



*Fluid notes*

**Second Exam**

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# ch #4:- The Bernoulli equation & pressure variation:

→ pressure variation occur due to: 1- elevation (Hydrostatic forces)  
2- acceleration

\* throughout the previous analysis, we were studying the fluid @ rest only; in these chapters; the pressure variation can occur due to acceleration "motion".

\* Fluid in motion: (Indicates velocity, acceleration and forces)

\* في المواضيع السابقة كنا ندرس الاختلاف في الضغط في حالة السكون --- والآن سندرس الجوانب في حالة الحركة!!

→ there are two approaches to study the moving body:

1. Lagrangian approach: describes the motion of the body of matter of fixed Identity, so it's suitable for "Dynamics"

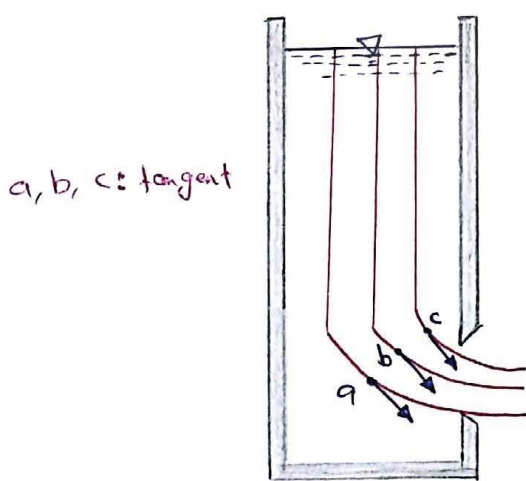
2. Eulerian approach: describes the motion of matter at spatial locations; So all mathematical equation in fluid mechanics written using this approach.

\* لهذا السبب نلاحظ انه (Eulerian approach) هو الأنسب في دراسة الجوانب الحركية، وهو يعتمد على مراقبة نقطة معينة يمر بها المائع أثناء مرور الزمن ---

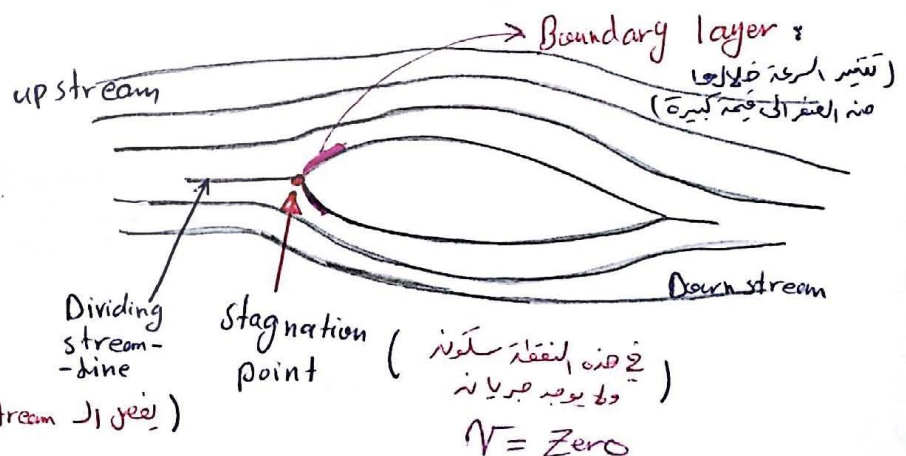
## → Fluid flow visualisation:

① Stream lines: the lines drawn through the flow field in such manner the local velocity vector is tangent to stream line at every point along the line at that instant.

\* هذه الخطوط وهمية وترسم حسب الجريانه او التدفقه حين يكونه متغير سريره جريانه المائع الحليته بحساسه للخط!!



(يفضل ان يكون ال upstream عن ال Downstream)



- \* Compressible and Incompressible flow: \*\* check  $E_v$  and Compressibility!!
- compressible flow: density of flowing fluid varies significantly. [ $\rho \neq ct$ ]
- Incompressible flow: density of flowing fluid assume constant. [ $\rho = ct$ ]

### \* Laminar, Transition and Turbulent flow:

- \* Laminar flow: well ordered flow in which fluid layers moved smoothly with respect to each other.
- \* Turbulent flow: the flow in which the mixing effect active.
- \* Transitional flow: the transition stage between (laminar to turbulent)

→ For flow in pipes (conduits), Reynolds number used as an index to differ types of flow:

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

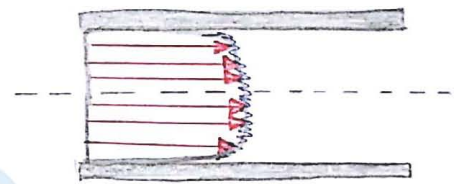
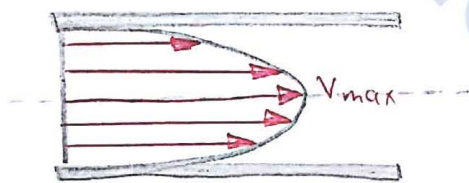
$\rho$ : density of fluid.

$V$ : velocity of fluid.

$D$ : Hydrodynamic diameter

$$\left\{ \begin{array}{l} \text{for plate} = \frac{L}{\nu} \\ \text{for circular} = \frac{4A}{P} \end{array} \right.$$

- If  $Re < 2000$ , so it's laminar flow.
- If  $Re > 3000$ , so it's Turbulent flow.
- If  $2000 < Re < 3000$ , so it's Transition flow.



\* Velocity profile for fully developed laminar flow; for Turbulent flow

$$\bar{V} = \frac{1}{2} V_{max}$$

$$\bar{V} = (0.8-0.9) V_{max}$$

### \* Viscous and non-viscous flow:

→ Viscous flow: the flow in which the fluid has large viscous forces enough to effect the motion of the fluid.

### \* Bounded, non-Bounded and semi Bounded flows:

\* Bounded: the flow which bounded by a physical system such as "pipe"

\* Semi-Bounded: the flow which partially bounded by a physical system "open channel"

\* non-Bounded: the flow with no physical bounded system "sprays"

### \* One - two and three dimensional flow:

↳ x-axis    ↳ x-axis y-axis    ↳ x-axis y-axis z-axis



- now; according to streamlines and Eulerian approach we can conclude:

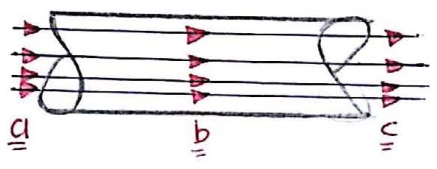
$$V = V(s, t) \quad ; \quad \text{Soe}$$

↑ Position      ↑ time

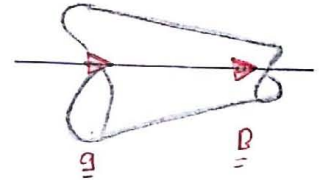
magnitude      Direction

① Uniform and non uniform flow:

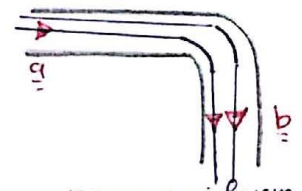
\* Uniform flow: the flow in which velocity vector does not change from a point to another along streamline.  $\left[ \frac{\partial V}{\partial s} = 0.0 \right]$



\* Uniform flow (No change neither in magnitude nor direction)  
 $\frac{\partial V}{\partial s} = 0.0$



\* non-uniform flow  
 "change in magnitude"  
 $\frac{\partial V}{\partial s} \neq 0.0$



\* non-uniform flow  
 "change in direction"  
 $\frac{\partial V}{\partial s} \neq 0.0$

② Steady and unsteady flow:

\* steady flow: the velocity at a given point on a fluid path does not change with time  $\left[ \frac{\partial V}{\partial t} = 0.0 \right]$

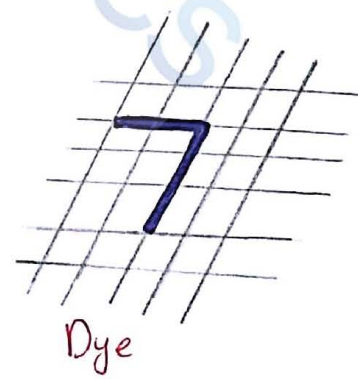
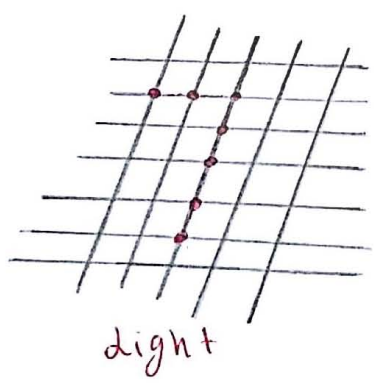
and if unsteady  $\rightarrow \left[ \frac{\partial V}{\partial t} \neq 0.0 \right]$

\* Pathlines & streaklines: also used commonly in fluid flow visualisation!!

- Pathline: the path of a fluid particle as it moves through the flow field.

... *دلالة على المسار الذي يتبعه جسيم سائل في حقل الجريان، وتتغير...*

- Streakline: the path (line) generated by tracing fluid; such as dye which injected continuously in flow



\* Acceleration: it's the rate of change in velocity with respect to time

$$\textcircled{1} \quad \mathbf{V} = V(s, t) \mathbf{e}_t$$

$$\begin{aligned} \mathbf{a} &= \frac{\partial \mathbf{V}}{\partial t} = V(s, t) \frac{\partial \mathbf{e}_t}{\partial t} + \frac{\partial V}{\partial t} \mathbf{e}_t \\ &= V(s, t) \frac{\partial \mathbf{e}_t}{\partial t} + \left( \frac{\partial V}{\partial s} \frac{\partial s}{\partial t} + \frac{\partial V}{\partial t} \frac{\partial t}{\partial t} \right) \mathbf{e}_t \\ \vec{\mathbf{a}} &= \left[ \underbrace{\frac{V^2}{r}}_{\text{centripetal acceleration}} \mathbf{e}_n + \underbrace{V \frac{\partial V}{\partial s}}_{\text{convective acceleration}} + \underbrace{\frac{\partial V}{\partial t}}_{\text{local acceleration}} \right] \mathbf{e}_t \end{aligned}$$

→ chain Rule derivative

note:  $\frac{\partial \mathbf{e}_t}{\partial t} = \frac{V}{r} \mathbf{e}_n$   
 radius of curvature  $\nwarrow$   
 unit vector directed to the center.  $\uparrow$

→ If the flow is uniform  $\rightarrow \left[ \frac{\partial V}{\partial s} = 0.0 \right]$ , convective acc. cancelled  
 → If the flow is steady  $\rightarrow \left[ \frac{\partial V}{\partial t} = 0.0 \right]$ , local acc. cancelled



also: "Eulerian approach"

$$u = f(x, y, z, t)$$

$$v = f(x, y, z, t)$$

$$w = f(x, y, z, t)$$

$$\left. \begin{aligned} a_x &= \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial u}{\partial z} \frac{\partial z}{\partial t} + \frac{\partial u}{\partial t} \\ a_y &= \frac{dv}{dt} = \frac{\partial v}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial v}{\partial z} \frac{\partial z}{\partial t} + \frac{\partial v}{\partial t} \\ a_z &= \frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial t} + \frac{\partial w}{\partial t} \end{aligned} \right\} \text{chain Rule derivatives}$$

$$\Rightarrow a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Convective acc.

local acc.

→ Convective acceleration: associated with the variation of velocity along streamline.

→ Local acceleration: associated with the change in velocity with respect to time at a certain point on the streamline.

Problem 4-17: the velocity along a path line is given by  $[V(m/s) = s^2 t^{1/2}]$ , where "s" in meters and "t" in seconds, the radius of curvature is (0.5 m), evaluate the acceleration along and normal to the path @  $[s = 2m]$  and  $[t = 0.5s]$

Sol:

$$a = \left( v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} \right) e_t + \left( \frac{v^2}{r} \right) e_n$$

$$|v| = s^2 t^{1/2} = (2)^2 (0.5)^{1/2} = 2.83 \text{ m/s}$$

$$\frac{\partial v}{\partial s} = 2s t^{1/2} \Big|_{\substack{s=2m \\ t=0.5s}} = 2(2)(0.5)^{1/2} = 2.83 \text{ 1/s}$$

$$\frac{\partial v}{\partial t} = \frac{1}{2} s^2 t^{-1/2} \Big|_{\substack{s=2m \\ t=0.5s}} = \frac{1}{2}(2)(0.5)^{-1/2} = 2.83 \text{ m/s}^2$$

$$a = \left[ (2.83 \times 2.83) + 2.83 \right] e_t + \left( \frac{2.83^2}{0.5} \right) e_n$$

$$a = 10.8 e_t + 16 e_n \text{ m/s}^2$$

\* Example for a certain flow field, the velocity of fluid is given by:

$$V = (3xy^2 - 4xt) \hat{i} + (2x^2y - 5t) \hat{j}$$

determine acceleration @  $x=1m, y=1m, t=2s$  !!

Sol:  $u = 3xy^2 - 4xt \Rightarrow u = 3(1)(1)^2 - 4(1)(2) = -5 \text{ m/s}$

$$\frac{\partial u}{\partial x} = 3y^2 - 4t \Rightarrow \frac{\partial u}{\partial x} = 3(1)^2 - 4(2) = -5$$

$$v = 2x^2y - 5t \Rightarrow v = 2(1)^2(1) - 5(2) = -8 \text{ m/s}$$

$$\frac{\partial u}{\partial y} = 6xy \Rightarrow \frac{\partial u}{\partial y} = 6(1)(1) = 6$$

$$* \frac{\partial v}{\partial t} = -4x = -4(1) = -4$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_x = \left[ (-5 \times -5) + (-8 \times 6) + (-4) \right] = \left[ 25 - 48 - 4 \right] = -27 \text{ m/s}^2$$

$a_y =$  لكل بنفس الطريقة 

#### Chapter 4: acceleration

- ✓ Q20 The nozzle in the figure is shaped such that the velocity of flow varies linearly from the base of the nozzle to its tip. Assuming quasi-one-dimensional flow, what is the convective acceleration midway between the base and the tip if the velocity is 0.5 m/s at the base and 1.5 m/s at the tip? Nozzle length is 45 cm

Since the velocity is linear, so:

$$V = ax + b$$

$$\text{@ } x = 0 \Rightarrow V = 0.5 \text{ m/s} \Rightarrow 0.5 = a(0) + b \Rightarrow \boxed{b = 0.5}$$

$$\text{@ } x = 0.45 \Rightarrow V = 1.5 \text{ m/s} \Rightarrow 1.5 = a(0.45) + 0.5 \Rightarrow \boxed{a = 2.22\bar{2}}$$

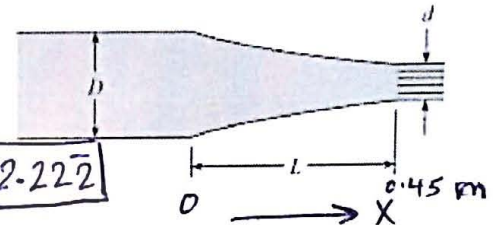
$$V \text{ @ midway } (x = \frac{0.45}{2}) \Rightarrow V = 2.22\bar{2} \left(\frac{0.45}{2}\right) + 0.5$$

$$\boxed{V = 1 \text{ m/s}}$$

$$\frac{\partial V}{\partial s} = 2.22\bar{2} \text{ s}^{-1}$$

$$a_c = V \frac{\partial V}{\partial s} = 1 * 2.22\bar{2} = \boxed{2.22 \text{ m/s}^2}$$

ملاحظة: لأن توزيع السرعة (linear) فبإمكان حساب السرعة في العلاقة:  $\frac{V_{tip} + V_{base}}{2}$  أيضاً !!

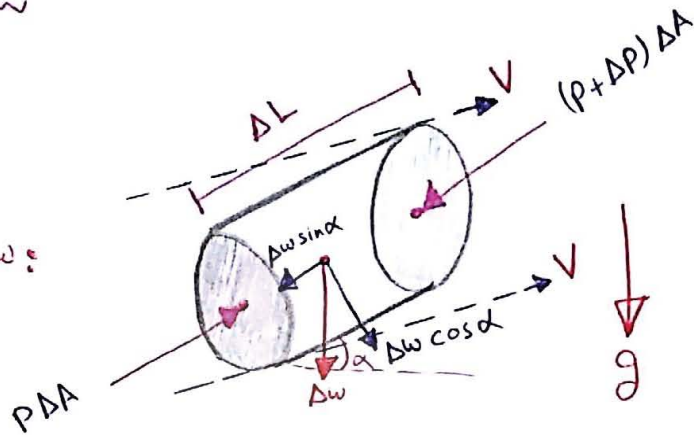


\* Euler's equation:

\* element of fluid;

do your analysis ---

apply newton's 2<sup>nd</sup> law:



\*  $\Delta w = \gamma \Delta V = \gamma \Delta A \Delta L$

\*  $\sin \alpha = \frac{\Delta z}{\Delta L}$

\*  $\rho = \frac{m}{V}$

\*  $m = \rho \Delta A \Delta L$

$\sum F_L = m a_L$

$F_{\text{pressure}} + F_{\text{gravity}} = m a_L$

$P \Delta A - [(P + \Delta P) \Delta A] - [\gamma \Delta A \Delta L \frac{\Delta z}{\Delta L}] = (\rho \Delta A \Delta L) a_L$

$\div \Delta A \rightarrow -\Delta P \Delta A - \gamma \Delta A \Delta L \frac{\Delta z}{\Delta L} = (\rho \Delta A \Delta L) a_L$

$-\frac{\Delta P}{\Delta L} - \gamma \frac{\Delta z}{\Delta L} = \rho a_L$

limit

$-\frac{\partial P}{\partial L} - \gamma \frac{\partial z}{\partial L} = \rho a_L \Rightarrow \frac{\partial}{\partial L} (P + \gamma z) = \rho a_L$  for incompressible flow ( $\gamma = \text{const}$ )

Euler's law

$\frac{\partial}{\partial L} (P + \gamma z) = -\rho a_L$

لا صفا انه ازدياد الضغط يكون  
بحسب اتجاه السريان

\* example: the tank shown is filled completely with gasoline which has specific weight of  $(6.6 \text{ kN/m}^3)$ , the truck is decelerating at a rate of  $(3.05 \text{ m/s}^2)$

a) If the tank trailer is  $(6.1 \text{ m})$  long and if the pressure @ the top rear of the tank is atmospheric, what is the pressure @ A??

b) If the tank is  $(1.83 \text{ m})$  high, what is the maximum pressure in the tank?

a) from atm to A:

$\frac{\partial}{\partial L} (P + \gamma z) = -\rho a_L$

$\int \frac{\partial P}{\partial L} + \frac{\partial z}{\partial L} \gamma = -\rho a_L$

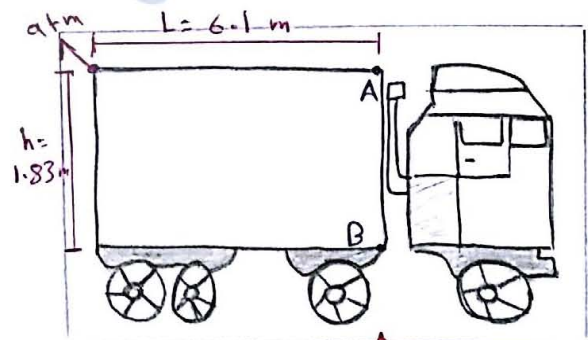
$\frac{P_A - P_{\text{atm}}}{L} = -\rho a_L$

بانه السريان سالب في الاتجاه  
من A الى atm

$P_A - 0 = -\frac{\gamma}{g} \cdot L \cdot (-3.05)$

$P_A = -\frac{6.6 \times 10^3}{9.81} \cdot 6.1 \cdot (-3.05) = 12,517 \text{ kPa gage}$

(B) more A  $\rightarrow$  B , ans  $P_B = 24.59 \text{ kPa}$  "gage" [7]



"بتشبر كل اشي، الا الشاكلة: 3:"

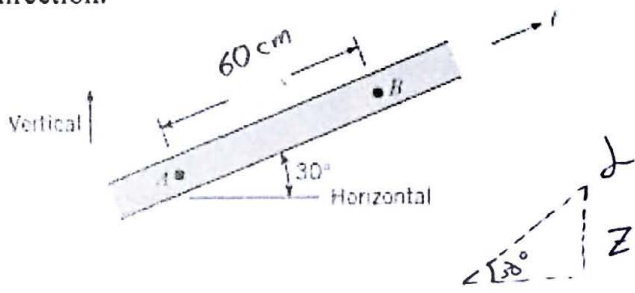


Euler's equation

ملاحظة: اتجاه الجريان (السرعة) لا يمكن تحديده من هذه المعطيات  
لذا الجواب الوحيد الصحيح هو D

- ✓ Q35 A liquid with a specific weight of 15717 N/m<sup>3</sup> is in the conduit. This is a special kind of liquid that has zero Viscosity. The pressures at points A and B are 9 kpa and 5 kpa, respectively. Which one (or more) of the following conclusions can one draw with certainty?
- (a) The velocity is in the positive  $l$  direction.
  - (b) The velocity is in the negative  $l$  direction.
  - (c) The acceleration is in the positive  $l$  direction.
  - (d) The acceleration is in the negative  $l$  direction.

apply Euler equation between A & B



$$\frac{\partial}{\partial L} (p + \gamma z) = -\rho a_L$$

$$\frac{\partial p}{\partial L} + \frac{\partial z}{\partial L} \gamma = -\rho a_L$$

$$\frac{P_B - P_A}{L} + 0.5 \gamma = -\rho a_L$$

$$P_B - P_A = [-\rho a_L - 0.5 \gamma] * L$$

$$(5 \times 10^3) - (9 \times 10^3) = [-\rho a_L - 0.5 \gamma] L \Rightarrow a_L = \frac{-4 \times 10^3 + 0.5 \gamma L}{-\rho L} = \frac{4 \times 10^3}{-\rho (0.6)} - 0.5 g$$

هذه القيمة سالبة لأنها في الاتجاه العكسي (d)

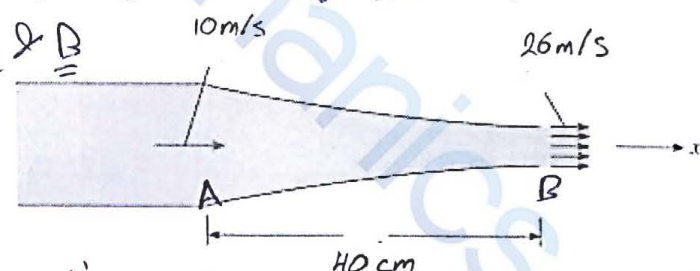
$$\frac{\partial z}{\partial L} = \sin 30^\circ = 0.5$$

- ✓ Q36 If the velocity varies linearly with distance through this water nozzle, what is the pressure gradient,  $dp/dx$ , halfway through the nozzle? ( $\gamma = 9810 \text{ N/m}^3$ )

apply Euler equation between A & B

$$\frac{\partial}{\partial x} (p + \gamma z) = -\rho a_x$$

$$\frac{\partial p}{\partial x} = -\rho a_x$$



لأنه افقي - ضغط

OR:  $V = ax + b$

$$a_x = a = v \frac{\partial v}{\partial x} \Big|_{\text{halfway}}$$

$$V_{\text{mid}} = \frac{10 + 26}{2} = 18 \text{ m/s}$$

$$\frac{\partial v}{\partial x} = 40 \text{ s}^{-1}$$

$$\text{@ } x=0 \Rightarrow v=10 = a(0) + b \Rightarrow b=10$$

$$\text{@ } x=0.4 \Rightarrow v=26 = a(0.4) + 10$$

$$a = 40$$

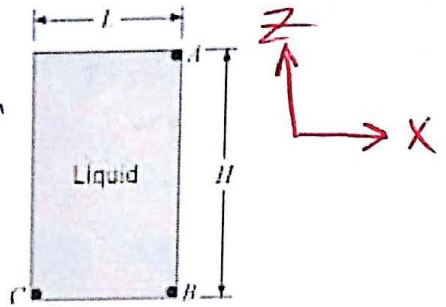
$$V = 40x + 10$$

velocity Distribution --

$$\frac{\partial p}{\partial x} = -\rho \left( v \frac{\partial v}{\partial x} \right)$$

$$\frac{\partial p}{\partial x} = -\rho (18 * 40) = -1000 (18 * 40) = 720 * 10^3 \text{ Pa/m}$$

Q37 The closed tank shown, which is full of liquid, is accelerated downward at  $1.5g$  and to the right at  $0.9g$ . Here  $L = 1\text{m}$ ,  $H = 5\text{m}$ , and the specific gravity of the liquid is  $1$ . Determine  $p_C - p_A$  and  $p_B - p_A$ .



Sol<sup>n</sup> ①  $p_B - p_A$ ; apply Euler equation in  $z$ -direction

$$\frac{\partial}{\partial z} (p + \gamma z) = -\rho a_z$$

$$\frac{\partial p}{\partial z} + \frac{\partial z}{\partial z} \gamma = -\rho a_z$$

$$\frac{p_B - p_A}{5} = -\rho (1.5g) - \gamma$$

$$p_B - p_A = 5 \left[ (1000 \times 1.5 \times 9.81) + 9810 \right]$$

$$p_B - p_A = 24525 \text{ Pa} =$$

(  $B$  ضغط  $<$   $A$  ضغط )

②  $p_C - p_A$ ; apply Euler equation in  $x$ -direction  $p_C - p_A = \overset{\text{Hor}}{(p_C - p_B)} + \overset{\text{Ver}}{(p_B - p_A)}$

$$\frac{\partial}{\partial x} (p + \gamma z) = -\rho a_x$$

$$\frac{\partial p}{\partial x} + \frac{\partial z}{\partial x} \gamma = -\rho a_x$$

$$\frac{p_C - p_B}{1} = -\rho (0.9g)$$

التسارع بـ  $0.9g$  إلى اليمين

$$p_C - p_B = 1 \times -1000 \times 0.9 \times 9.81 = +8829 \text{ Pa}$$

$$p_C - p_A = \overset{\text{Hor}}{(p_C - p_B)} + \overset{\text{Ver}}{(p_B - p_A)}$$

$$= 8829 + -24525 =$$



\* pressure distribution in rotating fluid:

→ it's situation in which a fluid rotates as a solid body are found in many engineering applications. "e.g: centrifugal separator"

\* from Euler's equation:  $\frac{\partial}{\partial L}(p + \gamma Z) = -\rho a_L$ , but in rotation the L-direction is replaced by  $r$  which indicates the distance between center of rotation and any point; it's become:  $\frac{\partial}{\partial r}(p + \gamma Z) = -\rho a_r \rightarrow a_r = \frac{-V^2}{r}$

$$\rightarrow -\frac{\partial}{\partial r}(p + \gamma Z) = -\rho \frac{V^2}{r} \quad (V = \omega r)$$

$$\rightarrow -\frac{\partial}{\partial r}(p + \gamma Z) = -\rho \frac{\omega^2 r^2}{r}$$

$$\rightarrow \frac{\partial}{\partial r}(p + \gamma Z) = -\rho \omega^2 r$$

Integrate

$$\rightarrow p + \gamma Z = \frac{\rho r^2 \omega^2}{2} + \text{constant}$$

$$p + \gamma Z - \frac{\rho r^2 \omega^2}{2} = C$$

$$\text{OR } \frac{p}{\gamma} + Z - \frac{\omega^2 r^2}{2g} = C$$

⊖ve cause it's away from the center of curvature.

Rotational equation's

\* example: a cylindrical tank of liquid shown is rotating at a rate of (4 rad/s) the tank diameter is (0.5 m), find the elevation difference between the liquid at the center and the wall during rotation

Sol:

$$\frac{p_1}{\gamma} + z_1 - \frac{\omega^2 r_1^2}{2g} = \frac{p_2}{\gamma} + z_2 - \frac{\omega^2 r_2^2}{2g}$$

atm       $r_1=0$       atm

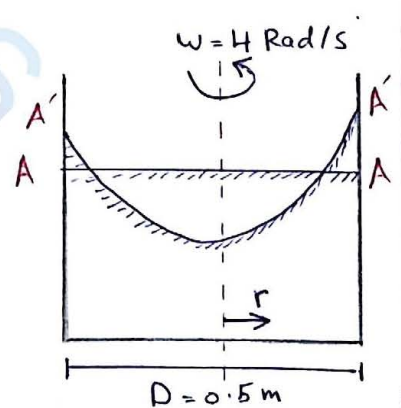
$$z_1 = z_2 - \frac{\omega^2 r_2^2}{2g}$$

$$\Delta z = z_2 - z_1 = \frac{\omega^2 r_2^2}{2g}$$

$$= \frac{(4)^2 \times (0.25)^2}{2 \times 9.81}$$

$$= 0.0509 \text{ m}$$

$$= 5.09 \text{ cm}$$



A-A: before rotation.

A'-A': after rotation. 10

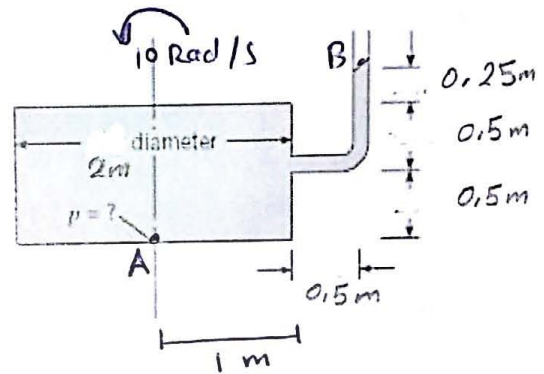
**Pressure distribution in rotating flows**

**Q41** This closed tank, which is 2m in diameter, is filled with water ( $\rho = 1000 \text{ kg/m}^3$ ) and is spun around its vertical centroidal axis at a rate of 10 rad/s. An open piezometer is connected to the tank as shown so that it is also rotating with the tank. For these conditions, what is the pressure at the center of the bottom of the tank?

apply Rotational equation between A & B

$$P_A + \gamma z_A - \frac{\rho r_A^2 \omega^2}{2} = P_B + \gamma z_B - \frac{\rho r_B^2 \omega^2}{2}$$

Zero (واضع على مركز الارتفاع)      Zero (atm)



$$P_A = \gamma(z_B - z_A) - \frac{\rho r_B^2 \omega^2}{2}$$

$$= 9810(1.25 - 0) - \left(1000 \frac{1.5^2 \cdot 10^2}{2}\right)$$

$$= 12,2625 - 112.5$$

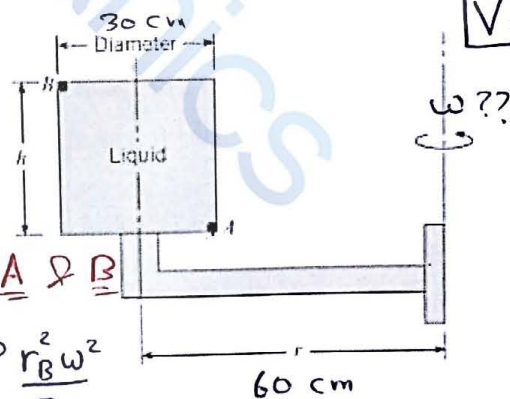
$P_A = -100.2375 \text{ kPa}$

**Q42** A tank of liquid ( $S = 0.80$ ) that is 30 cm in diameter and 30 cm high ( $h = 30 \text{ cm}$ ) is rigidly fixed (as shown) to a rotating arm having a 60 cm radius. The arm rotates such that the speed at point A is 6 m/s. If the pressure at A is 1200 Pa, what is the pressure at B?

note  
 $V = \omega r$

$V_A = 6 \text{ m/s}$ ,  $P_A = 1200 \text{ Pa}$ ,  $P_B$ ??

$V_A = \omega r_A \Rightarrow \omega = \frac{V_A}{r_A} = \frac{6}{0.6 \cdot \frac{0.3}{2}} = 13.33 \text{ Rad/s}$



apply Rotational equation Between A & B

$$P_A + \gamma z_A - \frac{\rho r_A^2 \omega^2}{2} = P_B + \gamma z_B - \frac{\rho r_B^2 \omega^2}{2}$$

$$1200 - \left(1000 \cdot \frac{0.45^2 \cdot 13.33^2}{2}\right) = P_B + (9810 \cdot 0.3) - \frac{1000 \cdot (0.6 + 0.15)^2 \cdot 13.33^2}{2}$$

$P_B =$

## Euler + Rotational 😊

!!

Q44 A closed tank of liquid ( $S = 1.2$ ) is rotated about a vertical axis (see the figure), and at the same time the entire tank is accelerated upward at  $4 \text{ m/s}^2$ . If the rate of rotation is  $10 \text{ rad/s}$ , what is the difference in pressure between points  $A$  and  $B$  ( $p_B - p_A$ )? Point  $B$  is at the bottom of the tank at a radius of  $0.5 \text{ m}$  from the axis of rotation, and point  $A$  is at the top on the axis of rotation.

$\omega = 10 \text{ Rad/s}$ ,  $r_B = 0.5 \text{ m}$ ,  $a_z = 4 \text{ m/s}^2 \uparrow$

$P_A - P_B = ??$  "  $P_A - P_B = (P_C - P_B) + (P_A - P_C)$

\* apply Rotational equation between  $B \text{ \& } C$ :

$P_B + \gamma z_B - \rho \frac{r_B^2 \omega^2}{2} = P_C + \gamma z_C - \rho \frac{r_C^2 \omega^2}{2}$

$\downarrow$  zero
 $\downarrow$  zero
 $\downarrow$  zero
 $\downarrow$  zero

$P_C - P_B = \frac{\rho \omega^2 r_B^2}{2} = \frac{1000 \times (1.2) \times 10^2 \times 0.5^2}{2} = -15,000 \text{ Pa}$

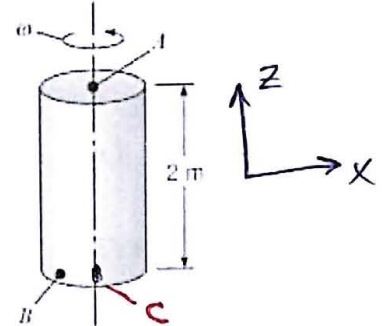
\* apply Euler equation between  $A \text{ \& } C$ :

$\frac{\partial}{\partial z} (p + \gamma z) = -\rho a_z$

$\frac{\partial p}{\partial z} + \frac{\partial z}{\partial z} \gamma = -\rho a_z \Rightarrow \frac{P_A - P_C}{2} + \gamma = -\rho \times 4 \Rightarrow P_A - P_C = -33144 \text{ Pa}$

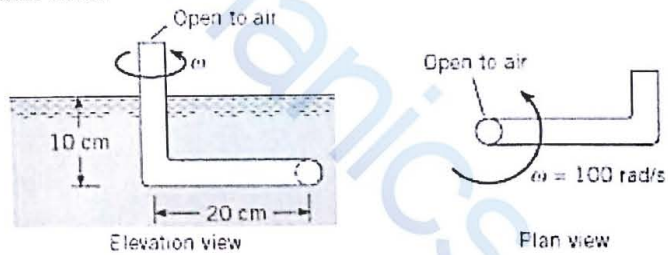
$\Rightarrow P_A - P_B = -15000 - 33144 = -48144 \text{ Pa}$

بعض الـ  $\gamma$   $\uparrow$



Q46 An arm with a stagnation tube on the end is rotated at  $100 \text{ rad/s}$  in a horizontal plane  $10 \text{ cm}$  below a liquid surface as shown. The arm is  $20 \text{ cm}$  long, and the tube at the center of rotation extends above the liquid surface. The liquid in the tube is the same as that in the tank and has a specific weight of  $10,000 \text{ N/m}^3$ . Find the location of the liquid surface in the central tube.

$\omega = 100 \text{ Rad/s}$   
 $r = 0.2 \text{ m}$   
 $\gamma = 10 \text{ kN/m}^3$



ans:  $L = 0$ ; (liquid surface in the tube is the same as elevation as outside surface)

والله مالي طالع \* 😊

\* Rotation of fluid: it's a vector quantity which defined as the average angular velocity of two initially mutually perpendicular forces in a flowing fluid.

( $\Omega$ )

" $\dot{\theta} = \frac{1}{2} \text{curl } \vec{V}$ "  $\rightarrow$  check the derivatives from text book

$$\begin{aligned} \rightarrow \Omega_z &= \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \\ \rightarrow \Omega_x &= \frac{1}{2} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \\ \rightarrow \Omega_y &= \frac{1}{2} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \end{aligned} \quad \left. \begin{array}{l} \text{So the rate of Rotation vector is given by:} \\ \Omega = \Omega_x \hat{i} + \Omega_y \hat{j} + \Omega_z \hat{k} \\ \Omega = \frac{1}{2} \left[ \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \hat{i} + \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \hat{j} + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k} \right] \end{array} \right\}$$

\* these equations used to check whether the flow is rotational or not!!

So: If  $\rightarrow \frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$  &  $\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$  &  $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$  the flow is -irrotational

\* example: the vector  $\vec{V} = 10x \hat{i} - 10y \hat{j}$ , represents a two dimensional velocity field, is the flow irrotational??

Sol: velocity components are:

$$\begin{aligned} x \rightarrow u &= 10x \rightarrow \frac{\partial u}{\partial y} = 0 \\ y \rightarrow v &= -10y \rightarrow \frac{\partial v}{\partial x} = 0 \end{aligned} \quad \text{so; } \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \quad \text{"irrotational"}$$

\* example: the velocity components for a two-dimensional flow are:

$$u = \frac{cx}{(y^2+x^2)}, \quad v = \frac{cy}{(x^2+y^2)}; \quad \text{check if the flow is rotational}$$

Sol:  $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \Rightarrow \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

$$\frac{-2xcy}{(x^2+y^2)^2} - \frac{-2ycx}{(y^2+x^2)^2} = 0, \quad \text{the flow is irrotational}$$

\* example: a two-dimensional flow field is defined by  $[u = x^2 - y^2]$  and  $[v = -2xy]$ , is the flow irrotational?

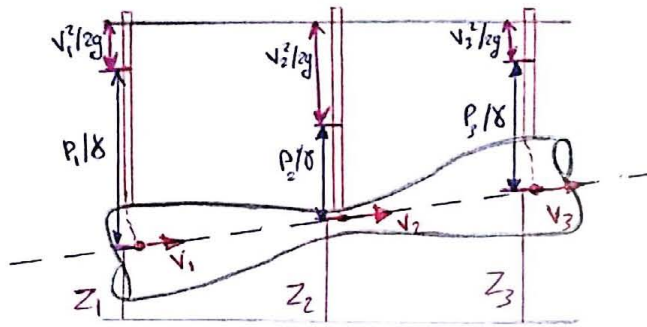
Sol:  $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -2y - (-2y) = 0$ ; so the flow is irrotational

\* ^ jaw

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\* Bernoulli equation: the integration of Euler's equation along pathline with many assumptions such:

- 1- steady flow.
- 2- Incompressible flow.
- 3- Irrotational flow.
- 4- non-viscous flow.
- 5- no heat transfer.
- 6- no shaft work.



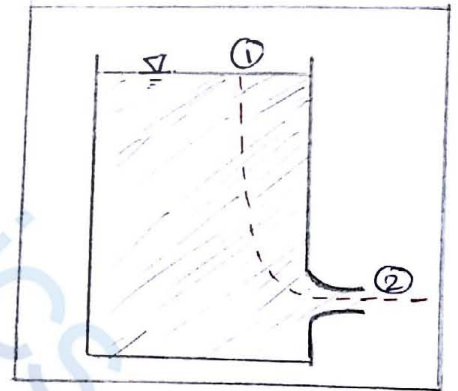
$Z$ : elevation head  
 $\frac{v}{2g}$ : velocity head  
 $\frac{P}{\rho}$ : pressure head

"streamline" ←  $\left[ \frac{P}{\rho} + \frac{v^2}{2g} + Z \right]$  يبقى ثابتاً عند أي نقطة على الـ

$$\left( \begin{array}{c} \text{pressure} \\ \text{head} \end{array} \right) + \left( \begin{array}{c} \text{velocity} \\ \text{head} \end{array} \right) + \left( \begin{array}{c} \text{elevation} \\ \text{head} \end{array} \right) = \text{constant}$$

example: an open tank is filled with water and drains through a port at the bottom of the tank; the elevation of the water in the tank is (10 m) above the drain; the drain port is at atmospheric pressure; find the velocity of the liquid in the drain port.

Sol: apply Bernoulli equation between ① & ②



$$\underbrace{\frac{P_1}{\rho}}_{\text{atm}} + \underbrace{\frac{v_1^2}{2g}}_{\text{Zero}} + z_1 = \underbrace{\frac{P_2}{\rho}}_{\text{atm}} + \frac{v_2^2}{2g} + z_2$$

$$\frac{v_2^2}{2g} = z_1 - z_2$$

$$v_2 = \sqrt{2g \Delta z}$$

$$= \sqrt{2 \times 9.81 \times 10} = \sqrt{196.2} \approx 14 \text{ m/s}$$

\* ملاحظة - السرعة عند نقطة 1 صغيرة جداً مقارنةً بالسرعة عند نقطة 2  
 Chapter 5 "gage" Zero = فتوانة للخلاف الجوي، لذلك الضغط  $\frac{P_2}{\rho} = \frac{P_1}{\rho}$  \*

### Bernoulli equation along streamline

**Q61** Water flows through a vertical contraction (venturi) section. Piezometers are attached to the upstream pipe and minimum area section as shown. The velocity in the pipe is 4 m/s. The difference in elevation between the two water levels in the piezometers is 20 cm. The water temperature is 293 K. What is the velocity (m/s) at the minimum area?

apply Bernoulli equation between 1 & 2

$$P_1 + \rho g Z_1 + \rho \frac{V_1^2}{2} = P_2 + \rho g Z_2 + \rho \frac{V_2^2}{2}$$

$$\rho \frac{V_2^2}{2} = \rho \frac{V_1^2}{2} + (P_1 + \rho g Z_1) - (P_2 + \rho g Z_2)$$

$\downarrow$  piezometric pressure       $\downarrow$  pressure

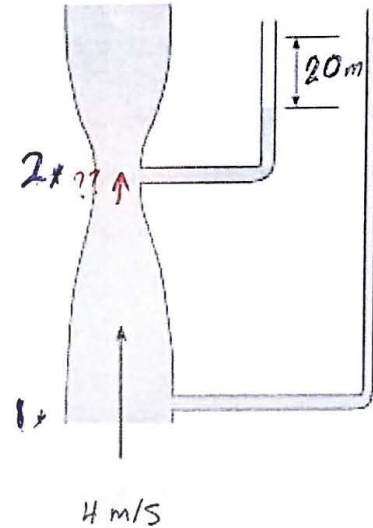
$$(P_1 + \rho g Z_1) - (P_2 + \rho g Z_2) = \rho g \Delta h \text{ "Piezometer equation"}$$

$$\text{So; } \Delta P_z = 9810 \times 0.2 = 1962$$

$$1000 \times \frac{V_2^2}{2} = \rho \frac{(4)^2}{2} + [1962]$$

$$\text{solving for } V_2 \Rightarrow V_2^2 = \left( \frac{1000 \times 16}{2} + 1962 \right) / 500$$

$$V_2^2 = 19.924 \text{ m/s} \Rightarrow V_2 = 4.46 \text{ m/s}$$



**Q62** Kerosene at 20°C flows through a contraction section as shown. A pressure gage connected between the upstream pipe and throat section shows a pressure difference of 20 kPa. The gasoline velocity in the throat section is 10 m/s. What is the velocity (m/s) in the upstream pipe?  $\rho = 814 \text{ kg/m}^3$

$$P_1 + \rho g Z_1 + \rho \frac{V_1^2}{2} = P_2 + \rho g Z_2 + \rho \frac{V_2^2}{2}$$

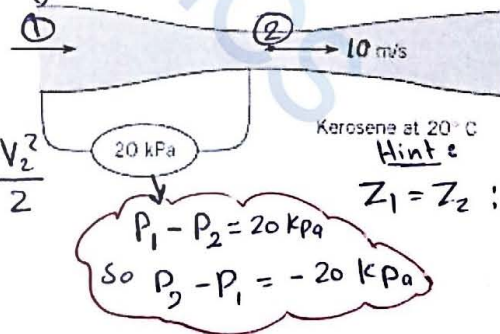
$$\rho \frac{V_1^2}{2} = (P_2 + \rho g Z_2) - (P_1 + \rho g Z_1) + \rho \frac{V_2^2}{2}$$

$$\rho \frac{V_1^2}{2} = 20 \times 10^3 + 814 \frac{(10)^2}{2}$$

$$V_1^2 = \frac{-20 \times 10^3 + 40700}{407}$$

$$V_1^2 = 50.86 \text{ (m/s)}^2$$

$$V_1 = \sqrt{50.86} = 7.13 \text{ m/s}$$



$P_1 - P_2 = 20 \text{ kPa}$   
 So  $P_2 - P_1 = -20 \text{ kPa}$

Hint:  $Z_1 = Z_2$ ; Horizontal



\* Applications of the Bernoulli equations:

→ Velocity measurement devices:

① Stagnation tube "Total head tube":

\* it's an open ended tube directed upstream in a flow and connected to a pressure sensor.

$$P_0 + \rho \frac{V_0^2}{2} = P_1 + \rho \frac{V_1^2}{2}$$

$$V_0^2 = \frac{2}{\rho} (P_1 - P_0)$$

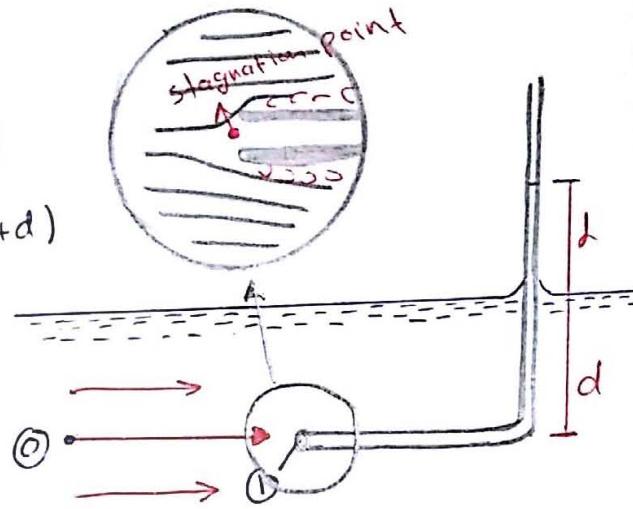
$$V_0^2 = \frac{2}{\rho} (\gamma(d+d) - \gamma d)$$

$$V_0 = \sqrt{2gd} \rightarrow \text{stagnation tube equation}$$

\*  $P_0 = \gamma d$

\*  $P_1 = \gamma(d+d)$

↑  
سبب ارتفاع السائل  
نسبة المرن



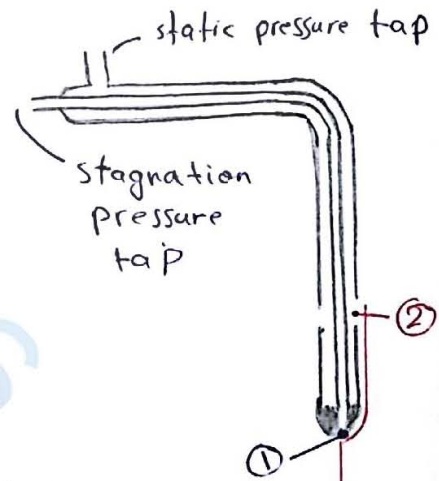
② Pitot-static tube: → the tube which based on same principle of stagnation tube, but it's much more versatile than the stagnation tube.

$$P_1 + \gamma z_1 + \frac{\rho V_1^2}{2} = P_2 + \gamma z_2 + \frac{\rho V_2^2}{2}$$

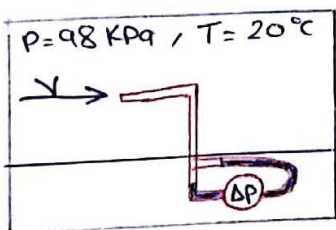
zero

$$V_2 = \left[ \frac{2}{\rho} (P_{2,1} - P_{2,2}) \right]^{1/2}$$

$$V = \sqrt{\frac{2\Delta P}{\rho}} \rightarrow \text{pitot-static tube equation}$$



example 4.9: a differential pressure gage is connected across the taps of a pitot-static tube, when it's used in wind tunnel test; the gage  $\rightarrow (\Delta P = 730 \text{ Pa})$ , what is the air velocity in the Tunnel? ( $T = 20^\circ\text{C}$ ,  $P = 98 \text{ kPa}$ )



+sol:  $\rho = \frac{P}{RT} = \frac{98 \times 10^3}{287 \times 293} = 1.17 \text{ kg/m}^3$

$$V = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{\frac{2 \times 730}{1.17}} = 35.3 \text{ m/s}$$

**Q64** A glass tube is inserted into a flowing stream of water with one opening directed upstream and the other end vertical. If the water velocity is 4 m/s, how high will the water rise in the vertical leg relative to the level of the water surface of the stream?

[Stagnation tube & water surface] =>

$$P_{\text{stagnation}} = \gamma (d + d)$$

=> now, apply Bernoulli between  $\underline{S}$  and free-stream.

$$\frac{P_s}{\gamma} + \frac{V_s^2}{2g} + z_s = \frac{P_{\text{free}}}{\gamma} + \frac{V_{\text{free}}^2}{2g} + z_{\text{free}}$$

*Hydrostatic equation !! نقطة !!*

$$\frac{\gamma (d+d)}{\gamma} = \frac{\gamma d}{\gamma} + \frac{(4)^2}{2g} \Rightarrow d+d = d + \frac{16}{19.62}$$

$$d = \frac{16}{19.62} = 0.81549 \text{ m}$$

سيرتفع الماء ٠٨١٥٤٩ متر من هذه القيمة !!

**Bernoulli equation in irrotational flow**

**Q99** Water ( $\rho = 1000 \text{ kg/m}^3$ ) flows from the large orifice at the bottom of the tank as shown. Assume that the flow is irrotational. Point B is at zero elevation, and point A is at 30 cm elevation. If  $V_A = 3 \text{ m/s}$  at an angle of  $45^\circ$  with the horizontal and if  $V_B = 36 \text{ m/s}$  vertically downward, what is the value of  $p_A - p_B$ ?

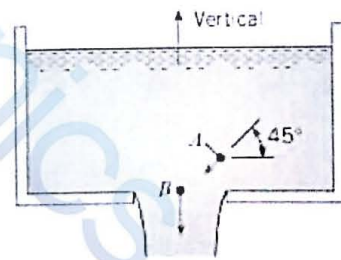
apply Bernoulli equation between  $\underline{A}$  &  $\underline{B}$

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$P_A - P_B = \gamma \left[ \frac{V_B^2 - V_A^2}{2g} + z_B - z_A \right]$$

$$P_A - P_B = 9810 \left[ \frac{36^2 - 3^2}{19.62} - 0.3 \right]$$

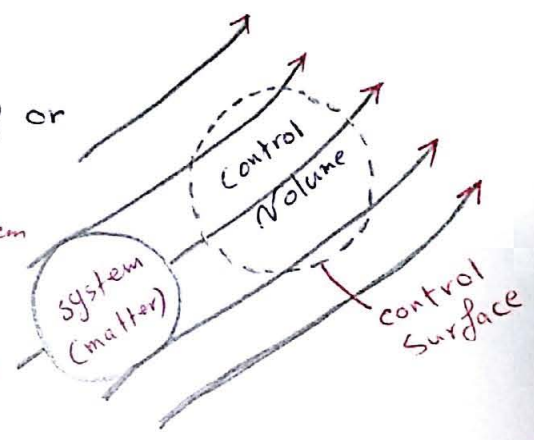
$$P_A - P_B =$$



\* Ch \* 5 : Control Volume approach and continuity equation:

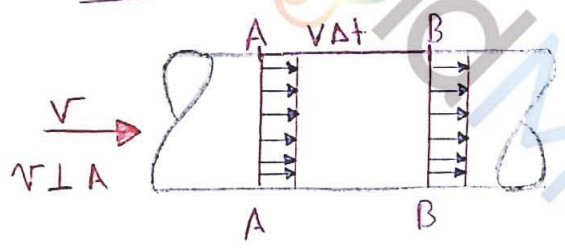
→ Control volume: Volume located in space and through which matter can pass and the selection of the control volume position and shape is problem-dependent; the control volume is enclosed by the control surface.

- \* Control surface can be stationary or moveable.
- \* Control surface separate the system from it's surrounding.



note: Check extensive and intensive properties which mentioned before!!

\* flow in pipes: Discharge "Volumetric flow rate" [Q]

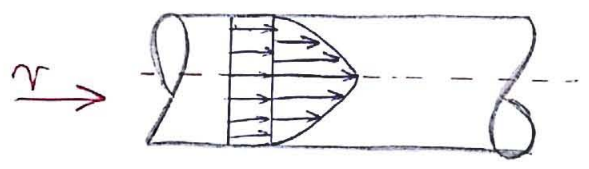


: the volume of fluid passing section A-A =  $\frac{V}{\Delta t}$

$$Q = \frac{V}{\Delta t} \quad (\text{m}^3/\text{s}, \text{L}^3/\text{s}, \text{ft}^3/\text{s} \dots)$$

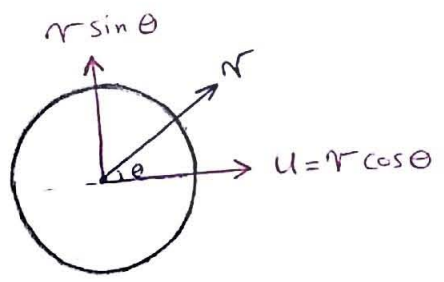
$$Q = VA \quad \text{uniform velocity distribution discharge}$$

\* for variable velocity distribution



$$Q = \int v \cdot dA$$

$$\text{mean (avg) velocity} = \frac{Q}{A} = \int \frac{Q}{A} = \int \frac{v \, dA}{A}$$



$$Q = \int v \cdot dA$$

$$= v \cos \theta \, dA$$

$$= u \cdot dA$$

$\theta$ : the angle between the outward norm of the surface and the velocity vector

\* mass flow Rate:  $\dot{m} = \rho Q = \rho VA$  (kg/s, slug/s, lbm/s ...)

\* also; @ variable velocity distribution:  $\dot{m} = \int \rho v \, dA$

- example: for the hypothetical velocity in the pipe shown;  $\frac{v}{v_{max}} = (1 - \frac{r}{R})$

determine  $\frac{\bar{v}}{v_{max}}$  ??

$$A = \bar{v} r^2$$

$$dA = 2\pi r dr \Rightarrow \bar{v} = \frac{\int v dA}{A} = \frac{\int_{max} v (1 - \frac{r}{R}) 2\pi r dr}{\pi R^2}$$

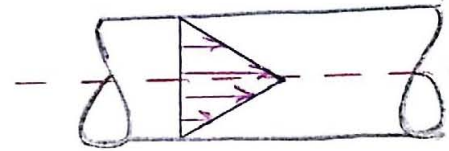
$$= \frac{2\pi}{\pi R^2} \int (v_{max} - \frac{v_{max}}{R} r) r dr$$

$$= \frac{2\pi}{\pi R^2} \int (v r - \frac{v_{max}}{R} r^2) dr$$

$$= \frac{2}{R^2} \left[ \frac{v_{max} r^2}{2} - \frac{v_{max}}{3R} r^3 \right]_0^R = \frac{2v_{max}}{R^2} \left( \frac{R^2}{2} - \frac{R^3}{3R} \right)$$

$$\bar{v} = \frac{2v_{max}}{R^2} \left( \frac{R^2}{2} - \frac{2R^2}{3} \right) = \frac{2v_{max}}{R^2} \frac{R^2}{6} = \frac{1}{3} v_{max}$$

$$\boxed{\frac{\bar{v}}{v_{max}} = \frac{1}{3}}$$



\* example: for the hypothetical velocity in the pipe, determine  $\frac{\bar{v}}{v_{max}}$  ??

$$v = ar + b$$

$$@ r=0 \Rightarrow v=0 \Rightarrow b=0$$

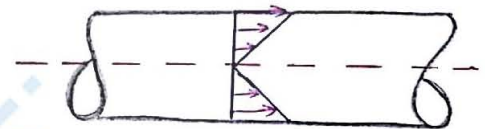
$$@ r=R \Rightarrow v=v_{max} \Rightarrow a = \frac{v_{max}}{R}$$

$$v = v_{max} \frac{r}{R} \Rightarrow \frac{v}{v_{max}} = \frac{r}{R}$$

$$\Rightarrow \bar{v} = \frac{\int v dA}{A} = \frac{\int v_{max} \frac{r}{R} \times 2\pi r dr}{\pi R^2} = \frac{2\pi v_{max}}{\pi R^2} \int_0^R \frac{r^2}{R} dr$$

$$\bar{v} = \frac{2v_{max}}{R^2} \frac{r^3}{3R} \Big|_0^R = \frac{2v_{max}}{R^2} \frac{R^3}{3R} = \frac{2}{3} v_{max}$$

$$\boxed{\frac{\bar{v}}{v_{max}} = \frac{2}{3}}$$



(فوقه وحتا)

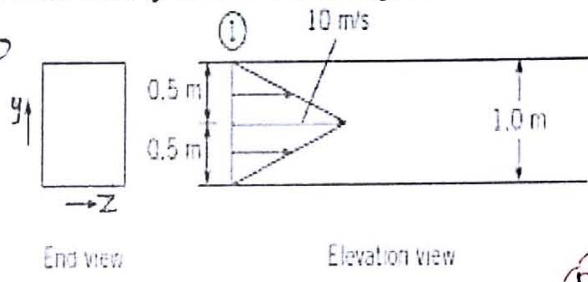
\* لاحظوا انه اقتارنا السرعة عمائد حول خط المركزنا لذلك باصكان صاب  $Q$  على ايمر كائينه وضربها بـ 2

الفكرة كيمون \*

**Q16** Air enters this square duct at section 1 with the velocity distribution as shown. Note that the velocity varies in the  $y$  direction only (for a given value of  $y$ , the velocity is the same for all values of  $z$ ).

- a. What is the volume rate of flow?
- b. What is the mean velocity in the duct?
- c. What is the mass rate of flow if the mass density of the air is  $1.2 \text{ kg/m}^3$ ?

$\rightarrow V = ay + b \Rightarrow 10 @ y = 0.5, 0 @ y = 0$   
 $\Rightarrow 10 = a(0.5) + b \Rightarrow a = 20$   
 $\Rightarrow 0 = a(0) + b \Rightarrow b = 0$



$V = 20y$

a)  $Q = \int V dA = \int_0^{0.5} 20y dy$   
 $= \frac{20}{2} y^2 \Big|_0^{0.5} = \frac{20}{2} (0.5)^2 = 2.5 \text{ m}^3/\text{s}$  (للمنوالايفل)

$A = y \times 1$   
 $dA = dy$   
 always 1m  
 هذا البعد كائنه

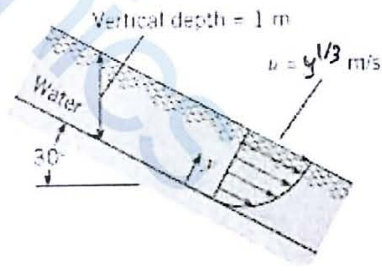
So  $Q = 2 \times 2.5 = 5 \text{ m}^3/\text{s}$

b)  $V_{mean} = \bar{V} = \frac{Q}{A} = \frac{5}{1 \times 1} = 5 \text{ m/s}$

c)  $\dot{m} = \rho Q = 1.2 \times 5 = 6 \text{ kg/s}$

**Q18** The rectangular channel shown is 1.5 m wide. What is the discharge in the channel?

$u \equiv v = y^{1/3}$



$Q = \int V dA$

$Q = \int_0^{0.866} y^{1/3} \times 1.5 dy$

$Q = 1.5 \left[ \frac{y^{4/3}}{4/3} \right]_0^{0.866}$

$Q = 1.5 \frac{(0.866)^{4/3}}{4/3}$

$Q = 0.9286 \text{ m}^3/\text{s}$

$A = \text{width} \times \text{depth}$   
 $A = 1.5 \times y$   
 هذا البعد كائنه

$y = 1 \cos 30^\circ = 0.866 \text{ m}$

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\* Reynolds transport theorem:

Lagrangian approach  $\rightarrow$  Eulerian approach

\* this theorem is derived by considering the rate of change of an extensive property of a system as it passes through the control volume.

$$\Rightarrow \underbrace{\frac{dB}{dt}}_{\text{Lagrangian approach}} \Big|_{\text{sys}} = \underbrace{\frac{d}{dt} \int_{C.V} b \rho dV + \int_{C.S} b \rho V \cdot dA}_{\text{Eulerian approach}}$$

B: any extensive prop.  
b: the corresponding intensive property.

← تلكه فكرة هذه النظرية في اختيار (C.V) مناسب ومراقبة هائله وحاجه (C.S) وبناء عليه يكونه البتير الحاصل في الخاصية (B) مساوية للتغيرات الساعه في المعادله على (C.V) وال (C.S) !!  
← تعتبر هذه النظرية من أهم وأقوى النظريات المتخذة في ميكانيكا الموائع والديناميكا الحرارية وانتقال الحرارة !!

→ let us now, use (mass) as "B" so, B = mass, b =  $\frac{\text{mass}}{\text{mass}} = 1$

$$\frac{dm}{dt} \Big|_{\text{sys}} = \frac{d}{dt} \int_{C.V} \rho dV + \int_{C.S} \rho V \cdot dA, \text{ now } \frac{dm}{dt} \Big|_{\text{sys}} = 0 \text{ so:}$$

$$\frac{d}{dt} \int_{C.V} \rho dV + \int_{C.S} \rho V \cdot dA = 0.0$$

$$\boxed{\frac{d}{dt} m_{C.V} + \sum_{C.S} \dot{m}_{out} - \sum_{C.S} \dot{m}_{in} = 0.0} \text{ the continuity equation}$$

- from continuity equation:
- ①  $\frac{dm_{C.V}}{dt} > 0.0 \rightarrow \sum \dot{m}_{in} > \sum \dot{m}_{out}$  "filling"
  - ②  $\frac{dm_{C.V}}{dt} = 0.0 \rightarrow \sum \dot{m}_{in} = \sum \dot{m}_{out}$  "stead state cond."
  - ③  $\frac{dm_{C.V}}{dt} < 0.0 \rightarrow \sum \dot{m}_{in} < \sum \dot{m}_{out}$  "emptying"

⇒ @ steady-state conditions:  $\sum \dot{m}_{in} = \sum \dot{m}_{out}$  and assuming Incompressible flow. (P=ct)

$$\rho \dot{m}_{in} = \rho \dot{m}_{out}$$

$$Q_{in} = Q_{out}$$

$$V_1 A_1 = V_2 A_2$$

Conceptual problem, doesn't need nums and calculations  
it's need perfect comprehension only!!

### Control volume approach

Q37 Gas flows into and out of the chamber as shown. For the conditions shown, which of the following statement(s) are true of the application of the control volume equation to the continuity principle?

- $B_{sys} = 0$
- $\frac{dB_{sys}}{dt} = 0$
- $\sum_{c.s} b \rho V \cdot A = 0$
- $\frac{d}{dt} \int_{c.v} \rho dV = 0$
- $b = 0$

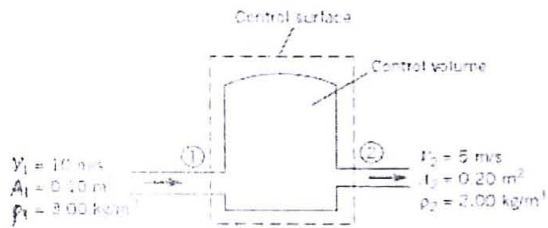
Sol:

$$\begin{aligned} \dot{m}_{input} &= \rho_{in} V_{in} A_{in} \\ &= 3 \times 10 \times 0.1 = 3 \text{ Kg/s} \end{aligned}$$

$$\begin{aligned} \dot{m}_{output} &= \rho_{out} V_{out} A_{out} \\ &= 2 \times 5 \times 0.2 = 2 \text{ Kg/s} \end{aligned}$$

$\Rightarrow$  mass flow Rate not equal !!

So; the flow is unsteady!!

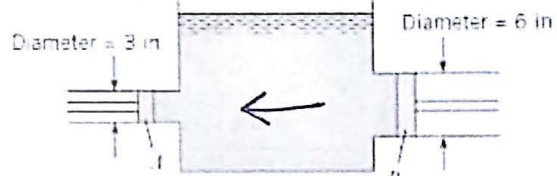


- $B_{sys} = 0$ ; false, there is matter (property)
- $\frac{dB_{sys}}{dt} = 0$ ; True; the matter will go on and out without change.
- $\sum_{c.s} b \rho V \cdot A = 0$ ; false; mass flow rate in not equal to  $\dot{m}_{out}$
- $\frac{d}{dt} \int_{c.v} \rho dV = 0$ ; false; the volume will change in control volume
- $b = 0$ ; false; equal to  $\frac{1}{m}$  ( $\frac{m}{m} = 1$ )

Continuity equation

Q44 Both pistons are moving to the left, but piston A has a speed twice as great as that of piston B. Then the water level in the tank is (a) rising, (b) not moving up or down, or (c) falling?

apply Reynolds transport theorem



zero  $\frac{dB}{dt}_{sys} = \frac{\partial}{\partial t} \int_{c.v} \rho b dV + \int_{c.s} \rho b V \cdot dA$

$\frac{\partial}{\partial t} \int_{c.v} \rho dV = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

$\frac{\partial}{\partial t} \int_{c.v} \rho dV = \rho \left[ \frac{\pi}{4} (6)^2 V_B - \frac{\pi}{4} (3)^2 (2V_B) \right]$

$\frac{\partial}{\partial t} \int_{c.v} \rho dV = \rho \times 14.13 V_B$

$\rho \frac{\partial V}{\partial t} = \rho \times 14.13 V_B$

$\dot{m}_{in} = \rho A_{in} V_{in} @ B$   
 $= \rho \frac{\pi}{4} (6)^2 V_B$

$\dot{m}_{out} = \rho A_{out} V_{out} @ A$   
 $= \rho \frac{\pi}{4} (3)^2 (2V_B)$

المستوى يرتفع  
 ارتفاع السطح

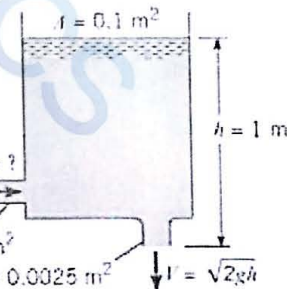
$A \frac{\partial h}{\partial t} = 14.13 V_B \Rightarrow \frac{\partial h}{\partial t} = \frac{14.13 V_B}{A_{tank}}$  this value is positive so it's Rising

Q49 tank has a hole in the bottom with a cross-sectional area of 0.0025 m<sup>2</sup> and an inlet line on the side with a cross-sectional area of 0.0025 m<sup>2</sup>, as shown. The cross-sectional area of the tank is 0.1 m<sup>2</sup>. The velocity of the liquid flowing out the bottom hole is  $v = \sqrt{2gh}$  where  $h$  is the height of the water surface in the tank above the outlet. At a certain time the surface level in the tank is 1 m and rising at the rate of 0.1 cm/s. The liquid is incompressible. Find the velocity of the liquid through the inlet

\* apply Reynolds transport theorem:

$\frac{\partial}{\partial t} \int_{c.v} \rho dV = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

$\rho \frac{\partial h}{\partial t} \times A = \rho (V_{in} \times 0.0025) - [\sqrt{2gh} \times 0.0025]$



take  $h = 1 \text{ m} \Rightarrow \frac{\partial h}{\partial t} A = 0.0025 V_{in} - \sqrt{2 \times 9.81 \times 1} \times 0.0025$

$0.01107 + 0.1 \times 10^{-2} \times 0.1 = 0.0025 V_{in}$

$V_{in} = 4.47 \text{ m/s}$



Q61 Is the tank in the figure filling or emptying? At what rate is the water level rising or falling in the tank?

\* apply Reynolds transport theorem:

$$\frac{\partial}{\partial t} \int_{c.v} \rho dV = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$$\frac{\partial v}{\partial t} \times \rho = \rho [0.02355 - 0.0391715] = 3 \text{ m/s}$$

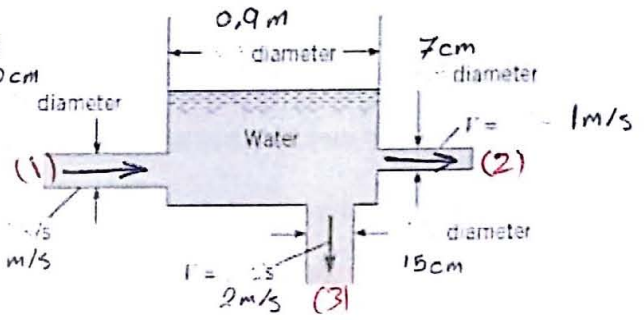
$$\frac{\partial h}{\partial t} \times A_{\text{tank}} = -0.0156215$$

$$\frac{dh}{dt} = -0.02456 \text{ m/s}$$

Falling (-)

سقوطی سطح آبی \*

$$\sum \dot{m}_{out} = \dot{m}_{out(2)} + \dot{m}_{out(3)} = 0.0391715 \rho$$



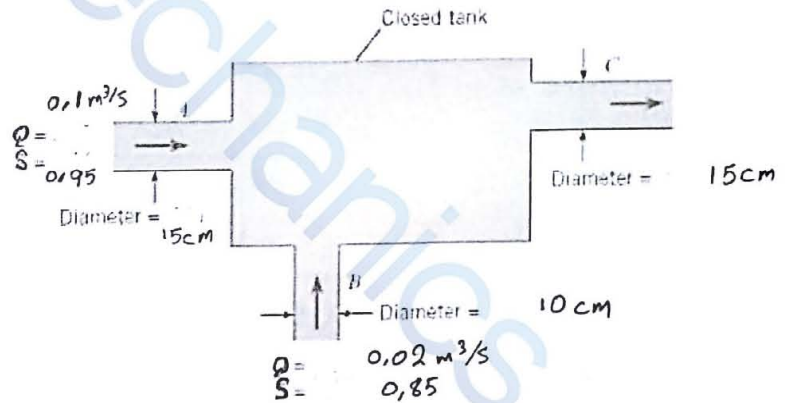
$$\dot{m}_{in(1)} = V_1 A_1 \rho = 3 \times \frac{\pi}{4} \times 0.1^2 \times \rho = 0.02355 \rho$$

$$\dot{m}_{out(2)} = V_2 A_2 \rho = 1 \times \frac{\pi}{4} \times 0.07^2 \times \rho = 3.8465 \times 10^{-3} \rho$$

$$\dot{m}_{out(3)} = V_3 A_3 \rho = 2 \times \frac{\pi}{4} \times 0.15^2 \times \rho = 0.035325 \rho$$

Q66 Assuming that complete mixing occurs between the two inflows before the mixture discharges from the pipe at C, find the mass rate of flow, the velocity, and the specific gravity of the mixture in the pipe at C.

$$\text{ans: } S = 0.925$$



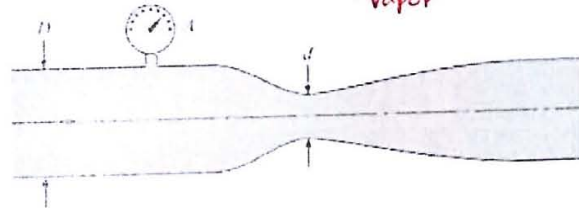
## Cavitation

**Q96** When gage *A* indicates a pressure of 120 kPa gage, then cavitation just starts to occur in the venturi meter.

If  $D = 40$  cm and  $d = 10$  cm, what is the water discharge in the system for this condition of incipient

cavitation? The atmospheric pressure is 100 kPa gage, and the water temperature is  $10^\circ\text{C}$ . Neglect gravitational effects

$$P_{\text{vapor}} = 1230 \text{ Pa (abs)}$$



\* Cavitation will occur when the pressure reaches vapor pressure of the flowing fluid "check 1st"

→ apply Bernoulli equation

$$P_A + \frac{\rho V_A^2}{2} + \frac{z_A}{\text{Hor}} = P_B + \frac{\rho V_B^2}{2} + \frac{z_B}{\text{Hor}}$$

$$P_A + \frac{\rho V_A^2}{2} = P_B + \frac{\rho V_B^2}{2} \quad / \quad V_A = \frac{Q}{A_A} = \frac{Q}{\frac{\pi}{4} (0.4)^2}$$

$$V_{\text{throat}} = \frac{Q}{A_{\text{throat}}} = \frac{Q}{\frac{\pi}{4} \cdot 0.1^2}$$

$$\frac{\rho Q^2}{2} \left[ \frac{1}{\left[ \frac{\pi}{4} \cdot (0.1)^2 \right]^2} - \frac{1}{\left[ \frac{\pi}{4} \cdot (0.4)^2 \right]^2} \right] = 220000 - 1230$$

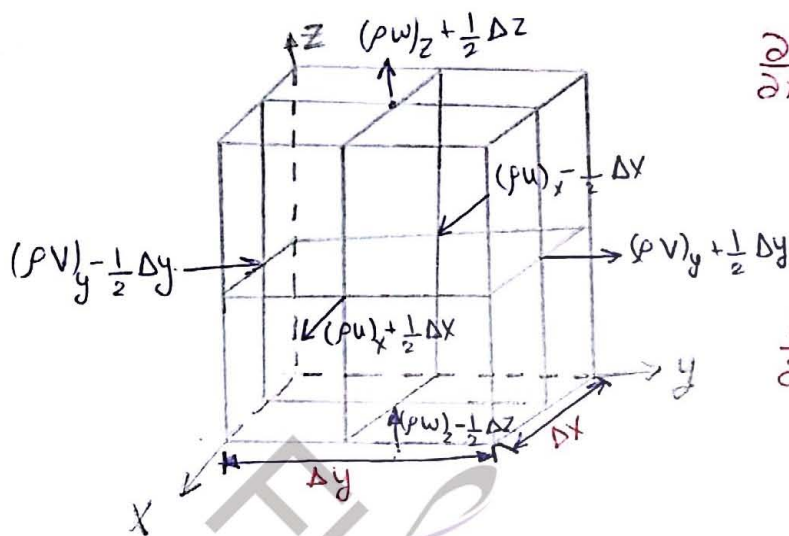
$\uparrow$  abs pressure  
 $\uparrow$  abs pressure  
 $(P_{\text{gage}} + P_{\text{atm}})$

solving for  $Q$ :

$$\boxed{Q = 0.165 \text{ m}^3/\text{s}}$$

\* Differential form of the continuity equations "3D"

→ this equation is derived by applying the integral form of the continuity equation to a small control volume and taking the limit as the volume approaches zero!!



$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0.0$$

\* check full derivative from text-book

\* If the flow is steady:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0.0$$

\* If the flow is incompressible:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.0$$

\* for steady and unsteady:

$$\nabla \cdot \mathbf{V} = 0$$

$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

\* Example:

→ the expression  $(\mathbf{V} = 10x\hat{i} - 10y\hat{j})$  is said to represent the velocity for a two-dimensional flow (planar) incompressible flow, check to see if the continuity equation satisfied

Sol: two dimensional, incompressible so; equation reduced to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \stackrel{??}{=} 0.0 \rightarrow \text{check}$$

$$10 + -10 \stackrel{??}{=} 0.0$$

$$0 = 0$$

$$u = 10x \rightarrow \frac{\partial u}{\partial x} = 10$$

$$v = -10y \rightarrow \frac{\partial v}{\partial y} = -10$$

So, it's satisfied!!

الخطوة

→ check if it's irrotational:

$$\Omega = \frac{1}{2} \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 10x & -10y & 0 \end{bmatrix} = \frac{1}{2} [0 - 0] = 0.0$$

So it's irrotational too!!

\* ركن القوانين صحيح \*

\* Chapter \* 6: Momentum principle: سرعة الحركة = سرعة اللقطة

← لاستقارة علاقات الزخم، منطبقه خانة نيوتن الثاني ( $F_n = m a_n$ ) استخدام (Reynolds Trans. the.)

← ببساطة: التغيير في زخم الجسم يساوي القوة التي تؤثر على الجسم نفسه ...

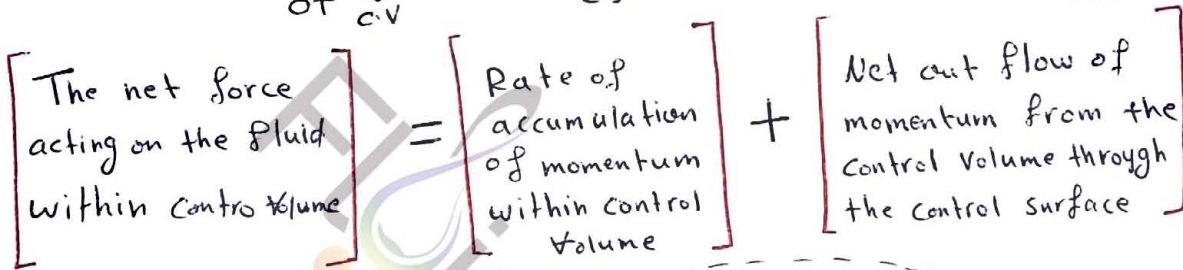
$$\frac{dB}{dt}_{sys} = \frac{\partial}{\partial t} \int_{c.v} \rho b dV + \int_{c.s} \rho b v \cdot dA$$

$B = m\vec{v}$  (زخم)  
 $b = \frac{m\vec{v}}{m} = \vec{v}$  (momentum per unit mass)

$$\frac{d(m\vec{v})}{dt}_{sys} = \frac{\partial}{\partial t} \int_{c.v} \rho \vec{v} dV + \int_{c.s} \rho \vec{v} \vec{v} \cdot dA \Rightarrow m \frac{d\vec{v}}{dt} = m a = \sum \text{Force}$$

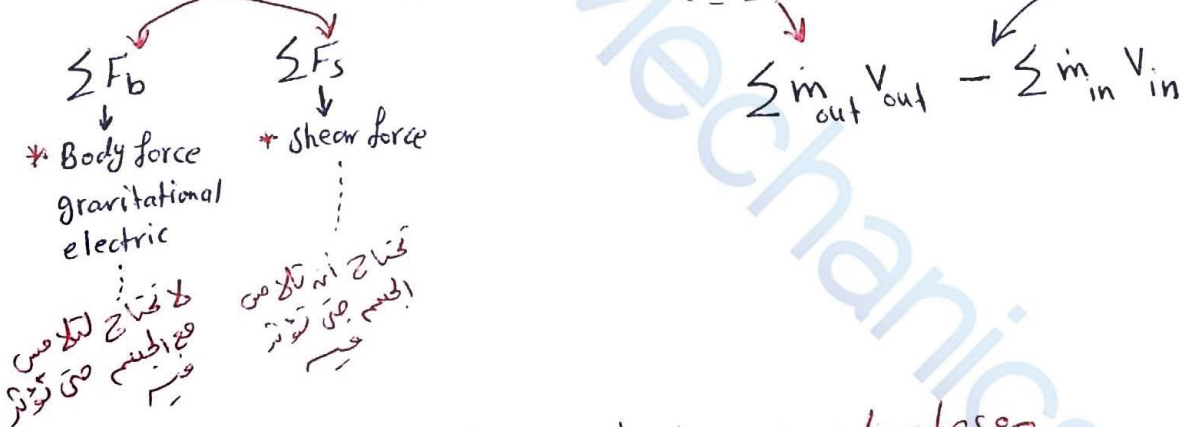
Newton's 2<sup>nd</sup> law

$$\sum F = \frac{\partial}{\partial t} \int_{c.v} \rho \vec{v} dV + \int_{c.s} \rho \vec{v} \vec{v} \cdot dA \Rightarrow \vec{v}, \vec{v} : \text{will be same if control Volume non accelerated}$$



$$\sum F = \frac{d}{dt} \int_{c.v} \rho \vec{v} dV + \int_{c.s} \rho \vec{v} \vec{v} \cdot dA$$

dot products



\* Momentum in Cartesian coordinates

$$\sum F_x = \frac{d}{dt} \int_{c.v} \rho \vec{v} dV + \sum \dot{m}_{out_x} v_{out_x} - \sum \dot{m}_{in_x} v_{in_x}$$

$$\sum F_y = \frac{d}{dt} \int_{c.v} \rho \vec{v} dV + \sum \dot{m}_{out_y} v_{out_y} - \sum \dot{m}_{in_y} v_{in_y}$$

$$\sum F_z = \frac{d}{dt} \int_{c.v} \rho \vec{v} dV + \sum \dot{m}_{out_z} v_{out_z} - \sum \dot{m}_{in_z} v_{in_z}$$

\* ملاحظة: أغلب الحالات التي ندرسها تكون (steady state) وهذا يعني أنه الزخم لا يتغير مع تغير الزمنة في ال (Control Volume) ولهذا السبب خانة نيوتن:

$$\frac{d}{dt} \int_{c.v} \rho \vec{v} dV = 0.0$$

## \* Mechanical components "fittings" \*

\* الهدف من دراسة الزخم هو تحديد القوى اللازمة لتثبيت أي قطعة ميكانيكية عند التقييم أو دراسة تأثير الموائع على هذه القطع ----

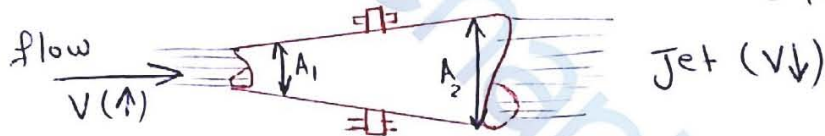
- ① Vanes: structural component that may be used to change the direction of flow, its used either to subtract energy from flow (Turbines) or add energy to the flow (pumps).



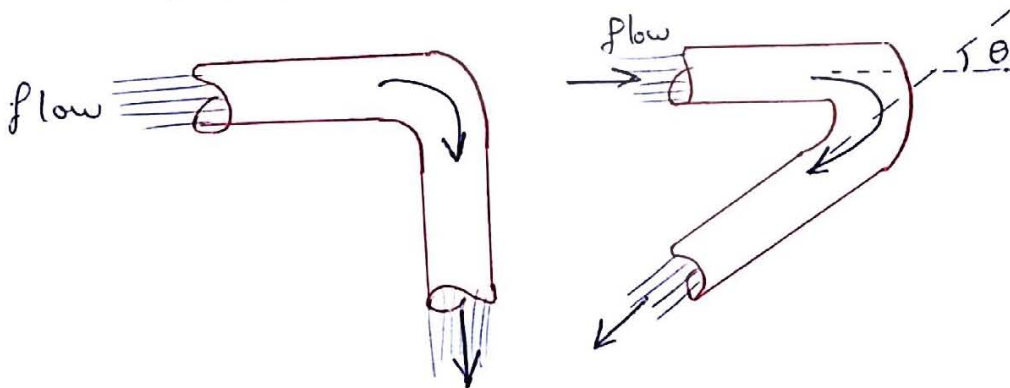
- ② Nozzle: mechanical component use to increase velocity of flow by reducing - the flow area.



- ③ diffuser: mechanical component use to decrease flow velocity by increasing - the flow area.  
(Nozzle) عكس



- ④ Bends: mechanical component use mainly for change direction of flow usually it's found between two pipes, every one has a different direction



\* كيف نحل أسئلة الزخم؟

1- اختر (control volume) مناسب

2- ارسم مخططاً ، واصل للقوى (forces diagram) وآخر للزخم (Momentum diagram) ---

3- ملل الوزن ، رد الفعل ، القوى الناتجة عن الضغط مراعيًا الاتجاهات ، وعيّن على الـ (force diagram)

3- حدد السرعات الداخلة والخارجة ، والتدفق الكلي الداخلي والخارج مراعيًا الاتجاهات ،  
ثم عيّن على الـ (momentum diagram)

4- استخدم علاقات الزخم ، واحسب المطلوب .

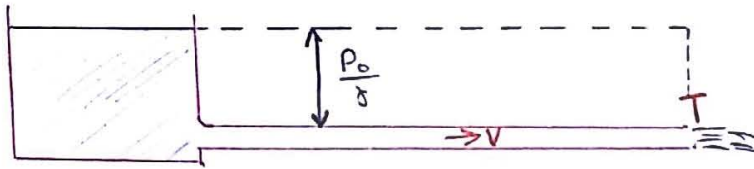
5- قد يتطلب السؤال استخدام معادلات التدفق الجمعي (Q) لحساب سرعة أو (m) وهكذا ---

\* يوجد في هذا الترخيص اللبّ من الأسئلة المتعلقة بموضوع الزخم ---

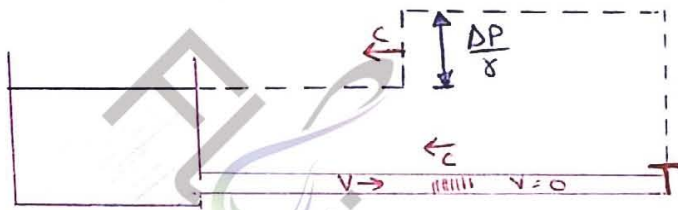
سأترك لك بعض الأسئلة خارجة --- جرّب الحل بنفسك ، ستفقد الاستفادة  
أكبر --- \*

# \* Water Hammer

\* Whenever a valve is closed in a pipe, a positive pressure wave is created upstream of the valve and travels up the pipe at speed of sound!!  
 In this context a positive pressure wave is created as one for which the pressure is greater than the existing steady-state pressure.  
 → this pressure wave may be great enough to cause a pipe failure.

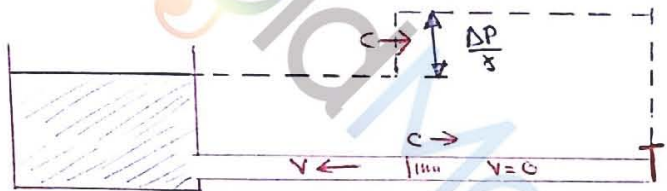


$$t = 0, P = P_0, V \rightarrow$$



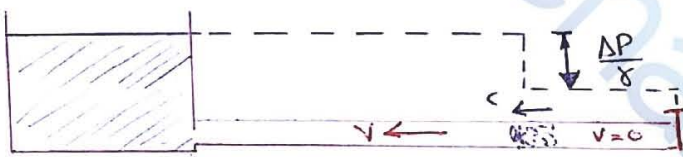
على هذا يكون الخلل في النظام للضغط و ينتج صدمة تسمى لذلك تسمى في الخزانة بسرعة الصوت بحيث يكون سرعة المائع قبلها = صفراً أما بعدها فهي تقلل عن السرعة الابتدائية

$$[0 < t < L/c]$$



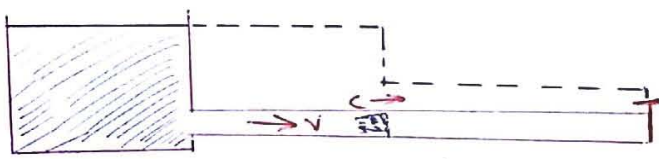
\* بعد وصول الموجة الى الخزانة و بدائرة عودتها سبب زيادة في الضغط مقدار ΔP أما ما قبلها فهو يبقى بنفس الضغط الابتدائي (P\_0).  
 ← هذا يحدث في زمنه قدره [L/c < t < 2L/c]

ونتيجة لذلك الخلل في الخزانة يعود الى الخزانة مرة أخرى.



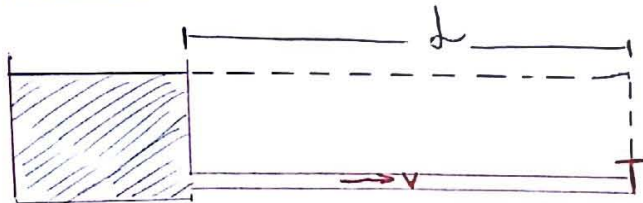
\* بعد وصول الموجة الى الخزانة فانها تعود الى الخزانة مرة أخرى بسبب اختلاف في الضغط بحيث تكون الموجة مع اتجاه السائل سيرانا باتجاه الخزانة وهذه المرة تسمى (B)

$$[2L/c < t < 3L/c]$$



\* هنا تعود الموجة للمرة الأخيرة في حتمية و بهذا يكون الضغط قد سادى

$$[3L/c < t < 4L/c]$$



$$t = 4L/c$$

\* Time to complete an cycle =  $\frac{4L}{c}$

\* Time critical =  $\frac{2L}{c}$

$$\Delta P = \rho V c$$

velocity of flowing fluid  
↑  
↓ density      speed of sound-wave

$$c = \sqrt{\frac{E_v}{\rho}}$$

$t_{closing} < t_{cr}$        $t_{closing} > t_{cr}$   
 water Hammer exist // water Hammer not exist

\* water Hammer can be avoided by:

- 1- installing an accumulator near the valve.
- 2- prevent sudden closing of the valve.
- 3- use safety Relief valve.

\* example: a rigid pipe leading from a reservoir is 900 m long, and water is flowing through it with ( $V=1.5$  m/s); if the initial pressure at downstream end is 280 kPa, what maximum pressure will develop at the downstream end when a rapid acting valve at that end is closed in 1 s ?? ( $\rho=1000$  kg/m<sup>3</sup>,  $E_v=2.2 \times 10^9$  Pa)

Sol: \* check whether water hammer existed by calculating  $t_{cr}$

$$t_{cr} = \frac{2L}{c} = \frac{2 \times 900}{1483} = 1.21 \text{ s}, \quad c = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{2.2 \times 10^9}{1000}} = 1483 \text{ m/s}$$

$t_{cr} > t_{\text{closing}}$ , so water hammer existed

$$\Delta P = \rho V c$$

$$P_{\text{max}} - P_0 = 1000 \times 1.5 \times 1483 = 2224.5 \text{ kPa}$$

$$P_{\text{max}} = 280 \times 10^3 + 2224.5 \times 10^3 = \boxed{2504 \text{ kPa}}$$

\* example: estimate the maximum water hammer pressure that is generated in a rigid pipe if the initial water velocity ( $=4$  m/s) and the pipe is (10 km) long with a valve at the downstream end that is closed in 10 s ??

Sol: \* check  $c = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{2.2 \times 10^9}{1000}} = 1483 \text{ m/s}$

$$t_{cr} = \frac{2L}{c} = \frac{2 \times 10 \times 10^3}{1483} = 13.48 \text{ s} > t_{\text{closing}}, \text{ so water hammer exist}$$

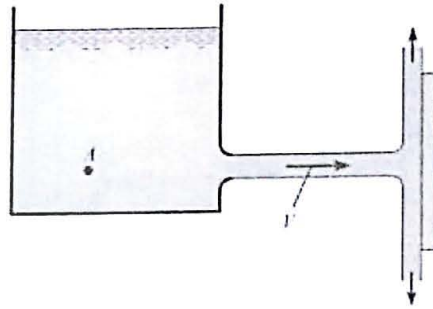
$$\Delta P = \rho V c = 1000 \times 4 \times 1483 = 5,933 \text{ kPa}$$



**Chapter 6 : jets**

**Q10** A horizontal water jet at 20°C impinges on a vertical-perpendicular plate. The discharge is 0.05 m<sup>3</sup>/s. If the external force required to hold the plate in place is 900 N, what is the velocity of the water?

$$Q = 0.05 \text{ m}^3/\text{s}$$



Momentum equation (x-direction)  
momentum out @ x-axis = 0

$$\sum F_x = \sum \dot{m}_{out} V_{out_x} - \sum \dot{m}_{in} V_{in_x}$$

$$+F_x = +\sum \dot{m}_{in} V_{in_x}$$

$$900 = \rho Q V_{in_x}$$

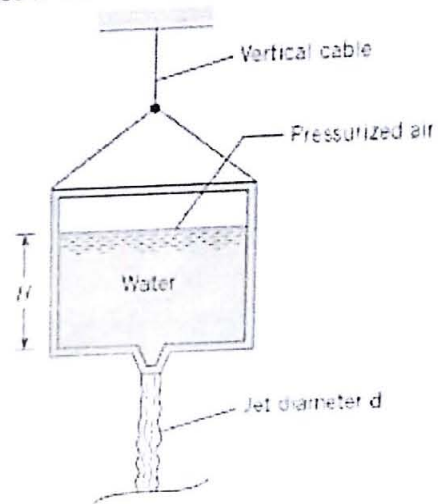
$$V_{in_x} = \frac{900}{\rho Q} = \frac{900}{1000 \times 0.05} = \boxed{18 \text{ m/s}}$$

**Q15** A tank of water (15°C) with a total weight of 200 N (water plus the container) is suspended by a vertical cable. Pressurized air drives a water jet ( $d = 12$  mm) out the bottom of the tank such that the tension in the vertical cable is 10 N. If  $H = 425$  mm, find the required air pressure in units of atmospheres (gage). Assume the flow of water is irrotational.

$$A_2 = \frac{\pi d^2}{4}$$

$$= \frac{\pi (0.012 \text{ m})^2}{4}$$

$$= 1.131 \times 10^{-4} \text{ m}^2$$



$$\Sigma F = m_0 v_0$$

$$-T + W = m v_2$$

$$(-10 + 200) \text{ N} = \rho A_2 v_2^2$$

$$190 \text{ N} = (999 \text{ kg/m}^3) (1.131 \times 10^{-4} \text{ m}^2) v_2^2$$

$$v_2 = 41.01 \text{ m/s}$$

$$P_{\text{air}} + \frac{\rho v_1^2}{2} + \rho g z_1 = P_2 + \frac{\rho v_2^2}{2} + \rho g z_2$$

Let  $v_1 \approx 0$ ,  $P_2 = 0$  gage and  $\Delta z = 0.425 \text{ m}$ .

$$P_{\text{air}} = \frac{\rho v_2^2}{2} - \rho g \Delta z$$

$$= \frac{(999 \text{ kg/m}^3) (41.01 \text{ m/s})^2}{2} - (999 \text{ kg/m}^3) (9.81 \text{ m/s}^2) (0.425)$$

$$= (835,900 \text{ Pa}) \left( \frac{1.0 \text{ atm}}{101.3 \text{ kPa}} \right)$$

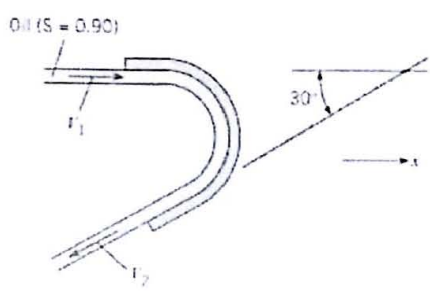
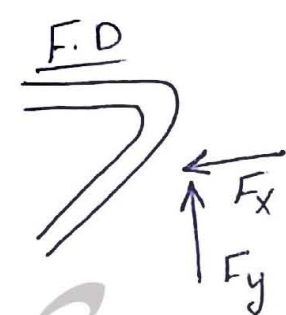
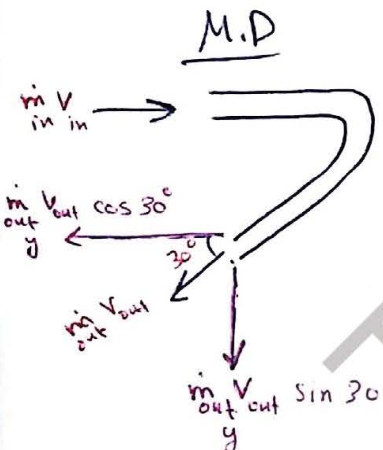
$$P_{\text{air}} = 8.25 \text{ atm}$$

vanes

6.20 Determine the external reactions in the x- and y-directions needed to hold this fixed vane, which turns the oil jet in a horizontal plane. Here  $V_1$  is 18 m/s,  $V_2 = 17$  m/s, and  $Q = 0.15$  m<sup>3</sup>/s.

$S.G. = 0.9$

6.21 Solve Prob. 6.20 for  $V_1 = 30$  m/s,  $V_2 = 25$  m/s, and  $Q = 0.05$  m<sup>3</sup>/s.



$\dot{m} = \rho Q = 0.9 \times 1000 \times 0.15 = 135 \text{ kg/s}$

$V_{in} = 18 \text{ m/s}$

$V_{out} = 17 \text{ m/s}$

$\sum F_x = \sum \dot{m}_{out} V_{out_x} - \sum \dot{m}_{in} V_{in_x}$   
 $-F_x = [135 \times (-17 \times \cos 30^\circ)] - [135 \times 18]$

$-F_x = -1987.53 - 2430$

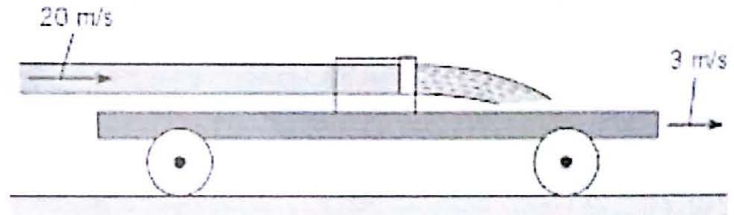
$F_x = 4.417 \text{ kN}$  ← (تجاه اليمين)

$\sum F_y = \sum \dot{m}_{out} V_{out_y} - \sum \dot{m}_{in} V_{in_y}$  (Zero)

$F_y = 135 \times (-17 \times \sin 30^\circ)$

$F_y = -1.1475 \text{ kN}$  ↓ (تجاه الاسفل)

**Q30** A vane on this moving cart deflects a 10 cm water ( $\rho = 1000 \text{ kg/m}^3$ ) jet as shown. The initial speed of the water in the jet is 20 m/s, and the cart moves at a speed of 3 m/s. If the vane splits the jet so that half goes one way and half the other, what force is exerted on the vane by the jet?



Elevation view

$$F_x = m_2 v_{2x} - m_1 v_1$$

$$\dot{m} = \rho A V = (1000 \text{ kg/m}^3) * \left(\frac{\pi}{4} (0.1 \text{ m})^2\right) * (17 \text{ m/s})$$

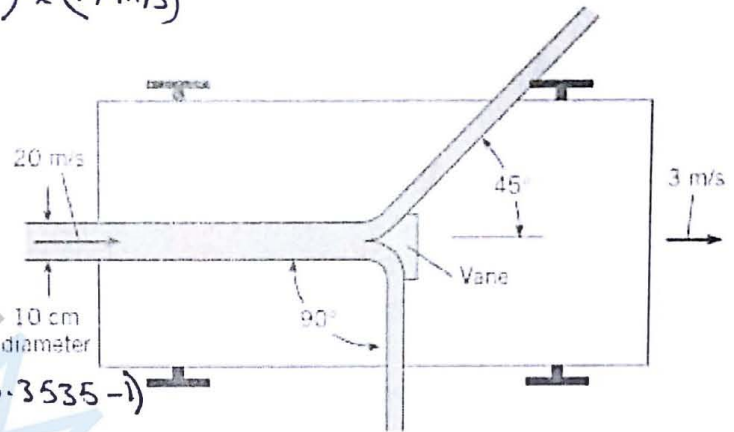
$$= 133.5 \text{ kg/s}$$

$$F_x = \left(\frac{\dot{m}}{2} v \cos 45^\circ - \dot{m} v\right)$$

$$= \dot{m} v \left(\frac{\cos 45^\circ}{2} - 1\right)$$

$$= (133.5 \text{ kg/s}) * (17 \text{ m/s}) * (0.3535 - 1)$$

$$= -1470 \text{ N}$$



Plan view

\* تذكر موضوع السرعة النسبية: "Relative Velocity."

$$F_y = m_2 v_{2y} - m_1 v_{1y}$$

$$= \frac{\dot{m}}{2} v \sin 45^\circ - \frac{\dot{m}}{2} v$$

$$= \frac{\dot{m}}{2} v (\sin 45^\circ - 1)$$

$$= \frac{133.5 \text{ kg/s}}{2} * 17 \text{ m/s} * (0.707 - 1)$$

$$= -332 \text{ N}$$

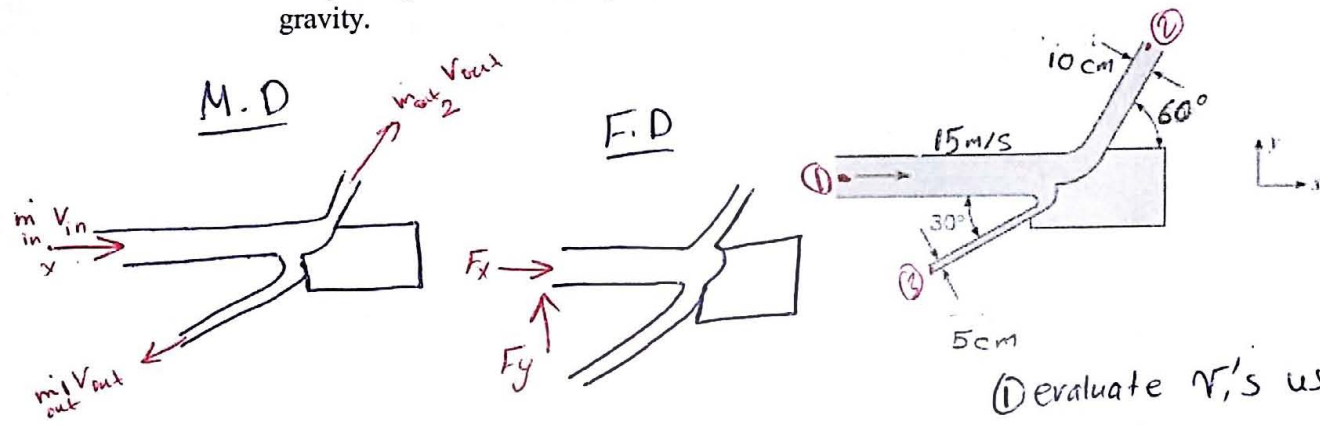
$$V = 20 - 3 = 17 \text{ m/s}$$

... وذلك لان الجسم انه يتحركه بنفسه لا يتحركه

... اذا كانا يتحركانه بنفسه الاياه اجمع السرعة

$$F(\text{water on vane}) = (1470 \text{ i} + 332 \text{ j}) \text{ N}$$

**Q22** This planar water jet (15°C) is deflected by a fixed vane. What are the x- and y-components of force per unit width needed to hold the vane stationary? Neglect gravity.



$$\sum F_x = \sum \dot{m}_{out} V_{out_x} - \sum \dot{m}_{in} V_{in_x}$$

$$F_y = \sum \dot{m}_{out} V_{out_y} - \sum \dot{m}_{in} V_{in_y}$$

complete alone

① evaluate  $V_i$ 's using Bernoulli:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2$$

$$= \frac{P_3}{\rho} + \frac{V_3^2}{2g} + z_3$$

→ all three points at same elevation  
→ all three points are atmospheric

$$\text{So: } V_1 = V_2 = V_3 = 15 \text{ m/s}$$

$$* Q_{total} = Q_2 + Q_3$$

$$Q_2 = V_2 A_2 = 15 \times \frac{\pi}{4} \times 0.1^2 = 0.1177 \text{ m}^3/\text{s}$$

$$Q_3 = V_3 A_3 = 15 \times \frac{\pi}{4} \times 0.05^2 = 0.0294 \text{ m}^3/\text{s}$$

$$Q_1 = Q_2 + Q_3 = 0.1471 \text{ m}^3/\text{s}$$

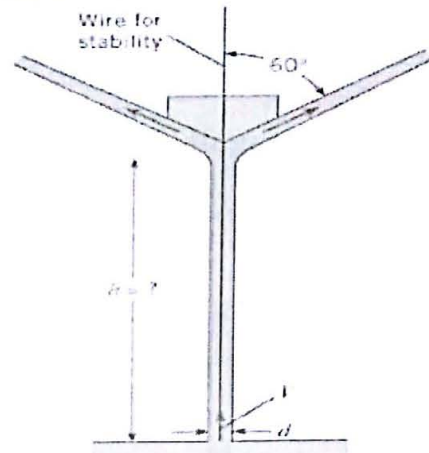
$$\dot{m}_1 = \rho Q_1 = 147.14 \text{ kg/s}$$

$$\dot{m}_2 = \rho Q_2 = 117.7 \text{ kg/s}$$

$$\dot{m}_3 = \rho Q_3 = 29.4 \text{ kg/s}$$

"continuity equation"

**Q28** A cone that is held stable by a wire is free to move in the vertical direction and has a jet of water (at 10°C) striking it from below. The cone weighs 30 N. The initial speed of the jet as it comes from the orifice is 15 m/s, and the initial jet diameter is 2 cm. Find the height to which the cone will rise and remain stationary. *note:* The wire is only for stability and should not enter into your calculations.



$$\frac{V_1^2}{2g} + 0 = \frac{V_2^2}{2g} + h$$

$$V_2^2 = (V_1)^2 - 2gh$$

$$V_2^2 = 225 - 19.62h$$

$$\sum F_y = \dot{m}_o V_{oy} - \dot{m}_i V_{iy}$$

$$-W = \dot{m} (V_{3y} - V_2)$$

$$-30 \text{ N} = (1000 \text{ Kg/m}^3) * (15 \text{ m/s}) * (\pi * (0.01)^2) * (V_2 \sin 30^\circ - V_2)$$

$$V_2 = 12.73 \text{ m/s}$$

$$V_2^2 = 225 - 19.62h$$

$$(12.73)^2 = 225 - 19.62h$$

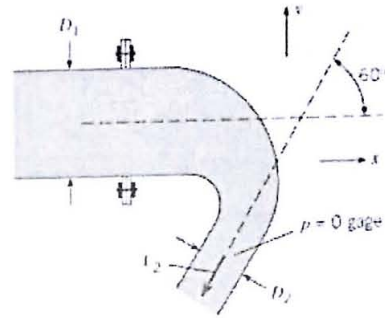
$$h = 3.21 \text{ m}$$

# Practice:

## pipes

Q45 This bend discharges water ( $\rho = 1000 \text{ kg/m}^3$ ) into the atmosphere. Determine the force components at the flange required to hold the bend in place. The bend lies in a horizontal plane. Assume viscous forces are negligible. The interior volume of the bend is  $0.25 \text{ m}^3$ ,  $D_1 = 60 \text{ cm}$ ,  $D_2 = 30 \text{ cm}$ , and  $V_2 = 10 \text{ m/s}$ . The mass of the bend material is  $250 \text{ kg}$ .

\* solve alone, same Idea, so easy

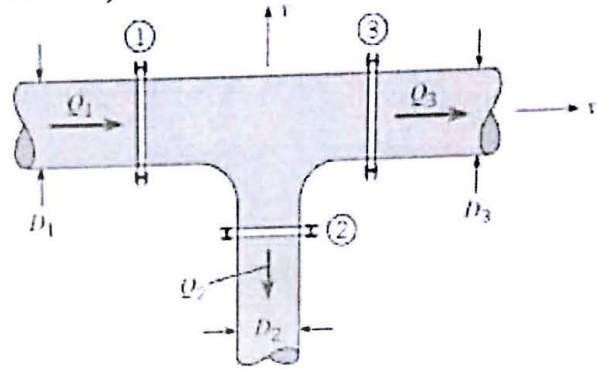


Kingdom of Fluid mechanics = Idea + trial + comprehension

best wishes --- ♥



**Q55** For this horizontal T through which water ( $\rho = 1000 \text{ kg/m}^3$ ) is flowing, the following data are given:  
 $Q_1 = 0.25 \text{ m}^3/\text{s}$ ,  $Q_2 = 0.10 \text{ m}^3/\text{s}$ ,  $p_1 = 100 \text{ kPa}$ ,  $p_2 = 70 \text{ kPa}$ ,  $p_3 = 80 \text{ kPa}$ ,  $D_1 = 15 \text{ cm}$ ,  $D_2 = 7 \text{ cm}$ , and  $D_3 = 15 \text{ cm}$ . For these conditions, what external force in the  $x$ - $y$  plane (through the bolts or other supporting devices) is needed to hold the T in place?



\* Solve this alone

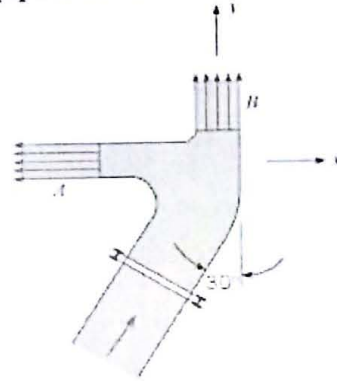
So easy; ^^

Fluid Mechanics; try yourself!!



## Nozzles

**Q63** This "double" nozzle discharges water ( $\rho = 1000 \text{ kg/m}^3$ ) into the atmosphere at a rate of  $0.8 \text{ m}^3/\text{s}$ . If the nozzle is lying in a horizontal plane, what  $x$ -component of force acting through the flange bolts is required to hold the nozzle in place? *note:* Assume irrotational flow, and assume the water speed in each jet to be the same. Jet  $A$  is  $20 \text{ cm}$  in diameter, jet  $B$  is  $22 \text{ cm}$  in diameter, and the pipe is  $60 \text{ cm}$  in diameter.



Solve this alone;

Practice!!

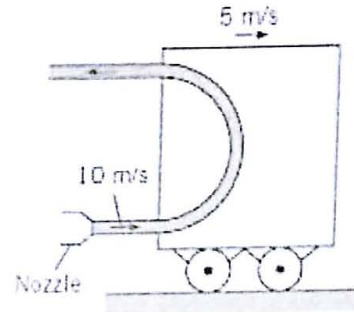
\* you have to solve some examples & problems ---

there are here --- Just Do it ^^

Applications " nonstationary "

Q86 A cart is moving along a track at a constant velocity of 5 m/s as shown. Water ( $\rho = 1000 \text{ kg/m}^3$ ) issues from a nozzle at 10 m/s and is deflected through  $180^\circ$  by a vane on the cart. The cross-sectional area of the nozzle is  $0.0012 \text{ m}^2$ . Calculate the resistive force on the cart.

Solve this alone 😊



Fluid Mechanics --- love it --- dive it

Best wishes all ---

Aboud Samee7