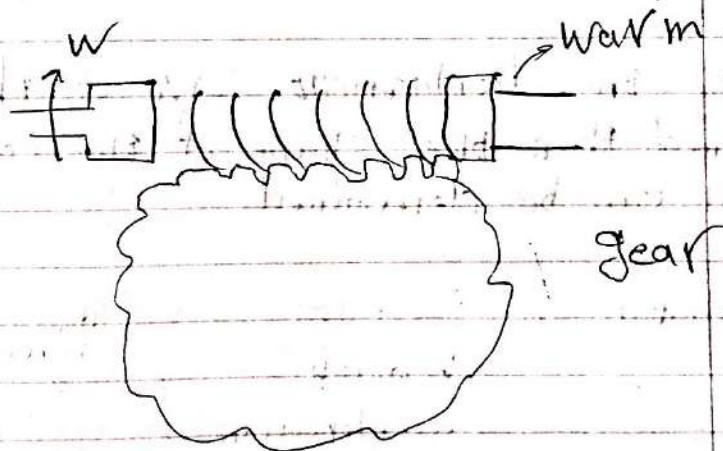
$$\frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}} = \frac{-48}{25}$$

$$\frac{-4 - \omega_{arm}}{2 - \omega_{arm}} = \frac{-48}{25} \Rightarrow \omega_{arm} =$$



Problem 32

Power Gear:



Lead: the distance that the warm gear advances when it rotates one complete revolution.

$$L = N_w \times P$$

$$V = \omega_w r_g$$

$\frac{2\pi}{s} \Rightarrow \frac{L}{s}$ كل لفّة كاملة بتغطي مسافة L للنقطة بتتكرر $\frac{rev}{s} \Rightarrow \frac{L}{s}$
 $\omega_{rpm} \Rightarrow V$ $\rightarrow V = \frac{\omega_w L}{2\pi}$

$\Rightarrow V = \omega_e r_e = \frac{\omega_w L}{2\pi}$

$\Rightarrow \boxed{\frac{\omega_w}{\omega_e} = \frac{\pi p_o}{L} = \frac{N_g p}{N_w p} = \frac{N_g}{N_w}}$

but $p = \frac{\pi p_o}{N_g}$

هنا معطيات السؤال تكون N_g, N_w, p_o أو p, N_w, p_o يعني احرف انقال معطى

Cam & Follower:

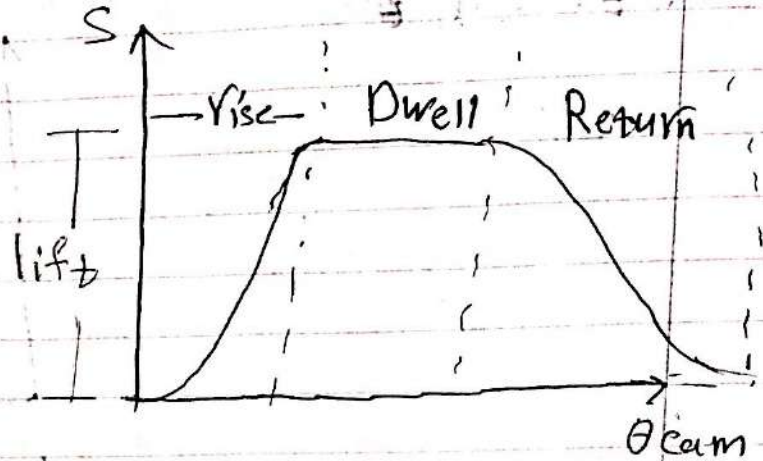
Disk Cam with
 \swarrow flat free follower
 \searrow Radial Roller follower

Firstly the displacement curve of the follower must be designed then the shape of the cam that produces this motion can be determined

Continuous in	} Finite
Position	
Velocity	
acceleration	Jake

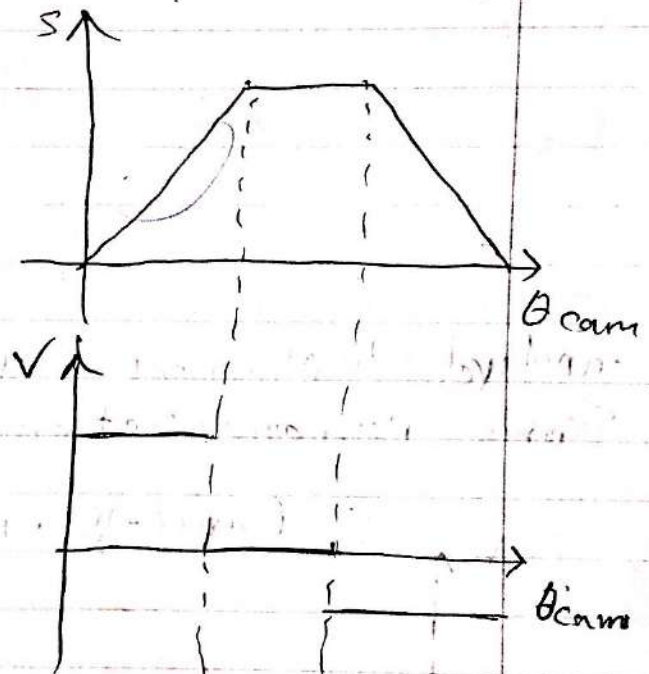
$J = \frac{da}{dt}$

Follower Displacement Curve:



Types of displacement curves:

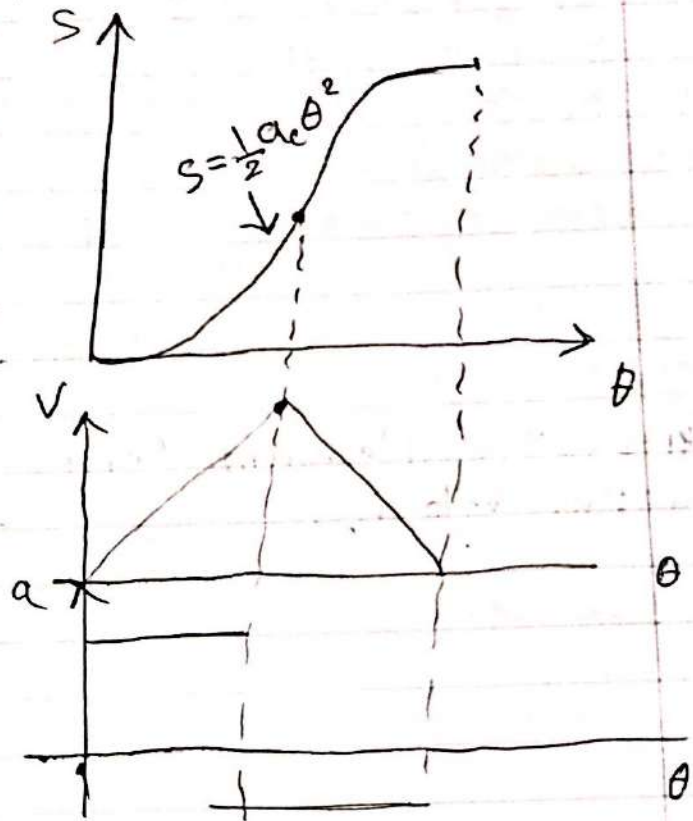
① Constant Velocity:



2) Constant acceleration decelerations

$$S = S_0 + V_0 \theta + \frac{1}{2} a_c \theta^2$$

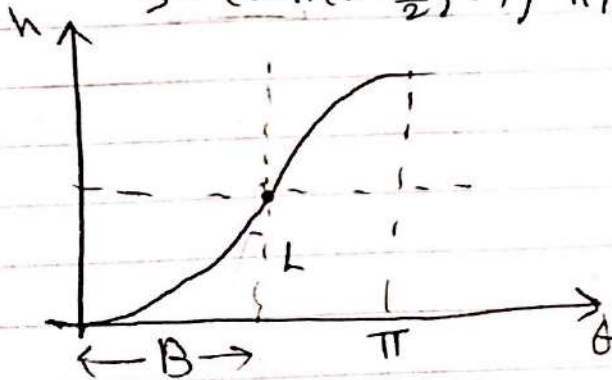
$$S = \frac{1}{2} a_c \theta^2$$



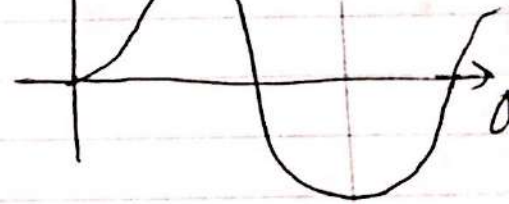
Standard displacement Curves:

1) Simple harmonic motion curve:

$$S = (\sin(\theta - \frac{\pi}{2}) + 1) h/2$$



$$S = \sin \theta$$



Concave (up or Down)

* Characteristics of Harmonic motions

- Lowest peak acceleration
- Lowest pressure angle
- Can't be matched with dwell $\rightarrow V=0, a=0$
- Can't be used in high velocity application

* Cycloidal Motions

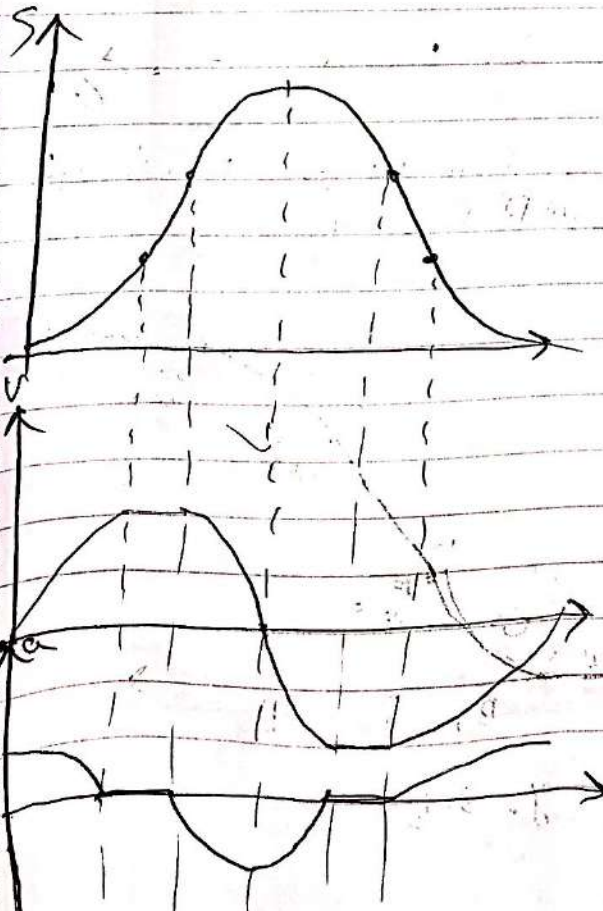
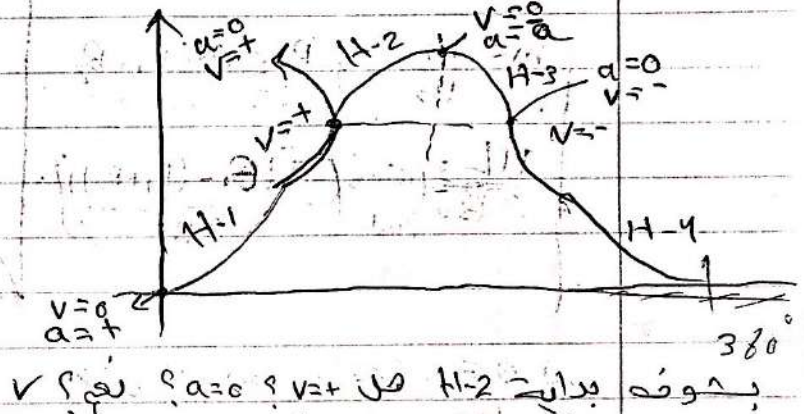
- Highest acceleration
- \neq pressure angle
- Can be matched with dwell
- low velocity app

* Eighth Power Polynomial:

In intermediate

Problem 330

Conditions $\Delta \theta = 360^\circ$

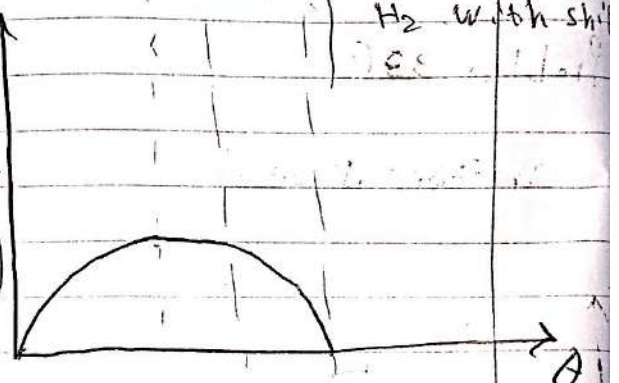


3. [17, 22, 38, 38, 40, 43, 44]

$$S = f(\theta) = \begin{cases} L_1 \left(1 - \cos \frac{\pi \theta}{2B_1} \right), & 0 \leq \theta \leq B_1 \\ \frac{L_2}{B_2} (\theta - B_1) + L_1, & B_1 \leq \theta \leq B_1 + B_2 \\ L_3 \left(\sin \pi \frac{(\theta - (B_1 + B_2))}{2B_3} \right) + L_1 + L_2, & B_1 + B_2 \leq \theta \leq B_1 + B_2 + B_3 \end{cases}$$

$$V = V(\theta) = \int \frac{\pi L_1}{2B_1} \sin\left(\frac{\pi\theta}{2B_1}\right), 0 \leq \theta \leq B_1$$

$m = \frac{L_2}{B_2}$, $B_1 \leq \theta \leq B_1 + B_2$
 3.14/2 B_2 \Rightarrow $\frac{\pi L_3}{2 B_3} (\cos(\frac{\pi}{2} - (B_1 + B_2)))$
 180 degrees



حاجي امام حقیقیہ، ناقدینہ فی لائہ

$$v = \frac{ds}{dt} = \frac{ds}{d\theta} \frac{d\theta}{dt} = v(\theta) \dot{\theta}$$

اذا جدي اراها لتفهم معنى بضرتها اد W

Prob. 3.23

Harmonic Motion with dwell? No

إذا برز H أو C يختار H .

بفهم بداية ونهاية كل C_1, C_2
وبوضوح حال تغطيته

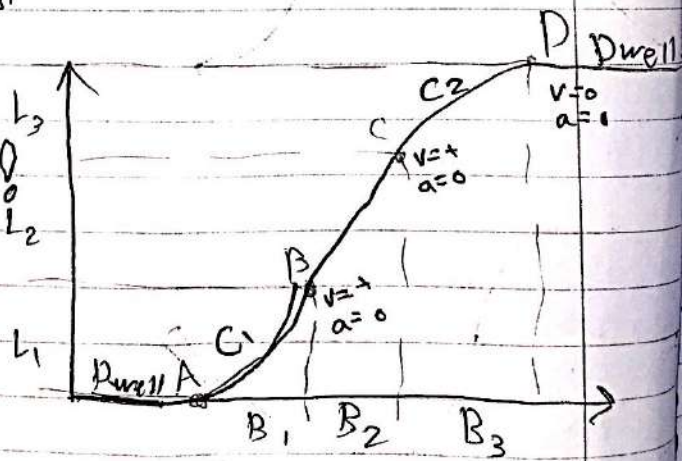
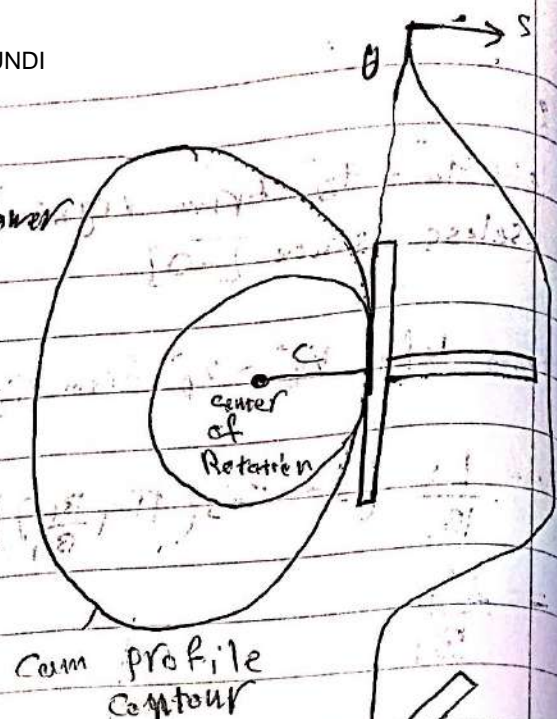


Fig. 3.48

$$a = \frac{d^2 s(t)}{dt^2} = \frac{d^2 f(\theta)}{d\theta^2} \cdot \frac{d\theta^2}{dt^2}$$

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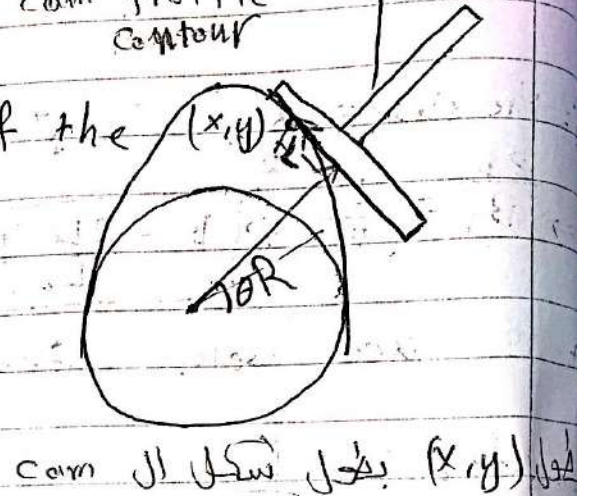
$C \equiv$ minimum Cam radius
 \equiv distance between the follower
 and the center of rotation of
 the Cam.



$$R = C + f(\theta)$$

at $\theta = 0 \Rightarrow R = C$

$L \equiv$ distance between the axis of the
 follower & the contact point

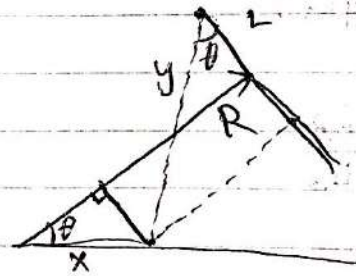


$$\begin{cases} R = x \cos \theta + y \sin \theta \\ L = y \cos \theta - x \sin \theta \end{cases}$$

$$L = \frac{dR}{d\theta} = \frac{df(\theta)}{d\theta} = V$$

من المعادلات

$$\Rightarrow \boxed{L = V}$$



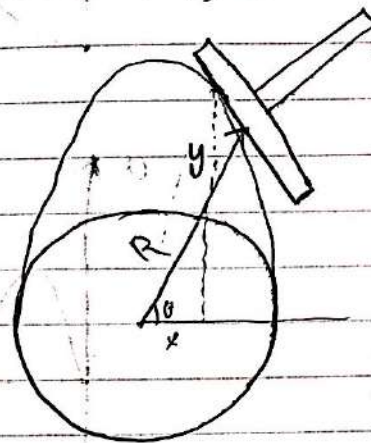
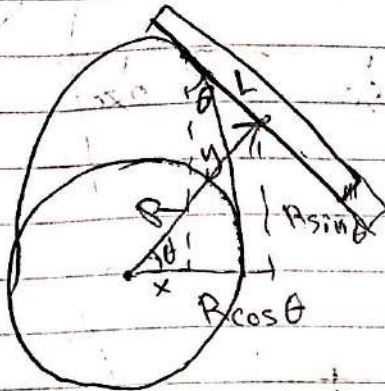
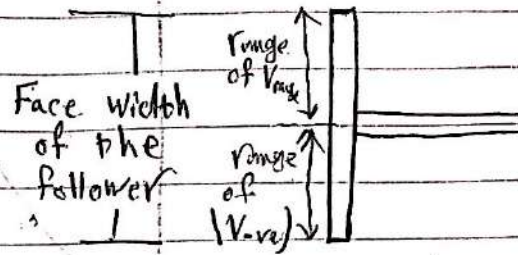
If $V = \text{Positive} \Rightarrow$ Contact Point is ~~located~~ on the upper part
 the axis of the follower.

$V = \text{negative} \Rightarrow$ Contact Point is on the lower part
 the follower axis.

Symmetry for Local !

UPLOADED BY AHMAD JUNDI

Theoretical face width of the follower = $V_{max} + |V_{negative}|$



$$x = R \cos \theta - L \sin \theta$$

$$y = R \sin \theta + L \cos \theta$$

cust: θ

إذا كانت ال circle مسطرة ←

$$\frac{dx}{d\theta} = 0$$

$$\frac{dy}{d\theta} = 0$$

Cust / Cam pointing

أكل C بتعطي تذبذب بال Cam بال Contact Point بتغير تقريباً

$$x = (c + f(\theta)) \cos \theta - f'(\theta) \sin \theta$$

$$\frac{dx}{d\theta} = f'(\theta) \cos \theta - (c + f(\theta)) \sin \theta - (f''(\theta) \sin \theta + f'(\theta) \cos \theta)$$

$$\frac{dx}{d\theta} = -(c + f(\theta) + f''(\theta)) \sin \theta = 0$$

0 = cos theta = sin theta
التي هي ال circle مسطرة

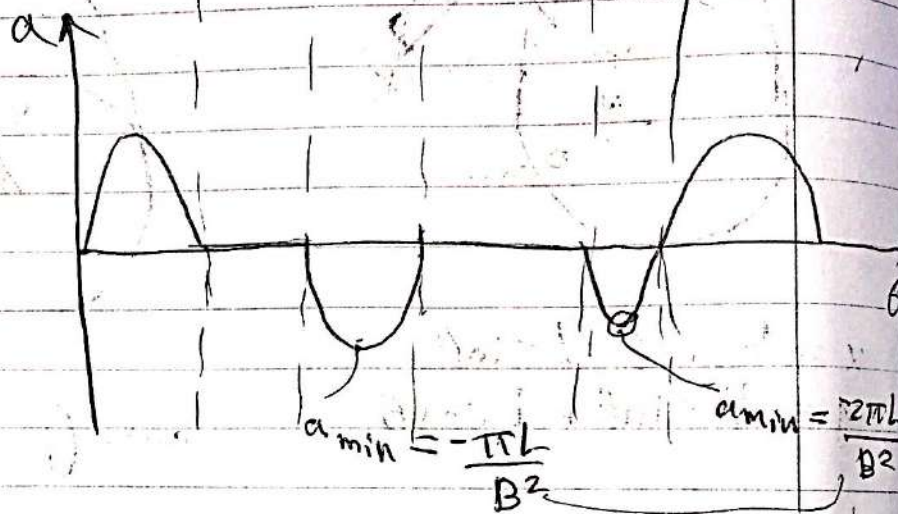
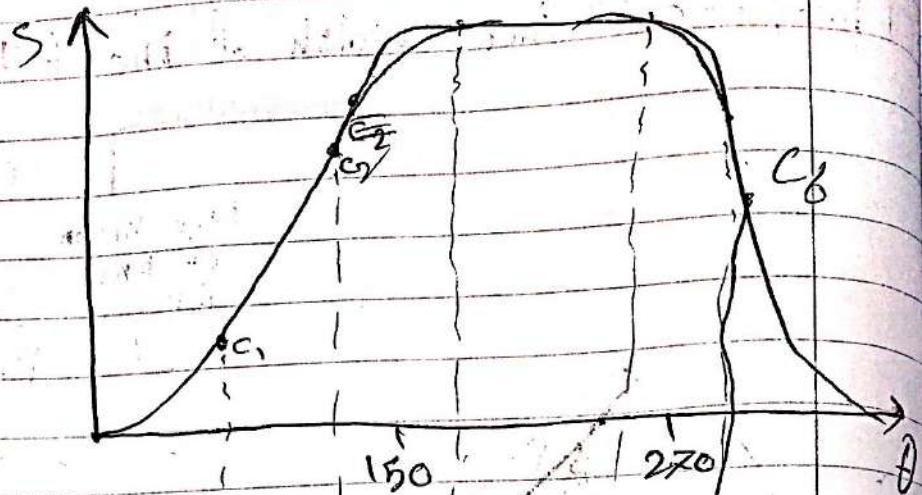
$$\frac{dy}{d\theta} = + (c + f(\theta) + f''(\theta)) \cos \theta = 0$$

$$\Rightarrow [c + f(\theta) + f''(\theta)] = 0 \Rightarrow \text{Cust occurs}$$

find minimum negative value then chose $c > 1$ to avoid cust



معنا انواع متاثرات ال cam بال circle (في هيك



local
 $f(150) = \frac{\pi L}{B^2}$ 1st min

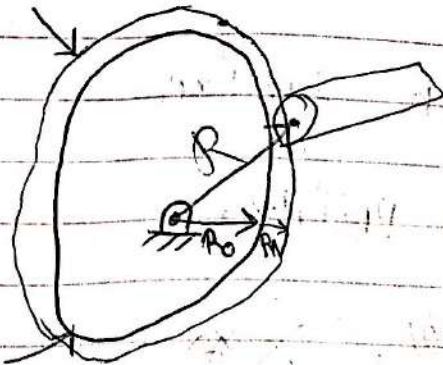
sinoidal amplitude
 $L = \sin$ كون

$f(270) = \frac{2\pi L}{B^2}$ 2nd min

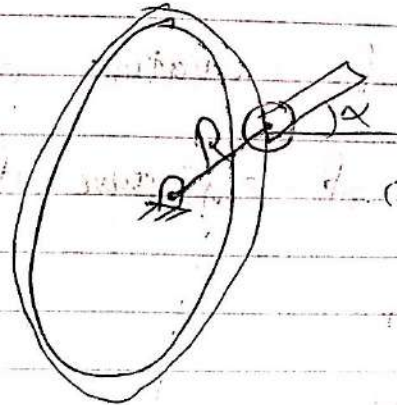
if ~~$c > 1$~~ $c > |f(0) + f''(0)|_{\min 1 \& 2} \Rightarrow \text{No Cust}$

Disk Cam with Radial roller follower:

Pitch Surface



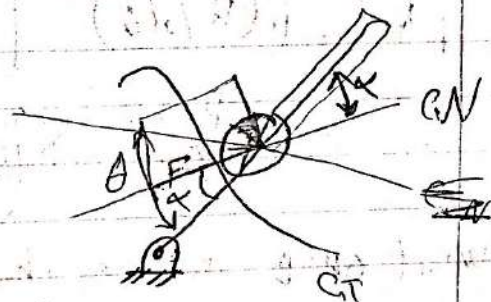
Cam Profile



$R_0 \equiv$ minimum distance between roller center & center of rotation of the cam

$R_r \equiv$ radius of the roller

$$R = R_0 + f(\theta)$$



$\alpha \equiv$ pressure angle
the angle between the axis of the follower and the CN direction

$$x_A = R \cos \theta$$

$$y_A = R \sin \theta$$

$$x = R \cos \theta - R_r \cos(\theta - \alpha)$$

$$y = R \sin \theta - R_r \sin(\theta - \alpha)$$

radius

$$\alpha = \tan^{-1} \left(\frac{f'(\theta)}{R_0 + f(\theta)} \right) = \tan^{-1} \left(\frac{V}{R} \right)$$

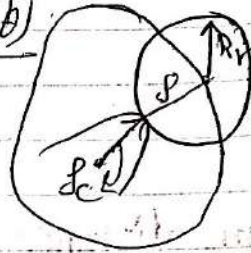
$$R = R_0 + f(\theta)$$

f = Radius of curvature of pitch surface

f_c = Radius of curvature of cam profile

$$f = f_c + R_r$$

$$R = R_0 + f(\theta)$$



$$R = R_0 + f(\theta)$$

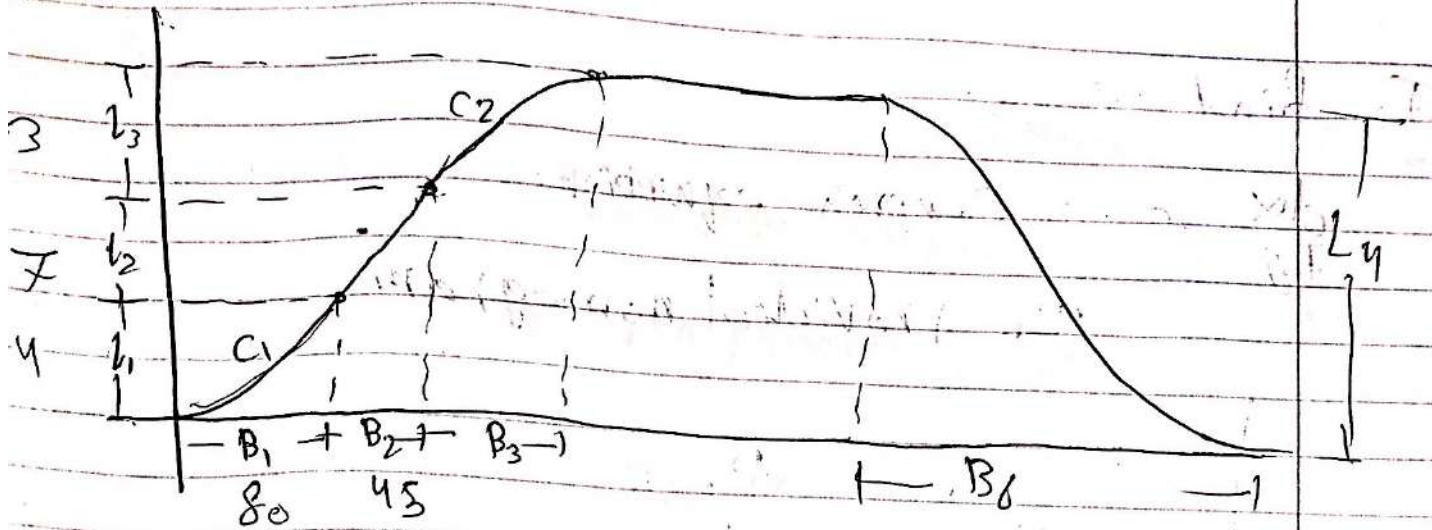
$$\text{as } R_r = f_c \downarrow$$

$R_r = f \Rightarrow f_c = 0 \Rightarrow \text{cusp, Cam Pointing}$

$$f = \frac{[R^2 + (dR/d\theta)^2]^{3/2}}{R^2 + 2 \left(\frac{dR}{d\theta} \right)^2 - R \left(\frac{d^2R}{d\theta^2} \right)}$$

to avoid cusp insure that $R_r < f_{\min}$ to find f_{\min}

$$\frac{df}{d\theta} = 0 \Rightarrow \text{complex relation} \Rightarrow \text{use provided case}$$



$$R_0 = 5 \text{ cm}$$

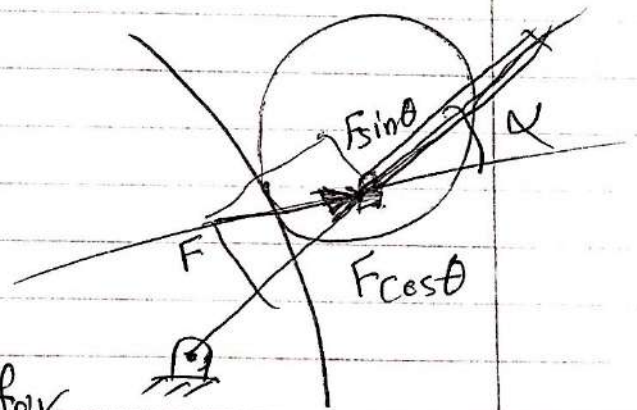
$$C_1: \frac{L}{R_0} = \frac{4}{5} = 0.8 \Rightarrow \frac{s_{\min}}{R_0} = 0.75$$

$$\Rightarrow s_{\min} = 0.75 R_0 = 0.75(5) = 3.75$$

$$\Rightarrow \text{maximum } R_r < 3.75$$

$\alpha \equiv$ Pressure angle

$F \sin \theta$ perpendicular to the axis of the follower



Keep α as low as possible for design conditions

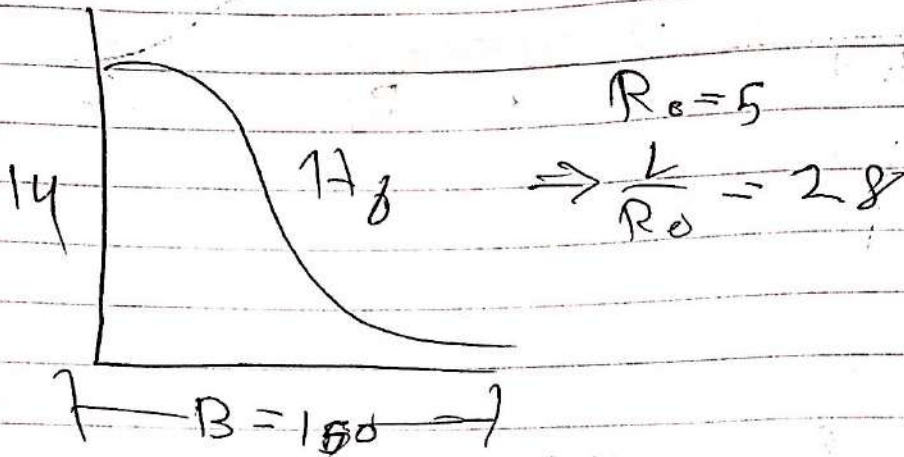
$$\text{Keep } \alpha \leq 30^\circ$$

$$\alpha = \tan^{-1} \left(\frac{f'(\theta)}{R_0 + f(\theta)} \right)$$

To find α_{max}

$$\frac{d\alpha}{d\theta} = 0 \Rightarrow \text{Complex equation}$$

use provided nomogram



$$\Rightarrow \alpha_{max} = 44^\circ > 30$$

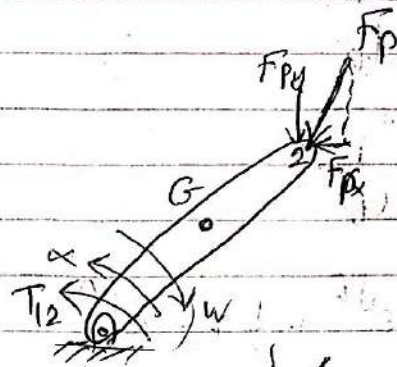
high pressure angle not recommended

* Force analysis:

Solution using matrix forces:

* The weight is neglected.

Equation of kinetic equilibrium:



$$\sum \vec{F}_x = m \vec{a}_{Gx}$$

$$\sum \vec{F}_y = m \vec{a}_{Gy}$$

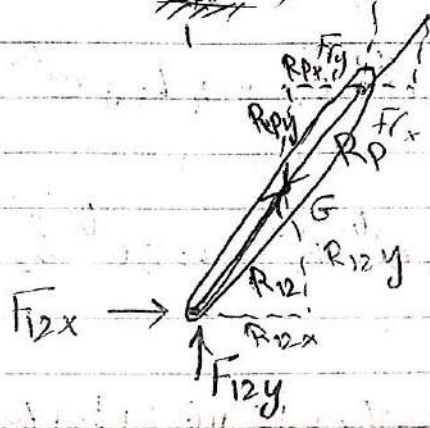
$$\sum M_G = I_G \alpha$$

$$F_{12x} - F_{px} = m_2 a_{G2x} \quad \text{--- (1)}$$

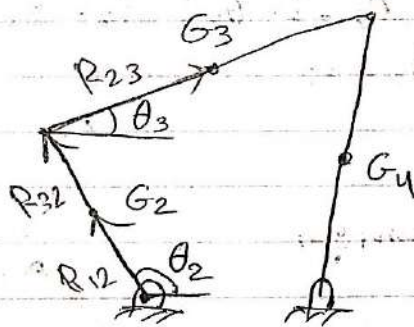
$$F_{12y} - F_{py} = m_2 a_{G2y} \quad \text{--- (2)}$$

F_{23} = Reaction force from link 2 on link 3

R_{23} = Position from link 2 to G_3



$$\begin{aligned} R_{12x} &= R_{12} \cos \theta_2 \\ R_{12y} &= R_{12} \sin \theta_2 \\ R_{px} &= R_p \cos \theta_2 \\ R_{py} &= R_p \sin \theta_2 \end{aligned}$$



F_{12x}, F_{12y}, T_{12} are unknowns

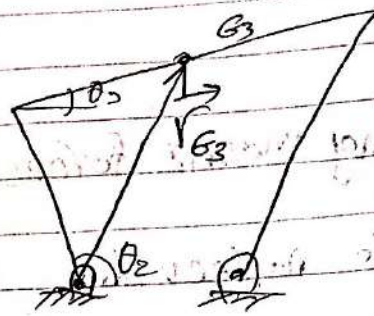
the moments:

$$F_{12x} R_{12y} - F_{12y} R_{12x} + F_{px} R_{py} - F_{py} R_{px} + T_{12} = I_{G2} \alpha_2 \quad \text{--- (3)}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ R_{12y} & -R_{12x} & 1 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G2x} + F_{px} \\ m_2 a_{G2y} + F_{py} \\ I_{G2} \alpha_2 - F_{px} R_{py} + F_{py} R_{px} \end{bmatrix}$$

$$\vec{a}_{G_2} = \frac{d^2 \vec{r}_{G_2}}{dt^2}$$

$$\vec{a}_{G_3} = \frac{d^2 \vec{r}_{G_3}}{dt^2}$$



$$\vec{r}_{G_3} = L_2 e^{j\theta_2} + \frac{1}{2} L_3 e^{j\theta_3}$$

$$\frac{d\vec{r}_{G_3}}{dt} = L_2 j e^{j\theta_2} \dot{\theta}_2 + \frac{1}{2} L_3 j e^{j\theta_3} \dot{\theta}_3$$

$$\frac{d^2 \vec{r}_{G_3}}{dt^2} = -L_2 e^{j\theta_2} \dot{\theta}_2^2 + L_2 j e^{j\theta_2} \ddot{\theta}_2 - \frac{1}{2} L_3 e^{j\theta_3} \dot{\theta}_3^2 + \frac{1}{2} L_3 j e^{j\theta_3} \ddot{\theta}_3$$

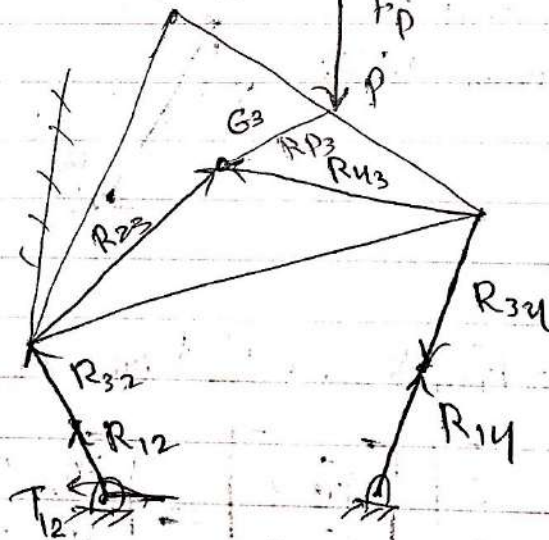
$$= -L_2 \dot{\theta}_2^2 (\cos\theta_2 + j \sin\theta_2) + L_2 j \ddot{\theta}_2 (\cos\theta_2 + j \sin\theta_2)$$

$$- \frac{L_3}{2} \dot{\theta}_3^2 (\cos\theta_3 + j \sin\theta_3) + \frac{L_3}{2} j \ddot{\theta}_3 (\cos\theta_3 + j \sin\theta_3)$$

$$a_{G_3x} = -L_2 \dot{\theta}_2^2 \cos\theta_2 - L_2 \ddot{\theta}_2 \sin\theta_2 - \frac{L_3}{2} \dot{\theta}_3^2 \cos\theta_3 - \frac{L_3}{2} \ddot{\theta}_3 \sin\theta_3$$

$$a_{G_3y} = -L_2 \dot{\theta}_2^2 \sin\theta_2 + L_2 \ddot{\theta}_2 \cos\theta_2 + \frac{L_3}{2} \dot{\theta}_3^2 \sin\theta_3 + \frac{L_3}{2} \ddot{\theta}_3 \cos\theta_3$$

Ex: four bar mechanism



نفس F_{12} + تبعد M حول G_2 ليا (موضن بالارقام θ_2 ربع ثاني بالثاني) $\cos \theta = -$ وبيعطيلي $M(-)$ عنان القاعدي الشكل، يحولها للربع الاول
وبكثبة المعادلات تبعدوني وياخر θ ذي ما هي، بدون مشاكل

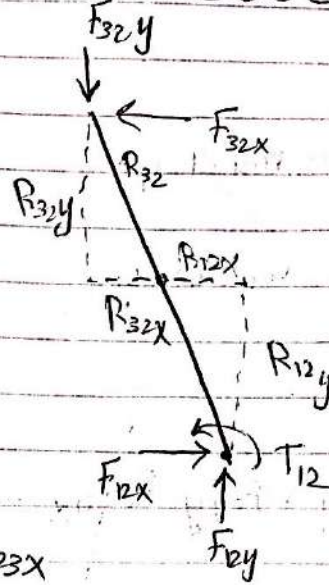
link 2:

$$\sum F_x = m_2 a_{G_2x}$$

~~F_{12x}~~

$$F_{12x} - F_{32x} = m_2 a_{G_2x}$$

$$F_{12y} - F_{32y} = m_2 a_{G_2y}$$



$$F_{12x} R_{12y} - F_{12y} R_{12x} + F_{32x} R_{23y} - F_{32y} R_{23x} + T_{12} = I_{G_2} \alpha_2$$

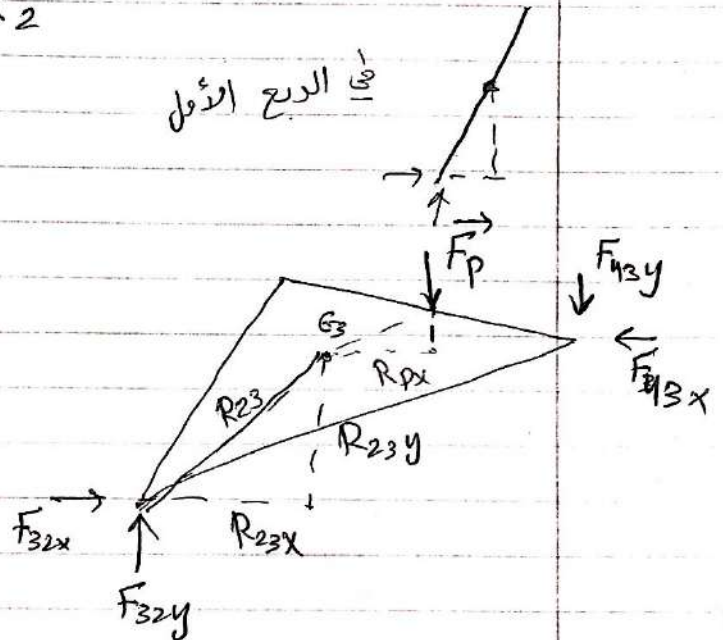
link 3:

$$\sum F_x ; \sum F_y ; \sum M_G$$

$$F_{32x} - F_{43x} = m_3 a_{G_3x}$$

$$F_{32y} - F_{43y} = m_3 a_{G_3y}$$

$$\sum M_{G_3} = I_{G_3} \alpha_3$$



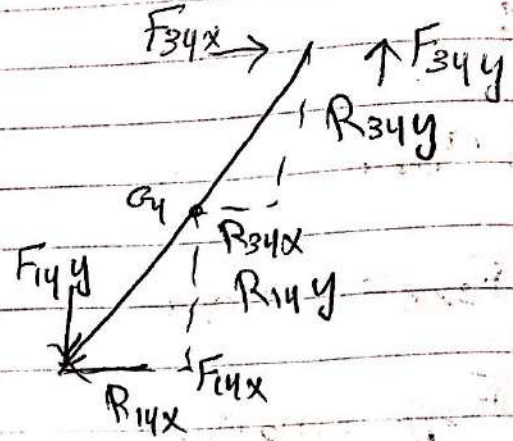
$$F_{32x} R_{23y} - F_{32y} R_{23x} - F_{43x} R_{43y} - F_{43y} R_{43x} - F_p R_{px} = I_{G_3} \alpha_3$$

F_{32} نفس F_{23} بشرط بعد الاتجاهات مع

link 4

$$F_{34x} - F_{14x} = m_4 a_{G4x}$$

$$F_{34y} - F_{14y} = m_4 a_{G4y}$$



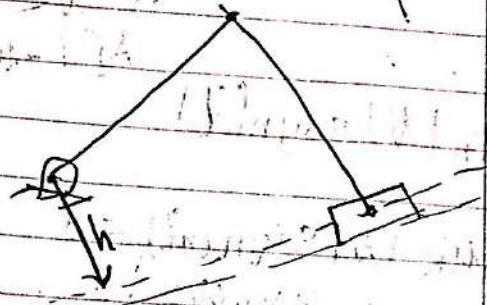
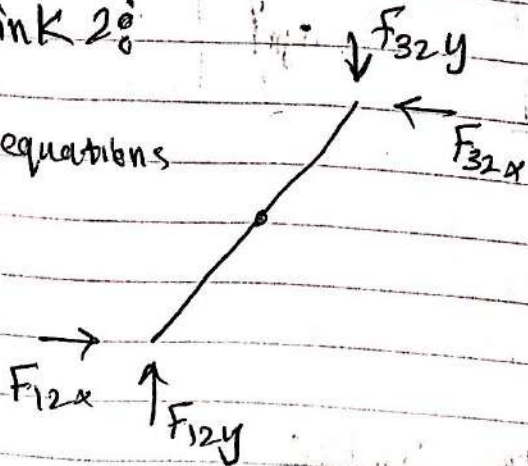
$$-F_{34x} R_{34y} + F_{34y} R_{34x} - F_{14x} R_{14y} + F_{14y} R_{14x} = I_{G4} \alpha_4$$

Remarks:

- Slider crank with friction:

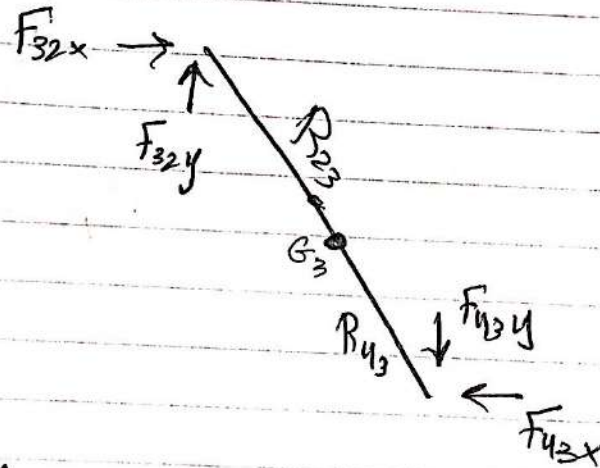
- link 2:

(3) equations

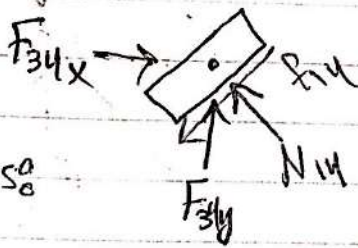


- link 3:

(3) equations



Slider:



(2) equations

slider // cases also are

& No moment equation

for slider

& friction can be neglected

$$\sum F_x = m_4 a_{G4x}$$

$$\sum F_x = F_{34x} \cos \theta_1 - N_{14} \sin \theta_1 = m_4 a_{G4x}$$

$$\sum F_y = m_4 a_{G4y}$$

$$\sum F_y = F_{34y} + N_{14} \cos \theta_1 - f_{14} \sin \theta_1 = m_4 a_{G4y}$$

$$f = \mu_k N$$

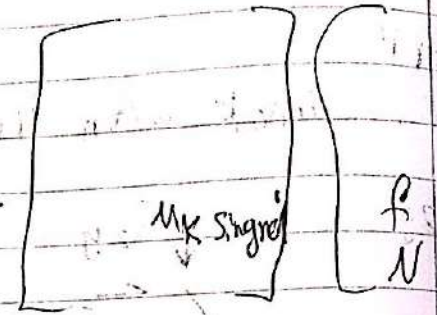
دالة انجاز f (مركبة)

check:

if $N < 0$

$A[1 - \mu_k \text{sign}(d)]$

\Leftarrow



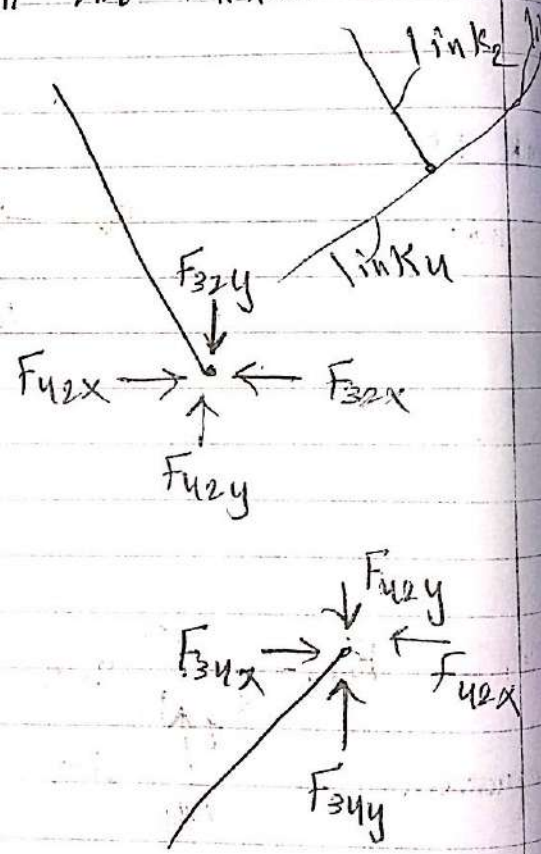
$$f = (-\mu_k |N| \text{sign}(d))$$

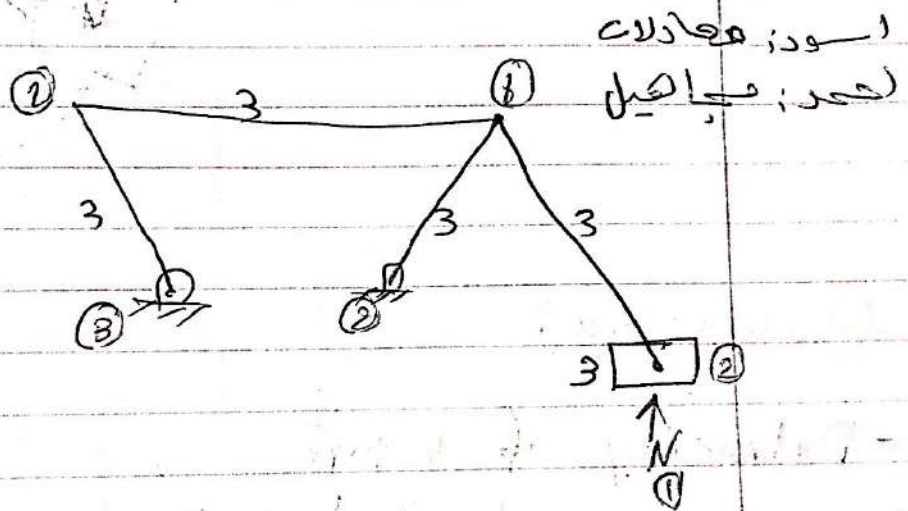
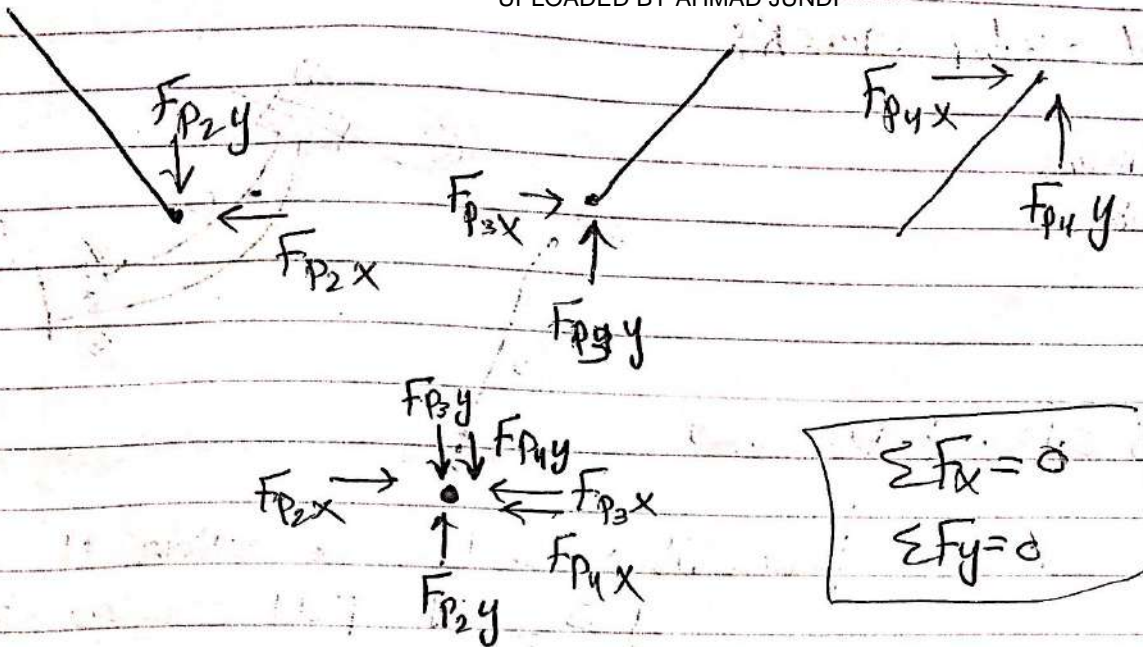
$$f + \mu_k |N| \text{sign}(d) = 0$$

$$f + \mu_k N \text{sign}(d) = 0, N \geq 0$$

$$f - \mu_k N \text{sign}(d) = 0, N < 0$$

At pin joint between more than two links



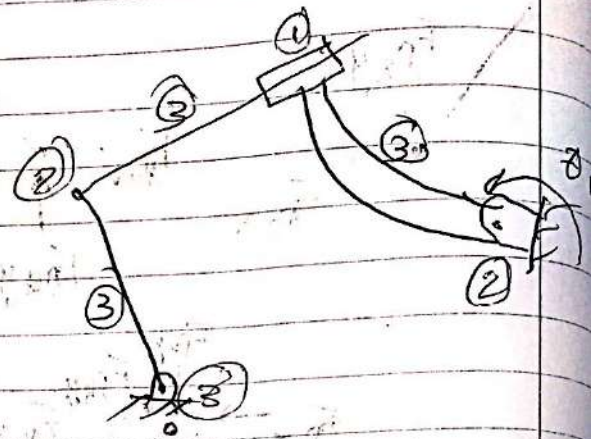


ال Time على ال mechanics ال input

③ Inverted slider crank:

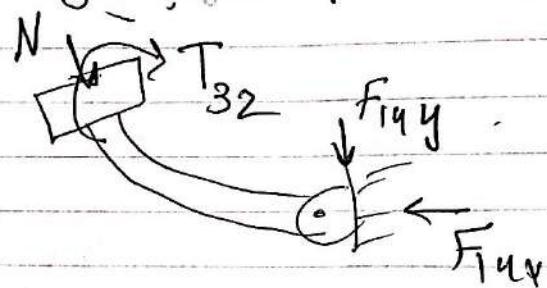
8 unknowns

9 equations



نحر صر في مقلبة كراكة

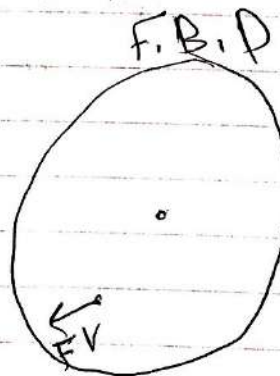
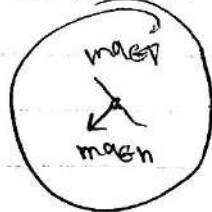
ال slider في مقلبة دوران على في مقلبة
بضيف T للبال



Balancing:

- Balancing of rotors
- of 4-bar mechanism

① Rotors



$$a_{cg} = \alpha R = 0 \quad (W \text{ constant})$$

$$a_{cn} = \omega^2 R$$

$$\Sigma \vec{F} = m \vec{a}_{cn} = m \vec{a}_{cn} + m \vec{a}_{ct}$$

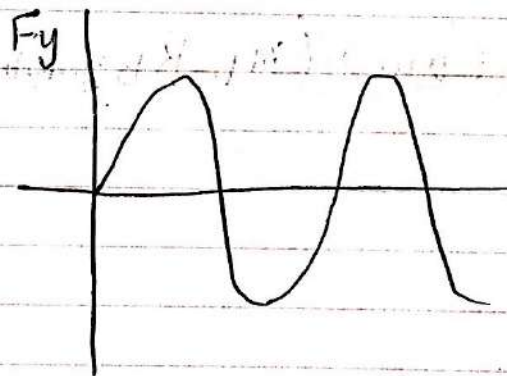
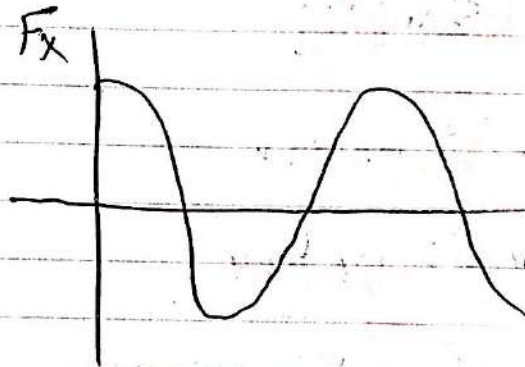
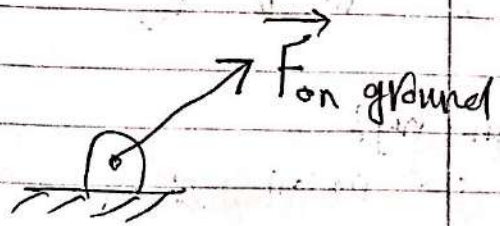
$$\vec{F}_{\text{on disk}} = -m \omega^2 \vec{R}$$

$$\vec{F}_{\text{on ground}} = m \omega^2 \vec{R}$$

$$= m \omega^2 R [\cos \theta \hat{i} + \sin \theta \hat{j}]$$

$$F_x = m \omega^2 R \cos \theta$$

$$F_y = m \omega^2 R \sin \theta$$



more than one mass on the same shaft:

$$\vec{F}_1 = m_1 \omega^2 \vec{R}_1$$

$$\vec{F}_2 = m_2 \omega^2 \vec{R}_2$$

balancing

Static balancing

$$\sum \vec{F} = 0$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_b = 0$$

$$m_1 \vec{R}_1 \omega^2 + m_2 \vec{R}_2 \omega^2 + m_b \vec{R}_b \omega^2 = 0$$

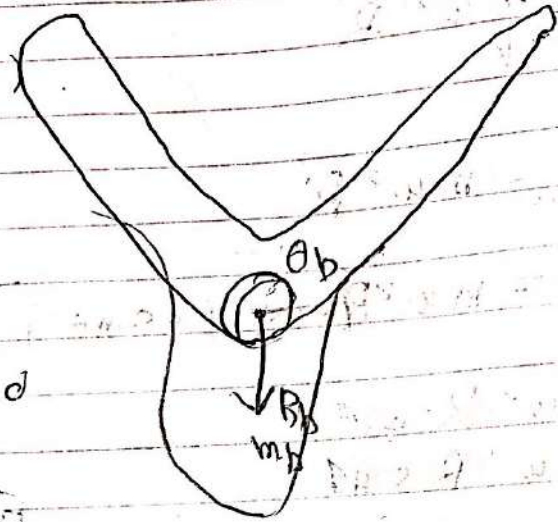
$$m_b \vec{R}_b = -\sum m_i \vec{R}_i$$

$$m_b R_{bx} = -\sum m_i R_{ix}$$

$$m_b R_{by} = -\sum m_i R_{iy}$$

$$m_b R_b = \sqrt{m_b^2 R_{bx}^2 + m_b^2 R_{by}^2}$$

$$\theta_b = \tan^{-1} (m_b R_{by} / m_b R_{bx}) \Rightarrow \text{given } R_b \Rightarrow m_b$$



Exo: 3 masses
 $R_u = 1$

$$\sum m_i R_i \cos \theta_i = -0.459$$

$$\sum m_i R_i \sin \theta_i = +0.872$$

$$m_b R_{bx} = +0.13$$

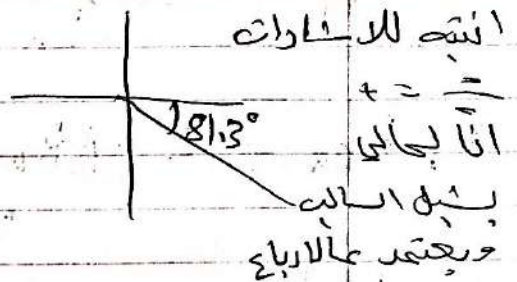
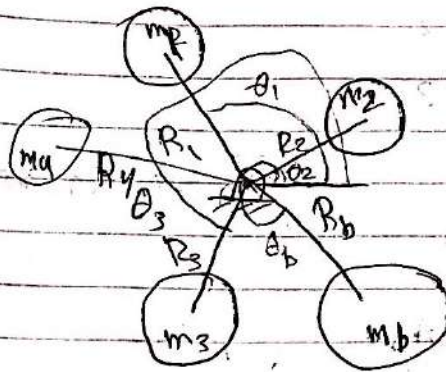
$$m_b R_{by} = -0.872$$

$$\Rightarrow m_b R_b = \sqrt{(m_b R_{bx})^2 + (m_b R_{by})^2}$$

$$m_b R_b = 0.882, \text{ Suppose } R_b = 1$$

$$\Rightarrow m_b = 0.882$$

$$\theta_b = \tan^{-1} \left(\frac{0.872}{0.133} \right) = 81.3^\circ$$



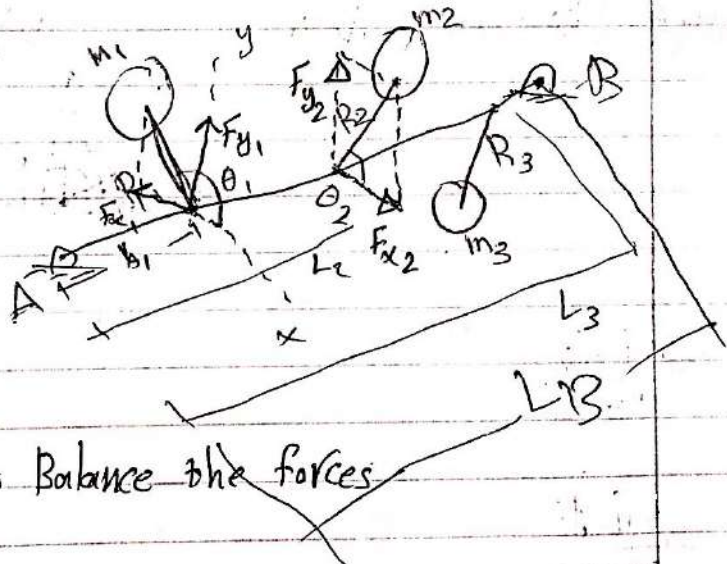
Dynamic Balancing:

Masses rotate in different planes

Exo:

\Rightarrow Shaking Forces

add mass b in the indicated plane to balance the ~~the~~ moments about the axis of plane A.



add new mass in plane A to balance the forces

My L, case 1

$$\begin{aligned} \text{for } M_{2x} &= F_{y2} L_2 & F_{y1} &= \\ M_{2y} &= -F_{x2} L_2 \end{aligned}$$

$$M_{1x} = F_{y1} L_1 =$$

$$M_{3x} = F_{y3} L_3$$

$$M_{3y} = F_{x3} L_3$$

$$F_{x1} = m_1 R_1 \cos \theta_1$$

$$M_{1y} = -F_{x1} L_1$$

$$\sum M_x = 0 \quad M_{bx}$$

$$\sum M_y = 0$$

$$m_B R_{By} L_B = -\sum m_i R_i \sin \theta_i L_i$$

$$m_B R_{By} = \frac{-\sum m_i R_i \sin \theta_i L_i}{L_B}$$

$$m_B R_{Bx} = \frac{-\sum m_i R_i \cos \theta_i L_i}{L_B}$$

II) Balancing Mechanisms 4-bar mechanism

ϕ = angle between L & R



$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i}$$

$$m_{tot} \bar{x} = m_2 R_2 \cos(\theta_2 + \phi_2)$$

$$m_{tot} \bar{R} = m_2 R_2 e^{j(\theta_2 + \phi_2)} + m_3 (L_2 e^{j\theta_2} + R_3 e^{j(\theta_3 + \phi_3)}) + m_4 (L_1 e^{j\theta_1} + R_4 e^{j(\theta_4 + \phi_4)})$$

$$= m_2 R_2 e^{j\theta_2} e^{j\phi_2} + m_3 L_2 e^{j\theta_2} + m_3 R_3 e^{j\theta_3} e^{j\phi_3} + m_4 L_1 e^{j\theta_1} + m_4 R_4 e^{j\theta_4} e^{j\phi_4} \quad (1)$$

Loop equations

$$L_2 e^{j\theta_2} + L_3 e^{j\theta_3} + L_4 e^{j\theta_4} - L_1 e^{j\theta_1} = 0$$

$$e^{j\theta_3} = \frac{L_1 e^{j\theta_1} + L_4 e^{j\theta_4} - L_2 e^{j\theta_2}}{L_3} \quad (2) \quad \text{substitute in (1)}$$

$$= \dots + m_3 R_3 \left(\frac{L_1 e^{j\theta_1} + L_4 e^{j\theta_4} + L_2 e^{j\theta_2}}{L_3} \right) e^{j\phi_3}$$

$$= \dots + m_2 R_2 e^{j\phi_2} + m_3 L_3 \left(\frac{m_3 R_3 L_2}{L_3} e^{j\phi_3} \right) e^{j\theta_2} + \left(m_4 R_4 e^{j\phi_4} + m_3 R_3 \frac{L_4}{L_3} e^{j\phi_3} \right) e^{j\theta_4}$$

Page 619

go to the book, and collect terms

$$\text{if } \left(\frac{\quad}{\cos} \right) e^{j\theta_2} + \left(\frac{\quad}{\cos} \right) e^{j\theta_4} \Rightarrow \text{variables} = 0$$

I have to add a term here and there

$$m_b R_{4x} + j m_{ub} R_{4by}$$

$$\tan^{-1} \left(\frac{m_{ub} R_{4by}}{m_b R_{4bx}} \right) = \phi$$

$$m_b R_{4b} = \sqrt{(m_b R_{4x})^2 + (m_{ub} R_{4by})^2}$$

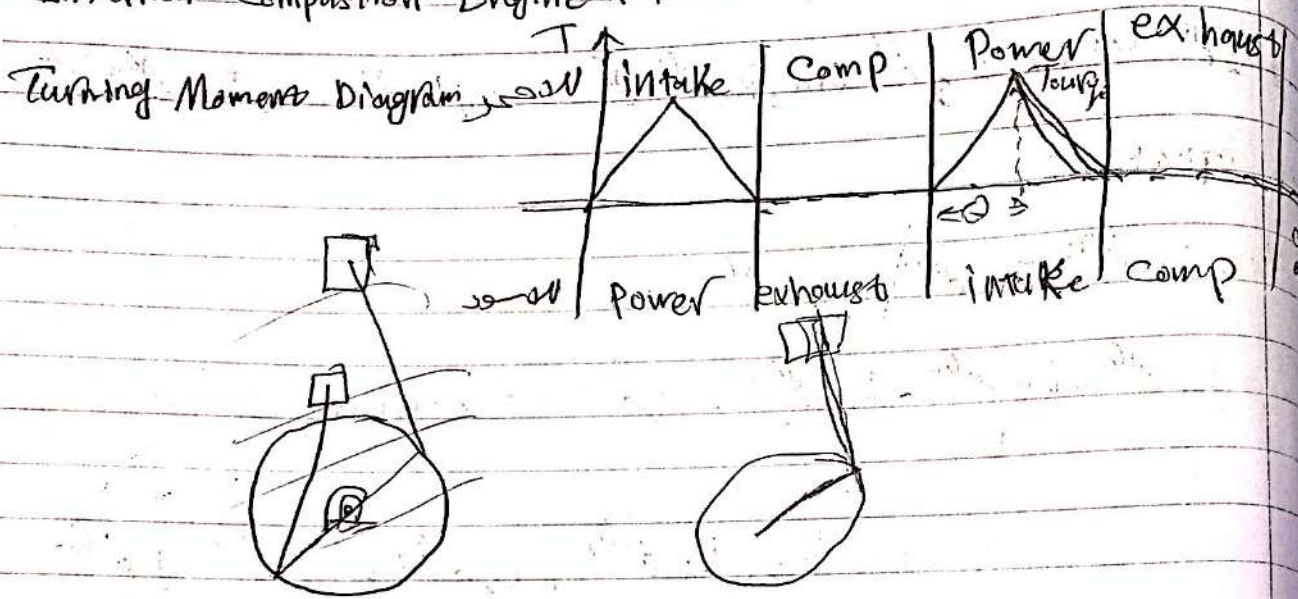
$$\phi_{4b} = \tan^{-1} \left(\frac{(\quad)_y}{(\quad)_x} \right)$$

الزاوية لـ ϕ_{4b}
بالنسبة لـ ϕ

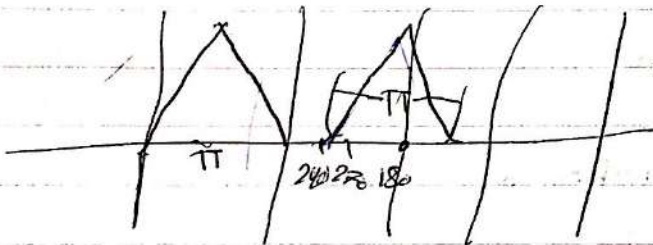
Fly Wheels:

Types of engines:

Internal Combustion Engine (4-stroke engine)



$$\text{Crank angle} = \frac{4\pi}{\text{number of cylinders}} = \text{Duration of complete \# of cylinder}$$



240° angle ← 3 150° 40°

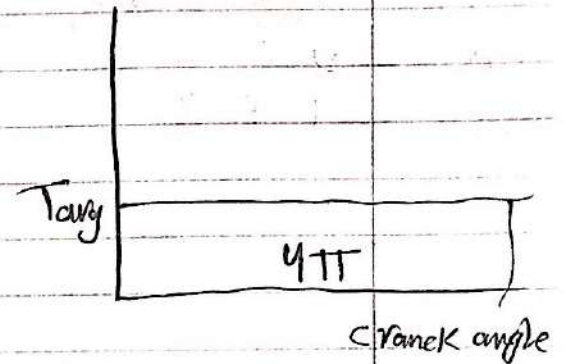
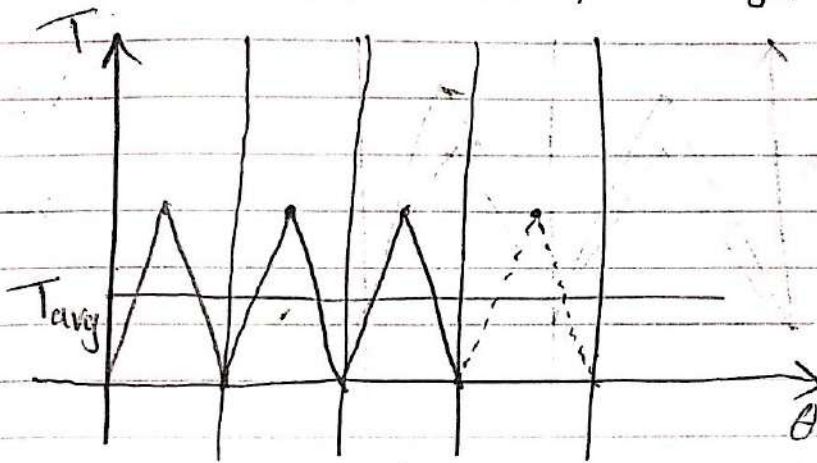
Total energy in one cycle = total work of the engine

$$U_m = \int M d\theta = \int T d\theta$$

= Total area under the curve

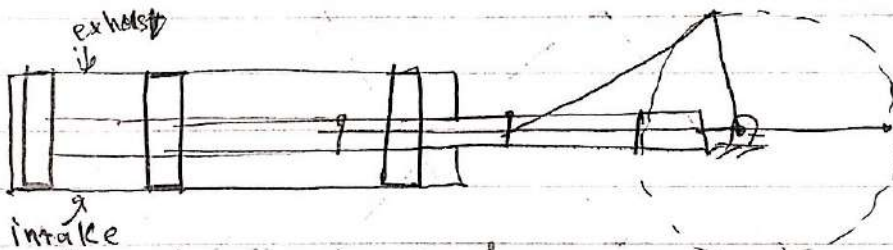
= Area under T- θ diagram of one cylinder \times # of cylinders

$$T_{avg} = \frac{\text{Total area under the curve}}{\text{duration of one cycle}}$$

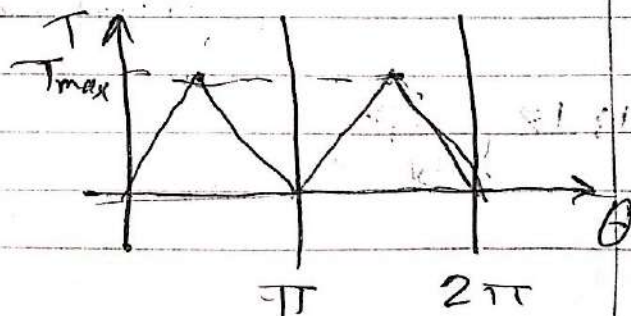


Steam Engines:

① Single acting cylinder

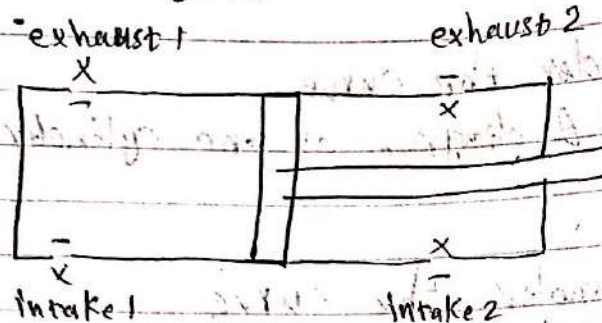


Duration of one cycle = 2π

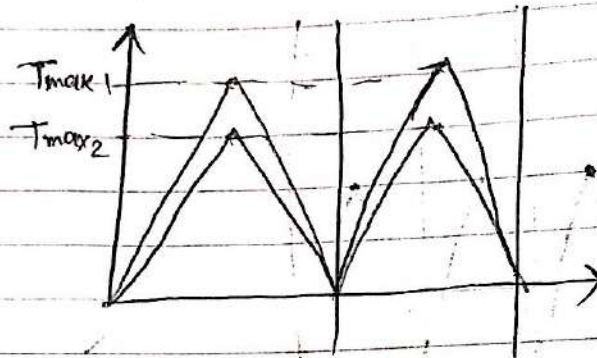


② Double acting cylinder

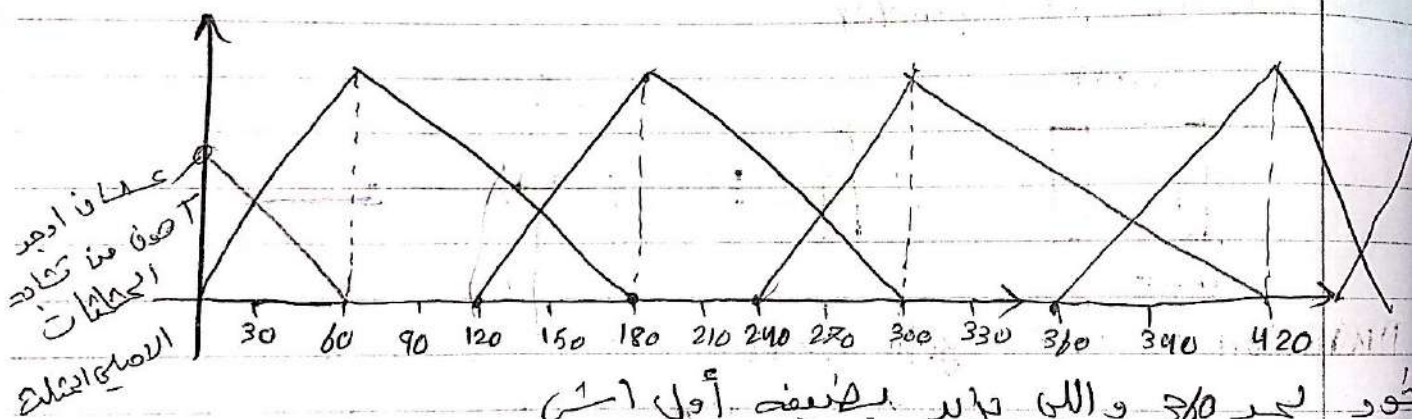
Duration of one cycle = 2π



ت و هو راجع $T > T$ وهو راجع
 بسبب اختلافات الـ A
 $F = \frac{P}{A}$ ق ق عا ليجي



3 Cylinders Crank angle = $\frac{2\pi}{3} = 120^\circ$



عدد 3 والي يزيد بضعف أول اسي
 عدد كل ضل 180°
 Crank angle = 120°

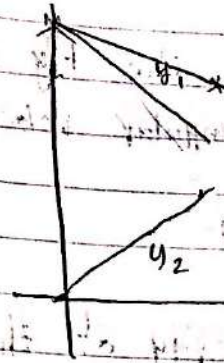
Ex 13.17
 في الابط

$$y_1 = m_1 x_1 + b_1$$

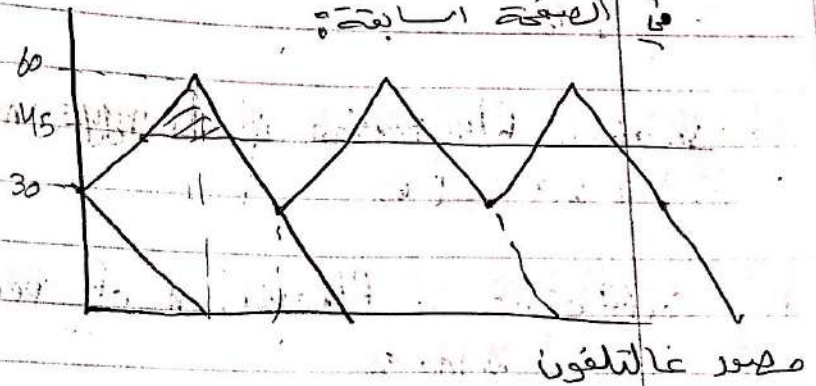
$$y_2 = m_2 x_2 + b_2$$

$$y_{total} = y_1 + y_2 = (m_1 + m_2)x + (b_1 + b_2)$$

$$y_{tot} = m_{tot}x + b_{tot}$$



اجتبه باللفظ عند 30
البدائية 50 والرقابة 180 وجاية
120 بين ربعي اللفظ بالتالي $\frac{1}{2}$ اللفظ



Find the maximum fluctuation of energy = e (Energy provided by flywheel)
Identify the points of intersection between the Targ & Tro +

المحركات بالبخار والديزل، التي تعمل على سرعة ثابتة، تكون حليتها الأولى
 الرخاوة، فبعض القوانين المطلوبة لتكون حليتها الأولى

UPLOADED BY AHMAD JUNDI

$I \equiv$ Inertia of the fly wheel

$\omega_1 \equiv$ maximum angular velocity of the flywheel (max speed)

$\omega_2 \equiv$ minimum " " " " " " (min speed)

$\omega \equiv$ mean speed

$E \equiv$ Kinetic energy of the flywheel at mean speed

$$= \frac{1}{2} I \omega^2$$

$e \equiv$ maximum fluctuation of energy \equiv energy supplied by the flywheel

$$= \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2$$

$K \equiv$ Coefficient of fluctuation of speed

$$= \frac{\omega_1 - \omega_2}{\omega} \times 100\%$$

Coefficient of fluctuation of energy = $\frac{e}{E}$

Total work in one cycle (under T.D. curve)

E & K & e دالة في حل المسألة

13.18

(ii)
13.18

$$e = \frac{1}{2} I (\omega_1^2 - \omega_2^2) = I \left(\frac{\omega_1 - \omega_2}{\omega} \right) \left(\frac{\omega_1 + \omega_2}{2} \right) \times \omega$$

$$e = I K \omega^2 = \left(\frac{1}{2} I \omega^2 \right) K (2) \Rightarrow e = 2EK$$

$$\Rightarrow \left[K = \frac{e}{2E} = \frac{e}{I \omega^2} \right] = \frac{2.5 \pi}{(\pi K^2) \omega^2} = \frac{2.5 \pi}{(6) (0.088)^2 (400 \times \frac{2\pi}{60})^2}$$

K is radius of gyration

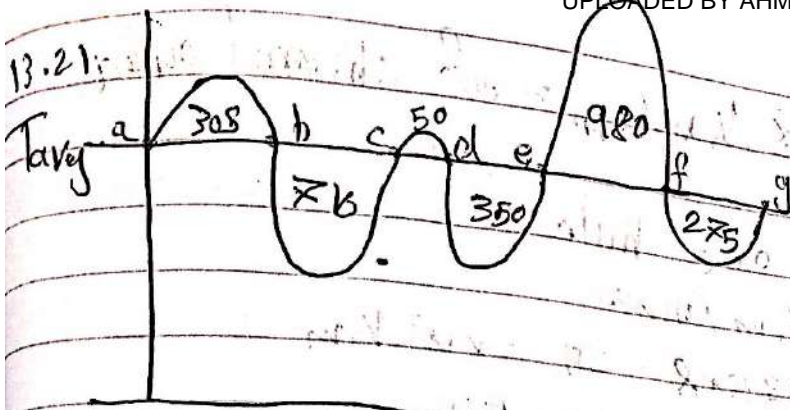
Rpm to rad/s

$$(iii) \text{ coef. of fluc. of energy} = \frac{e}{\text{Total work}} = \frac{2.5 \pi}{3 \times \frac{1}{2} \times \pi \times 60}$$

(iv) maximum Torque

الحد الأقصى
للمoment

Ex 13.21



$$W = 1500 \text{ rpm}$$

$$K = ?$$

$$I = m K \omega^2 = 40 \times (0.14)^2$$

$$K = \frac{e}{I \omega^2}$$

to find e:

assume the energy of a = H

$$b = H + 305 \text{ Max energy}$$

$$c = H + 305 - 710 = H - 405$$

$$d = H - 405 + 50 = H - 355$$

$$e = H - 355 - 350 = H - 705 \text{ min energy}$$

$$f = H - 705 + 980 = H + 275$$

$$g = H + 275 - 275 = H \text{ check f}$$

$$e = H_{\text{max}} - H_{\text{min}}$$

$$= H + 305 - (H - 705) = 1010 \text{ mm}^2$$

$$1 \text{ mm} = 8 \text{ N.m}$$

$$= 1010 \text{ mm}^2 \left(\frac{6 \text{ N.m}}{\text{mm}} \right) \times \left(1^\circ \times \frac{\pi}{180^\circ} \right)$$

(ارتفاع) (زاوية)

$$1^\circ = 1 \text{ mm} / 5$$

$$= 105.8 \text{ N.m}$$

$$K = \frac{105.8}{40 (0.14)^2 \times \left(1500 \times \frac{2\pi}{60} \right)^2} = 8.454 \times 10^{-3} = 0.55\%$$

Ex 13.27

13.27: The Plate requires 8 N.m/mm^2 of sheared area

Energy Required to shear one hole

$$= \text{Shear area} \times 8 \text{ N.m/mm}^2$$

$$= 2\pi r \times \text{thickness} \times \delta = 2\pi \times 20 \times 35 \times 8 = 35.2 \times 10^3 \text{ N.m}$$

$$= 35.2 \text{ kJ}$$

% Without the flywheel the power of the required motor = 35.2 kW

time to shear one hole = $\frac{1 \text{ min}}{6} = \frac{60 \text{ s}}{6} = 10 \text{ s}$

$$\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{35.2 \text{ kJ}}{10 \text{ s}} = 3.52 \text{ kW} \quad (\text{with flywheel})$$

actual cutting time = $\frac{35}{2 \times 95} \times 10 = 1.84 \text{ Sec}$
 تقطع به 35 من ال 95

يقطع بي 35 و 36
ويقطع وهو نازل بي

Power without flywheel = $\frac{35.2 \text{ kJ}}{1.84} = 19.13 \text{ kW}$

0 2999999999

The energy supplied by the motor in 1.54 s = $35.2 \times 1.54 = 248$ J
 = = = = fly wheel = $35.2 - 6.48 = 28.7$ J
 KJ =

$$V = 20 \text{ m/s} \quad , \quad m = ?$$

~~$V = W$~~

I = m

$$K = \frac{w_1 - w_2}{w}$$

$$K = \frac{e}{I \omega^2} = \frac{e}{m v^2}$$

$$m = 2390 \text{ kg}$$

$$-(0.03)(w) = (\Delta w + w) - \cancel{(w - \Delta w)}$$

Force Analysis Using Polygons

$$\sum \vec{F} = m\vec{a} \Rightarrow \sum \vec{F} + \vec{F}_0 = 0$$

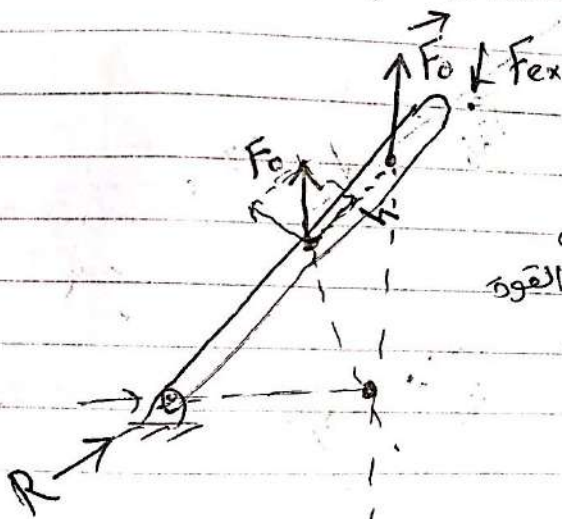
$$\vec{F}_0 = -m\vec{a}$$

مساكن بكر ال Polygon

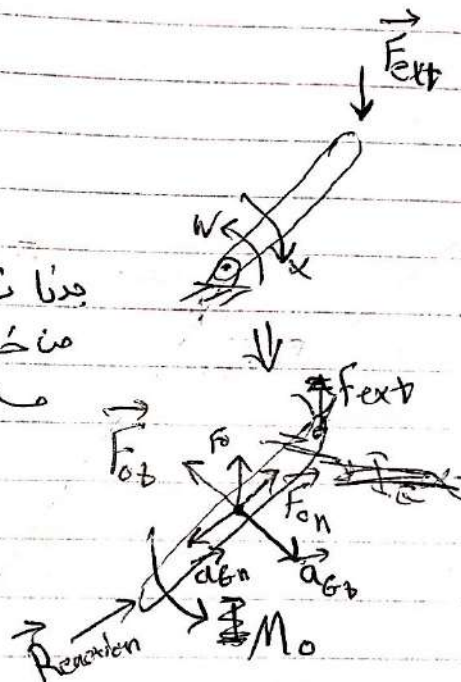
$$\sum M_G = I_G \alpha \Rightarrow \sum M_G + M_0 = 0$$

$$\sum M_G - I_G \alpha = 0 \quad M_0 = -I_G \alpha$$

1) EX: Find reaction force



بدون نضيق الموصلة
من خلال مركز القوة
ماسة معينة



$$h = \frac{I_G \alpha}{m a_G}$$

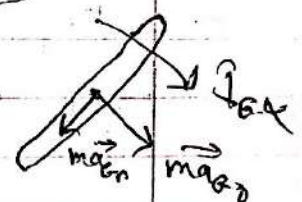
Suppose $\omega = 0 \Rightarrow \alpha = 0$
 $h = \frac{I_G \alpha}{m r \alpha} \Rightarrow h = \frac{I_G}{m r}$

$$F_0 h = M_0$$

$$m a_G h = I_G \alpha$$

$$h = \frac{I_G \alpha}{m a_G}$$

مساكن بكر
ويساوي صفو



Force Analysis Using Polygons

$$\sum \vec{F} = m\vec{a} \Rightarrow \sum \vec{F} + \vec{F}_0 = 0$$

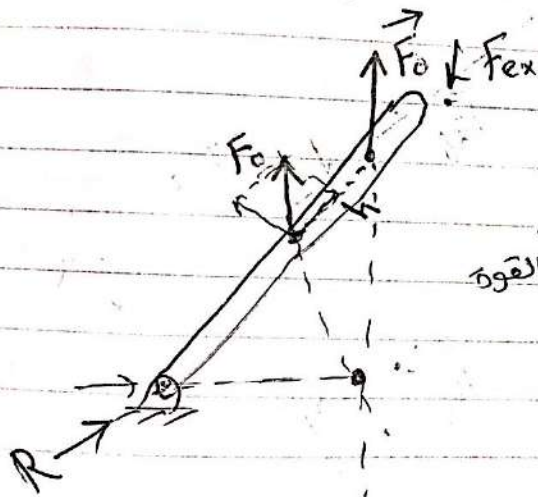
$$\vec{F}_0 = -m\vec{a}$$

سكان بكر ال Polygon

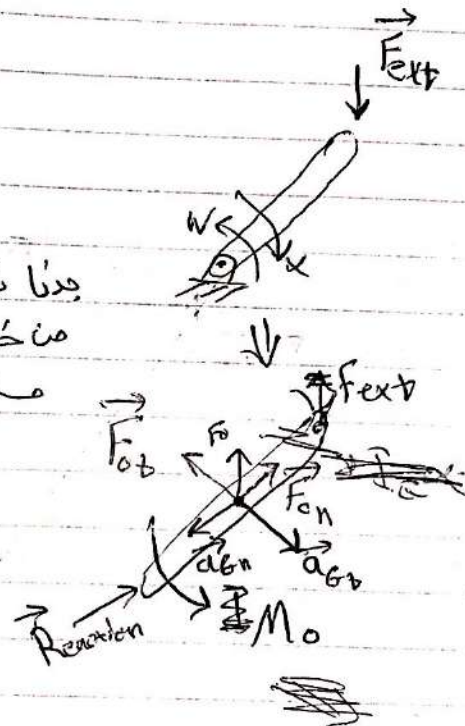
$$\sum M_G = I_G \alpha \Rightarrow \sum M_G + M_0 = 0$$

$$\sum M_G - I_G \alpha = 0 \quad M_0 = -I_G \alpha$$

Ex: Find reaction force



بدون نضيق الموصلة
من خلال مركز القوة
ساعة معك



Suppose $\omega = 0 \Rightarrow \alpha = 0$

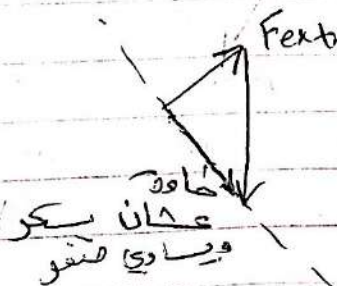
$$h = \frac{I_G \alpha}{m r \alpha} \Rightarrow h = \frac{I_G}{m r}$$

$$h = \frac{I_G \alpha}{m a \alpha}$$

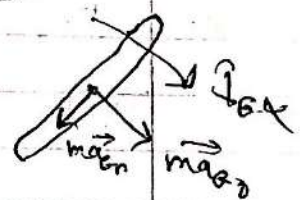
$$F_0 h = M_0$$

$$m a h = I_G \alpha$$

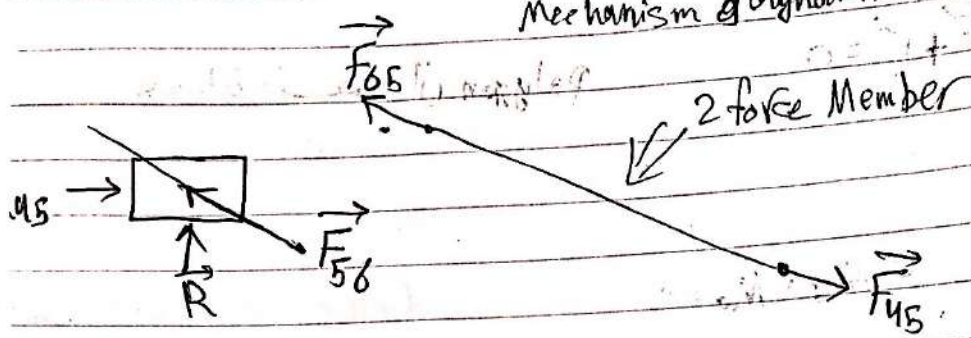
$$h = \frac{I_G \alpha}{m a \alpha}$$



الزاوية
سكان بكر
ويشدد في

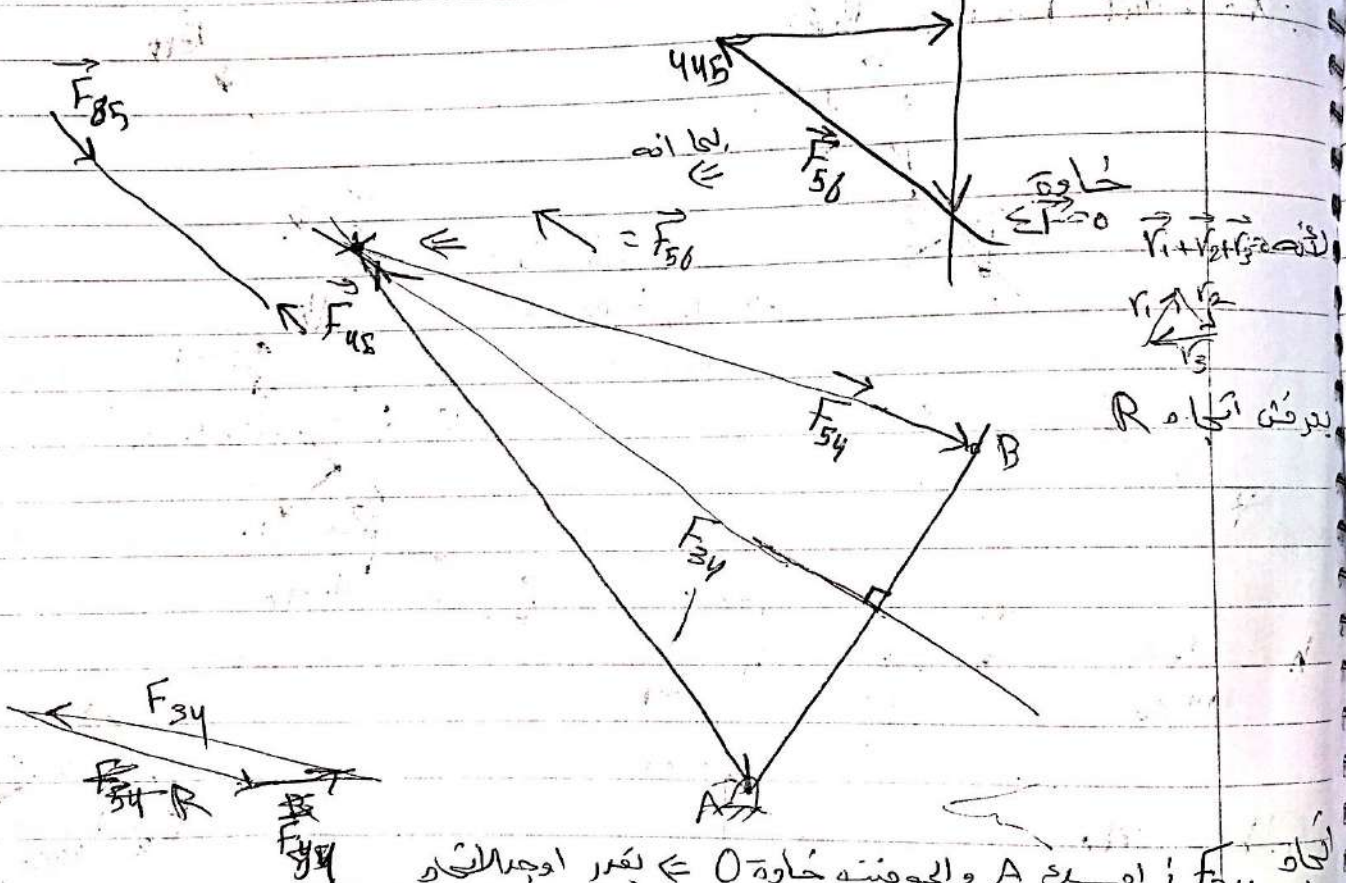


ميكانيكا الديناميكية Mechanism & Dynamic Machinery



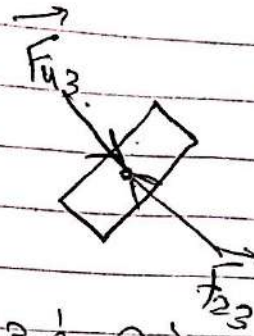
No a.e.
No x

$\sum F = 0$
 \Rightarrow force polygon



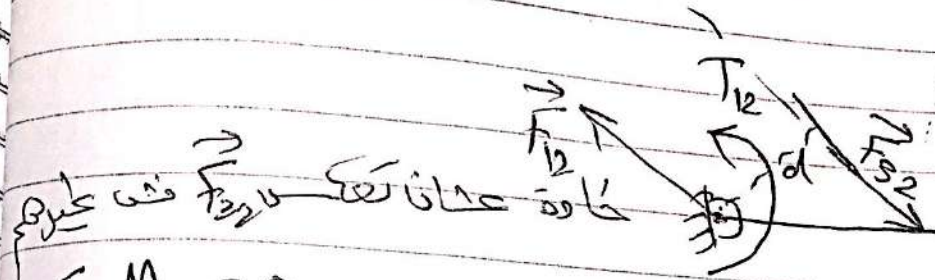
يعرف اتجاه R

اتجاه F_{34} : افك A والوجهه خارجه \Leftarrow يعرف اتجاهه
اتجاه R : افك B \Leftarrow يعرف اتجاهه



كافة لانه في حين فيهما تقاطعا هانا Pin ممكن ان يكون في اتجاه افقي
وانما البنيان الافقي لا يطبق الفورس افقي

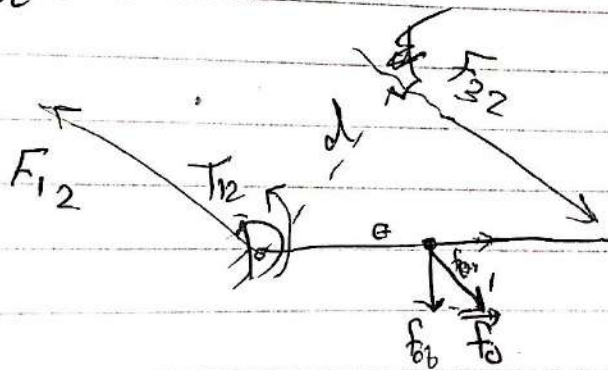
~~في~~



$$\sum M_o = 0$$

$$T_{12} + (F_{32}d) = 0$$

$$+T_{12} - F_{32}d = 0$$

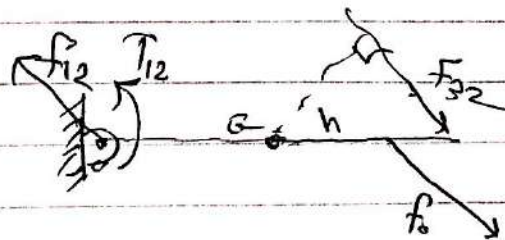


اذا كان في
Dynamic

3 forces without Torque (external moment)
use h

$$\sum M_o = 0$$

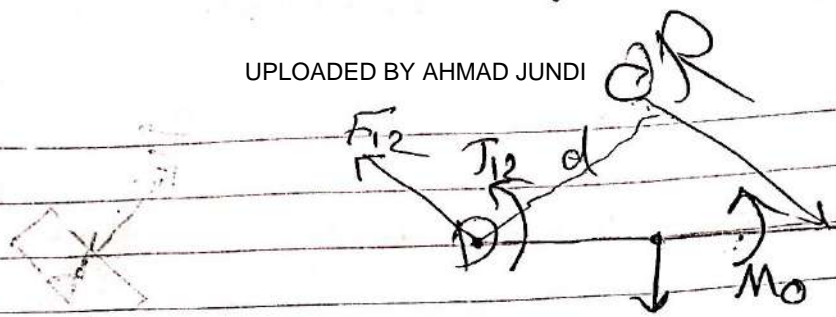
$$\Rightarrow T_{12} + (f_0 h) + F_{32}d = 0$$



$$T_{12} \oplus f_0 h \oplus F_{32}d = 0$$

معجل حركه معجل

تدريك f0 الى Mo التي قد تكون



لأنه ما يتحرك إلى اليمين فمبدأ M_o ونجعله بالحادثة

$$T_{12} - M_o - F_{32}d - F_{32}d = 0$$