Chapter 5

5-1 S_y = 350 MPa.

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

$$n = \frac{S_y}{\sigma'}$$

(a) MSS: σ_1 = 100 MPa, σ_2 = 100 MPa, σ_3 = 0

$$n = \frac{350}{100 - 0} = 3.5 \quad \textit{Ans.}$$

DE: $\sigma' = (100^2 - 100(100) + 100^2)^{1/2} = 100$ MPa, $\quad n = \dfrac{350}{100} = 3.5 \quad$ *Ans.*

(b) MSS: σ_1 = 100 MPa, σ_2 = 50 MPa, σ_3 = 0

$$n = \frac{350}{100 - 0} = 3.5 \quad \textit{Ans.}$$

DE: $\sigma' = (100^2 - 100(50) + 50^2)^{1/2} = 86.6$ MPa, $\quad n = \dfrac{350}{86.6} = 4.04 \quad$ *Ans.*

(c) $\sigma_A,\ \sigma_B = \dfrac{100}{2} \pm \sqrt{\left(\dfrac{100}{2}\right)^2 + (-75)^2} = 140,\ -40$ MPa

$\sigma_1 = 140,\ \sigma_2 = 0,\ \sigma_3 = -40$ MPa

MSS: $n = \dfrac{350}{140 - (-40)} = 1.94 \quad$ *Ans.*

DE: $\sigma' = \left[100^2 + 3\left(-75^2\right)\right]^{1/2} = 164$ MPa, $\quad n = \dfrac{350}{164} = 2.13 \quad$ *Ans.*

(d) $\sigma_A, \sigma_B = \dfrac{-50 - 75}{2} \pm \sqrt{\left(\dfrac{-50 + 75}{2}\right)^2 + (-50)^2} = -11.0, -114.0$ MPa

$\sigma_1 = 0,\ \sigma_2 = -11.0,\ \sigma_3 = -114.0$ MPa

MSS: $n = \dfrac{350}{0 - (-114.0)} = 3.07 \quad$ *Ans.*

DE: $\sigma' = [(-50)^2 - (-50)(-75) + (-75)^2 + 3(-50)^2]^{1/2} = 109.0$ MPa

$$n = \frac{350}{109.0} = 3.21 \quad \textit{Ans.}$$

(e) $\sigma_A, \sigma_B = \dfrac{100 + 20}{2} \pm \sqrt{\left(\dfrac{100 - 20}{2}\right)^2 + \left(-20\right)^2} = 104.7,\ 15.3$ MPa

$\sigma_1 = 104.7,\ \sigma_2 = 15.3,\ \sigma_3 = 0$ MPa

MSS: $n = \dfrac{350}{104.7 - 0} = 3.34$ *Ans.*

DE: $\sigma' = \left[100^2 - 100(20) + 20^2 + 3\left(-20\right)^2\right]^{1/2} = 98.0$ MPa

$$n = \frac{350}{98.0} = 3.57 \quad \textit{Ans.}$$

5-2 $S_y = 350$ MPa.

MSS: $\sigma_1 - \sigma_3 = S_y / n \quad \Rightarrow \quad n = \dfrac{S_y}{\left(\sigma_1 - \sigma_3\right)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \dfrac{S_y}{n} \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma'}$

(a) MSS: $\sigma_1 = 100$ MPa, $\sigma_3 = 0 \Rightarrow n = \dfrac{350}{100 - 0} = 3.5$ *Ans.*

DE: $n = \dfrac{350}{[100^2 - (100)(100) + 100^2]^{1/2}} = 3.5$ *Ans.*

(b) MSS: $\sigma_1 = 100\ ,\ \sigma_3 = -100$ MPa $\Rightarrow n = \dfrac{350}{100 - (-100)} = 1.75$ *Ans.*

DE: $n = \dfrac{350}{\left[100^2 - (100)(-100) + \left(-100\right)^2\right]^{1/2}} = 2.02$ *Ans.*

(c) MSS: $\sigma_1 = 100$ MPa, $\sigma_3 = 0 \Rightarrow n = \dfrac{350}{100 - 0} = 3.5$ *Ans.*

DE: $n = \dfrac{350}{\left[100^2 - (100)(50) + 50^2\right]^{1/2}} = 4.04$ *Ans.*

(d) MSS: $\sigma_1 = 100,\ \sigma_3 = -50$ MPa $\Rightarrow n = \dfrac{350}{100 - (-50)} = 2.33$ *Ans.*

DE: $n = \dfrac{350}{\left[100^2 - (100)(-50) + \left(-50\right)^2\right]^{1/2}} = 2.65$ *Ans.*

(e) MSS: $\sigma_1 = 0,\ \sigma_3 = -100$ MPa $\Rightarrow n = \dfrac{350}{0 - (-100)} = 3.5$ *Ans.*

DE: $n = \dfrac{350}{\left[\left(-50\right)^2 - (-50)(-100) + \left(-100\right)^2\right]^{1/2}} = 4.04$ *Ans.*

5-3 From Table A-20, $S_y = 37.5$ kpsi

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

$n = \dfrac{S_y}{\sigma'}$

(a) MSS: $\sigma_1 = 25$ kpsi, $\sigma_3 = 0 \quad \Rightarrow \quad n = \dfrac{37.5}{25-0} = 1.5$ *Ans.*

DE: $n = \dfrac{37.5}{\left[25^2 - (25)(15) + 15^2\right]^{1/2}} = 1.72$ *Ans.*

(b) MSS: $\sigma_1 = 15$ kpsi, $\sigma_3 = -15 \quad \Rightarrow \quad n = \dfrac{37.5}{15-(-15)} = 1.25$ *Ans.*

DE: $n = \dfrac{37.5}{\left[15^2 - (15)(-15) + (-15)^2\right]^{1/2}} = 1.44$ *Ans.*

(c) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1$ kpsi

$\sigma_1 = 24.1,\ \sigma_2 = 0,\ \sigma_3 = -4.1$ kpsi

MSS: $n = \dfrac{37.5}{24.1-(-4.1)} = 1.33$ *Ans.*

DE: $\sigma' = \left[20^2 + 3\left(-10^2\right)\right]^{1/2} = 26.5$ kpsi $\quad \Rightarrow \quad n = \dfrac{37.5}{26.5} = 1.42$ *Ans.*

(d) $\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7$ kpsi

$\sigma_1 = 17.7,\ \sigma_2 = 0,\ \sigma_3 = -14.7$ kpsi

MSS: $n = \dfrac{37.5}{17.7-(-14.7)} = 1.16$ *Ans.*

DE: $\sigma' = \left[(-12)^2 - (-12)(15) + 15^2 + 3(-9)^2\right]^{1/2} = 28.1$ kpsi

$n = \dfrac{37.5}{28.1} = 1.33$ *Ans.*

(e) $\sigma_A, \sigma_B = \dfrac{-24-24}{2} \pm \sqrt{\left(\dfrac{-24+24}{2}\right)^2 + (-15)^2} = -9,\ -39$ kpsi

$\sigma_1 = 0,\ \sigma_2 = -9,\ \sigma_3 = -39$ kpsi

MSS: $n = \dfrac{37.5}{0-(-39)} = 0.96$ *Ans.*

DE: $\sigma' = \left[(-24)^2 - (-24)(-24) + (-24)^2 + 3(-15)^2\right]^{1/2} = 35.4 \text{ kpsi}$

$$n = \frac{37.5}{35.4} = 1.06 \quad \textit{Ans.}$$

5-4 From Table A-20, S_y = 47 kpsi.

MSS: $\sigma_1 - \sigma_3 = S_y / n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \dfrac{S_y}{n} \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma'}$

(a) MSS: $\sigma_1 = 30 \text{ kpsi}, \ \sigma_3 = 0 \ \Rightarrow \ n = \dfrac{47}{30-0} = 1.57 \quad \textit{Ans.}$

DE: $n = \dfrac{47}{[30^2 - (30)(30) + 30^2]^{1/2}} = 1.57 \quad \textit{Ans.}$

(b) MSS: $\sigma_1 = 30 \ , \ \sigma_3 = -30 \text{ kpsi} \ \Rightarrow \ n = \dfrac{47}{30-(-30)} = 0.78 \quad \textit{Ans.}$

DE: $n = \dfrac{47}{\left[30^2 - (30)(-30) + (-30)^2\right]^{1/2}} = 0.90 \quad \textit{Ans.}$

(c) MSS: $\sigma_1 = 30 \text{ kpsi}, \ \sigma_3 = 0 \ \Rightarrow \ n = \dfrac{47}{30-0} = 1.57 \quad \textit{Ans.}$

DE: $n = \dfrac{47}{\left[30^2 - (30)(15) + 15^2\right]^{1/2}} = 1.81 \quad \textit{Ans.}$

(d) MSS: $\sigma_1 = 0, \ \sigma_3 = -30 \text{ kpsi} \ \Rightarrow \ n = \dfrac{47}{0-(-30)} = 1.57 \quad \textit{Ans.}$

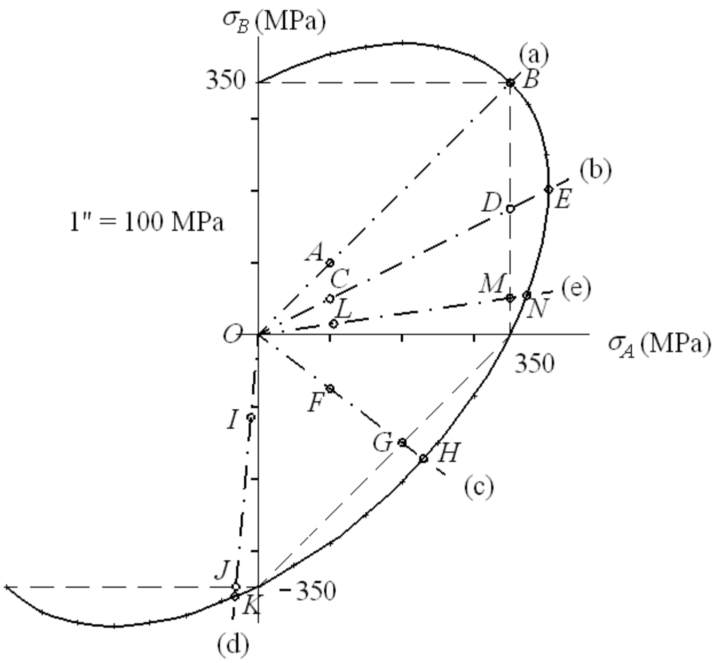
DE: $n = \dfrac{47}{\left[(-30)^2 - (-30)(-15) + (-15)^2\right]^{1/2}} = 1.81 \quad \textit{Ans.}$

(**e**) MSS: $\sigma_1 = 10, \ \sigma_3 = -50 \text{ kpsi} \ \Rightarrow \ n = \dfrac{47}{10-(-50)} = 0.78 \quad \textit{Ans.}$

DE: $n = \dfrac{47}{\left[(-50)^2 - (-50)(10) + 10^2\right]^{1/2}} = 0.84 \quad \textit{Ans.}$

5-5 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

(**a**) MSS and DE:



$$n = \frac{OB}{OA} = \frac{4.95"}{1.41"} = 3.51 \quad Ans.$$

(**b**) MSS:

$$n = \frac{OD}{OC} = \frac{3.91"}{1.12"} = 3.49 \quad Ans.$$

DE:

$$n = \frac{OE}{OC} = \frac{4.51"}{1.12"} = 4.03 \quad Ans.$$

(**c**) MSS:

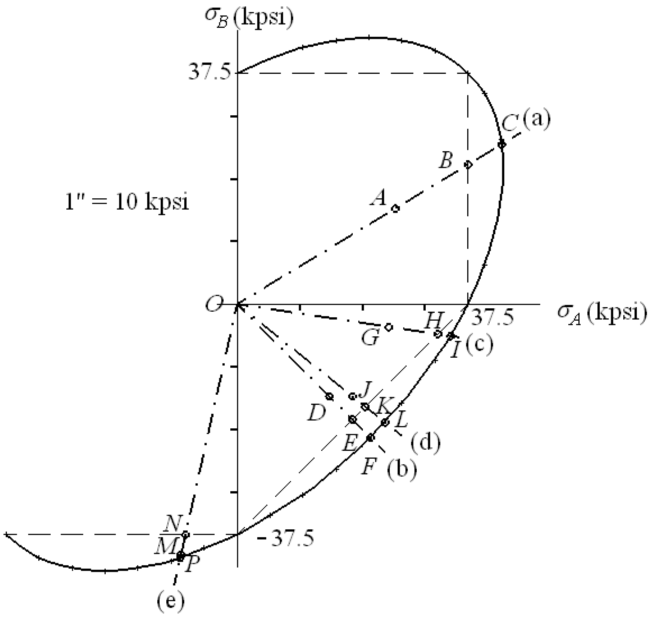
$$n = \frac{OG}{OF} = \frac{2.50"}{1.25"} = 2.00 \quad Ans.$$

DE: $n = \frac{OH}{OF} = \frac{2.86"}{1.25"} = 2.29 \quad Ans.$

(**d**) MSS: $n = \frac{OJ}{OI} = \frac{3.51"}{1.15"} = 3.05 \quad Ans.$, DE: $n = \frac{OK}{OI} = \frac{3.65"}{1.15"} = 3.17 \quad Ans.$

(**e**) MSS: $n = \frac{OM}{OL} = \frac{3.54"}{1.06"} = 3.34 \quad Ans.$, DE: $n = \frac{ON}{OL} = \frac{3.77"}{1.06"} = 3.56 \quad Ans.$

5-6 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.



(**a**) σ_A = 25 kpsi, σ_B = 15 kpsi

MSS:

$$n = \frac{OB}{OA} = \frac{4.37''}{2.92''} = 1.50 \quad Ans.$$

DE:

$$n = \frac{OC}{OA} = \frac{5.02''}{2.92''} = 1.72 \quad Ans.$$

(**b**) σ_A = 15 kpsi, σ_B = – 15 kpsi

MSS:

$$n = \frac{OE}{OD} = \frac{2.66''}{2.12''} = 1.25 \quad Ans.$$

DE:

$$n = \frac{OF}{OD} = \frac{3.05''}{2.12''} = 1.44 \quad Ans.$$

(**c**) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1$ kpsi

MSS: $n = \dfrac{OH}{OG} = \dfrac{3.25''}{2.43''} = 1.34 \quad Ans.$ DE: $n = \dfrac{OI}{OG} = \dfrac{3.46''}{2.43''} = 1.42 \quad Ans.$

(**d**) $\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7$ MPa

MSS: $n = \dfrac{OK}{OJ} = \dfrac{2.67''}{2.30''} = 1.16 \quad Ans.$ DE: $n = \dfrac{OL}{OJ} = \dfrac{3.06''}{2.30''} = 1.33 \quad Ans.$

(**e**) $\sigma_A, \sigma_B = \dfrac{-24-24}{2} \pm \sqrt{\left(\dfrac{-24+24}{2}\right)^2 + (-15)^2} = -9,\ -39$ kpsi

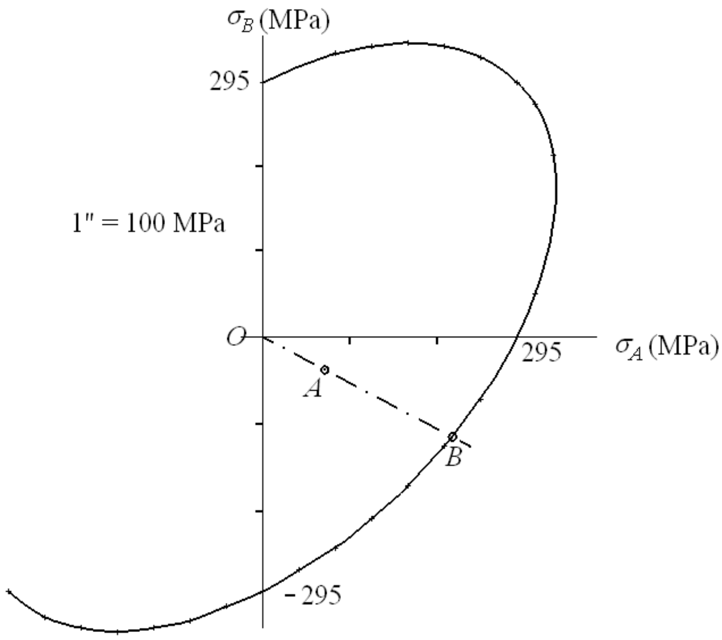
MSS: $n = \dfrac{ON}{OM} = \dfrac{3.85''}{4.00''} = 0.96 \quad Ans.$ DE: $n = \dfrac{OP}{OM} = \dfrac{4.23''}{4.00''} = 1.06 \quad Ans.$

5-7 S_y = 295 MPa, σ_A = 75 MPa, σ_B = – 35 MPa,

(**a**) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[75^2 - 75(-35) + (-35)^2\right]^{1/2}} = 3.03 \quad Ans.$$

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.50"}{0.83"} = 3.01 \quad Ans.$$

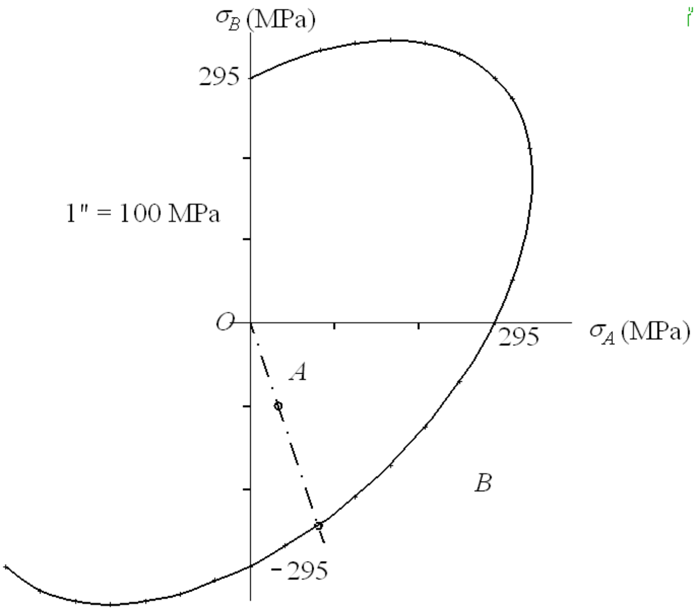


5-8 S_y = 295 MPa, σ_A = 30 MPa, σ_B = – 100 MPa,

(**a**) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[30^2 - 30(-100) + (-100)^2\right]^{1/2}} = 2.50 \quad Ans.$$

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.50"}{0.83"} = 3.01 \quad Ans.$$

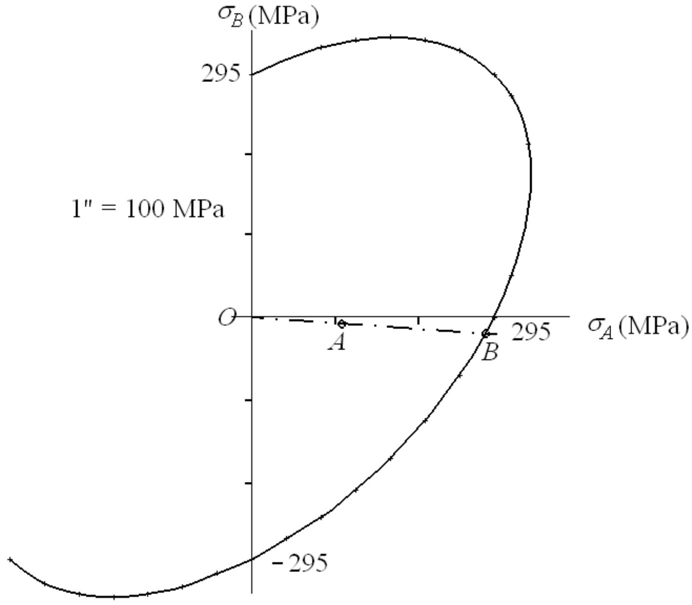


5-9 $S_y = 295$ MPa, $\sigma_A,\ \sigma_B = \dfrac{100}{2} \pm \sqrt{\left(\dfrac{100}{2}\right)^2 + (-25)^2} = 105.9,\ -5.9$ MPa

(**a**) $n = \dfrac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \dfrac{295}{\left[105.9^2 - 105.9(-5.9) + (-5.9)^2\right]^{1/2}} = 2.71$ *Ans.*

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$n = \dfrac{OB}{OA} = \dfrac{2.87"}{1.06"} = 2.71$ *Ans.*

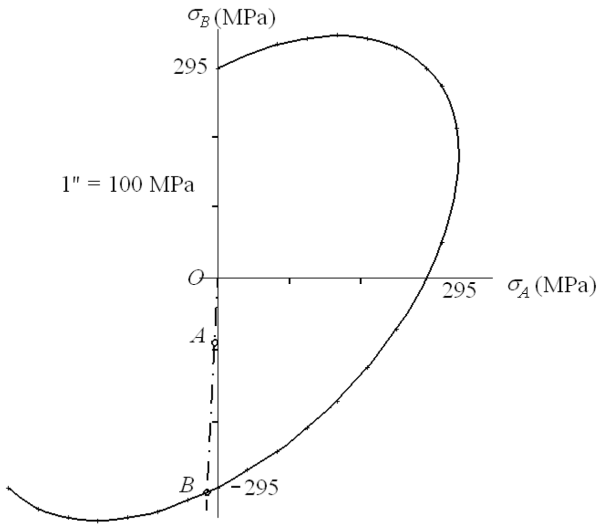


5-10 S_y = 295 MPa, $\sigma_A, \sigma_B = \dfrac{-30-65}{2} \pm \sqrt{\left(\dfrac{-30+65}{2}\right)^2 + 40^2} = -3.8, -91.2$ MPa

(**a**) $$n = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{295}{\left[(-3.8)^2 - (-3.8)(-91.2) + (-91.2)^2\right]^{1/2}} = 3.30 \quad \textit{Ans.}$$

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{3.00''}{0.90''} = 3.33 \quad \textit{Ans.}$$

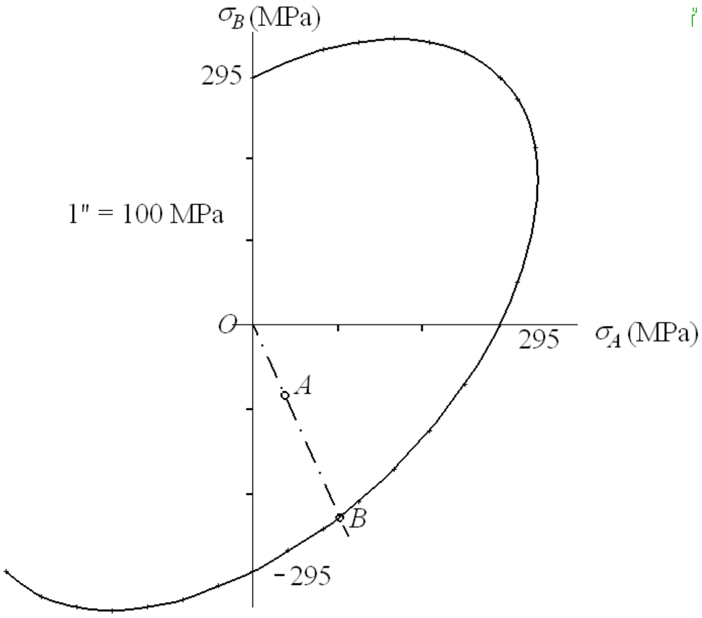


5-11 S_y = 295 MPa, $\sigma_A, \sigma_B = \dfrac{-80+30}{2} \pm \sqrt{\left(\dfrac{-80-30}{2}\right)^2 + (-10)^2} = 30.9, -80.9$ MPa

(**a**) $n = \dfrac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \dfrac{295}{\left[30.9^2 - 30.9(-80.9) + (-80.9)^2\right]^{1/2}} = 2.95$ *Ans.*

(**b**) Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.55"}{0.87"} = 2.93 \quad Ans.$$



5-12 S_{yt} = 60 kpsi, S_{yc} = 75 kpsi. Eq. (5-26) for yield is

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1}$$

(**a**) $\sigma_1 = 25$ kpsi, $\sigma_3 = 0 \quad \Rightarrow \quad n = \left(\dfrac{25}{60} - \dfrac{0}{75}\right)^{-1} = 2.40$ *Ans.*

(**b**) $\sigma_1 = 15$, $\sigma_3 = -15$ kpsi $\quad \Rightarrow \quad n = \left(\dfrac{15}{60} - \dfrac{-15}{75}\right)^{-1} = 2.22$ *Ans.*

(c) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1 \text{ kpsi}$,

$\sigma_1 = 24.1,\ \sigma_2 = 0,\ \sigma_3 = -4.1 \text{ kpsi} \ \Rightarrow \quad n = \left(\dfrac{24.1}{60} - \dfrac{-4.1}{75}\right)^{-1} = 2.19 \quad$ *Ans.*

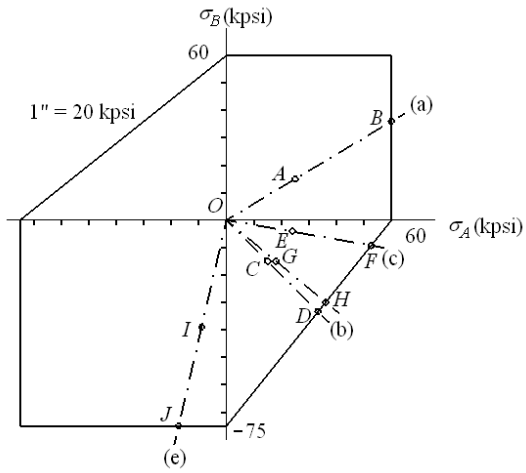
(d) $\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ kpsi}$

$\sigma_1 = 17.7,\ \sigma_2 = 0,\ \sigma_3 = -14.7 \text{ kpsi} \quad \Rightarrow \quad n = \left(\dfrac{17.7}{60} - \dfrac{-14.7}{75}\right)^{-1} = 2.04 \quad$ *Ans.*

(e) $\sigma_A, \sigma_B = \dfrac{-24-24}{2} \pm \sqrt{\left(\dfrac{-24+24}{2}\right)^2 + \left(-15\right)^2} = -9,\ -39 \text{ kpsi}$

$\sigma_1 = 0,\ \sigma_2 = -9,\ \sigma_3 = -39 \text{ kpsi} \ \Rightarrow \quad n = \left(\dfrac{0}{60} - \dfrac{-39}{75}\right)^{-1} = 1.92 \quad$ *Ans.*

5-13 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.



(a) $\sigma_A = 25,\ \sigma_B = 15 \text{ kpsi}$

$n = \dfrac{OB}{OA} = \dfrac{3.49"}{1.46"} = 2.39 \quad$ *Ans.*

(b) $\sigma_A = 15,\ \sigma_B = -15 \text{ kpsi}$

$n = \dfrac{OD}{OC} = \dfrac{2.36"}{1.06"} = 2.23 \quad$ *Ans.*

(c)

$\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1,\ -4.1 \text{ kpsi}$

$n = \dfrac{OF}{OE} = \dfrac{2.67"}{1.22"} = 2.19 \quad$ *Ans.*

(d)

$\sigma_A, \sigma_B = \dfrac{-12+15}{2} \pm \sqrt{\left(\dfrac{-12-15}{2}\right)^2 + (-9)^2} = 17.7, -14.7 \text{ kpsi}$

$$n = \frac{OH}{OG} = \frac{2.34"}{1.15"} = 2.03 \quad Ans.$$

(**e**)

$$\sigma_A, \sigma_B = \frac{-24-24}{2} \pm \sqrt{\left(\frac{-24+24}{2}\right)^2 + (-15)^2} = -9, \ -39 \text{ kpsi}$$

$$n = \frac{OJ}{OI} = \frac{3.85"}{2.00"} = 1.93 \quad Ans.$$

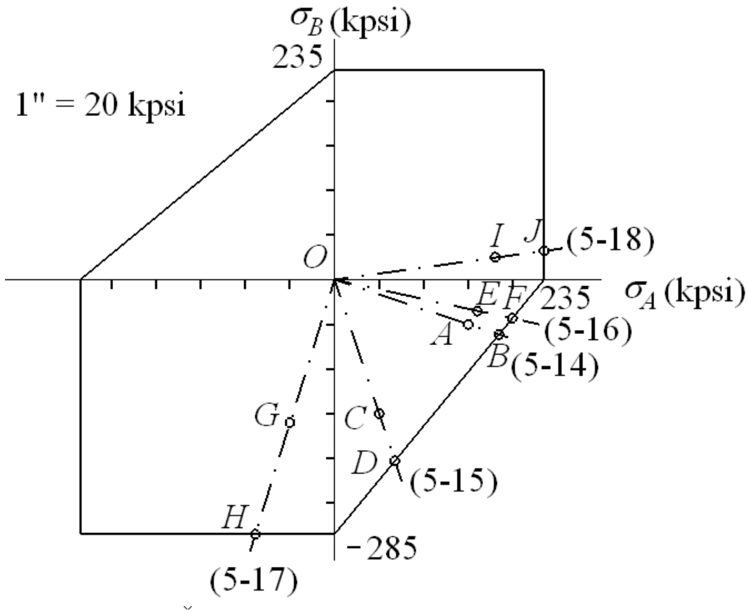
5-14 Since $\varepsilon_f > 0.05$, and $S_{yt} \neq S_{yc}$, the Coulomb-Mohr theory for ductile materials will be used.

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{150}{235} - \frac{-50}{285}\right)^{-1} = 1.23 \quad Ans.$$

(**b**) Plots for Problems 5-14 to 5-18 are found here. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{1.94"}{1.58"} = 1.23 \quad Ans.$$



5-15 (**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{50}{235} - \frac{-150}{285}\right)^{-1} = 1.35 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OD}{OC} = \frac{2.14''}{1.58''} = 1.35 \quad Ans.$$

5-16 $$\sigma_A,\ \sigma_B = \frac{125}{2} \pm \sqrt{\left(\frac{125}{2}\right)^2 + (-75)^2} = 160,\ -35 \text{ kpsi}$$

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{160}{235} - \frac{-35}{285}\right)^{-1} = 1.24 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OF}{OE} = \frac{2.04''}{1.64''} = 1.24 \quad Ans.$$

5-17 $$\sigma_A,\ \sigma_B = \frac{-80-125}{2} \pm \sqrt{\left(\frac{-80+125}{2}\right)^2 + 50^2} = -47.7,\ -157.3 \text{ kpsi}$$

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}\right)^{-1} = \left(\frac{0}{235} - \frac{-157.3}{285}\right)^{-1} = 1.81 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OH}{OG} = \frac{2.99''}{1.64''} = 1.82 \quad Ans.$$

5-18 $$\sigma_A,\ \sigma_B = \frac{125+80}{2} \pm \sqrt{\left(\frac{125-80}{2}\right)^2 + (-75)^2} = 180.8,\ 24.2 \text{ kpsi}$$

(**a**) From Eq. (5-26),

$$n = \left(\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} \right)^{-1} = \left(\frac{180.8}{235} - \frac{0}{285} \right)^{-1} = 1.30 \quad Ans.$$

(**b**) The plot for this problem is found on the page for Prob. 5-14. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OJ}{OI} = \frac{2.37''}{1.83''} = 1.30 \quad Ans.$$

5-19 S_{ut} = 30 kpsi, S_{uc} = 90 kpsi

BCM: Eqs. (5-31), p. 250
MM: see Eqs. (5-32), p. 250

(**a**) σ_A = 25 kpsi, σ_B = 15 kpsi

BCM : Eq. (5-31*a*), $n = \frac{S_{ut}}{\sigma_A} = \frac{30}{25} = 1.2 \quad Ans.$

MM: Eq. (5-32*a*), $n = \frac{S_{ut}}{\sigma_A} = \frac{30}{25} = 1.2 \quad Ans.$

(**b**) σ_A = 15 kpsi, σ_B = −15 kpsi,

BCM: Eq. (5-31*a*), $n = \left(\frac{15}{30} - \frac{-15}{90} \right)^{-1} = 1.5 \quad Ans.$

MM: $\sigma_A \ge 0 \ge \sigma_B$, and $|\sigma_B / \sigma_A| \le 1$, Eq. (5-32a), $n = \frac{S_{ut}}{\sigma_A} = \frac{30}{15} = 2.0 \quad Ans.$

(**c**) $\sigma_A,\ \sigma_B = \frac{20}{2} \pm \sqrt{\left(\frac{20}{2}\right)^2 + (-10)^2} = 24.14,\ -4.14$ kpsi

BCM: Eq. (5-31*b*), $n = \left(\frac{24.14}{30} - \frac{-4.14}{90} \right)^{-1} = 1.18 \quad Ans.$

MM: $\sigma_A \ge 0 \ge \sigma_B$, and $|\sigma_B / \sigma_A| \le 1$, Eq. (5-32*a*), $n = \frac{S_{ut}}{\sigma_A} = \frac{30}{24.14} = 1.24 \quad Ans.$

(**d**) $\sigma_A,\ \sigma_B = \frac{-15+10}{2} \pm \sqrt{\left(\frac{-15-10}{2}\right)^2 + (-15)^2} = 17.03,\ -22.03$ kpsi

BCM: Eq. (5-31*b*), $n = \left(\dfrac{17.03}{30} - \dfrac{-22.03}{90} \right)^{-1} = 1.23 \quad Ans.$

MM: $\sigma_A \geq 0 \geq \sigma_B$, and $|\sigma_B / \sigma_A| \geq 1$, Eq. (5-32*b*),

$$n = \left[\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} \right]^{-1} = \left[\frac{(90-30)17.03}{90(30)} - \frac{-22.03}{90} \right]^{-1} = 1.60 \quad Ans.$$

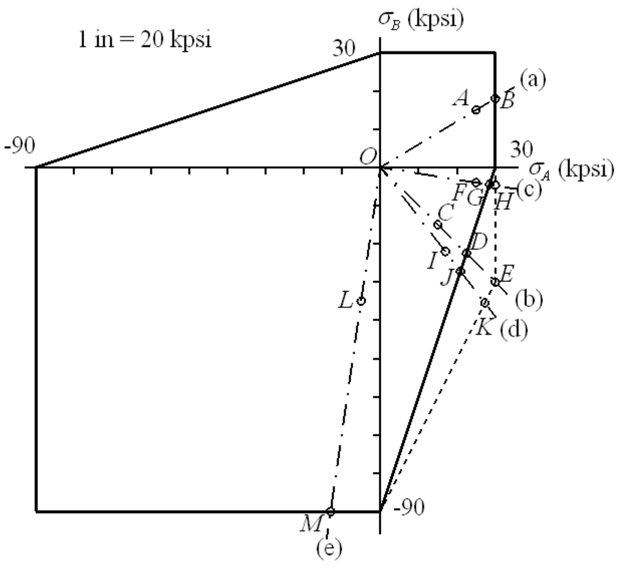
(**e**) $\sigma_A,\ \sigma_B = \dfrac{-20-20}{2} \pm \sqrt{\left(\dfrac{-20+20}{2}\right)^2 + (-15)^2} = -5,\ -35$ kpsi

BCM: Eq. (5-31*c*), $n = -\dfrac{S_{uc}}{\sigma_B} = -\dfrac{90}{-35} = 2.57 \quad Ans.$

MM: Eq. (5-32*c*), $n = -\dfrac{S_{uc}}{\sigma_B} = -\dfrac{90}{-35} = 2.57 \quad Ans.$

5-20 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

(**a**) σ_A = 25, σ_B = 15 kpsi



BCM & MM:

$n = \dfrac{OB}{OA} = \dfrac{1.74"}{1.46"} = 1.19 \quad Ans.$

(**b**) σ_A = 15, σ_B = – 15 kpsi

BCM: $n = \dfrac{OC}{OD} = \dfrac{1.59"}{1.06"} = 1.5 \quad Ans.$

MM: $n = \dfrac{OE}{OC} = \dfrac{2.12"}{1.06"} = 2.0 \quad Ans.$

(**c**) $\sigma_A,\ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2}$

$= 24.14,\ -4.14$ kpsi

BCM: $n = \dfrac{OG}{OF} = \dfrac{1.44"}{1.22"} = 1.18 \quad Ans.$

MM: $n = \dfrac{OH}{OF} = \dfrac{1.52"}{1.22"} = 1.25 \quad Ans.$

(**d**) $\sigma_A,\ \sigma_B = \dfrac{-15+10}{2} \pm \sqrt{\left(\dfrac{-15-10}{2}\right)^2 + (-15)^2} = 17.03,\ -22.03$ kpsi

BCM: $n = \dfrac{OJ}{OI} = \dfrac{1.72"}{1.39"} = 1.24$ *Ans.*

MM: $n = \dfrac{OK}{OI} = \dfrac{2.24"}{1.39"} = 1.61$ *Ans.*

(**e**) $\sigma_A,\ \sigma_B = \dfrac{-20-20}{2} \pm \sqrt{\left(\dfrac{-20+20}{2}\right)^2 + (-15)^2} = -5,\ -35$ kpsi

BCM and MM: $n = \dfrac{OM}{OL} = \dfrac{4.55"}{1.77"} = 2.57$ *Ans.*

5-21 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eqs. (5-31), MM: Eqs. (5-32)

(**a**) σ_A = 15, σ_B = 10 kpsi.

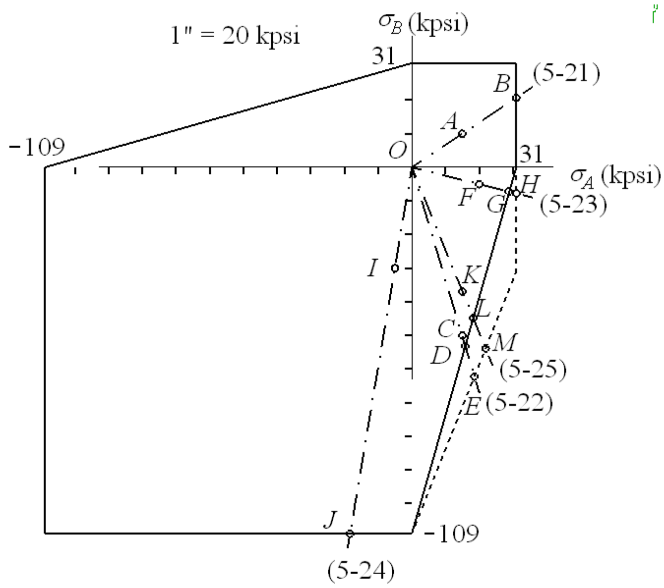
BCM: Eq. (5-31*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{31}{15} = 2.07$ *Ans.*

MM: Eq. (5-32*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{31}{15} = 2.07$ *Ans.*

(**b**), (**c**) The plot is shown below is for Probs. 5-21 to 5-25. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

BCM and MM:

$$n = \frac{OB}{OA} = \frac{1.86"}{0.90"} = 2.07 \quad Ans.$$



5-22 S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

(**a**) σ_A = 15, σ_B = – 50 kpsi, $|\sigma_B / \sigma_A| > 1$

BCM: Eq. (5-31*b*), $n = \left(\dfrac{15}{31} - \dfrac{-50}{109} \right)^{-1} = 1.06$ *Ans.*

MM: Eq. (3-32*b*), $n = \left[\dfrac{(S_{uc} - S_{ut})\sigma_A}{S_{uc} S_{ut}} - \dfrac{\sigma_B}{S_{uc}} \right]^{-1} = \left[\dfrac{(109-31)15}{109(31)} - \dfrac{-50}{109} \right]^{-1} = 1.24$ *Ans.*

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM: $n = \dfrac{OD}{OC} = \dfrac{2.78"}{2.61"} = 1.07$ *Ans.*

MM: $n = \dfrac{OE}{OC} = \dfrac{3.25"}{2.61"} = 1.25$ *Ans.*

5-23 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A,\ \sigma_B = \frac{15}{2} \pm \sqrt{\left(\frac{15}{2}\right)^2 + (-10)^2} = 20,\ -5 \text{ kpsi}$$

(**a**) BCM: Eq. (5-32*b*), $n = \left(\dfrac{20}{31} - \dfrac{-5}{109} \right)^{-1} = 1.45$ *Ans.*

MM: Eq. (5-32*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{31}{20} = 1.55$ *Ans.*

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM: $n = \dfrac{OG}{OF} = \dfrac{1.48"}{1.03"} = 1.44$ *Ans.*

MM: $n = \dfrac{OH}{OF} = \dfrac{1.60"}{1.03"} = 1.55$ *Ans.*

5-24 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A,\ \sigma_B = \frac{-10-25}{2} \pm \sqrt{\left(\frac{-10+25}{2}\right)^2 + (-10)^2} = -5,\ -30 \text{ kpsi}$$

(**a**) BCM: Eq. (5-31*c*), $n = -\dfrac{S_{uc}}{\sigma_B} - \dfrac{109}{-30} = 3.63$ *Ans.*

MM: Eq. (5-32*c*), $n = -\dfrac{S_{uc}}{\sigma_B} = -\dfrac{109}{-30} = 3.63$ *Ans.*

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM and MM: $n = \dfrac{OJ}{OI} = \dfrac{5.53''}{1.52''} = 3.64$ *Ans.*

5-25 From Table A-24, S_{ut} = 31 kpsi, S_{uc} = 109 kpsi

BCM: Eq. (5-31), MM: Eqs. (5-32)

$$\sigma_A,\ \sigma_B = \frac{-35+13}{2} \pm \sqrt{\left(\frac{-35-13}{2}\right)^2 + (-10)^2} = 15,\ -37 \text{ kpsi}$$

(**a**) BCM: Eq. (5-31*b*), $n = \left(\dfrac{15}{31} - \dfrac{-37}{109}\right)^{-1} = 1.21$ *Ans.*

MM: Eq. (5-32*b*), $n = \left[\dfrac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \dfrac{\sigma_B}{S_{uc}}\right]^{-1} = \left[\dfrac{(109-31)15}{109(31)} - \dfrac{-37}{109}\right]^{-1} = 1.46$ *Ans.*

(**b**), (**c**) The plot is shown in the solution to Prob. 5-21.

BCM: $n = \dfrac{OL}{OK} = \dfrac{2.42''}{2.00''} = 1.21$ *Ans.*

MM: $n = \dfrac{OM}{OK} = \dfrac{2.91''}{2.00''} = 1.46$ *Ans.*

5-26 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

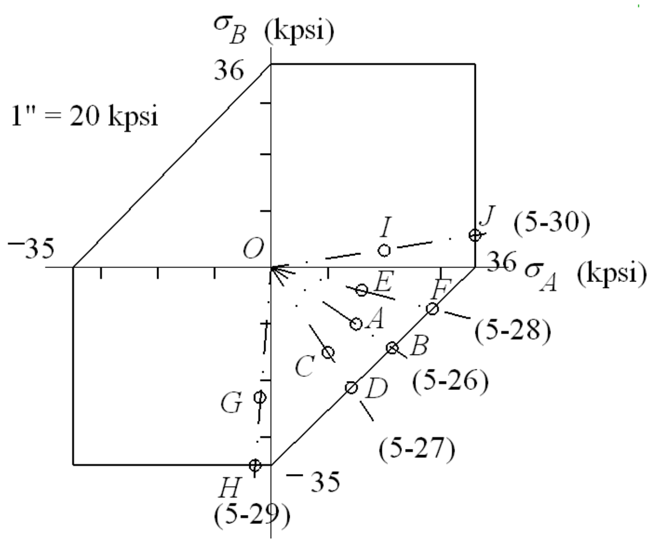
BCM: Eq. (5-31),

(**a**) σ_A = 15, σ_B = – 10 kpsi.

BCM: Eq. (5-31*b*), $n = \left(\dfrac{15}{36} - \dfrac{-10}{35}\right)^{-1} = 1.42$ *Ans.*

(**b**) The plot is shown below is for Probs. 5-26 to 5-30. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{1.28"}{0.90"} = 1.42 \quad Ans.$$



5-27 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) σ_A = 15, σ_B = – 15 kpsi.

BCM: Eq. (5-31*b*), $n = \left(\frac{10}{36} - \frac{-15}{35}\right)^{-1} = 1.42 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OD}{OC} = \frac{1.28"}{0.90"} = 1.42 \quad Ans.$$

5-28 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) $\sigma_A,\ \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16,\ -4$ kpsi

BCM: Eq. (5-31*b*), $n = \left(\frac{16}{36} - \frac{-4}{35}\right)^{-1} = 1.79 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OF}{OE} = \frac{1.47"}{0.82"} = 1.79 \quad Ans.$$

5-29 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) $$\sigma_A,\ \sigma_B = \frac{-10-15}{2} \pm \sqrt{\left(\frac{-10+15}{2}\right)^2 + 10^2} = -2.2,\ -22.8 \text{ kpsi}$$

BCM: Eq. (5-31*c*), $n = -\dfrac{35}{-22.8} = 1.54 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

$$n = \frac{OH}{OG} = \frac{1.76"}{1.15"} = 1.53 \quad Ans.$$

5-30 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi

BCM: Eq. (5-31),

(**a**) $$\sigma_A,\ \sigma_B = \frac{15+8}{2} \pm \sqrt{\left(\frac{15-8}{2}\right)^2 + (-8)^2} = 20.2,\ 2.8 \text{ kpsi}$$

BCM: Eq. (5-31*a*), $n = \dfrac{36}{20.2} = 1.78 \quad Ans.$

(**b**) The plot is shown in the solution to Prob. 5-26.

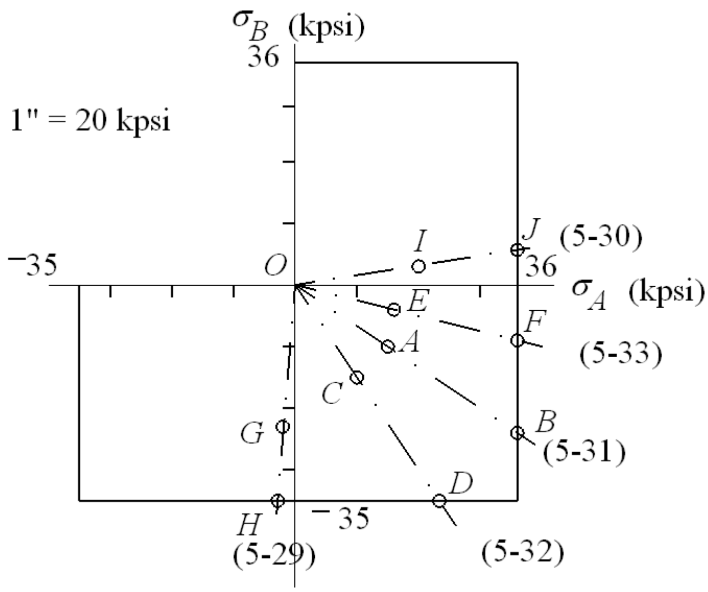
$$n = \frac{OJ}{OI} = \frac{1.82"}{1.02"} = 1.78 \quad Ans.$$

5-31 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32). For this problem, MM reduces to the MNS theory.

(**a**) σ_A = 15, σ_B = – 10 kpsi. Eq. (5-32*a*), $n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{36}{15} = 2.4 \quad Ans.$

(**b**) The plot on the next page is for Probs. 5-31 to 5-35. Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

$$n = \frac{OB}{OA} = \frac{2.16"}{0.90"} = 2.43 \quad Ans.$$



5-32 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(**a**) σ_A = 10, σ_B = – 15 kpsi. Eq. (3-32b) is not valid and must use Eq, (3-32c),

$$n = -\frac{S_{uc}}{\sigma_B} = -\frac{35}{-15} = 2.33 \quad Ans.$$

(**b)** The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OD}{OC} = \frac{2.10"}{0.90"} = 2.33 \quad Ans.$$

5-33 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(**a**) $\sigma_A,\ \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16,\ -4 \text{ kpsi}$

$$n = \frac{S_{ut}}{\sigma_A} = \frac{36}{16} = 2.25 \quad Ans.$$

(**b**) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OF}{OE} = \frac{1.86"}{0.82"} = 2.27 \quad Ans.$$

5-34 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(**a**) $\sigma_A,\ \sigma_B = \dfrac{-10-15}{2} \pm \sqrt{\left(\dfrac{-10+15}{2}\right)^2 + 10^2} = -2.2,\ -22.8 \text{ kpsi}$

$$n = -\frac{S_{uc}}{\sigma_B} = -\frac{35}{-22.8} = 1.54 \quad \textit{Ans.}$$

(**b**) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OH}{OG} = \frac{1.76''}{1.15''} = 1.53 \quad \textit{Ans.}$$

5-35 S_{ut} = 36 kpsi, S_{uc} = 35 kpsi. MM: Use Eq. (5-32).For this problem, MM reduces to the MNS theory.

(**a**) $\sigma_A,\ \sigma_B = \dfrac{15+8}{2} \pm \sqrt{\left(\dfrac{15-8}{2}\right)^2 + (-8)^2} = 20.2,\ 2.8 \text{ kpsi}$

$$n = \frac{S_{ut}}{\sigma_A} = \frac{36}{20.2} = 1.78 \quad \textit{Ans.}$$

(**b**) The plot is shown in the solution to Prob. 5-31.

$$n = \frac{OJ}{OI} = \frac{1.82''}{1.02''} = 1.78 \quad \textit{Ans.}$$

5-36 Given: AISI 1006 CD steel, F = 0.55 kN, P = 4.0 kN, and T = 25 N·m. From Table A-20, S_y =280 MPa. Apply the DE theory to stress elements A and B

A: $$\sigma_x = \frac{4P}{\pi d^2} = \frac{4(4)10^3}{\pi(0.015^2)} = 22.6(10^6) \text{ Pa} = 22.6 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4}{3}\frac{V}{A} = \frac{16(25)}{\pi(0.015^3)} + \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)0.015^2}\right] = 41.9(10^6) \text{ Pa} = 41.9 \text{ MPa}$$

$$\sigma' = \left[22.6^2 + 3(41.9^2)\right]^{1/2} = 76.0 \text{ MPa}$$

$$n = \frac{280}{76.0} = 3.68 \qquad \textit{Ans.}$$

B: $$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)10^3(0.1)}{\pi(0.015^3)} + \frac{4(4)10^3}{\pi(0.015^2)} = 189(10^6) \text{ Pa} = 189 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(25)}{\pi(0.015^3)} = 37.7(10^6) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \left[189^2 + 3\left(37.7^2\right)\right]^{1/2} = 200 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{200} = 1.4 \qquad \textit{Ans.}$$

5-37 From Prob. 3-44, the critical location is at the top of the beam at x = 27 in from the left end, where there is only a bending stress of σ = – 7 456 psi. Thus, σ' = 7 456 psi and

$(S_y)_{\min} = n\sigma' = 2(7\ 456) = 14\ 912$ psi

Choose $(S_y)_{\min}$ = 15 kpsi *Ans.*

5-38 From Table A-20 for 1020 CD steel, S_y = 57 kpsi. From Eq. (3-42), p.116,

$$T = \frac{63\ 025H}{n} \qquad (1)$$

where n is the shaft speed in rev/min. From Eq. (5-3), for the MSS theory,

$$\tau_{\max} = \frac{S_y}{2n_d} = \frac{16T}{\pi d^3} \qquad (2)$$

where n_d is the design factor. Substituting Eq. (1) into Eq. (2) and solving for d gives

$$d = \left[\frac{32\left(63\ 025\right)Hn_d}{n\pi S_y}\right]^{1/3} \qquad (3)$$

Substituting H = 20 hp, n_d = 3, n = 1750 rev/min, and $S_y = 57(10^3)$ psi results in

$$d_{\min} = \left[\frac{32\left(63\ 025\right)20\left(3\right)}{1750\pi\left(57\right)10^3}\right]^{1/3} = 0.728 \text{ in} \qquad \textit{Ans.}$$

5-39 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-68, in the plane of analysis

σ_1 = 16.5 kpsi, σ_2 = – 1.19 kpsi, and $\tau_{\max}$ = 8.84 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 16.5 kpsi, σ_2 = 0, and σ_3 = – 1.19 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{16.5-(-1.19)} = 3.05$ *Ans.*

Note: Whenever the two principal stresses of a plane stress state are of opposite sign, the maximum shear stress found in the analysis is the *true* maximum shear stress. Thus, the factor of safety could have been found from

$$n = \frac{S_y}{2\tau_{\max}} = \frac{54}{2(8.84)} = 3.05 \quad Ans.$$

DE: The von Mises stress can be found from the principal stresses or from the stresses found in part (d) of Prob. 3-68. That is,

Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[16.5^2 - 16.5(-1.19) + (-1.19)^2\right]^{1/2}}$$
$$= 3.15 \quad Ans.$$

or, Eqs. (5-15) and (5-19) using the results of part (d) of Prob. 3-68

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma^2 + 3\tau^2\right)^{1/2}} = \frac{54}{\left[15.3^2 + 3\left(4.43^2\right)\right]^{1/2}}$$
$$= 3.15 \quad Ans.$$

5-40 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-69, in the plane of analysis

σ_1 = 275 MPa, σ_2 = – 12.1 MPa, and $\tau_{\max}$ = 144 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 275 MPa, σ_2 = 0, and σ_3 = – 12.1 MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{275-(-12.1)} = 1.29$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[275^2 - 275(-12.1) + (-12.1)^2\right]^{1/2}}$$
$$= 1.32 \quad Ans.$$

5-41 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-70, in the plane of analysis

σ_1 = 22.6 kpsi, σ_2 = – 1.14 kpsi, and τ_{max} = 11.9 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 22.6 kpsi, σ_2 = 0, and σ_3 = – 1.14 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{22.6 - (-1.14)} = 2.27$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[22.6^2 - 22.6(-1.14) + (-1.14)^2\right]^{1/2}}$$

$= 2.33$ *Ans.*

5-42 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-71, in the plane of analysis

σ_1 = 78.2 MPa, σ_2 = – 5.27 MPa, and τ_{max} = 41.7 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 78.2 MPa, σ_2 = 0, and σ_3 = – 5.27 MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{78.2 - (-5.27)} = 4.43$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[78.2^2 - 78.2(-5.27) + (-5.27)^2\right]^{1/2}}$$

$= 4.57$ *Ans.*

5-43 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-72, in the plane of analysis

σ_1 = 36.7 kpsi, σ_2 = – 1.47 kpsi, and τ_{max} = 19.1 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 36.7 kpsi, σ_2 = 0, and σ_3 = – 1.47 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{36.7 - (-1.47)} = 1.41$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[36.7^2 - 36.7(-1.47) + (-1.47)^2\right]^{1/2}}$$

$= 1.44$ *Ans.*

5-44 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-73, in the plane of analysis

σ_1 = 376 MPa, σ_2 = – 42.4 MPa, and τ_{max} = 209 MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 376 MPa, σ_2 = 0, and σ_3 = – 42.4 MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{376 - (-42.4)} = 0.88$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[376^2 - 376(-42.4) + (-42.4)^2\right]^{1/2}}$$

$= 0.93$ *Ans.*

5-45 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-74, in the plane of analysis

σ_1 = 7.19 kpsi, σ_2 = – 17.0 kpsi, and τ_{max} = 12.1 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 7.19 kpsi, σ_2 = 0, and σ_3 = – 17.0 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{7.19 - (-17.0)} = 2.23$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[7.19^2 - 7.19(-17.0) + (-17.0)^2\right]^{1/2}}$$

$= 2.51$ *Ans.*

5-46 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-76, in the plane of analysis

σ_1 = 1.72 kpsi, σ_2 = – 35.9 kpsi, and $\tau_{\max}$ = 18.8 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 1.72 kpsi, σ_2 = 0, and σ_3 = – 35.9 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{1.72 - (-35.9)} = 1.44$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[1.72^2 - 1.72(-35.9) + (-35.9)^2\right]^{1/2}}$$
$$= 1.47 \quad \textit{Ans.}$$

5-47 From Table A-20, S_y = 370 MPa. From the solution of Prob. 3-77,
Bending: σ_B = 68.6 MPa, Torsion: τ_B = 37.7 MPa

For a plane stress analysis it was found that $\tau_{\max}$ = 51.0 MPa. With combined bending and torsion, the plane stress analysis yields the true $\tau_{\max}$.

MSS: From Eq. (5-3), $n = \dfrac{S_y}{2\tau_{\max}} = \dfrac{370}{2(51.0)} = 3.63$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_B^2 + 3\tau_B^2\right)^{1/2}} = \frac{370}{\left[68.6^2 + 3(37.7^2)\right]^{1/2}}$$
$$= 3.91 \quad \textit{Ans.}$$

5-48 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-79,
Bending: σ_C = 3460 psi, Torsion: τ_C = 882 kpsi

For a plane stress analysis it was found that $\tau_{\max}$ = 1940 psi. With combined bending and torsion, the plane stress analysis yields the true $\tau_{\max}$.

MSS: From Eq. (5-3), $n = \dfrac{S_y}{2\tau_{\max}} = \dfrac{54(10^3)}{2(1940)} = 13.9$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_C^2+3\tau_C^2\right)^{1/2}}=\frac{54\left(10^3\right)}{\left[3460^2+3\left(882^2\right)\right]^{1/2}}$$

$$=14.3 \qquad Ans.$$

5-49 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-80, in the plane of analysis

σ_1 = 17.8 kpsi, σ_2 = – 1.46 kpsi, and $\tau_{\max}$ = 9.61 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 17.8 kpsi, σ_2 = 0, and σ_3 = – 1.46 kpsi

MSS: From Eq. (5-3), $n=\dfrac{S_y}{\sigma_1-\sigma_3}=\dfrac{54}{17.8-(-1.46)}=2.80 \qquad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{54}{\left[17.8^2-17.8(-1.46)+(-1.46)^2\right]^{1/2}}$$

$$=2.91 \qquad Ans.$$

5-50 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-81, in the plane of analysis

σ_1 = 17.5 kpsi, σ_2 = – 1.13 kpsi, and $\tau_{\max}$ = 9.33 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 17.5 kpsi, σ_2 = 0, and σ_3 = – 1.13 kpsi

MSS: From Eq. (5-3), $n=\dfrac{S_y}{\sigma_1-\sigma_3}=\dfrac{54}{17.5-(-1.13)}=2.90 \qquad Ans.$

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{54}{\left[17.5^2-17.5(-1.13)+(-1.13)^2\right]^{1/2}}$$

$$=2.98 \qquad Ans.$$

5-51 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-82, in the plane of analysis

σ_1 = 21.5 kpsi, σ_2 = – 1.20 kpsi, and $\tau_{\max}$ = 11.4 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 21.5 kpsi, σ_2 = 0, and σ_3 = – 1.20 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{21.5-(-1.20)} = 2.38$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{54}{\left[21.5^2 - 21.5(-1.20) + (-1.20)^2\right]^{1/2}}$$

$$= 2.44 \quad \textit{Ans.}$$

5-52 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-83, the concern was failure due to twisting of the flat bar where it was found that $\tau_{\max}$ = 14.3 kpsi in the middle of the longest side of the rectangular cross section. The bar is also in bending, but the bending stress is zero where $\tau_{\max}$ exists.

MSS: From Eq. (5-3), $n = \dfrac{S_y}{2\tau_{\max}} = \dfrac{54}{2(14.3)} = 1.89$ *Ans.*

DE: From Eqs. (5-15) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(3\tau_{\max}^2\right)^{1/2}} = \frac{54}{\left[3\left(14.3^2\right)\right]^{1/2}} = 2.18 \quad \textit{Ans.}$$

5-53 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-84, in the plane of analysis

σ_1 = 34.7 kpsi, σ_2 = – 6.7 kpsi, and $\tau_{\max}$ = 20.7 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 34.7 kpsi, σ_2 = 0, and σ_3 = – 6.7 kpsi

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{54}{34.7-(-6.7)} = 1.30$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{54}{\left[34.7^2-34.7\left(-6.7\right)+\left(-6.7\right)^2\right]^{1/2}}$$

$$=1.40 \qquad Ans.$$

5-54 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-85, in the plane of analysis

σ_1 = 51.1 kpsi, σ_2 = – 4.58 kpsi, and $\tau_{\max}$ = 27.8 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 51.1 kpsi, σ_2 = 0, and σ_3 = – 4.58 kpsi

MSS: From Eq. (5-3), $n=\dfrac{S_y}{\sigma_1-\sigma_3}=\dfrac{54}{51.1-\left(-4.58\right)}=0.97$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{54}{\left[51.1^2-51.1\left(-4.58\right)+\left(-4.58\right)^2\right]^{1/2}}$$

$$=1.01 \qquad Ans.$$

5-55 From Table A-20, S_y = 54 kpsi. From the solution of Prob. 3-86, in the plane of analysis

σ_1 = 59.7 kpsi, σ_2 = – 3.92 kpsi, and $\tau_{\max}$ = 31.8 kpsi

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

σ_1 = 59.7 kpsi, σ_2 = 0, and σ_3 = – 3.92 kpsi

MSS: From Eq. (5-3), $n=\dfrac{S_y}{\sigma_1-\sigma_3}=\dfrac{54}{59.7-\left(-3.92\right)}=0.85$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n=\frac{S_y}{\sigma'}=\frac{S_y}{\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_B^2\right)^{1/2}}=\frac{54}{\left[59.7^2-59.7\left(-3.92\right)+\left(-3.92\right)^2\right]^{1/2}}$$

$$=0.87 \qquad Ans.$$

5-56 For Prob. 3-84, from Prob. 5-53 solution, with 1018 CD, DE theory yields, $n = 1.40$.

From Table A-21, for 4140 Q&T @400°F, $S_y = 238$ kpsi. From Prob. (3-87) solution which considered stress concentrations for Prob. 3-84

$\sigma_1 = 53.0$ kpsi, $\sigma_2 = -8.48$ kpsi, and $\tau_{\max} = 30.7$ kpsi

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{238}{\left[53.0^2 - 53.0(-8.48) + (-8.48)^2\right]^{1/2}}$$

$$= 4.12 \qquad \textit{Ans.}$$

Using the 4140 versus the 1018 CD, the factor of safety increases by a factor of 4.12/1.40 = 2.94. *Ans.*

5-57 Design Decisions Required:

- Material and condition
- Design factor
- Failure model
- Diameter of pin

Using $F = 416$ lbf from Ex. 5-3,

$$\sigma_{\max} = \frac{32M}{\pi d^3}$$

$$d = \left(\frac{32M}{\pi \sigma_{\max}}\right)^{\frac{1}{3}}$$

Decision 1: Select the same material and condition of Ex. 5-3 (AISI 1035 steel, $S_y = 81$ kpsi)

Decision 2: Since we prefer the pin to yield, set n_d a little larger than 1. Further explanation will follow.

Decision 3: Use the Distortion Energy static failure theory.

Decision 4: Initially set $n_d = 1$

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{S_y}{1} = 81\,000 \text{ psi}$$

$$d = \left(\frac{32(416)(15)}{\pi(81\,000)}\right)^{\frac{1}{3}} = 0.922 \text{ in}$$

Choose preferred size of $d = 1.000$ in

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.27$$

Set design factor to $n_d = 1.27$

Adequacy Assessment:

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{81\,000}{1.27} = 63\,800 \text{ psi}$$

$$d = \left(\frac{32(416)(15)}{\pi(63\,800)}\right)^{\frac{1}{3}} = 1.00 \text{ in (OK)}$$

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.27 \text{ (OK)}$$

5-58 From Table A-20, for a thin walled cylinder made of AISI 1020 CD steel, S_{yt} = 57 kpsi, S_{ut} = 68 kpsi.

Since r/t = 7.5/0.0625 = 120 > 10, the shell can be considered thin-wall. From the solution of Prob. 3-93 the principal stresses are

$$\sigma_1 = \sigma_2 = \frac{pd}{4t} = \frac{p(15)}{4(0.0625)} = 60p, \qquad \sigma_3 = -p$$

From Eq. (5-12)

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[(60p - 60p)^2 + (60p + p)^2 + (-p - 60p)^2\right]^{1/2} = 61p$$

For yield, $\sigma' = S_y \;\Rightarrow\; 61p = 57\,(10^3) \;\Rightarrow\; p = 934$ psi *Ans.*
For rupture, $61p = 68 \quad\Rightarrow\quad p = 1.11$ kpsi *Ans.*

5-59 For AISI 1020 HR steel, from Tables A-5 and A-20, w = 0.282 lbf/in^3, S_y = 30 kpsi, and ν = 0.292. Then, $\rho = w/g$ = 0.282/386 lbf·s^2/in. For the problem, r_i = 3 in, and r_o = 5 in. Substituting into Eqs. (3-55), p. 129, gives

$$\sigma_t = \frac{0.282}{386}\omega^2\left(\frac{3+0.292}{8}\right)\left[9+25+\frac{9(25)}{r^2}-\frac{1+3(0.292)}{3+0.292}r^2\right]$$

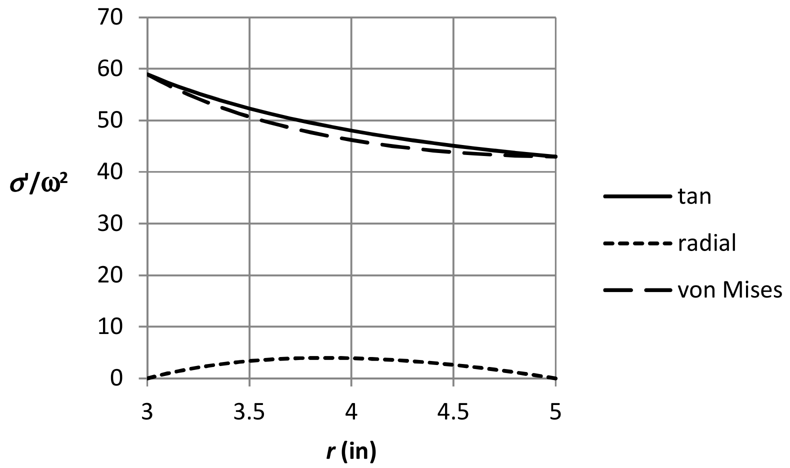
$$= 3.006\left(10^{-4}\right)\omega^2\left(34+\frac{225}{r^2}-0.5699r^2\right) = F(r)\,\omega^2 \qquad (1)$$

$$\sigma_r = 3.006\left(10^{-4}\right)\omega^2\left(34-\frac{225}{r^2}-r^2\right) = G(r)\,\omega^2 \qquad (2)$$

For the distortion-energy theory, the von Mises stress will be

$$\sigma' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_2^2\right)^{1/2} = \omega^2\left[F^2(r) - F(r)G(r) + G^2(r)\right]^{1/2} \qquad (3)$$

Although it was noted that the maximum radial stress occurs at $r = (r_o r_i)^{1/2}$ we are more interested as to where the von Mises stress is a maximum. One could take the derivative of Eq. (3) and set it to zero to find where the maximum occurs. However, it is much easier to plot σ'/ω^2 for $3 \le r \le 5$ in. Plotting Eqs. (1) through (3) results in



It can be seen that there is no maxima, and the greatest value of the von Mises stress is the tangential stress at $r = r_i$. Substituting r = 3 in into Eq. (1) and setting $\sigma' = S_y$ gives

$$\omega = \left[\frac{30\left(10^3\right)}{3.006\left(10^{-4}\right)\left(34+\frac{225}{3^2}-0.5699\left(3^2\right)\right)}\right]^{1/2} = 1361 \text{ rad/s}$$

$$n = \frac{60\omega}{2\pi} = \frac{60(1361)}{2\pi} = 13\,000 \text{ rev/min} \qquad Ans.$$

5-60 Since r/t = 1.75/0.065 = 26.9 > 10, we can use thin-walled equations. From Eqs. (3-53) and (3-54), p. 128,

$$d_i = 3.5 - 2(0.065) = 3.37 \text{ in}$$

$$\sigma_t = \frac{p(d_i + t)}{2t}$$

$$\sigma_t = \frac{500(3.37 + 0.065)}{2(0.065)} = 13\,212 \text{ psi} = 13.2 \text{ kpsi}$$

$$\sigma_l = \frac{pd_i}{4t} = \frac{500(3.37)}{4(0.065)} = 6481 \text{ psi} = 6.48 \text{ kpsi}$$

$$\sigma_r = -p_i = -500 \text{ psi} = -0.5 \text{ kpsi}$$

These are all principal stresses, thus, from Eq. (5-12),

$$\sigma' = \frac{1}{\sqrt{2}}\left\{(13.2 - 6.48)^2 + [6.48 - (-0.5)]^2 + (-0.5 - 13.2)^2\right\}^{1/2}$$

$$= 11.87 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{46}{11.87}$$

$$n = 3.88 \qquad Ans.$$

5-61 From Table A-20, $S_y = 320$ MPa

With $p_i = 0$, Eqs. (3-49), p. 127, are

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right) = c\left(1 + \frac{b^2}{r^2}\right) \qquad (1)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right) = c\left(1 - \frac{b^2}{r^2}\right)$$

For the distortion-energy theory, the von Mises stress is

$$\sigma' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = c\left[\left(1 + \frac{b^2}{r^2}\right)^2 - \left(1 + \frac{b^2}{r^2}\right)\left(1 - \frac{b^2}{r^2}\right) + \left(1 - \frac{b^2}{r^2}\right)^2\right]^{1/2}$$

$$= c\left(1 + 3\frac{b^4}{r^4}\right)^{1/2}$$

We see that the maximum von Mises stress occurs where r is a minimum at $r = r_i$. Here, $\sigma_r = 0$ and thus $\sigma' = -\sigma_t$. Setting $-\sigma_t = S_y = 320$ MPa at $r = 0.1$ m in Eq. (1) results in

$$-\sigma_t\big|_{r=r_i} = \frac{2r_o^2 p_o}{r_o^2 - r_i^2} = \frac{2\left(0.15^2\right) p_o}{0.15^2 - 0.1^2} = 3.6 p_o = 320 \;\Rightarrow\; p_o = 88.9 \text{ MPa} \qquad \textit{Ans.}$$

5-62 From Table A-24, $S_{ut} = 31$ kpsi for grade 30 cast iron. From Table A-5, $\nu = 0.211$ and $w = 0.260$ lbf/in^3. In Prob. 5-59, it was determined that the maximum stress was the tangential stress at the inner radius, where the radial stress is zero. Thus at the inner radius, Eq. (3-55), p. 129, gives

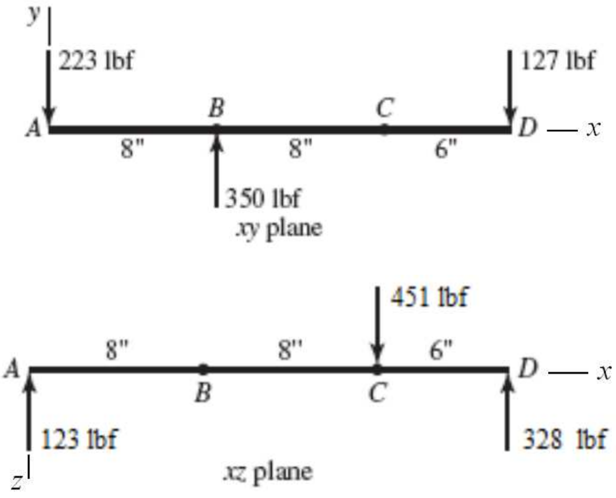
$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(2r_o^2 + r_i^2 - \frac{1+3\nu}{3+\nu} r_i^2\right) = \frac{0.260}{386}\omega^2\left(\frac{3.211}{8}\right)\left[2\left(5^2\right) + 3^2 - \frac{1+3(0.211)}{3.211} 3^2\right]$$

$$= 0.01471\omega^2 = 31\left(10^3\right) \qquad \Rightarrow \qquad 1452 \text{ rad/sec}$$

$n = 60(1452)/(2\pi) = 13\ 870$ rev/min *Ans.*

5-63 From Table A-20, for AISI 1035 CD, $S_y = 67$ kpsi.

From force and bending-moment equations, the ground reaction forces are found in two planes as shown.



The maximum bending moment will be at B or C. Check which is larger. In the xy plane,

$M_B = 223(8) = 1784$ lbf · in and $M_C = 127(6) = 762$ lbf · in.

In the xz plane, $M_B = 123(8) = 984$ lbf · in and $M_C = 328(6) = 1968$ lbf · in.

$$M_B = [(1784)^2 + (984)^2]^{\frac{1}{2}} = 2037 \text{ lbf} \cdot \text{in}$$

$$M_C = [(762)^2 + (1968)^2]^{\frac{1}{2}} = 2110 \text{ lbf} \cdot \text{in}$$

So point C governs. The torque transmitted between B and C is T = (300 − 50)(4) = 1000 lbf·in. The stresses are

$$\tau_{xz} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5093}{d^3} \text{ psi}$$

$$\sigma_x = \frac{32M_C}{\pi d^3} = \frac{32(2110)}{\pi d^3} = \frac{21\,492}{d^3} \text{ psi}$$

For combined bending and torsion, the maximum shear stress is found from

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2\right]^{1/2} = \left[\left(\frac{21.49}{2d^3}\right)^2 + \left(\frac{5.09}{d^3}\right)^2\right]^{1/2} = \frac{11.89}{d^3} \text{ kpsi}$$

Max Shear Stress theory is chosen as a conservative failure theory. From Eq. (5-3)

$$\tau_{\max} = \frac{S_y}{2n} = \frac{11.89}{d^3} = \frac{67}{2(2)} \qquad \Rightarrow \qquad d = 0.892 \text{ in} \qquad \textit{Ans.}$$

5-64 As in Prob. 5-63, we will assume this to be a statics problem. Since the proportions are unchanged, the bearing reactions will be the same as in Prob. 5-63 and the bending moment will still be a maximum at point C. Thus

xy plane: $M_C = 127(3) = 381 \text{ lbf} \cdot \text{in}$

xz plane: $M_C = 328(3) = 984 \text{ lbf} \cdot \text{in}$

So

$$M_{\max} = \left[(381)^2 + (984)^2\right]^{1/2} = 1055 \text{ lbf} \cdot \text{in}$$

$$\sigma_x = \frac{32M_C}{\pi d^3} = \frac{32(1055)}{\pi d^3} = \frac{10\,746}{d^3} \text{ psi} = \frac{10.75}{d^3} \text{ kpsi}$$

Since the torsional stress is unchanged,

$$\tau_{xz} = \frac{5.09}{d^3} \text{ kpsi}$$

For combined bending and torsion, the maximum shear stress is found from

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2\right]^{1/2} = \left[\left(\frac{10.75}{2d^3}\right)^2 + \left(\frac{5.09}{d^3}\right)^2\right]^{1/2} = \frac{7.40}{d^3} \text{ kpsi}$$

Using the MSS theory, as was used in Prob. 5-63, gives

$$\tau_{\max} = \frac{S_y}{2n} = \frac{7.40}{d^3} = \frac{67}{2(2)} \qquad \Rightarrow \qquad d = 0.762 \text{ in} \qquad \textit{Ans.}$$

5-65 For AISI 1018 HR, Table A-20 gives S_y = 32 kpsi. Transverse shear stress is a maximum at the neutral axis, and zero at the outer radius. Bending stress is a maximum at the outer radius, and zero at the neutral axis.

Model (*c*): From Prob. 3-40, at outer radius,

$$\sigma' = \sigma = 17.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{17.8} = 1.80$$

At neutral axis,

$$\sigma' = \sqrt{3\tau^2} = \sqrt{3(3.4)^2} = 5.89 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{5.89} = 5.43$$

The bending stress at the outer radius dominates. n = 1.80 *Ans.*

Model (*d*): Assume the bending stress at the outer radius will dominate, as in model (*c*). From Prob. 3-40,

$$\sigma' = \sigma = 25.5 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{25.5} = 1.25 \qquad \textit{Ans.}$$

Model (*e*): From Prob. 3-40,

$$\sigma' = \sigma = 17.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{17.8} = 1.80 \qquad \textit{Ans.}$$

Model (*d*) is the most conservative, thus safest, and requires the least modeling time. Model (*c*) is probably the most accurate, but model (*e*) yields the same results with less modeling effort.

5-66 For AISI 1018 HR, from Table A-20, S_y = 32 kpsi. Model (*d*) yields the largest bending moment, so designing to it is the most conservative approach. The bending moment is M = 312.5 lbf·in. For this case, the principal stresses are

$$\sigma_1 = \frac{32M}{\pi d^3}, \ \sigma_2 = \sigma_3 = 0$$

Using a conservative yielding failure theory use the MSS theory and Eq. (5-3)

$$\sigma_1 - \sigma_3 = \frac{S_y}{n} \quad \Rightarrow \quad \frac{32M}{\pi d^3} = \frac{S_y}{n} \quad \Rightarrow \quad d = \left(\frac{32Mn}{\pi S_y}\right)^{1/3}$$

Thus, $d = \left[\frac{32(312.5)2.5}{\pi(32)10^3}\right]^{1/3} = 0.629 \text{ in} \quad \therefore \quad \text{Use } d = \frac{11}{16} \text{ in}$ *Ans.*

5-67 When the ring is set, the hoop tension in the ring is equal to the screw tension.

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

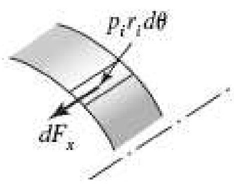
The differential hoop tension dF at r for the ring of width w, is $dF = w\sigma_t dr$. Integration yields

$$F = \int_{r_i}^{r_o} w\sigma_t dr = \frac{wr_i^2 p_i}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(1 + \frac{r_o^2}{r^2}\right)dr = \frac{wr_i^2 p_i}{r_o^2 - r_i^2}\left(r - \frac{r_o^2}{r}\right)\Bigg|_{r_i}^{r_o} = wr_i p_i \quad (1)$$

The screw equation is

$$F_i = \frac{T}{0.2d} \qquad (2)$$

From Eqs. (1) and (2)



$$p_i = \frac{F}{wr_i} = \frac{T}{0.2d\,wr_i}$$

$$dF_x = fp_i r_i d\theta$$

$$F_x = \int_0^{2\pi} fp_i wr_i d\theta = \frac{f\,Tw}{0.2d\,wr_i} r_i \int_0^{2\pi} d\theta$$

$$= \frac{2\pi f\,T}{0.2d} \quad \textit{Ans.}$$

5-68 T = 20 N·m, S_y = 450 MPa

(a) From Prob. 5-67, $T = 0.2\, F_i\, d$

$$F_i = \frac{T}{0.2d} = \frac{20}{0.2\left[6\left(10^{-3}\right)\right]} = 16.7\left(10^3\right)\text{ N} = 16.7\text{ kN} \qquad Ans.$$

(b) From Prob. 5-67, $F = w r_i\, p_i$

$$p_i = \frac{F}{wr_i} = \frac{F_i}{wr_i} = \frac{16.7\left(10^3\right)}{\left[12\left(10^{-3}\right)\right]\left[(25/2)\left(10^{-3}\right)\right]} = 111.3\left(10^6\right)\text{ Pa} = 111.3\text{ MPa} \qquad Ans.$$

(c)
$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r}\right)_{r=r_i} = \frac{p_i\left(r_i^2 + r_o^2\right)}{r_o^2 - r_i^2}$$

$$= \frac{111.3\left(0.0125^2 + 0.025^2\right)}{0.025^2 - 0.0125^2} = 185.5\text{ MPa} \qquad Ans.$$

$$\sigma_r = -p_i = -111.3\text{ MPa}$$

(d)

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_t - \sigma_r}{2}$$
$$= \frac{185.5 - (-111.3)}{2} = 148.4\text{ MPa} \qquad Ans.$$
$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}$$

$$= \left[185.5^2 - (185.5)(-111.3) + (-111.3)^2\right]^{1/2}$$

$$= 259.7\text{ MPa} \qquad Ans.$$

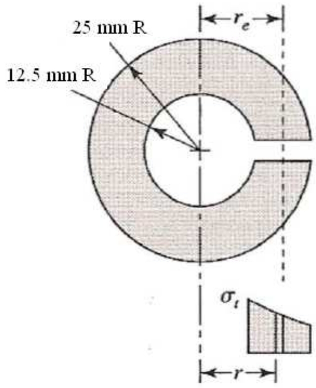
(e) Maximum Shear Stress Theory

$$n = \frac{S_y}{2\tau_{\max}} = \frac{450}{2(148.4)} = 1.52 \qquad Ans.$$

Distortion Energy theory

$$n = \frac{S_y}{\sigma'} = \frac{450}{259.7} = 1.73 \qquad Ans.$$

5-69 The moment about the center caused by the force F is $F\,r_e$ where r_e is the effective radius. This is balanced by the moment about the center caused by the tangential (hoop) stress. For the ring of width w



$$Fr_e = \int_{r_i}^{r_o} r\sigma_t w\,dr$$

$$= \frac{w p_i r_i^2}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(r + \frac{r_o^2}{r}\right)dr$$

$$r_e = \frac{w p_i r_i^2}{F\left(r_o^2 - r_i^2\right)}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

From Prob. 5-67, $F = wr_i\,p_i$. Therefore,

$$r_e = \frac{r_i}{r_o^2 - r_i^2}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

For the conditions of Prob. 5-67, r_i = 12.5 mm and r_o = 25 mm

$$r_e = \frac{12.5}{25^2 - 12.5^2}\left(\frac{25^2 - 12.5^2}{2} + 25^2 \ln\frac{25}{12.5}\right) = 17.8 \text{ mm} \qquad \textit{Ans.}$$

5-70 (**a**) The nominal radial interference is δ_{nom} = (2.002 – 2.001) /2 = 0.0005 in.

From Eq. (3-57), p. 130,

$$p = \frac{E\delta}{2R^3}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{r_o^2 - r_i^2}\right]$$

$$= \frac{30\left(10^6\right)0.0005}{2\left(1^3\right)}\left[\frac{\left(1.5^2 - 1^2\right)\left(1^2 - 0.625^2\right)}{1.5^2 - 0.625^2}\right] = 3072 \text{ psi} \quad \textit{Ans.}$$

Inner member: $p_i = 0$, $p_o = p$ = 3072 psi. At fit surface $r = R$ = 1 in,

Eq. (3-49), p. 127, $$\sigma_t = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -3072\left(\frac{1^2 + 0.625^2}{1^2 - 0.625^2}\right) = -7\,010 \text{ psi}$$

$$\sigma_r = -p = -3072 \text{ psi}$$

Eq. (5-13)

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$
$$= \left[(-7010)^2 - (-7010)(-3072) + (-3072)\right]^{1/2} = 6086 \text{ psi} \quad \textit{Ans.}$$

Outer member: $p_i = p = 3072$ psi, $p_o = 0$. At fit surface $r = R = 1$ in,

Eq. (3-49), p. 127, $$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 3072\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 7987 \text{ psi}$$

$$\sigma_r = -p = -3072 \text{ psi}$$

Eq. (5-13)

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$
$$= \left[7987^2 - 7987(-3072) + (-3072)\right]^{1/2} = 9888 \text{ psi} \quad \textit{Ans.}$$

(**b**) For a solid inner tube,

$$p = \frac{30(10^6)0.0005}{2(1^3)}\left[\frac{(1.5^2 - 1^2)(1^2)}{1.5^2}\right] = 4167 \text{ psi} \quad \textit{Ans.}$$

Inner member: $\sigma_t = \sigma_r = -p = -4167$ psi

$$\sigma' = \left[(-4167)^2 - (-4167)(-4167) + (-4167)^2\right]^{1/2} = 4167 \text{ psi} \quad \textit{Ans.}$$

Outer member: $p_i = p = 4167$ psi, $p_o = 0$. At fit surface $r = R = 1$ in,

Eq. (3-49), p. 127, $$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 4167\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 10834 \text{ psi}$$

$$\sigma_r = -p = -4167 \text{ psi}$$

Eq. (5-13)

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2\right)^{1/2}$$
$$= \left[10\,834^2 - 10\,834(-4167) + (-4167)\right]^{1/2} = 13\,410 \text{ psi} \quad \textit{Ans.}$$

5-71 From Table A-5, E = 207 (10^3) MPa. The nominal radial interference is δ_{nom} = (40 – 39.98) /2 = 0.01 mm.

From Eq. (3-57), p. 130,

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right]$$
$$= \frac{207(10^3)0.01}{2(20^3)}\left[\frac{(32.5^2 - 20^2)(20^2 - 10^2)}{32.5^2 - 10^2}\right] = 26.64 \text{ MPa} \quad \textit{Ans.}$$

Inner member: p_i = 0, p_o = p = 26.64 MPa. At fit surface r = R = 20 mm,

Eq. (3-49), p. 127, $$\sigma_t = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -26.64\left(\frac{20^2 + 10^2}{20^2 - 10^2}\right) = -44.40 \text{ MPa}$$

$$\sigma_r = -p = -26.64 \text{ MPa}$$

Eq. (5-13)
$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2)^{1/2}$$
$$= \left[(-44.40)^2 - (-44.40)(-26.64) + (-26.64)\right]^{1/2} = 38.71 \text{ MPa} \quad \textit{Ans.}$$

Outer member: p_i = p = 26.64 MPa, p_o = 0. At fit surface r = R = 20 mm,

Eq. (3-49), p. 127, $$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 26.64\left(\frac{32.5^2 + 20^2}{32.5^2 - 20^2}\right) = 59.12 \text{ MPa}$$

$$\sigma_r = -p = -26.64 \text{ MPa}$$

Eq. (5-13)
$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_A^2)^{1/2}$$
$$= \left[59.12^2 - 59.12(-26.64) + (-26.64)\right]^{1/2} = 76.03 \text{ MPa} \quad \textit{Ans.}$$

5-72 From Table A-5, E = 207 (10^3) MPa. The nominal radial interference is δ_{nom} = (40.008 – 39.972) /2 = 0.018 mm.

From Eq. (3-57), p. 130,

$$p=\frac{E\delta}{2R^3}\left[\frac{\left(r_o^2-R^2\right)\left(R^2-r_i^2\right)}{r_o^2-r_i^2}\right]$$

$$=\frac{207\left(10^3\right)0.018}{2\left(20^3\right)}\left[\frac{\left(32.5^2-20^2\right)\left(20^2-10^2\right)}{32.5^2-10^2}\right]=47.94\text{ MPa} \quad \textit{Ans.}$$

Inner member: $p_i = 0$, $p_o = p = 47.94$ MPa. At fit surface $r = R = 20$ mm,

Eq. (3-49), p. 127, $$\sigma_t=-p\frac{R^2+r_i^2}{R^2-r_i^2}=-47.94\left(\frac{20^2+10^2}{20^2-10^2}\right)=-79.90\text{ MPa}$$

$$\sigma_r = -p = -47.94\text{ MPa}$$

Eq. (5-13)

$$\sigma'=\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_A^2\right)^{1/2}$$

$$=\left[\left(-79.90\right)^2-\left(-79.90\right)\left(-47.94\right)+\left(-47.94\right)\right]^{1/2}=69.66\text{ MPa} \quad \textit{Ans.}$$

Outer member: $p_i = p = 47.94$ MPa, $p_o = 0$. At fit surface $r = R = 20$ mm,

Eq. (3-49), p. 127, $$\sigma_t=p\frac{r_o^2+R^2}{r_o^2-R^2}=47.94\left(\frac{32.5^2+20^2}{32.5^2-20^2}\right)=106.4\text{ MPa}$$

$$\sigma_r = -p = -47.94\text{ MPa}$$

Eq. (5-13)

$$\sigma'=\left(\sigma_A^2-\sigma_A\sigma_B+\sigma_A^2\right)^{1/2}$$

$$=\left[106.4^2-106.4\left(-47.94\right)+\left(-47.94\right)\right]^{1/2}=136.8\text{ MPa} \quad \textit{Ans.}$$

5-73 From Table A-5, for carbon steel, $E_s = 30$ kpsi, and $\nu_s = 0.292$. While for $E_{ci} = 14.5$ Mpsi, and $\nu_{ci} = 0.211$. For ASTM grade 20 cast iron, from Table A-24, $S_{ut} = 22$ kpsi.

For midrange values, $\delta = (2.001 - 2.0002)/2 = 0.0004$ in.

Eq. (3-50), p. 127,

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2+R^2}{r_o^2-R^2}+\nu_o\right)+\frac{1}{E_i}\left(\frac{R^2+r_i^2}{R^2-r_i^2}-\nu_i\right)\right]}$$

$$= \frac{0.0004}{1\left[\frac{1}{14.5\left(10^6\right)}\left(\frac{2^2+1^2}{2^2-1^2}+0.211\right)+\frac{1}{30\left(10^6\right)}\left(\frac{1^2}{1^2}-0.292\right)\right]} = 2613 \text{ psi}$$

At fit surface, with $p_i = p$ =2613 psi, and $p_o = 0$, from Eq. (3-50), p. 127

$$\sigma_t = p\frac{r_o^2+R^2}{r_o^2-R^2} = 2613\left(\frac{2^2+1^2}{2^2-1^2}\right) = 4355 \text{ psi}$$

$\sigma_r = -p = -2613$ psi

From Modified-Mohr theory, Eq. (5-32a), since $\sigma_A > 0 > \sigma_B$ and $|\sigma_B/\sigma_A| < 1$,

$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{4.355} = 5.05 \qquad Ans.$$

5-74 E = 207 GPa
Eq. (3-57), p. 130, can be written in terms of diameters,

$$p = \frac{E\delta_d}{2D^3}\left[\frac{(d_o^2-D^2)(D^2-d_i^2)}{(d_o^2-d_i^2)}\right] = \frac{207\left(10^3\right)(0.062)}{2\left(45\right)^3}\left[\frac{(50^2-45^2)(45^2-40^2)}{(50^2-40^2)}\right]$$

$$p = 15.80 \text{ MPa}$$

Outer member: From Eq. (3-50),

Outer radius: $\left(\sigma_t\right)_o = \dfrac{45^2(15.80)}{50^2-45^2}(2) = 134.7$ MPa

$$\left(\sigma_r\right)_o = 0$$

Inner radius: $\left(\sigma_t\right)_i = \dfrac{45^2(15.80)}{50^2-45^2}\left(1+\dfrac{50^2}{45^2}\right) = 150.5$ MPa

$$\left(\sigma_r\right)_i = -15.80 \text{ MPa}$$

Bending (no slipping): $I = (\pi/64)(50^4 - 40^4) = 181.1\ (10^3)\ \text{mm}^4$

At r_o: $$(\sigma_x)_o = \pm\frac{Mc}{I} = \pm\frac{675(0.05/2)}{181.1\ (10^{-9})} = \pm 93.2(10^6)\ \text{Pa} = \pm 93.2\ \text{MPa}$$

At r_i: $$(\sigma_x)_i = \pm\frac{675(0.045/2)}{181.1\ (10^{-9})} = \pm 83.9\left(10^6\right)\ \text{Pa} = \pm 83.9\ \text{MPa}$$

Torsion: $J = 2I = 362.2\ (10^3)\ \text{mm}^4$

At r_o: $$(\tau_{xy})_o = \frac{Tc}{J} = \frac{900(0.05/2)}{362.2\ (10^{-9})} = 62.1\left(10^6\right)\ \text{Pa} = 62.1\ \text{MPa}$$

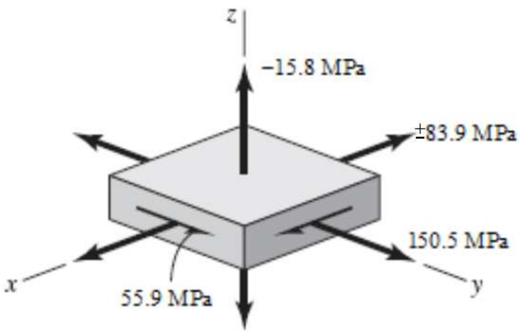
At r_i: $$(\tau_{xy})_i = \frac{900(0.045/2)}{362.2\ (10^{-9})} = 55.9\left(10^6\right)\ \text{Pa} = 55.9\ \text{MPa}$$

Outer radius, is plane stress. Since the tangential stress is positive the von Mises stress will be a maximum with a negative bending stress. That is,

$$\sigma_x = -93.2\ \text{MPa},\ \sigma_y = 134.7\ \text{MPa},\ \tau_{xy} = 62.1\ \text{MPa}$$

Eq. (5-15) $$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$
$$= \left[(-93.2)^2 - (-93.2)(134.7) + 134.7^2 + 3(62.1)^2\right]^{1/2} = 226\ \text{MPa}$$
$$n_o = \frac{S_y}{\sigma'} = \frac{415}{226} = 1.84 \qquad \textit{Ans.}$$

Inner radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$ and $\sigma_x = +\ 83.9$ MPa
$$\sigma' = \frac{1}{\sqrt{2}}\left[(83.9-150.5)^2 + (150.5+15.8)^2 + (-15.8-83.9)^2 + 6(55.9)^2\right]^{1/2} = 174\ \text{MPa}$$
With $\sigma_x = -\ 83.9$ MPa
$$\sigma' = \frac{1}{\sqrt{2}}\left[(-83.9-150.5)^2 + (150.5+15.8)^2 + (-15.8+83.9)^2 + 6(55.9)^2\right]^{1/2} = 230\ \text{MPa}$$

$$n_i = \frac{S_y}{\sigma'} = \frac{415}{230} = 1.80 \qquad Ans.$$

5-75 From the solution of Prob. 5-74, p = 15.80 MPa

Inner member: From Eq. (3-50),

Outer radius: $$(\sigma_t)_o = -\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\, p_o = -\frac{45^2 + 40^2}{45^2 - 40^2}(15.80) = -134.8 \text{ MPa}$$

$$(\sigma_r)_o = -p = -15.80 \text{ MPa}$$

Inner radius: $$(\sigma_t)_i = -\frac{2r_o^2}{r_o^2 - r_i^2}\, p_o = -\frac{2\left(45^2\right)}{45^2 - 40^2}(15.80) = -150.6 \text{ MPa}$$

$$(\sigma_r)_i = 0$$

Bending (no slipping): $I = (\pi/64)(50^4 - 40^4) = 181.1\ (10^3)\ \text{mm}^4$

At r_o: $$(\sigma_x)_o = \pm\frac{Mc}{I} = \pm\frac{675(0.045/2)}{181.1\left(10^{-9}\right)} = \pm 83.9(10^6)\ \text{Pa} = \pm 83.9 \text{ MPa}$$

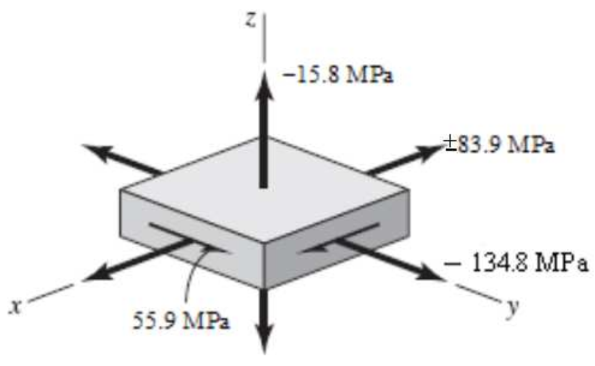
At r_i: $$(\sigma_x)_i = \pm\frac{675(0.040/2)}{181.1\left(10^{-9}\right)} = \pm 74.5\left(10^6\right)\ \text{Pa} = \pm 74.5 \text{ MPa}$$

Torsion: $J = 2I = 362.2\ (10^3)\ \text{mm}^4$

At r_o: $$(\tau_{xy})_o = \frac{Tc}{J} = \frac{900(0.045/2)}{362.2\left(10^{-9}\right)} = 55.9\left(10^6\right)\ \text{Pa} = 55.9 \text{ MPa}$$

At r_i: $$(\tau_{xy})_i = \frac{900(0.040/2)}{362.2\left(10^{-9}\right)} = 49.7\left(10^6\right)\ \text{Pa} = 49.7 \text{ MPa}$$

Outer radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$ and $\sigma_x = +83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}}\left[(83.9+134.8)^2+(-134.8+15.8)^2+(-15.8-83.9)^2+6(55.9)^2\right]^{1/2} = 213\text{ MPa}$$

With $\sigma_x = -83.9$ MPa

$$\sigma' = \frac{1}{\sqrt{2}}\left[(-83.9+134.8)^2+(-134.8+15.8)^2+(-15.8+83.9)^2+6(55.9)^2\right]^{1/2} = 142\text{ MPa}$$

$$n_o = \frac{S_y}{\sigma'} = \frac{415}{213} = 1.95 \qquad \textit{Ans.}$$

Inner radius, plane stress. Worst case is when σ_x is positive

$$\sigma_x = 74.5\text{ MPa},\ \sigma_y = -150.6\text{ MPa},\ \tau_{xy} = 49.7\text{ MPa}$$

Eq. (5-15)

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

$$= \left[74.5^2 - 74.5(-150.6) + (-150.6)^2 + 3(49.7)^2\right]^{1/2} = 216\text{ MPa}$$

$$n_i = \frac{S_y}{\sigma'} = \frac{415}{216} = 1.92 \qquad \textit{Ans.}$$

5-76 For AISI 1040 HR, from Table A-20, $S_y = 290$ MPa.

From Prob. 3-110, $p_{\max} = 65.2$ MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 65.2\frac{50^2+25^2}{50^2-25^2} = 108.7\text{ MPa}$$

$$\sigma_r = -p = -65.2\text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[108.7^2 - 108.7(-65.2) + (-65.2)^2\right]^{1/2} = 152.2\text{ MPa}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{290}{152.2} = 1.91 \qquad \textit{Ans.}$$

5-77 For AISI 1040 HR, from Table A-20, $S_y = 42$ kpsi.

From Prob. 3-111, $p_{\max} = 9$ kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 9\frac{2^2 + 1^2}{2^2 - 1^2} = 15 \text{ kpsi}$$

$$\sigma_r = -p = -9 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[15^2 - 15(-9) + (-9)^2\right]^{1/2} = 21 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{42}{21} = 2 \qquad \textit{Ans.}$$

5-78 For AISI 1040 HR, from Table A-20, S_y = 290 MPa.

From Prob. 3-111, $p_{\max}$ = 91.6 MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 91.6\frac{50^2 + 25^2}{50^2 - 25^2} = 152.7 \text{ MPa}$$

$$\sigma_r = -p = -91.6 \text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[152.7^2 - 152.7(-91.6) + (-91.6)^2\right]^{1/2} = 213.8 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{290}{213.8} = 1.36 \qquad \textit{Ans.}$$

5-79 For AISI 1040 HR, from Table A-20, S_y = 42 kpsi.

From Prob. 3-111, $p_{\max}$ = 12.94 kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 12.94\frac{2^2 + 1^2}{2^2 - 1^2} = 21.57 \text{ kpsi}$$

$$\sigma_r = -p = -12.94 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[21.57^2 - 21.57(-12.94) + (-12.94)^2\right]^{1/2} = 30.20 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{42}{30.2} = 1.39 \qquad \textit{Ans.}$$

5-80 For AISI 1040 HR, from Table A-20, S_y = 290 MPa.

From Prob. 3-111, $p_{\max}$ = 134 MPa. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 134\frac{50^2 + 25^2}{50^2 - 25^2} = 223.3 \text{ MPa}$$

$$\sigma_r = -p = -134 \text{ MPa}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[223.3^2 - 223.3(-134) + (-134)^2\right]^{1/2} = 312.6 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{290}{312.6} = 0.93 \qquad \textit{Ans.}$$

5-81 For AISI 1040 HR, from Table A-20, S_y = 42 kpsi.

From Prob. 3-111, $p_{\max}$ = 19.13 kpsi. From Eq. (3-50) at the inner radius R of the outer member,

$$\sigma_t = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = 19.13\frac{2^2 + 1^2}{2^2 - 1^2} = 31.88 \text{ kpsi}$$

$$\sigma_r = -p = -19.13 \text{ kpsi}$$

These are principal stresses. From Eq. (5-13)

$$\sigma_o' = \left(\sigma_t^2 - \sigma_t\sigma_r + \sigma_r^2\right)^{1/2} = \left[31.88^2 - 31.88(-19.13) + (-19.13)^2\right]^{1/2} = 44.63 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_o'} = \frac{42}{44.63} = 0.94 \qquad \textit{Ans.}$$

5-82

$$\sigma_p = \frac{1}{2}\left(2\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\right) \pm \left[\left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2}\right)^2 + \left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)^2\right]^{1/2}$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left[\cos\frac{\theta}{2} \pm \left(\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\sin^2\frac{3\theta}{2} + \sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\cos^2\frac{3\theta}{2}\right)^{1/2}\right]$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left(\cos\frac{\theta}{2} \pm \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 \pm \sin\frac{\theta}{2}\right)$$

Plane stress: The third principal stress is zero and

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\right),\ \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right),\ \sigma_3 = 0 \quad \textit{Ans.}$$

Plane strain: Equations for σ_1 and σ_2 are still valid,. However,

$$\sigma_3 = \nu\left(\sigma_1 + \sigma_2\right) = 2\nu\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2} \qquad \textit{Ans.}$$

5-83 For $\theta = 0$ and plane strain, the principal stress equations of Prob. 5-82 give

$$\sigma_1 = \sigma_2 = \frac{K_I}{\sqrt{2\pi r}},\quad \sigma_3 = 2\nu\frac{K_I}{\sqrt{2\pi r}} = 2\nu\sigma_1$$

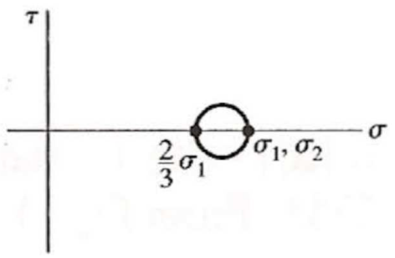
(**a**) DE: Eq. (5-18) $\frac{1}{\sqrt{2}}\left[\left(\sigma_1-\sigma_1\right)^2 + \left(\sigma_1 - 2\nu\sigma_1\right)^2 + \left(2\nu\sigma_1 - \sigma_1\right)^2\right]^{1/2} = S_y$

or, $\sigma_1 - 2\nu\sigma_1 = S_y$

For $\nu = \frac{1}{3}$, $\left[1-2\left(\frac{1}{3}\right)\right]\sigma_1 = S_y \quad \Rightarrow \quad \sigma_1 = 3S_y \quad$ *Ans.*

(**a**) MSS: Eq. (5-3) , with n =1 $\quad \sigma_1 - \sigma_3 = S_y \quad \Rightarrow \quad \sigma_1 - 2\nu\sigma_1 = S_y$

$$\nu = \frac{1}{3} \quad \Rightarrow \quad \sigma_1 = 3S_y \qquad \textit{Ans.}$$



$$\sigma_3 = \sigma_1 - S_y = 2S_y \quad \Rightarrow \quad \sigma_3 = \frac{2}{3}\sigma_1$$

Radius of largest circle

$$R = \frac{1}{2}\left(\sigma_1 - \frac{2}{3}\sigma_1\right) = \frac{\sigma_1}{6}$$

5-84 Given: a = 16 mm, K_{Ic} = 80 MPa$\cdot\sqrt{\text{m}}$ and $S_y = 950$ MPa

(a) Ignoring stress concentration

$F = S_yA$ =950(100 – 16)(12) = 958(10^3) N = 958 kN *Ans.*

(b) From Fig. 5-26: h/b = 1, a/b = 16/100 = 0.16, β = 1.3

Eq. (5-37) $K_I = \beta\sigma\sqrt{\pi a}$

$$80 = 1.3\frac{F}{100(12)}\sqrt{\pi(16)10^{-3}}$$

F = 329.4(10^3) N = 329.4 kN *Ans.*

5-85 Given: a = 0.5 in, K_{Ic} = 72 kpsi$\cdot\sqrt{\text{in}}$ and S_y = 170 kpsi, S_{ut} = 192 kpsi

r_o = 14/2 = 7 in, r_i = (14 – 2)/2 = 6 in

$$\frac{a}{r_o - r_i} = \frac{0.5}{7-6} = 0.5, \qquad \frac{r_i}{r_o} = \frac{6}{7} = 0.857$$

Fig. 5-30: $\beta \doteq 2.4$

Eq. (5-37): $K_{Ic} = \beta\sigma\sqrt{\pi a} \quad \Rightarrow \quad 72 = 2.4\sigma\sqrt{\pi(0.5)} \quad \Rightarrow \quad \sigma = 23.9$ kpsi

Eq. (3-50), p. 127, at $r = r_o$ = 7 in:

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}(2) \quad \Rightarrow \quad 23.9 = \frac{6^2 p_i}{7^2 - 6^2}(2) \quad \Rightarrow \quad p_i = 4.315 \text{ kpsi} \qquad \textit{Ans.}$$
